CS570
Analysis of Algorithms
Spring 2008
Exam II

Name: _____

Student ID: _____

	Maximum	Received
Problem 1	20	
Problem 2	15	
Problem 3	15	
Problem 4	15	
Problem 5	20	
Problem 6	15	
Total	100	

Note: The exam is closed book closed notes.

1) 20 pts
Mark the following statements as **TRUE**, **FALSE**. No need to provide any justification.

True **[TRUE/FALSE]**
If all capacities in a network flow are rational numbers, then the maximum flow will be a rational number, if exist.

False [TRUE/FALSE]
The Ford-Fulkerson algorithm is based on the greedy approach.

False [TRUE/FALSE]
The main difference between divide and conquer and dynamic programming is that divide and conquer solves problems in a top-down manner whereas dynamic-programming does this bottom-up.

False [TRUE/FALSE]
The Ford-Fulkerson algorithm has a polynomial time complexity with respect to the input size.

True [TRUE/FALSE]
Given the Recurrence, $T(n) = T(n/2) + \theta(1)$, the running time would be $O(\log(n))$

True [TRUE/FALSE]
If all edge capacities of a flow network are increased by k, then the maximum flow will be increased by at least k.

True [TRUE/FALSE]
A divide and conquer algorithm acting on an input size of n can have a lower bound less than $\Omega(n \log n)$.

True [TRUE/FALSE]
One can actually prove the correctness of the Master Theorem.

True [TRUE/FALSE]
In the Ford Fulkerson algorithm, choice of augmenting paths can affect the number of iterations.

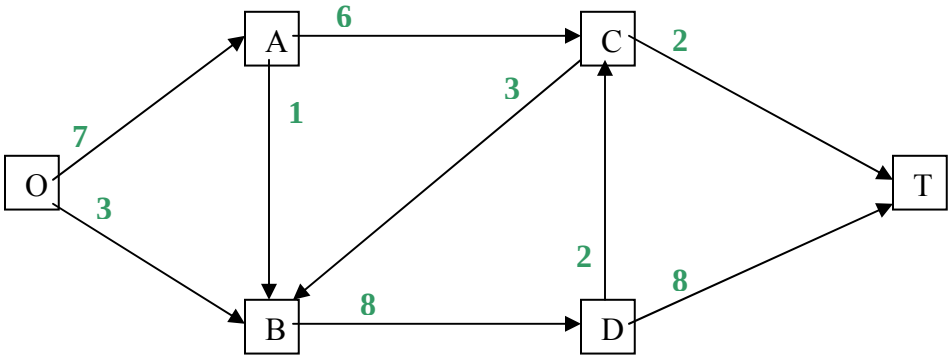
False [TRUE/FALSE]
In the Ford Fulkerson algorithm, choice of augmenting paths can affect the min cut.

2) 15 pts
Present a divide-and-conquer algorithm that determines the minimum difference between any two elements of a sorted array of real numbers.

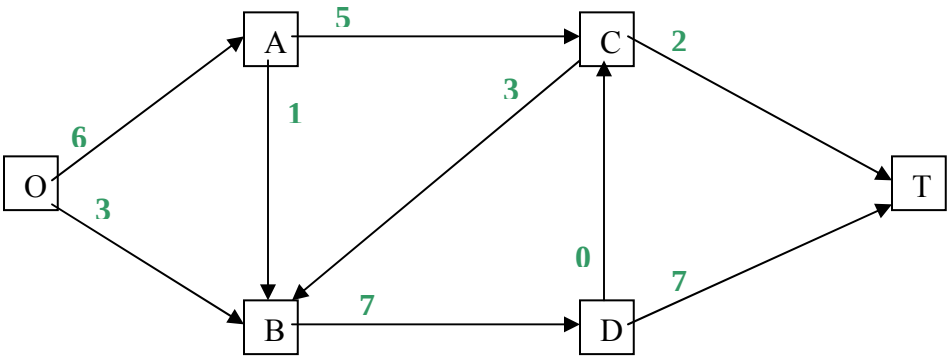
Let us assume the array is A[1…n]

```
LargestDifference(n)
{
        If n<=1, return Infinity;
        Else,    return min(LargestDifference(n-1), abs(A[n]-A[n-1]));
}
```

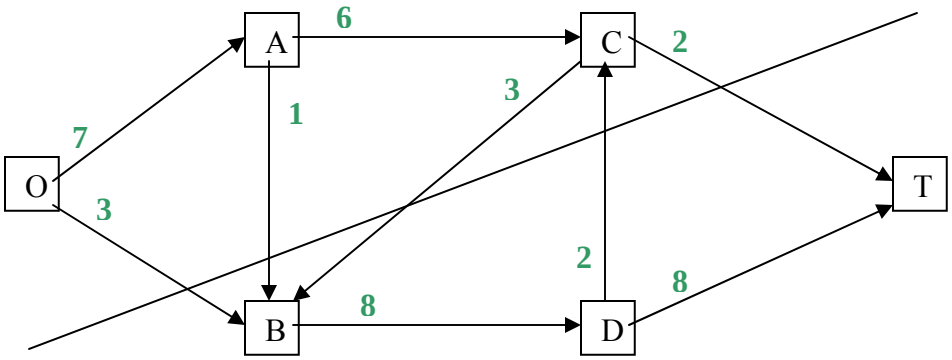
3) 15 pts
You are given the following directed network with source O and sink T.



a) Find a maximum flow from O to T in the network.



b) Find a minimum cut. What is its capacity?



