3.(f) f1) Assuming fast equilibrium for eq^{n}s 3-5
We are left with

$$\frac{dR_i}{dt} = \frac{\beta^n}{K^n + N_i^{*n}} - \gamma_R R_i \quad -(6)$$

∴ For cell 1

$$\frac{dR_1}{dt} = \frac{\beta^n}{K^n + N_1^{*n}} - \gamma_R R_1$$

$$\frac{dR_2}{dt} = \frac{\beta^n}{K^n + N_i^{o*n}} - \gamma_R R_2$$

From (3)-(5) at fast eq^m, we get

$$K_f L R_i - k_r R_i^* = 0 \quad \rightarrow (3)$$

$$K_f^{ND} N_i D_j^o - k_r^{ND} N_i^{o*} = 0 \quad - (4)$$

$$K_D R_i - \gamma_D D_i = 0 \Rightarrow R_i = \frac{\gamma_D D_i}{K_D} \Rightarrow D_i = \frac{k_D}{\gamma_D} R_i \quad -(5)$$

$$\therefore \cancel{N_i^{*} = \frac{K_f^{ND}\gamma_D N_j^o}{N_i K_D}}$$

∴ From (4)(3) we get

$$\boxed{N_i^{*} = \frac{K_f^{ND} N_i D_j^o}{K_r^{ND}} = \frac{k_f^{ND} N_i k_D}{k_r^{ND}\gamma_D} R_j^{*} = A_j R_j^o L}$$

where $A_j = \frac{k_f^{ND} N_i k_D}{k_r^{ND}\gamma_D}$

From (i)

$$\therefore \frac{dR_i}{dt} = \frac{\beta^n}{K^n + M_j^{\circ n} R_j^n L^n} - \gamma_R R_i$$

Now, let $u = \frac{R_1}{K}$, $v = \frac{R_2}{K}$, $t\gamma_R = \tau$

$$\therefore K\gamma_R \frac{du}{d\tau} = \frac{\beta^n}{K^n + K^n M_j^{\circ n} v^n L^n} - \gamma_R K u$$

$$\Rightarrow \frac{du}{d\tau} = \frac{\frac{\beta^n}{K^{n+1}\gamma_R}}{1 + M_j^{\circ n} v^n L^n} - u$$

$$\Rightarrow \boxed{\frac{du}{d\tau} = \frac{\alpha}{1 + A^n v^n} - u}$$ where $\alpha = \frac{\beta^n}{K^{n+1}\gamma_R}$

$A = M_j^\circ L$

$||^{ly}$ $$\boxed{\frac{dv}{d\tau} = \frac{\alpha}{1 + A^n u^n} - v}$$

For Stability we have Jacobian of the form

$$J = \begin{bmatrix} -1 & -nA^n(1+A^n v^n)^{-2} \\ -nA^n(1+A^n u^n)^{-2} & -1 \end{bmatrix}$$

$$\text{Det}|J| = 1 - n^2 A^{2n}(1+A^n v^n)^{-2} \cdot n^2 A^{2n}(1+A^n u^n)^{-2}$$

$$= 1 - \frac{n^2 M_1^{2n} L^{2n}}{1 + M_j^{\circ 2n} L^{2n} v^{2n}} \cdot \frac{n^2 M_2^{2n} L^{2n}}{1 + M_2^{2n} L^{2n} u^{2n}}$$

$\therefore$ For stability $\text{Det}|J| \propto L^{4n}$ and depends strongly on conc^n of Ligand for stability.

The degree of cooperativity, n can be manipulated for instability.

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