1. True or False [10%]

(F) $(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$ is unsatisfiable.

(F) $KB \vDash \alpha$ if and only if $KB \rightarrow \alpha$ is satisfiable.

(F) The key difference between Propositional Logic and First-order Logic is that semantics of Propositional Logic can refer to objects in the world.

(F) Predicates and Functions both represent relationships in the world.

(T) Truth Table is an effective way to divide the universe for model checking

(T) The frame problem is about how to state all the non-changed facts of an action without many duplicated rules.

(T) In FOL, => is a natural connective to use with Universal Quantifiers.

(T) A logic sentence is valid if and only if it is true in all models.

(T) A plan is complete if and only if every precondition of every step in the plan is achieved.

(F) This sentence is false: $KB \vDash \alpha$ if and only if $(KB \rightarrow \alpha)$ is valid.

2. Propositional Logic [20%]

1.

a. **[5%]** Construct the truth table for the propositional statement : (p ⋁ ~q) → ((~p ⋀ q) ⋁ ~q)

p	**q**	**(p ⋁ ~q)**	**(~p ⋀ q)**	**(~p ⋀ q) v ~q**
Row1(A1,A2,A3,A4,A5)				
Row2(A6,A7,A8,A9,A10)				
Row3(A11,A12,A13,A14,A15)				
Row4(A16,A17,A18,A19,A20)				

p	**q**	**(p ⋁ ~q)**	**(~p ⋀ q)**	**(~p ⋀ q) ⋁ ~q**
T	**T**	**T**	**F**	**F**
T	**F**	**T**	**F**	**T**
F	**T**	**F**	**T**	**T**

F	F	T	F	T

b. [1%] From the truth table above, does (~p ∧ q) entail (~p ∧ q) ∨ ~q?
Yes

2. Using Inference Rules [14%] [2% each]

Consider the following knowledge base that describes how a student works:

ExamNextWeek ⇒ Study	**1**
HomeworkDueNextWeek ^ HighWeitageofHW => WorkonHW	**2**
Knowledge => GoodGrades	**3**
Covid-19 ^ FallBreak	**4**
StudyBreak => ExamNextWeek	**5**
HomeworkDueNextWeek	**6**
HighWeightageofHW	**7**
Covid-19 => StayIndoors	**8**
LargeVacation V FallBreak	**9**
~FallBreak V ShortSemester	**10**
~GoodGrades	**11**

Using the various propositional logic inference rules studied in class, show how each of the following conclusions can be inferred: In each case, mention which inference rule is used [1%], and to which sentence(s) above it was applied [1%]. Every sentence can be proven using only sentences with smaller indices (Hint: All of the conclusions can be inferred in at most 2 steps).

11. WorkonHW	**R1**

12. Fall Break	**R2**
13. ShortSemester	**R3**
14. ~Knowledge	**R4**
15. StayIndoors	**R5**
16. LargeVacation V ShortSemester	**R6**
17. HomeworkDueNextWeek ^ ~GoodGrades	**R7**

11. WorkonHW	**And Introduction 6 and 7, Modus Ponens 2**
12. Fall Break	**And Elimination 4**
13. ShortSemester	**Unit Resolution 10, 12**
14. ~Knowledge	**Modus Tollens 11(1), 3**
15. StayIndoors	**AND Elimination 4, Modus Ponens 8**
16. LargeVacation V ShortSemester	**Resolution 9, 10**
17. HomeworkDueNextWeek ^ ~GoodGrades	**AND Introduction 6,11(1)**

3. First Order Logic [20%]

Consider a domain with the following relations and objects.

Color(x, y)	Object x has the color y
On(x, y)	Object x is on top of object y
Besides(x, y)	Object x and object y are beside each other
Expensive(x)	Object x is expensive
Block1, Block2, Block3, Block4	Constants denoting objects
White, Black, Red	Constants denoting colors

Formalize the following sentences for this domain.

2- [3%] Everything red is on top of something black.

$\forall x$ Color(x,Red) => $\exists y$ Color(y,Balck) $\wedge$ On(x,y)

3- [4%] Block3 is not on top of any red object.

$\forall x$ Color(x,Red) => ¬On(Block3,x)

6- [4%] All red things are expensive.

$\forall x$ Color(x,Red) => Expensive(x)

7- [5%] There exists a tower of 3 blocks (3 blocks on top of each other) which all have different colors.

$\exists x$ $\exists y$ $\exists z$ $\exists x1$ $\exists y1$ $\exists z1$ On(x,y) $\wedge$ On(y,z) $\wedge$ Color(x,x1) $\wedge$ Color(y,y1) $\wedge$ Color(z,z1) $\wedge$ (x1≠y1) $\wedge$ (y1≠z1) $\wedge$ (x1≠z1)

8- [4%] When there are two blocks on each other, the top of is expensive and the lower one is not.

