CS570

Analysis of Algorithms

Fall 2011

Exam I

Name: ______________________

Student ID: __________________

____Monday ____Friday ____DEN

	Maximum	Received
Problem 1	20	
Problem 2	16	
Problem 3	14	
Problem 4	20	
Problem 5	10	
Problem 6	20	
Total	100	

2 hr exam
Close book and notes

If a description to an algorithm is required please limit your description to within 150 words, anything beyond 150 words will not be considered.

1) 20 pts

Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]
If *T* is a spanning tree of *G* and *e* an edge in *G* which is not in *T*, then the graph *T+e* has a unique cycle.

[TRUE/FALSE]
Any BFS tree corresponding to a graph G=(V,E) will have exactly |V| edges.

[TRUE/FALSE]
A constant $n_0 \geq 1$ exists such that for any $n \geq n_0$, there exists an array of n elements such that insertion sort runs faster than merge sort on that input.

[TRUE/FALSE]
If *f*, *g*, and *h* are positive increasing functions with *f* in $O(h)$ and *g* in $\Omega(h)$, then the function *f+g* must be in $\Theta(h)$.

[TRUE/FALSE]
Delete operation takes $O(logn)$ time in the worst case in any of the 3 heaps discussed in class.

[TRUE/FALSE]
By running BFS at most n times (n is the number of vertexes in G) one can find all-pairs shortest path in an un-weighted graph G.

[TRUE/FALSE]
DFS Tree can never be the same as a BFS tree for a given graph.

[TRUE/FALSE]
The order of $T(n) = 3T\left(\frac{n}{4}\right) + \Theta(n^2)$ is $O(n^2 \text{Log}(n))$.

[TRUE/FALSE]
A greedy algorithm always makes the choice that looks best at the moment.

[TRUE/FALSE]
Dijkstra’s algorithm will always fail in the presence of negative cost edges in the graph.

2) 16 pts

Rank the following functions in order from smallest asymptotic complexity to largest. Additionally, identify all pairs i, j where $f_i(n) = \theta(f_j(n))$.

(a) $f_a(n) = 4^{2n}$

(b) $f_b(n) = 1 + \frac{1}{\log n}$

(c) $f_c(n) = \log(n^n)$

(d) $f_d(n) = n^n$

(e) $f_e(n) = \log n^4$

(f) $f_f(n) = \sqrt{n}$

(g) $f_g(n) = 37$

(h) $f_h(n) = \log 2n$

3) 14 pts

Given an undirected graph G, the LINE GRAPH G_L is a graph where all edges in G are vertices in G_L. Two vertices in G_L (which were edges in G) have an edge between them if and only if their edges in G have a common endpoint.

Prove or disprove: The line graph of a bipartite graph is a bipartite graph.

4) 20 pts

Hardy decides to start running to work in San Francisco city to get in shape. He prefers a route that goes entirely uphill and then entirely downhill so that he could work up a sweat uphill and get a nice, cool breeze at the end of his run as he runs faster downhill. He starts running from his home and ends at his workplace. To guide his run, he prints out a detailed map of the roads between home and work with k intersections and m road segments (any existing road between two intersections). Each road segment has unit length, and each intersection has a distinct elevation. Assuming that every road segment is either fully uphill or fully downhill, give an efficient algorithm to find the shortest path (route) that meets Hardy's specifications. If no such path meets Hardy's specifications, your algorithm should determine this. Justify your answer.

5) 10 pts

How many times does "USC" get printed when the function Func() is called with input *m*, a power of 2. Your answer is required to be in $\Theta(\,)$ notation as a function of *m*.

```
Func(n) {
        if (n > 1) {
                Print(USC) n times
                Func(n/2)
                Func(n/2)
                }
        return}
```

6) 20 pts

The citizens of Hollywood vote for their favorite film of the year. Assume there are $n = 2^m$ citizens voting. The vote of the i^{th} citizen is for the film V(i). A film is said to win with majority if more than half the citizens vote for it. Assume you can answer if V (i) and V (j) are the same film in constant time. Describe an O(n log n) algorithm to find if a film wins with majority and if so to find that film.

Additional Space

Additional Space