CS570
Analysis of Algorithms
Summer 2006
Exam 2

Name: _____
Student ID: _____

	Maximum	Received
Problem 1	10	
Problem 2	20	
Problem 3	25	
Problem 4	25	
Problem 5	20	

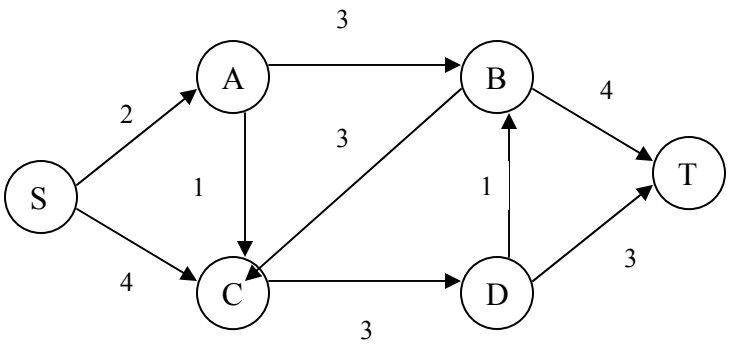
1) 10 pts
A divide and conquer algorithm is constructed the following way
Divide: Split the problem (originally of size n) into 8 equal pieces of size n/2. This takes $O(n^2)$ time.
Conquer: Solve the n subproblems recursively
Combine: Combine the subproblems. This takes O(nlgn) time.

a- What is the complexity of this algorithm?

b- What aspect of this algorithm has to improved in order to reduce the overall complexity of the algorithm.
1- time it takes to divide
2- number of subproblems at each step
3- size of the subproblems at each step
4- time it takes to combine
Mark all that apply and explain your answer.

2) 20 pts
Given an array P=(P(1),…,P(n)), use divide and conquer to find $\max_{0<i<j<n+1}$ (P(j)-P(i)) in O(n log n). (Note: this problem can be solved using other techniques but you are specifically asked to solved the problem using divide and conquer)

3) 25 pts
Question 1: In the flow network illustrated below, each directed edge is labeled with its capacity. We are using the Ford-Fulkerson algorithm to find the maximum flow. The first augmenting path is S-A-C-D-T, and the second augmenting path is S-A-B-C-D-T.



a) Draw the residual network after we have updated the flow using these two augmenting paths(in the order given)

b) List all of the augmenting paths that could be chosen for the third augmentation step.

c) What is the numerical value of the maximum flow? Draw a dotted line through the original graph to represent the minimum cut.

4) 25 pts

Let G=(V,E) be a flow network with source s, sink t, and suppose each edge e have capacity c(e)=1. For convenience, assume that E=O(V)

a) Suppose we implement the Ford-Fulkerson maximum-flow algorithm by using DFS to find augmenting paths in the residual graph. What is the worst-case running time of the algorithm on G?

b) Suppose a maximum flow for G has been computed, and a new edge with unit capacity is added to E. Describe how the maximum flow can be efficiently updated. Analyze your algorithm(Note: It is not the value of the flow that must be updated, but the flow itself)

c) Suppose a maximum flow for G has been computed, but an edge is now removed from E. Describe how the maximum flow can be efficiently updated. Analyze your algorithm

5) 20 pts
Given a graph G=(V,E) and two nodes X and Y in V, present an algorithm to find the maximum number of **node** disjoint paths from X to Y. Include complexity analysis and proof of correctness.