CS570

Analysis of Algorithms

Fall 2006

Exam 1

Name: ________________________

Student ID: ____________________

	Maximum	Received
Problem 1	20	
Problem 2	10	
Problem 3	10	
Problem 4	10	
Problem 5	10	
Problem 6	20	
Problem 7	20	

Note: The exam is closed book closed notes.

1) 20 pts
Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]
If T(n) is both **O**(f(n)) and **Ω**(f(n)), then T(n) is **Θ**(f(n)) .

[TRUE/FALSE]
For a graph G and a node v in that graph, the DFS and BFS trees of G rooted at v always contain the same number of edges

[TRUE/FALSE]
Complexity of the "Decrease_Key" operation is always O(lgn) for a priority queue.

[TRUE/FALSE]
For a graph with distinct edge weights there is a unique MST.

[TRUE/FALSE]
Dynamic programming considers all the possible solutions.

[TRUE/FALSE]
Consider an undirected graph G=(V, E) and a shortest path P from s to t in G. Suppose we add one 1 to the cost of each edge in G. P will still remain as a shortest path from s to t.

[TRUE/FALSE]
Consider an undirected graph G=(V, E) and its minimum spanning tree T. Suppose we add one 1 to the cost of each edge in G. T will still remain as an MST.

[TRUE/FALSE]
Problems solved using dynamic programming cannot be solved thru greedy algorithms.

[TRUE/FALSE]
The union-Find data structure can be used for an efficient implementation of the reverse delete algorithm to find an MST.

[TRUE/FALSE]

While there are different algorithms to find a minimum spanning tree of undirected connected weighted graph G, all of these algorithms produce the same result for a given G.

2) 10 pts

Indicate for each pair of expressions (A,B) in the table below, whether A is **O**, **Ω**, or **Θ** of B. Assume that k and c are positive constants. You can mark each box with Y (yes) and N (no).

A	B	**O**	**Ω**	**Θ**
$n^3 + n^2 + n + c$	n^3			
2^n	$2^{(n+k)}$			
n^2	$n \,.\, 2^{\log(n)}$			

3) 10 pts

a- What is the minimum and maximum numbers of elements in a heap of height h?

b- What is the number of leaves in a heap of size *n*?

c- Is the sequence < 23, 7, 14, 6, 13, 10, 1, 5, 17, 12 > a max-heap? If not, show how to heapify the sequence.

d- Where in a max-heap might the smallest element reside, assuming that all elements are distinct.

4) 10 pts
Prove or disprove the following:
The shortest path between any two nodes in the minimum spanning tree T = (V, E') of connected weighted undirected graph G = (V,E) is a shortest path between the same two nodes in G. Assume the weights of all edges in G are unique and larger than zero.

5) 10 pts

Suppose that you divided a graph G = (V,E) into two sub graphs G1 = (V1,E1) and G2 = (V2, E2). And, we can find M1 which is a MST of G1 and M2 which is MST of G2. Then, M1 U M2 U {minimum weight edge among those connecting two graph G1 and G2} always gives MST of G. Prove it or disprove it.

6) 20 pts

There are n workers in the factory with heights of $h_1, h_2, \ldots, h_n$, and n working-clothes with height sizes of $s_1, s_2, \ldots, s_n$. The problem is to find best matching strategy such that we minimize the following average differences.

$$\frac{1}{n}\Sigma\left|h_i - s_i\right|$$

Present an efficient algorithm to solve this problem along with its proof of correctness.

7) 20 pts

Given an unlimited supply of coins of denominations $x_1, x_2, \ldots, x_n$, we wish to make change for a value *v*, that is, we wish to find a set of coins whose total value is *v*. This might not be possible: for example, if the denominations are 5 and 10 then we can make change for 15 but not for 12. Give an *O(nv)* algorithm to determine if it is possible to make change for *v* using coins of denominations $x_1, x_2, \ldots, x_n$.

Additional Space

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