CS570
Analysis of Algorithms
Summer 2006
Final Exam

Name: _____
Student ID: _____

	Maximum	Received
Problem 1	20	
Problem 2	20	
Problem 3	20	
Problem 4	20	
Problem 5	20	

1) 20 pts
Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.
(a) Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; If f is a maximum s-t flow in G, then f saturates every edge out of s with flow (i,e.,for all edges e out of s, we have $f(e) = c_e$)

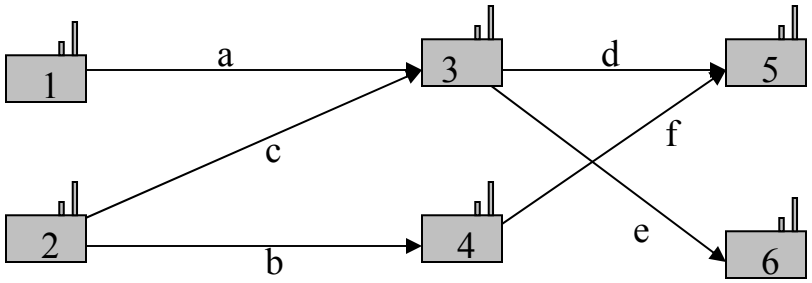
(b) Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e; and let (A,B) be a minimum s-t cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity; then (A,B) is still a minimum s-t cut with respect to these new capacities $\{1 + c_e : e \in E\}$.

2) 20 pts

There are n unit time tasks to be scheduled on a machine in the time interval $[0, \ldots, n]$. The i^{th} task has a deadline d_i, a profit p_i and a loss l_i. The machine can process only one task at a time, and each job must run uninterruptedly for one unit time. If the i^{th} task is complete by its deadline d_i, you receive a profit p_i, but if it it completed after its deadline, you suffer a loss l_i. Give a polynomial time algorithm to find a schedule that obtains the maximum amount of net profit. Analyze the time complexity of your algorithm.

Additional space

3) An instance of graph *G=(E,V)* given below represents a supply chain for a manufacturing company that produces products *P1* and *P2*. Nodes *V* represent manufacturing sites with total capacity *Cv* and edges *E* represent transportation routes with capacity *Ce*. Each product *Pi* takes away *Wiv* units of capacity when moving thru manufacturing site *v.* Each product Pi also takes away *Wie* units of capacity when moving thru transportation route *e.* Each product *Pi* brings a revenue of *Vi* and has manufacturing and transportation costs *Liv and Lie* at site *v* and route *e* respectively. Manufacturing on both products is started at nodes 1 and 2 and completed at nodes 5 and 6. The objective is to maximize the profit associated with this supply chain.



Graph G

Give a complete formulation of how this problem is solved and describe whether your solution is an exact solution or an approximate solution.

Additional space

4) Given an mxn array A of integers, let $f(i)$ denote, for $i = 1,...,m$, the index of the column containing the leftmost minimum element of row i. Suppose $f(m) \geq f(m-1) \geq ... \geq f(1)$.
   a) Describe in words (instead of pseudo code) a divide-and-conquer strategy that computes all the $f(i)$ in $O(m + n + n \log m)$ time.

b) Describe what your algorithm does when $m = 2$. Write the recurrence describing the running time of your algorithm.

c) Demonstrate that the minimum of the whole array can be computed in $O(m + n + n \log m)$ time.

5) 20 pts

In a certain town, there are many clubs, and every adult belongs to at least one club. The townspeople would like to simplify their social life by disbanding as many clubs as possible, but they want to make sure that afterwards everyone will still belong to at least one club.

Prove that the Redundant Clubs problem is NP-complete. You may make use of the known NP-completeness of the Independent Set, Set Cover, Vertex Cover, Hamiltonian Cycle, or Traveling Salesman Problems.

Formally the Redundant Clubs problem has the following input and output.

INPUT: List of people; list of clubs; list of members of each club; number *K*.

OUTPUT: Yes if there exists a set of *K* clubs such that, after disbanding all clubs in this set, each
person still belongs to at least one club. No otherwise.

6) Additional space

7) Additional space