1. [10%] Search
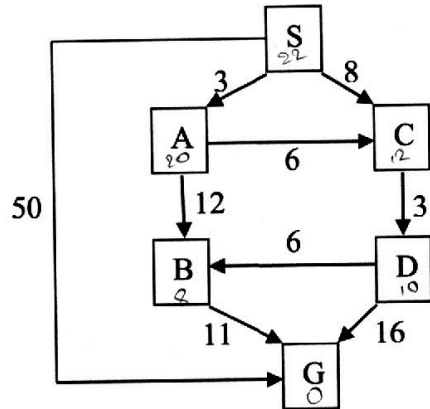
Consider the following search problem where **S** is the start state and **G** satisfies the goal test. Arcs are labeled with the cost of traversing them:

The heuristic estimates of the distance to G are:

from:	S	A	B	C	D	G
distance:	22	20	8	12	10	0

For each of the following search strategies, indicate which goal state is reached (if any) and list, in order, all the states of the nodes popped off of the OPEN queue, and the cost of the path found by the strategy to reach the goal state from S. When all else is equal, nodes should be removed from OPEN in <u>alphabetical order</u>.

<u>Please apply the "clean and robust" algorithm studied in class for loop detection.</u>

Note how the arcs in the figure are oriented, which means that you can only go from one state to another in the direction of the arrow.

a) [5%] Uniform cost Search

Goal state reached: __G__ States popped off OPEN: __S A C D B G__ Path Cost __26__
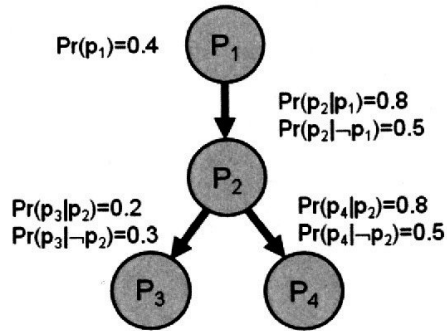
b) [5%] A* search

Goal state reached: __G__ States popped off OPEN: __S C D A B G__ Path Cost __26__

S. A_3 C_8 C_9 B_{15} B_7 G_{21} S A C D B G.
      D_{11} G_{26}

S D_{21} A_{23} C_{20} G_{50} S C D A B G.
     B_{13} B_{15} G_{27}
       G_{26}

2. [20%] Bayesian Networks

Consider the following Bayesian Network:

$Pr(p_1)=0.4$ — P_1

$Pr(p_2|p_1)=0.8$
$Pr(p_2|\neg p_1)=0.5$

P_2

$Pr(p_3|p_2)=0.2$
$Pr(p_3|\neg p_2)=0.3$

$Pr(p_4|p_2)=0.8$
$Pr(p_4|\neg p_2)=0.5$

P_3 P_4

Derive symbolic and numerical expressions for the following probabilities given the network, using **inference by enumeration**. Please first write symbolic expressions (e.g., Pr(p1) x Pr(p3|p2) + …) and then use the above probabilities values to write numerical expressions (e.g., 0.4 x 0.2 + …). You need not compute the final numerical result, a correct numerical expression (with sums and products of numerical values) is sufficient to gain full credits. You will lose marks if either the symbolic expression or the numerical expression is missing.

A. [10%] Compute Pr(¬p3):

$$Pr(\neg P_3) = Pr(\neg P_3 \, P_1 \, P_2) = \sum_{P_1} \sum_{P_2} Pr(\neg P_3 \, P_1 \, P_2) =$$
$$\{\neg P_1, P_1\} \{P_2, P_2\}$$

$$Pr(\neg P_3 \, P_1 \, P_2) + Pr(\neg P_3 \, \neg P_1 \, P_2) + Pr(\neg P_3 \, \neg P_2 \, P_1) + Pr(\neg P_3 \, \neg P_1 \, \neg P_2)$$

$$Pr(\neg P_3 | P_2) Pr(P_2 | P_1) + Pr(\neg P_3 | P_2) Pr(P_2 | \neg P_1) + Pr(\neg P_3 | \neg P_2) Pr(\neg P_2 | P_1) + Pr(\neg P_3 | \neg P_2) Pr(\neg P_2 | \neg P_1).$$

$$(1-0.2)(0.8) + (1-0.2)(0.5) + (1-0.3)(1-0.8) + (1-0.3)(1-0.5)$$

B. [10%] Compute Pr(p1|¬p3, p4):

$$Pr(P_1 | \neg P_3, P_4) = \frac{Pr(P_1 \neg P_3 \, P_4)}{Pr(\neg P_3 \, P_4)} = \frac{Pr(P_1 \, \neg P_3 \, P_4 \, P_2)}{Pr(\neg P_3) \, Pr(P_4)} = \frac{\sum_{P_2} Pr(P_1 \, \neg P_3 \, P_4 \, P_2)}{Pr(\neg P_3) \sum_{P_1 P_2} Pr(P_1 P_2 P_4)}$$

