CSCI 570 Spring 2025 Homework 2

Q1. What is the tight upper bound to the worst-case runtime performance of the procedure below? (8points)

```
c = 0
i = n
while i > 1 do
    for j = 1 to i do
        c = c + 1
    end for
    i = floor(i/2)
end while
return c
```

Q2. Given functions f_1, f_2, g_1, g_2 such that $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample. (12points)

(a) $f_1(n) \cdot f_2(n) = O(g_1(n) \cdot g_2(n))$

(b) $f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$

(c) $f_1(n)^2 = O(g_1(n)^2)$

(d) $\log_2 f_1(n) = O(\log_2 g_1(n))$

Q3. Given an undirected graph G with n nodes and m edges, design an $O(m+n)$ algorithm to detect whether G contains a cycle. Your algorithm should output a cycle if G contains one. (10points)

Q4. Design an algorithm which, given a directed graph G = (V, E) and a particular edge e $\in$ E going from node u to node v, determines whether G has a cycle containing e. The running time should be bounded by O(|V | + |E|). Explain why your algorithm runs in O(|V | + |E|) time. (10points)

Q5. For each of the following indicate if f = O(g) or f = Θ(g) or f = Ω(g). (10points)

(1). $f(n) = n^4/\log(n)$ $\quad g(n) = n(\log(n))^4$

(2). $f(n) = n*\log(n)$ $\quad g(n) = n^2\log(n^2)$

(3). $f(n) = \log(n)$ $\quad g(n) = \log(\log(5^n))$

(4). $f(n) = n^{1/3}$ $\quad g(n) = (\log(n))^3$

(5). $f(n) = 2^n$ $\quad g(n) = 2^{3n}$

Practice (Ungraded) Problems:

1. Solve Kleinberg and Tardos, **Chapter 3, Exercise 9.**

2. Solve Kleinberg and Tardos, **Chapter 3, Exercise 6.**