Random Variable, Probability, Probability Distribution, Normal Distribution, Standard Normal Distribution, and Expected Value

# Random Variable

## Overview

A random variable is a numerical outcome of a random phenomenon. It assigns a real number to each outcome in a sample space.

## Types

### Discrete Random Variable

Takes on a countable number of distinct values.  
Example: Number of heads in a series of coin tosses.

### Continuous Random Variable

Takes on an infinite number of possible values within a given range.  
Example: The exact height of individuals in a population.

## Properties

Probability Distribution: Describes how probabilities are distributed over the values of the random variable.  
Expected Value: The long-run average value of the random variable over many repetitions of the experiment.

# Probability

## Overview

Probability measures the likelihood that a particular event will occur. It is a fundamental concept in statistics used to quantify uncertainty.

## Basic Concepts

Experiment: A process that leads to one of several possible outcomes.  
Sample Space (S): The set of all possible outcomes of an experiment.  
Event (E): A subset of the sample space.

## Rules

Probability of an Event:  
P(E) = Number of favorable outcomes / Total number of possible outcomes  
  
Complementary Rule:  
P(E') = 1 - P(E)  
Where E' is the complement of event E.  
  
Addition Rule:  
For mutually exclusive events A and B:  
P(A ∪ B) = P(A) + P(B)  
  
Multiplication Rule:  
For independent events A and B:  
P(A ∩ B) = P(A) \* P(B)

# Probability Distribution

## Overview

A probability distribution describes how probabilities are assigned to the possible values of a random variable.

## Types

### Discrete Probability Distribution

Example: Binomial Distribution.  
Defined by a probability mass function (PMF).

### Continuous Probability Distribution

Example: Normal Distribution.  
Defined by a probability density function (PDF).

## Properties

Mean (μ): The average or expected value.  
Variance (σ²): Measures the spread of the random variable around the mean.

# Normal Distribution

## Overview

The normal distribution is a continuous probability distribution that is symmetric around the mean, depicting that data near the mean are more frequent in occurrence than data far from the mean.

## Characteristics

Bell-Shaped Curve: Symmetrical around the mean.  
Mean (μ): The central point.  
Standard Deviation (σ): Measures the dispersion.  
Empirical Rule:  
Approximately 68% of the data falls within one standard deviation of the mean.  
Approximately 95% falls within two standard deviations.  
Approximately 99.7% falls within three standard deviations.

## Probability Density Function (PDF)

f(x) = (1 / √(2πσ²)) \* e^(-(x-μ)² / (2σ²))

# Standard Normal Distribution (SND)

## Overview

The standard normal distribution is a special case of the normal distribution with a mean of 0 and a standard deviation of 1.

## Standardization

Any normal distribution can be transformed into a standard normal distribution using the z-score formula:  
z = (X - μ) / σ

## Properties

Z-Score: Represents the number of standard deviations a data point is from the mean.  
Z-Table: Used to find the probability of a z-score occurring in a standard normal distribution.

# Expected Value

## Overview

The expected value (or mean) of a random variable gives a measure of the center of the distribution of the variable.

## Calculation

### Discrete Random Variable

E(X) = Σ [x\_i \* P(x\_i)]  
Where x\_i are the values of the random variable and P(x\_i) are their corresponding probabilities.

### Continuous Random Variable

E(X) = ∫ x \* f(x) dx  
Where f(x) is the probability density function.

## Properties

Linearity: E(aX + b) = aE(X) + b  
Expectation of a Constant: E(c) = c