CS570

Analysis of Algorithms

Fall 2011

Exam II

Name: ______________________

Student ID: __________________

_____12:30 PM Session   _____ 2:00 PM Session

	Maximum	Received
Problem 1	20	
Problem 2	10	
Problem 3	10	
Problem 4	20	
Problem 5	20	
Problem 6	20	
Total	100	

2 hr exam
Close book and notes

If a description to an algorithm is required please limit your description to within 150 words, anything beyond 150 words will not be considered.

1) 20 pts

Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]
Maximum value of an s-t flow could be less than the capacity of a given s-t cut in a flow network.

[TRUE/FALSE]
In a flow network, if we increase the capacity of an edge that happens to be on a minimum cut, this will increase the max flow in the network .

[TRUE/FALSE]
The best worst-case time complexity to solve the max flow problem is $O(Cm)$ where C is the total capacity of the edges leaving the source and m is the number of edges in the network.

[TRUE/FALSE]
Bellman-Ford algorithm cannot solve the shortest path problem in graphs with negative cost edges in polynomial time.

[TRUE/FALSE]
If a dynamic programming solution is set up correctly, i.e. the recurrence equation is correct and the sub-problems are always smaller than the original problem, then the resulting algorithm will always find the optimal solution in polynomial time.

[TRUE/FALSE]
If a problem can be solved by dynamic programming, then it can always be solved by exhaustive search.

[TRUE/FALSE]
Fractional knapsack problem can be solved in polynomial time.

[TRUE/FALSE]
0/1 knapsack problem can be solved in polynomial time.

[TRUE/FALSE]
A flow network with unique edge capacities has a unique min cut.

[TRUE/FALSE]
The number of iterations it takes Bellman-Ford to converge can vary depending on the order of nodes updated within each iteration.

2) 10 pts

You are given a network with n vertices, m edges, a source s and a sink t. Suppose your friend presents to you an s-t flow for the network by assigning for every edge e, a flow f(e). Describe an $O(n + m)$ algorithm to test if your friend's flow assignment is a maximum s-t flow.

Run BFS on the residual graph. If we cannot find a path from s to t, then the flow is maximum.

Discussion and previous exams

3) 10 pts

You have successfully computed a maximum s-t flow f for a network $G = (V;E)$ with integer edge capacities. Your boss now gives you another network G' that is identical to G except that the capacity of exactly one edge is increased by one. You are also explicitly given the edge who's capacity was changed. Describe how you can compute a maximum flow for G' in $O(|V| + |E|)$ time, where $|V|$ is the number of vertices and $|E|$ is the number of edges.

4) 20 pts

Given a weighted directed graph, describe an algorithm that decides if there exists a negative weight cycle in the graph. The running time should be polynomial in the number of vertices.

Bellman – Ford Algorithm

5) 20 pts

We are given a string $S = s_1s_2s_3...s_n$, and we want to delete some characters such that what remains is a palindrome. Give an efficient dynamic programming algorithm to determine the length of the longest palindrome, and analyze the running time.

A palindrome is a string that reads the same backward or forward such as ACCBCCA.

Solution:

Let X[0..n-1] be the input sequence of length n and L(0, n-1) be the length of the longest palindromic subsequence of X[0..n-1].

If last and first characters of X are same, then L(0, n-1) = L(1, n-2) + 2.
Else L(0, n-1) = MAX (L(1, n-1), L(0, n-2)).

6) 20 pts
Determine if there is a feasible circulation in the following network. Demand values are in circles. Edge capacities are marked on each edge.
Show all your steps.