$\forall x,y$ On(x,y) => Expensive(x) $\wedge$ ¬Expensive(y)

4. Inference

Given the following knowledge base:

1. Anyone who is smart and not poor is happy
2. Anyone who reads is smart
3. John can read and is not poor.
4. Anyone who is happy has exciting life

Note- Use predicates: Poor, Smart, Happy, Reads, HasExcitingLife

A. Please translate the above statements into First Order Logic. You also need to convert it to CNF.[8%]

[2% for each statement. Alternative correct answers possible for FOL.
1% if FOL correct and CNF incorrect, 2% if CNF correct otherwise 0%]

i. FOL : ∀x ¬Poor(x) ∧ Smart(x) ⇒ Happy(x)
CNF: 1. Poor(x) ∨ ¬Smart(x) ∨ Happy(x)

ii. FOL: Reads(x) ⇒ Smart(x)

CNF: 2. ¬Reads(x) ⋁ Smart(x)

iii. FOL : Reads(John) ⋀ ¬Poor(John)
CNF: 3. Reads(John)
4. ¬Poor(John)

iv. FOL: ∀x Happy(x) ⇒ HasExcitingLife(x)
CNF: ¬Happy(z) ⋁ HasExcitingLife(z)

B. *Prove that John has an exciting life using resolution.Clearly mention which 2 statements you are resolving and what the result shall be. You also need to provide the correct reasoning for your proof.[12%]*

6. ¬HasExcitingLife(John)

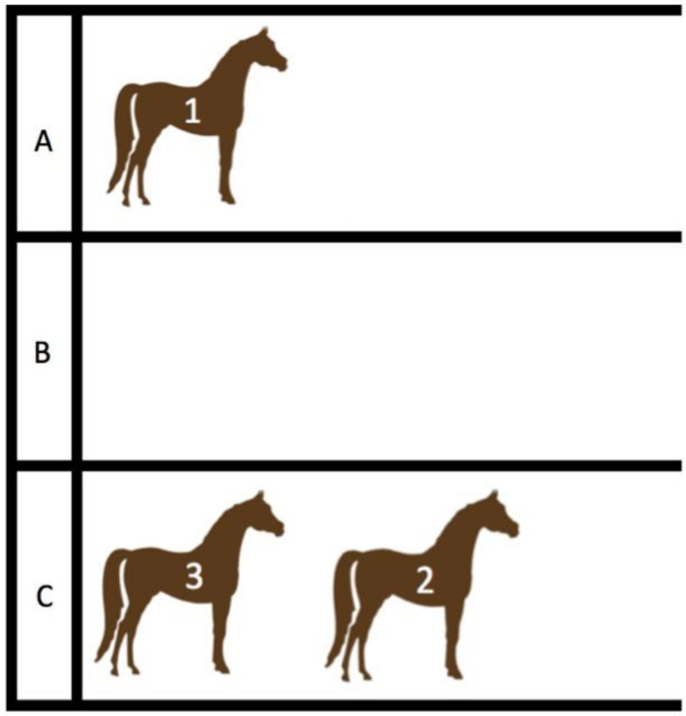
a. *Resolving 5 & 6 , Substitution [z/John] : 7. ¬Happy(John)*
b. *Resolving 7 & 1, Substitution [x/John]: 8. Poor(John) ⋁ ¬Smart(John)*
c. *Resolving 8 & 2, Substitution [y/John]: 9. Poor(John) ⋁ ¬Reads(John)*
d. *Resolving 9 & 3: 10. Poor(John)*
e. *Resolving 10 & 4:* ***Empty***

Since ¬HasExcitingLife(John) is unsatisfiable with KB, we can say John has an exciting life

[1% for negation, -2% for each wrong or extra resolution not contributing towards the answer util zero. Alternative correct resolution accepted.]

5. Planning

Consider a planning problem where there are three horses in a three-stall stable. These horses are known to be friendly and therefore can be arranged such that there could be more than one horse in a stall at the same time. However, because the stable is narrow, the horses can only be moved one at a time and in the opposite order in which they were ushered into or out of the stall, i.e. last in, first out (LIFO). For example, in the figure below, horse 2 must be moved out of C before horse 3 can be moved.



iTommy, a Los Angeles-based AI company, has teamed up with Amazon to build a robotic horse trainer that can move horses one at a time from stall to stall, obviously through the use of Alexa. In fact, “Alexa, move horse 1 from stall A to stall B” translates to the system’s STRIPS code moveFromTo(Horse1, A, B) or “Alexa, move horse 3 from stall C to stall B” would elicit a response, “Precondition not met. Sorry it’s being blocked.”