Numerator $= Pr(P_1 \, P_2 \, \neg P_3 \, P_4) + Pr(P_1 \, \neg P_2 \, \neg P_3 \, P_4) = P(\neg P_3 | P_2) P(P_4 | P_2)(P_2 | P_1) + P(\neg P_3 | \neg P_2) P(P_4 | \neg P_2)(\neg P_2 | P_1).$

$= (1-0.2)(0.8)(0.8) + (1-0.3)(0.5)(1-0.8).$

Denominator $= Pr(\neg P_3) \left(Pr(P_1 \, P_2 \, P_4) + Pr(\neg P_1 \, P_2 \, P_4) + Pr(P_1 \, \neg P_2 \, P_4) + Pr(\neg P_1 \, \neg P_2 \, P_4) \right)$

$= Pr(\neg P_3) \left(Pr(P_4 | P_2) Pr(P_2 | P_1) + P_-(P_4 | P_2) Pr(P_2 | \neg P_1) + Pr(P_4 | \neg P_2) Pr(\neg P_2 | P_1) + Pr(P_4 | \neg P_2) Pr(\neg P_2 | \neg P_1) \right)$

$= Pr(\neg P_3) \left((0.8)(0.8) + (0.8)(0.5) + (0.5)(1-0.8) + (0.5)(1-0.5) \right).$

↑
From Previous Question

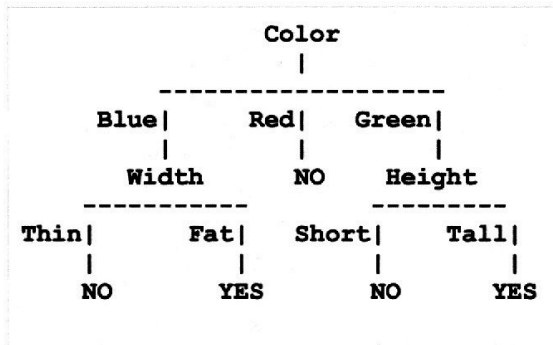$Ans = Pr(P_1 | \neg P_3 \, P_4) = \dfrac{Numerator}{Denominator}$

3. [10%] Decision trees

Given the following decision tree, show
how the new examples in the table would be
classified, by filling in the last column in the table.
If an example cannot be classified, enter *UNKNOWN* in the last column.
You receive 2% for each correct answer.

```
                    Color
                      |
        ----------------------------
    Blue|        Red|      Green|
        |           |          |
      Width         NO      Height
    -------------            ----------
Thin|       Fat|       Short|     Tall|
    |          |           |         |
   NO         YES         NO        YES
```

Example	Color	Height	Width	Class
A	Red	Short	Thin	No
B	Blue	Tall	Fat	YES
C	Green	Short	Fat	NO
D	Green	Tall	Thin	YES
E	Blue	Short	Thin	No

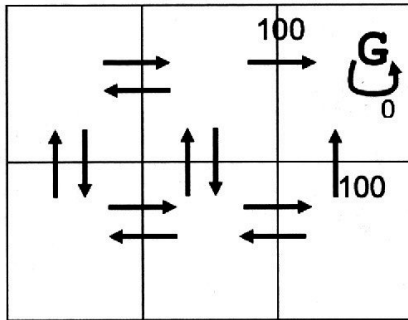(space below available for rough work)

4. [20%] Markov Decision Processes

Consider the following MDP problem:

Assume that the value function is initialized to 0 in every cell. Assume a discount factor γ=0.5 and assume that the immediate reward associated with actions is 1 everywhere except for: a) the two actions that lead to G, whose immediate reward is 100, and b) the action from G to G, whose immediate reward is 0, as shown above. Assume that the actions always succeed.

