CS570
Analysis of Algorithms
Fall 2006
Exam 2

Name: _____

Student ID: _____

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Problem 1	10	
Problem 2	20	
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Note: The exam is closed book closed notes.

1) 10 pts

By using Strassen's algorithm, we can computes the product of two n x n matrices in O(n^2.81) time. This was achieved because we found a way to multiply the n/2 x n/2 matrices with only 7 multiplications rather than 8. Suppose we came up with a Strassen-like algorithm as follows. Assume that instead of splitting each matrix into four n/2 x n/2 matrices, we split each matrix into sixteen n/4 x n/4 matrices, and that the result is computed with only *m* multiplications instead of the normal 64. How small should *m* be for this new algorithm to be asymptotically faster than the original Strassen algorithm?

2) 20 pts

a- Given an array of integers A[1…n], give a divide-and-conquer algorithm to find the minimum ratio of A[i]/A[i+1] for 0<i<n. Your answer should clearly say how to divide the problem into subproblems and how to merge the results.

b- Give a recurrence relation for the time complexity of the algorithm and solve it.

3) 10 pts

Let $G = (V,E)$ be a flow network with source s, sink t, and integer capacities. Suppose that we are given a maximum flow in G. Suppose that the capacity of a single edge (u, v) is increased by 1, Give an $O(V + E)$ time algorithm to update the maximum flow.

4) 20 pts

a- Prove or disprove the following:

If all edges in a flow network have distinct capacities, then there is a unique maximum flow.

b-If all of the edges have unique capacity values, the network has a unique min-cut.

5) 20 pts
Seven construction equipments are to be flown to a destination by five commercial planes. There are 4 units of each kind of construction equipment, and the 5 commercial planes can carry 8, 8, 5, 4, and 4 units, respectively. Can this construction equipment be loaded in such a way that no two units of the same kind are in the same plane?

6) 20 pts
Solve the following feasible circulation problem below. The numbers in parentheses are (*lowerbound, upperbound*) on flow.