The Capacity is 9.

4) 15 pts
Solve the following recurrences

a) $T(n) = 2T(n/2) + n \log n$

By the master theorem, $T(n) = n\log^2 n$

b) $T(n) = 2T(n/2) + \log n$

c) $T(n) = 2T(n-1) - T(n-2)$ for $n \geq 2$; $T(0) = 3$; $T(1) = 3$

$T(n) = 2T(n-1) – T(n-2) = 2(2T(n-2) – T(n-3)) –T(n-2) = 3T(n-2) – 2T(n-3)$
$=4T(n-3)-3T(n-4) = nT(1) – (n-1)T(0) = 3$

5) 20 pts
You are given a flow network with integer capacity edges. It consists of a directed graph G = (V, E), a source s and a destination t, both belong to V. You are also given a parameter k. The goal is to delete k edges so as to reduce the maximum flow in G as much as possible. Give a efficient algorithm to find the edges to be deleted. Prove the correctness of your algorithm and show the running time.

6) 15 pts

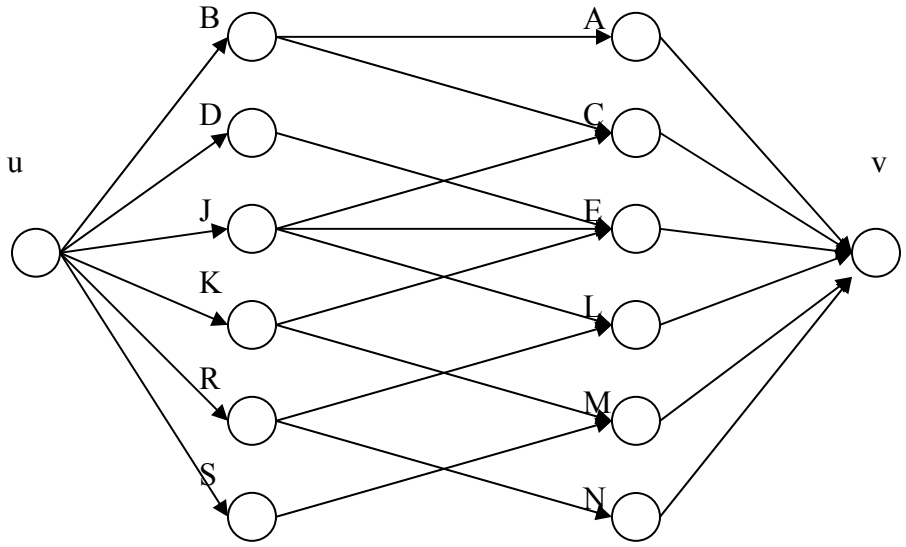
Six men and six women are at a dance. The goal of the matchmaker is to match each woman with a man in a way that maximizes the number of people who are matched with compatible mates. The table below describes the compatibility of the dancers.

	Ann	**Cindy**	**Erin**	**Liz**	**Mary**	**Nancy**
Bob	C	C	-	-	-	-
Dave	-	-	C	-	-	-
John	-	C	C	C	-	-
Kevin	-	-	C	-	C	-
Ron	-	-	-	C	-	C
Sam	-	-	-	-	C	-

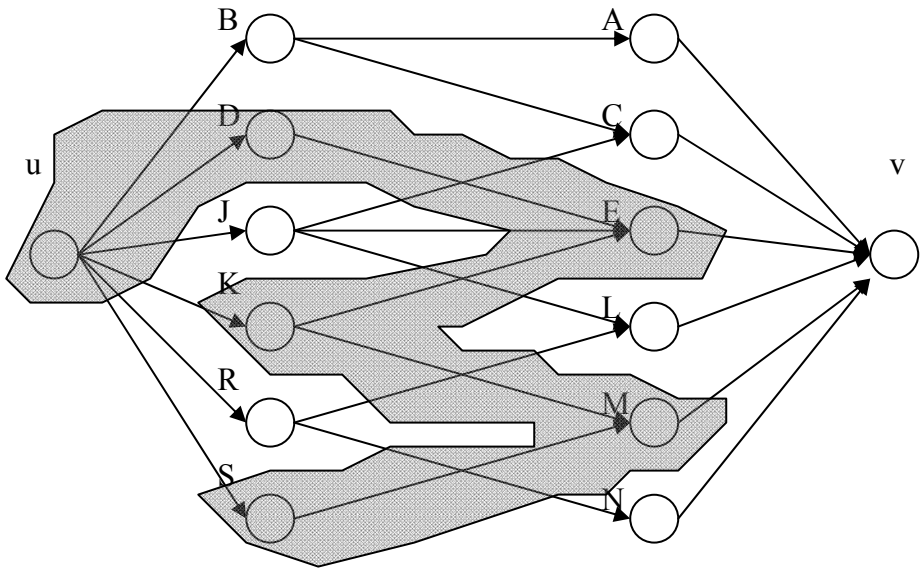
Note: C indicates compatibility.

a) Determine the maximum number of compatible pairs by reducing the problem to a max flow problem.

All edges have capacity of one.



b) Find a minimum cut for the network of part (a).



c) Give the list of pairs in the maximum pairs set.

(B, A), (J, C), (K, E), (R, N), (S, M)