Please fill in the values computed by the value iteration algorithm, at convergence, in the cells below:
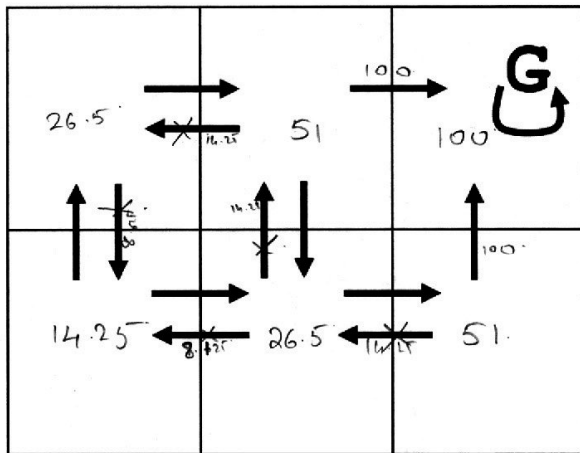
$$u(s, a') = r + r\, U(s, a').$$

5. [10%] Neural Networks
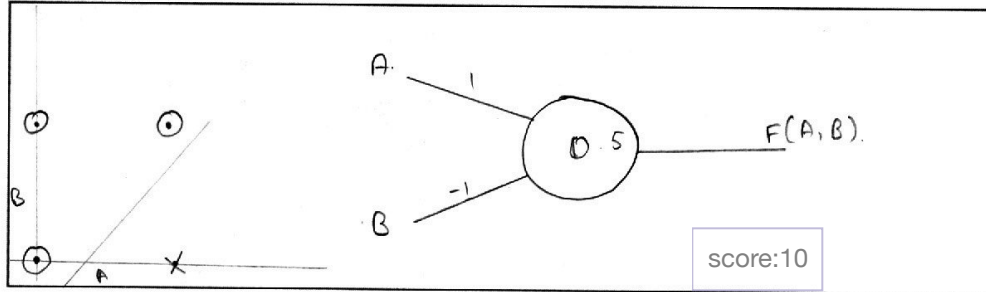
Can you represent the following boolean
function f(A, B) with a single artificial neuron?
If yes, show the weights and threshold. If not, explain why not in 1-2 sentences.

A	B	f(A,B)
1	1	0
0	0	0
1	0	1
0	1	0

A ·
1
0.5 → F(A, B)
-1
B

score:10

6. [10%] Bayes theorem

+OR.

I don't have a car. I come to work either by bike or by bus. If I take the bus, there is a 10% chance that I am
late. If I take the bike, there is a 2% chance that I am late. I take the bike 4 days out of 5. Today I was late.
What is the probability that I took the bus?

A [2%] Write down and explain the formula used in Bayes' theorem for this problem.

$P(\text{Late} \mid \text{Bus}) = 0.1$ → Chance of reaching late given I take the Bus.

Late|¬Bus $P(\text{Late} \mid \text{Bike}) = 0.02$ → Chance of reaching late given I take the Bike

$P(\text{Bike}) = 0.8 \mid P(\text{Bus}) = 0.2$ → chance of taking Bike   Bus = ¬Bike

Score:0 Bayes Formula not written and explained.

B. [3%] Use Bayes' theorem to calculate the probability that I took the bus today.

$$P(\text{Bus} \mid \text{Late}) = \frac{P(\text{Late} \mid \text{Bus}) \cdot P(\text{Bus})}{P(\text{Late})} = \frac{P(\text{Late} \mid \text{Bus}) P(\text{Bus})}{P(\text{Late} \mid \text{Bus}) P(\text{Bus}) + P(\text{Late} \mid \neg\text{Bus}) P(\neg\text{Bus})}$$

$$= \frac{0.1 \times 0.2}{0.1 \times 0.2 + 0.02 \times 0.8}$$

Score: 2.5

-0.5: final calculated answer not written.

C. [5%] Model the situation as a Bayesian network with 2 nodes, and give the conditional probability tables for
both nodes.

Bus → Late

$P(\text{Bus}) = 0.2$.

$P(\text{Late} \mid \text{Bus}) = 0.1$
$P(\text{Late} \mid \neg\text{Bus}) = 0.02$.
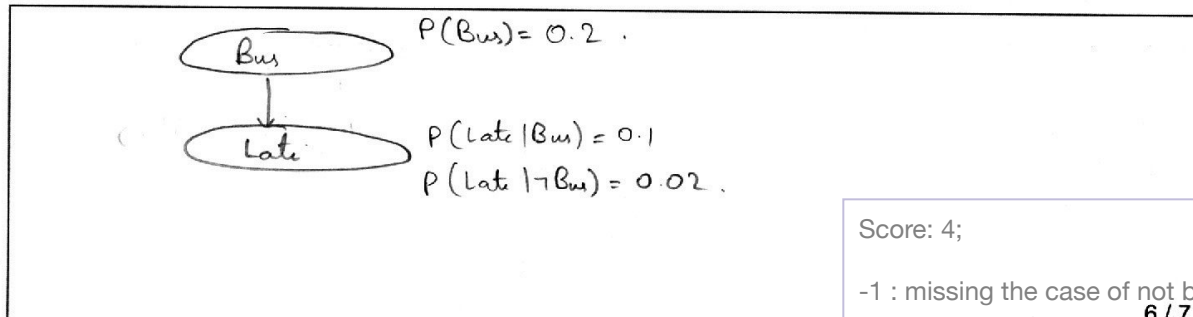
Score: 4;

-1 : missing the case of not being late on second node. See rubrics.