Please use the following literal definitions to answer the next 2 questions:
- rightmost(s1,h1)/rightmost(s1,fence): means horse h1 is at right most position in stall s1 (h1 could be a horse or the special Fence object)
- in(h1,s1): horse h1 is in the stall s1
- rightOf(h1,h2)/rightOf(h1,fence): h1 is to the right of h2 in the same stall (h2 could be a horse or the special Fence object)

1.[6%] STRIPS actions: Complete the STRIPS action, with their preconditions and effects:

Action: Horse h1 moves from s1 to s2:

moveFromTo(h1,s1,s2)

Precondition:
Effects:

[-0.5 for each incorrect action] No deduction for incorrect position of variables like h1,s1,s2 etc.
Preconditions:
rightmost(s1,h1), in(h1,s1)
Effects:
not rightmost(s1,h1), not in(h1,s1)
rightmost(s2,h1), in(h1,s2)

Partial credit: 1 points each for the precondition and the effects.

2. [6%] Initial Plan: if the robotic horse trainer plans to rearrange the horses from the starting state in the figure above so that Horse 3 goes into stall A, Horse 2 into stall B, and Horse 1 into stall C, write down the initial condition and goal of this planning problem using the STRIPS definitions from 6.1.

Initial conditions:

in(Horse1,A) in(Horse2,C) in(Horse3,C)
rightmost(A,Horse1) rightmost(B,Fence) rightmost(C,Horse2)
rightOf(Horse1,Fence) rightOf(Horse2,Horse3) rightOf(Horse3,Fence)

Goal condition:

in(Horse1,C) in(Horse2,B) in(Horse3,A)

Partial credit: 0.5 points for each of the initial condition and the goal.

3. [8%] Complete Plan: write down the plan to reach the goal from the initial condition, using the condition and goal specified in 2 part.

Plan using moveFromTo:

moveFromTo(Horse2,C,B)
moveFromTo(Horse3,C,B)
moveFromTo(Horse1,A,C)
moveFromTo(Horse3,B,A)

or

moveFromTo(Horse2,C,B)
moveFromTo(Horse1,A,B)
moveFromTo(Horse3,C,A)
moveFromTo(Horse1,B,C)

Partial credit: 2 points for each step.

50% marks if written in simple english and plan is correct

6. MCQ

1. In the discussions, we showed the universal and existential quantifiers, circle all that are true

a. $\forall x \exists y P(x, y)$ is the same as $\exists y \forall x P(x, y)$ (F)
b. $\forall x \exists y P\left(x, y\right)$ is the same as $\forall x(\exists y P\left(x, y\right))$ (T)
c. $\forall y \exists x P(x, y)$ is the same as $\exists y \forall x P(x, y)$ (F)
d. $\forall y P(x, y)$ is the same as $\neg \exists y \neg P(x, y)$ (T)
e. $\neg \forall y P(x, y)$ is the same as $\exists y \neg P(x, y)$ (T)

2. In the discussions, we showed the unification, please circle all that are true

   a. P(C, D, D) and P(x, y, z) are unifiable (T)
   b. H(y, G(A,B)) and H(G(x,x), y) are unifiable (F)
   c. Older(Mother(y), y) and Older(Mother(x), Mary) are unifiable (T)
   d. Knows(Mother(x), x) and Knows(y, y) are unifiable (F)
   e. P(x, y) and Q(x, y) are not unifiable (T)

3. In the discussions, we showed the proof by resolution, please circle all that are true

   a. Resolution is sound but not complete (F)
   b. Resolution can only be applied to Normal Conjunctive Forms (F)
   c. Prolog uses resolution to prove (F)
   d. To prove by resolution, one can start from a contradiction as root and grow a tree (T)
   e. To prove by resolution, one proves that is unsatisfiable (T)

4. In the discussions, we showed several planning mechanisms, please circle all that are true

   a. Situation Calculus uses FOL proofs to generate a plan of actions (T)
   b. STRIPS operators can be used to represent actions but not states (T)
   c. Planning is a completely different problem from search (F)

d. In Plan Space, the search is through plans not states (T)
e. Partial order plans can be executed in a linear fashion (F)

5. In the discussions, we showed many logic inference mechanisms, please circle all that are true

a. Skolemization must consider the scopes of the variables (T)
b. Resolution can only use Conjunctive Normal Form (F)
c. Proof by contradiction can only be done by resolution (F)
d. Neither forward chaining nor backward chaining is complete (T)
e. Backward chaining is more efficient than forward chaining (T)