CSCI 570 Fall 2025

Homework 8

Due: March 28

Note: In Questions 3, 4, and 5, we assume that the Ford-Fulkerson algorithm is provided. You do not need to rewrite the Ford-Fulkerson algorithm.

Question 1

Consider a flow network with source 0, sink 6, and the following edges:

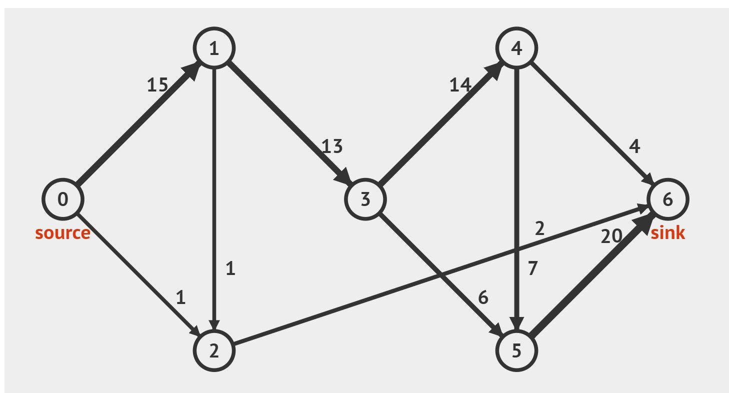
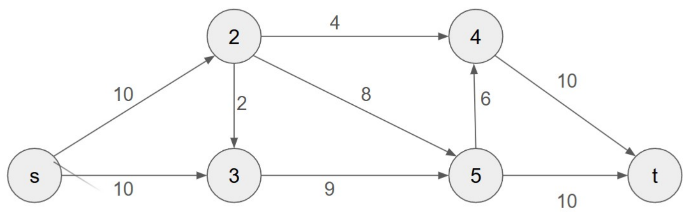


Figure 1: (*The capacity of the edge from 2 to 6 is two.*)

- Draw the **first, second, and final** residual graphs G_f using the Edmonds-Karp implementation of Ford-Fulkerson algorithm corresponding to the max flow problem.
- Determine the max-flow value.
- Provide **all** of the min-cuts.

Question 2

Consider a flow network with source s, sink t, and the following edges:



- Draw the **first, second, and final** residual graph G_f using the scaled version of Ford-Fulkerson algorithm corresponding to the max flow problem.
  - Give the value of Δ for the first iteration, and the residual graph G_f after the first iteration.
  - Give the value of Δ for the second iteration, and the residual graph G_f after the second iteration.
  - Give the final G_f.
- Find the max-flow value.
- Find **all** of the min-cuts.

Question 3

Part (a): Assignment Feasibility

You are given:

- n entities $e_1, e_2, \ldots, e_n$.
- k groups $g_1, g_2, \ldots, g_k$.
- For each entity e_j, a subset $p_j \subseteq \{g_1, g_2, \ldots, g_k\}$ with $|p_j| \geq m$, where m is a positive integer.
- For each group g_i, a capacity q_i (maximum number of entities it can contain).

You want to determine if it is possible to assign each entity e_j to **at least** m groups from its subset p_j, such that no group g_i contains more than q_i entities.

Task: Design an algorithm to check if such an assignment exists and **prove** its correctness.

(Hint: The prrof will have two directions. 1)Forward Direction: If feasible, then [statement]. 2) Backward Direction: If [statement] then feasible.)

Part (b): Selection with Constraints

Assume a feasible solution to part (a) exists, and each entity e_j is assigned to **exactly** m groups from p_j. Select exactly one entity as a "representative" for each group g_i from the entities assigned to it. No entity can be a representative for more than r groups, where $r < m$. Determine if such a selection of representatives is possible.

Task: Design an algorithm to check if this selection exists and prove its correctness.

(Hint: Use the solution from part (a) as a starting point.)

(Hint: The prrof will have two directions. 1)Forward Direction: If feasible, then [statement]. 2) Backward Direction: If [statement] then feasible.)

Question 4

Given a flow network $G = (V, E)$ with source s, sink t, and unit-capacity edges ($u_e = 1$ for all $e \in E$), and an integer k, find a set $F \subseteq E$, $|F| = k$, to minimize the max s-to-t flow in $G' = (V, E - F)$.

Task: Give an algorithm to solve this problem. Prove the correctness.

Question 5

A dating service receives data $\mathbf{D}$ from p men and q women. These data determine which pairs of men and women are mutually compatible and which are not. Since the dating service's commission is proportional to the number of dates it arranges, it would like to determine the maximum number of compatible couples that can be formed. (Note that each man or woman is assigned at most one date.)

Task: Design an algorithm to determine the maximum number of mutually compatible dates that can occur simultaneously. Proof is not required.

Ungraded

A vertex cover of an undirected graph $G = (V, E)$ is a subset $A \subseteq V$ such that every edge $e \in E$ has at least one of its incident vertices in A. Given a bipartite graph G' with vertex partition $V = L \cup R$ ($L \cap R = \emptyset$, edges only between L and R) and a positive integer k, design an algorithm to decide if G' has a vertex cover of size at most k.