CS570

Analysis of Algorithms
Summer 2008
Exam I

Name: ______________________

Student ID: __________________

____4:00 - 5:40 Section ____6:00 – 7:40 Section

	Maximum	Received
Problem 1	20	
Problem 2	20	
Problem 3	10	
Problem 4	15	
Problem 5	10	
Problem 6	10	
Problem 7	15	
Total	100	

2 hr exam
Close book and notes

1) 20 pts

Mark the following statements as **TRUE** or **FALSE**. No need to provide any justification.

[TRUE/FALSE]
There exist a perfect matching and a corresponding preference list such that every man is part of an instability, and every woman is part of an instability.

[TRUE/FALSE]
A greedy algorithm always returns the optimal solution.

[TRUE/FALSE]
The function 100n + 3 is O(n^2)

[TRUE/FALSE]
You are given n elements to insert into an empty heap. You could directly call heap insert n times. Or you could first sort the elements and then call heap insert n times. In either case, the asymptotic time complexity is the same.

[TRUE/FALSE]
If a problem can be solved correctly using the greedy strategy, there will only be one greedy choice (e.g. "choose the object with highest value to weight ratio") for that problem that leads to the optimal solution.

[TRUE/FALSE]
The Depth First Search algorithm can not be used to test whether a directed graph is strongly-connected.

[TRUE/FALSE]
Consider an undirected graph G=(V, E) with non-negative edge weights. Suppose all edge weights are different. Then the edge of maximum weight can be in the minimum spanning tree.

[TRUE/FALSE]
Consider a perfect matching S where Mike is matched to Susan. Suppose Mike prefers Winona to Rachel. Then the pair (Mike, Rachel) can never be an instability with respect to S.

[TRUE/FALSE]
Consider an undirected graph G=(V, E). Suppose all edge weights are different. Then the shortest path from A to B must be unique.

[TRUE/FALSE]
The number of elements in a heap must always be an integer power of 2.

2) 20 pts

Given two graphs G and G' that have the same sets of vertices V and edges E, however different weight functions (W and W' respectively) on their edges. Suppose for each graph the weights on the edges are distinct and satisfy the following relation: $W'(e)=W(e)^2$ (Note: all edge weights in G and G' are positive integers) for every edge e of E. Decide whether each of the following statements is true. Give either a short proof or a counter example.

a) The minimum spanning tree of G is the same as the minimum spanning tree of G'.

b) For a pair of vertices *a* and *b* in V, a shortest path between them in G is also a shortest path in G'.

3) 10 pts

Prove or give a counterexample: An array that is sorted in ascending order is a min-heap.

4) 15 pts

Let G = (V,E) be an undirected graph with maximum degree d. A coloring of G is an assignment to each vertex of G of a “color” such that adjacent vertices have distinct colors. Consider the greedy algorithm that colors as many vertices as possible with color “j” before moving to color “j+1.”

Prove or give a counterexample: This greedy algorithm never requires more than d+1 colors to color G.

5) 10 pts

You and your eight-year-old nephew Shrek decide to play a simple card game. At the beginning of the game, the cards are dealt face up in a long row. Each card is worth a different number of points. After all the cards are dealt, you and Shrek take turns removing either the leftmost or rightmost card from the row, until all the cards are gone. At each turn, you can decide which of the two cards to take. The winner of the game is the player that has collected the most points when the game ends.

Having never taken an algorithms class, Shrek follows the obvious greedy strategy—when it's his turn, Shrek always takes the card with the higher point value. Your task is to find a strategy that will beat Shrek whenever possible. (It might seem mean to beat up on a little kid like this, but Shrek absolutely hates it when grown-ups let him win.)

Prove that you should not also use the greedy strategy. That is, show that there is a game that you can win, but only if you do not follow the same greedy strategy as Shrek.

6) 10 pts

Arrange the following functions in increasing order of asymptotic complexity. If $f(n)=\Theta(g(n))$ then put f=g. Else, if $f(n)=O(g(n))$, put $f < g$.

$4n^2$, $\log_2(n)$, $20n$, 2, $\log_3(n)$, n^n, 3^n, $n\log(n)$, $2n$, 2^{n+1}, $\log(n!)$

7) 15 pts

Prove or give a counterexample: Let G be an undirected, connected, bipartite, weighted graph. If the weight of each edge in G is +1, and for every pair of vertices (u,v) in G there is exactly one shortest path, then G is a tree.

Additional Space

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