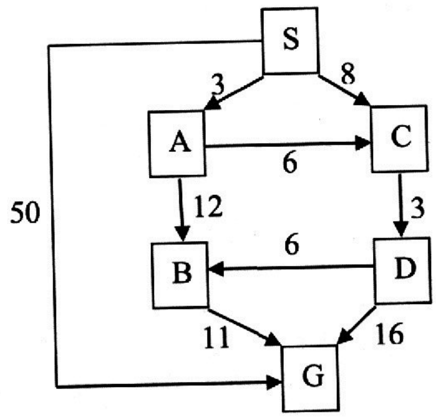
1. [10%] Search

Consider the following search problem where **S** is the start state and **G** satisfies the goal test. Arcs are labeled with the cost of traversing them:



The heuristic estimates of the distance to G are:

from:	S	A	B	C	D	G
distance:	22	20	8	12	10	0

For each of the following search strategies, indicate which goal state is reached (if any) and list, in order, all the states of the nodes popped off of the OPEN queue, and the cost of the path found by the strategy to reach the goal state from S. When all else is equal, nodes should be removed from OPEN in alphabetical order.

Please apply the "clean and robust" algorithm studied in class for loop detection.

Note how the arcs in the figure are oriented, which means that you can only go from one state to another in the direction of the arrow.

a) [5%] Uniform cost Search

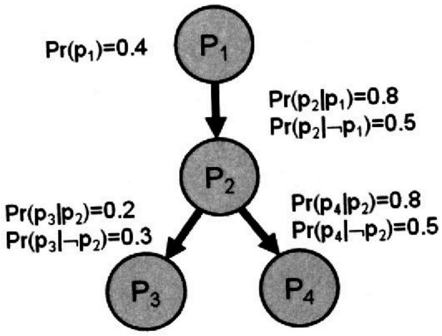
Goal state reached: G States popped off OPEN: S A C D B G Path Cost 26

b) [5%] A* search

Goal state reached: G States popped off OPEN: S C D A B G Path Cost 26

2. [20%] Bayesian Networks

Consider the following Bayesian Network:



Derive symbolic and numerical expressions for the following probabilities given the network, using **inference by enumeration**. Please first write symbolic expressions (e.g., Pr(p1) x Pr(p3|p2) + …) and then use the above probabilities values to write numerical expressions (e.g., 0.4 x 0.2 + …). You need not compute the final numerical result, a correct numerical expression (with sums and products of numerical values) is sufficient to gain full credits. You will lose marks if either the symbolic expression or the numerical expression is missing.

A. [10%] Compute Pr(¬p3):

$$Pr(\neg P_3) = Pr(\neg P_3 \mid P_2) * (Pr(P_2 \mid P_1) * Pr(P_1) + Pr(P_2 \mid \neg P_1) * Pr(\neg P_1))$$
$$+ Pr(\neg P_3 \mid \neg P_2) * (Pr(\neg P_2 \mid P_1) * Pr(P_1) + Pr(\neg P_2 \mid \neg P_1) * Pr(\neg P_1))$$
$$= 0.8 * (0.8 * 0.4 + 0.5 * 0.6) + 0.7 * (0.2 * 0.4 + 0.5 * 0.6)$$
$$= 0.8 * (0.62) + 0.7 * (0.38)$$
$$= 0.496 + 0.266 = 0.762$$

10

B. [10%] Compute Pr(p1|¬p3, p4):

$$Pr(p1 \mid \neg P3, p4) = \frac{Pr(\neg P3 \mid P1) * Pr(P4 \mid P1) * Pr(P1)}{Pr(\neg P3) * Pr(P4)}$$
$$= \frac{(0.8 * 0.8 + 0.7 * 0.2) * (0.8 * 0.8 + 0.5 * 0.2) * 0.4}{(0.8 * (0.8 * 0.4 + 0.5 * 0.6) + 0.7 * (0.2 * 0.4 + 0.5 * 0.6)) * (0.8 * (0.8 * 0.4 + 0.5 * 0.6) + 0.5 * (0.2 * 0.4 + 0.5 * 0.6))}$$

9

3. [10%] Decision trees

Given the following decision tree, show how the new examples in the table would be classified, by filling in the last column in the table.
If an example cannot be classified, enter *UNKNOWN* in the last column.
You receive 2% for each correct answer.

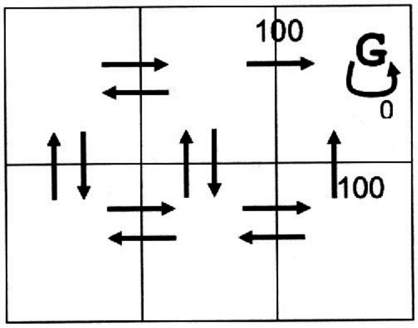
```
                    Color
                      |
            -------------------
       Blue|       Red|     Green|
           |          |          |
         Width        NO       Height
     -----------             ---------
Thin|         Fat|      Short|       Tall|
    |            |           |           |
    NO          YES          NO         YES
```

Example	Color	Height	Width	Class
A	Red	Short	Thin	NO
B	Blue	Tall	Fat	YES
C	Green	Short	Fat	NO
D	Green	Tall	Thin	YES
E	Blue	Short	Thin	NO

(space below available for rough work)