7. [20%] FOL Resolution Proof

Given: 1. $\forall_{S1,S2}$ subset$(S1, S2) \Leftrightarrow [\forall_X$ member$(X, S1) \Rightarrow$ member$(X, S2)]$.
Prove: H. $\forall_{S1,S2,S3}$ [subset$(S1, S2) \wedge$ subset$(S2, S3)] \Rightarrow$ subset$(S1, S3)$.

a. [9%] Convert sentence 1 and the negation of sentence H to CNF:

Assum
Member$(x, S_1) = m(x, S_1)$.
Subset$(S1, S2) = S(S1, S2)$

$\forall S_1 S_2 \; S(S_1, S_2) \Leftrightarrow [\forall x \; m(x, S_1) \Rightarrow m(x, S_2)]$

$\Rightarrow$ $\forall S_1 S_2 \; S(S_1, S_2) \Rightarrow [\forall x \; m(x, S_1) \Rightarrow m(x, S_2)] \wedge [m(x, S_1) \Rightarrow m(x, S_2)] \Rightarrow S(S_1, S_2)$

$\Rightarrow$ $\forall S_1 S_2 \; \neg S(S_1, S_2) \vee [\forall x \; \neg m(x, S_1) \vee m(x, S_2)] \wedge [\forall x \; \neg m(x, S_1) \vee m(x, S_2) \vee S(S_1, S_2)]$

$\forall S_1 S_2 (\neg S(S_1, S_2) \vee \neg m(x, S_1) \vee m(x, S_2)) \wedge ([\exists x \; m(x, S_1) \wedge \neg m(x, S_1)] \vee S(S_1, S_2))$

(∃E) $\forall S_1 S_2 (\neg S(S_1, S_2) \vee \neg m(x, S_1) \vee m(x, S_2)) \wedge (m(H(S_1), S_1) \wedge \neg m(H(S_1), S_2) \vee S(S_1, S_2))$

(∀E) $(\neg S(S_1, S_2) \vee \neg m(x, S_1) \vee m(x, S_2)) \wedge (m(H(S_1), S_1) \vee S(S_1, S_2)) \wedge$

$(\neg m(H(S_2), S_1) \vee S(S_1, S_2))$ +6

$\neg H \Rightarrow (S(S_1, S_2) \wedge S(S_2, S_3) \wedge \neg S(S_1, S_3))$ [See Behind] $\Rightarrow$ +2

b. [11%] Draw your resolution proof. Only use the resolution inference rule, as you will lose points if you use any other rule. Please clearly show which sentences are resolved and what results. If unification is used at any step, please show the substitution, or you will lose points for each missing substitution.

KB = $\neg S(S_1, S_2) \vee \neg m(x, S_1) \vee m(x, S_2) \rightarrow$ ①   Query $S(S_1, S_2) \rightarrow \alpha$
      $m(H(S_1), S_1) \vee S(S_1, S_2) \rightarrow$ ② $S(S_2, S_3) \rightarrow \beta$
      $\neg m(H(S_2), S_2) \vee S(S_1, S_2) \rightarrow$ ③ $S(S_1, S_3) \rightarrow \gamma$

α resolve ① $\Rightarrow S(S_1, S_2)$ $\neg S(S_1, S_2) \vee \neg m(x, S_1) \vee m(x, S_2)$
                              $S_1 \in S_1$
                              $S_2 \in S_2$ 0
      ②
$m(H(S_1), S_1) / \vee S(S_1, S_2)$ $\neg m(x, S_1) \vee m(x, S_2)$.
                                        $x \in H(S_1) \in S_1$.

$S(S_1, S_2) \vee m(x, S_2)$. $\neg H(H(S_1), S_1) \vee S(S_1, S_2)$.

                              $/ x \in H(S_1) \in S_2$

$S(S_1, S_2). \rightarrow$ Remain Loop ∴ Fail.

Fail ∧ B ∧ γ $\Rightarrow$ Fail.

7 / 7

Hence Disproved.