CS570
Analysis of Algorithms
Fall 2014
Exam I

Name: _____

Student ID: _____

Thursday Evening Section **DEN Yes / No**

	Maximum	Received
Problem 1	20	
Problem 2	15	
Problem 3	15	
Problem 4	20	
Problem 5	15	
Problem 6	15	
Total	100	

2 hr exam
Close book and notes

If a description to an algorithm is required please limit your description to within 150 words, anything beyond 150 words will not be considered.

1) 20 pts
Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]
Adding a number *w* on the weight of every edge of a graph might change the shortest path between two vertices u and v.

[TRUE/FALSE]
Suppose that for some graph G we have that the average edge weight is *A*. Then a minimum spanning tree of G will have weight at most $(n-1) \cdot A$.

[TRUE/FALSE]
DFS finds the longest paths from start vertex *s* to each vertex *v* in the graph.

[TRUE/FALSE]
If one can reach every vertex from a start vertex s in a directed graph, then the graph is strongly connected.

[TRUE/FALSE]
F(n) = *4n+3*$\sqrt{n}$ is both $O(n)$ and $\Omega(n)$.

[TRUE/FALSE]
In Fibonacci heaps, the decrease-key operation takes $O(1)$ time.

[TRUE/FALSE]
If the edge weights of a weighted graph are doubled, then the number of minimum spanning trees of the graph remains unchanged.

[TRUE/FALSE]
Given a binary max-heap with n elements, the time complexity of finding the smallest element is O(lg n).

[TRUE/FALSE]
An undirected graph $G = (V, E)$ must be connected if $|E| > |V| - 1$

[TRUE/FALSE]
If all edges in a connected undirected graph have unit cost, then you can find the MST using BFS.

2) 16 pts
At the Perfect Programming Company, programmers program in pairs in order to ensure that the highest quality code is produced. The productivity of each pair of programmers is the speed of the slower programmer. For an even number of programmers, give an efficient algorithm for pairing them up so that the sum of the productivity of all pairs is maximized. Analyze the running time and argue the correctness of your algorithm.

3) 16 pts

The graph K_n is defined to be an undirected graph with n vertices and all possible edges (a fully connected graph). That is, the vertices are named $\{1, \ldots, n\}$ and for any numbers i and j with $i \neq j$, there is an edge between vertex i and vertex j. Describe the result of a breadth-first search and a depth-first search of K_n. For each search, describe the resulting search tree.

4) 16 pts

A *d-ary heap* is like a binary heap, but instead of 2 children, nodes have *d* children.

(a) How would you represent a d-ary heap in an array? [4 points]

(b) What is the height of a d-ary heap of n elements in terms of *n* and *d*? [4 points]

(c) Give an efficient implementation of ExtractMin. Analyze its running time in terms of *d* and *n*. [8 points]

5) 16 pts

You are given a graph representing the several career paths available in industry. Each node represents a position and there is an edge from node v to node u if and only if v is a pre-requisite for u. Top positions are the ones which are not pre-requisites for any positions. Ivan wants to start a career and achieve a top position by performing minimum number of positions. Using the given graph can you provide a linear time algorithm to help him? Assume the graph is a DAG.

6) 16 pts

Suppose we are given an instance of the Minimum Spanning Tree problem on a graph G. Assume that all edges costs are distinct. Let T be a minimum spanning tree for this instance. Now suppose that we replace each edge cost c_e by its square, c_e^2 thereby creating a new instance of the problem with the same graph but different costs.
Prove or disprove: T is still a MST for this new instance.

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Additional Space