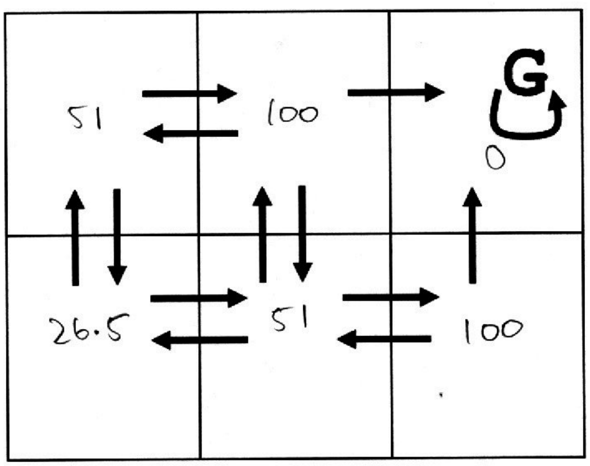
4. [20%] Markov Decision Processes

Consider the following MDP problem:



Assume that the value function is initialized to 0 in every cell. Assume a discount factor γ=0.5 and assume that the immediate reward associated with actions is 1 everywhere except for: a) the two actions that lead to G, whose immediate reward is 100, and b) the action from G to G, whose immediate reward is 0, as shown above. Assume that the actions always succeed.

Please fill in the values computed by the value iteration algorithm, at convergence, in the cells below:



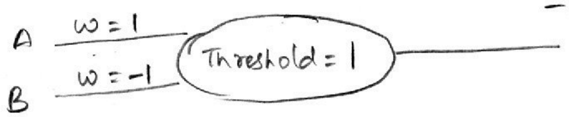
Q5+6 Score: **19.0**

5. [10%] Neural Networks

Can you represent the following boolean function f(A, B) with a single artificial neuron?
If yes, show the weights and threshold. If not, explain why not in 1-2 sentences.

A	B	f(A,B)
1	1	0
0	0	0
1	0	1
0	1	0

Yes. A has a weight of 1, B has a weight of -1 and the Threshold is 1.



score:10

6. [10%] Bayes theorem

I don't have a car. I come to work either by bike or by bus. If I take the bus, there is a 10% chance that I am late. If I take the bike, there is a 2% chance that I am late. I take the bike 4 days out of 5. Today I was late.
What is the probability that I took the bus?

A [2%] Write down and explain the formula used in Bayes' theorem for this problem.

$$P(Bus \mid Late) = \frac{P(Late \mid Bus) * P(Bus)}{P(Late \mid Bus) * P(Bus) + P(Late \mid Bike) * P(Bike)}$$

score:2

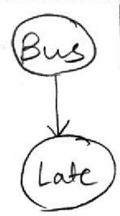
The LHS is the problem. To compute Prob. of Bus if late. The RHS is Baye's theorem, with numerator being Prob of being late if Bus and prob. of taking a bus and denominator being prob. of being late.

B. [3%] Use Bayes' theorem to calculate the probability that I took the bus today.

$$P(Bus \mid Late) = \frac{0.1 * 0.2}{0.1 * 0.2 + 0.02 * 0.8} = \frac{0.02}{0.02 + 0.016} = \frac{0.02}{0.036} = \frac{5}{9} = 0.55$$

score:3

C. [5%] Model the situation as a Bayesian network with 2 nodes, and give the conditional probability tables for both nodes.



P(Bus)
0.2

Bus	P(Late)
T	0.1
F	0.02

Score: 4; -1 : missing the case of not being late on second node. See rubrics.

7. [20%] FOL Resolution Proof

Given: 1. $\forall_{S1,S2}$ subset$(S1, S2) \Leftrightarrow [\forall_X$ member$(X, S1) \Rightarrow$ member$(X, S2)]$.
Prove: H. $\forall_{S1,S2,S3}$ [subset$(S1, S2) \wedge$ subset$(S2, S3)] \Rightarrow$ subset$(S1, S3)$.

a. [9%] Convert sentence 1 and the negation of sentence H to CNF:

1. can be split into two sentences -
$\forall S1, S2$ subset$(S1, S2) \Rightarrow [\forall x$ member$(x, S1) \Rightarrow$ member$(x, S2)]$
and
$\forall S1, S2 [\forall x$ member$(x, S1) \Rightarrow$ member$(x, S2)] \Rightarrow$ subset$(S1, S2)$

After CNF -
1. $\neg$subset$(y, z) \vee \neg$member$(x, y) \vee$ member(x, z)
2. member$(g(v, w), v) \vee$ subset(v, w)
3. $\neg$member$(g(t, u), u) \vee$ subset(t, u)

[g is a skollem function]

+6

Negation of H to CNF -
subset$(A, B) \wedge$ subset$(B, C) \wedge \neg$subset(A, C) where A, B, C are skollem constants

+3

b. [11%] Draw your resolution proof. Only use the resolution inference rule, as you will lose points if you use any other rule. Please clearly show which sentences are resolved and what results. If unification is used at any step, please show the substitution, or you will lose points for each missing substitution.

$\boxed{\neg\text{member}(g(t,u),u) \vee \text{subset}(t,u)}$ $\boxed{\neg\text{subset}(A,C)}$

$\theta = \{t/A, u/C\}$

$\boxed{\neg\text{subset}(y,z) \vee \neg\text{member}(x,y) \vee \text{member}(x,z)}$ $\boxed{\neg\text{member}(g(A,C),C)}$

$\theta = \{y/A, z/C, x/g(A,C)\}$

$\boxed{\text{member}(g(v,w),v) \vee \text{subset}(v,w)}$ $\boxed{\neg\text{subset}(A,C) \vee \neg\text{member}(g(A,C),A)}$

$\theta = \{v/A, w/C\}$

$\boxed{}$

Proof by contradiction

+11