A well mixed balance (material) around species i written with respect to an abstract volume basis β is given by:

$$\dot{n}_{X,acc,i} = \dot{n}_{X,in,i} - \dot{n}_{X,out,i} + \dot{n}_{X,gen,i} \qquad i = 1, 2, \cdots, M$$

$\because$ cell free system, no convective transport takes place

$$\dot{n}_{X,in,i} = \dot{n}_{X,out,i} = 0$$

$$\therefore \dot{n}_{X,acc,i} = \dot{n}_{X,gen,i}$$

In terms of β this can be written as

$$\underbrace{\frac{d}{dt}\int_\beta x_i d\beta}_{\text{accumulation}} = \int_\beta \underbrace{(\ldots) d\beta}_{\text{generation terms}} \qquad i = 1, 2, \ldots M.$$

After making the well mixed assumption

$$\frac{d}{dt}(x_i\beta) = (\ldots)\beta \qquad i = 1, 2, 3, \cdots M$$

$$\Rightarrow \dot{x}_i\beta + \dot{\beta}x_i = (\cdot\cdot)\beta$$

$$\Rightarrow \dot{x}_i = (\ldots)\beta\beta^{-1} - \dot{\beta}\beta^{-1}x_i \Rightarrow \dot{x}_i = (\ldots) - \dot{\beta}\beta^{-1}x_i$$

We know $\beta = X V_R$

$$\beta^{-1}\dot{\beta} = X^{-1}\dot{X} + V_R^{-1}\dot{V}_R$$

For cell free system, change in cell mass conc^n $= 0$ $\because$ no growth takes place

$$\Rightarrow \dot{X} = 0$$

Also change in volume $= 0$ $\because$ Volume of eppendorf remains const.

$$\Rightarrow \dot{V}_R = 0$$

$$\therefore \beta^{-1}\dot{\beta} = 0 \Rightarrow \mu = 0$$

$\therefore$ Material balance eq^{ns} governing conc^n of protein and mRNA

$$\dot{m}_i = r_{x,i} u_i - (\mu + \theta_{m,i}) m_i + \lambda_i \qquad i = 1, 2, \ldots, N$$

$$\dot{p}_i = r_{L,i} w_i - (\mu + \theta_{p,i}) p_i$$

reduce to

$$\dot{m}_i = r_{x,i} u_i - (\theta_{m,i}) m_i + \lambda_i$$

$$\dot{p}_i = r_{L,i} w_i - (\theta_{p,i}) p_i$$

OR

$$\boxed{\begin{aligned} \dot{m} &= r_x u - \theta_m m + \lambda \\ \dot{p} &= r_L w - \theta_p p \end{aligned}}$$