Homework 3

1. Solve Kleinberg and Tardos, Chapter 4, Exercise 3. (15 points)

2. Solve Kleinberg and Tardos, Chapter 4, Exercise 5. (15 points)

3. There are N tasks that need to be completed using 2 computers A and B. Each task i has 2 parts that take time: ai (first part) and bi (second part) to be completed. The first part must be completed before starting the second part. Computer A does the first part of all the tasks while computer B does the second part of all the tasks. Computer A can only do one task at a time, while computer B can do any amount of tasks at the same time. Find an $O(n \log n)$ algorithm that minimizes the time to complete all the tasks, and give a proof of why the solution obtained by the algorithm is optimal. (15 points)

4. (a) Consider the problem of making change for n cents using the fewest number of coins. Describe a greedy algorithm to make change consisting of quarters(25 cents), dimes(10 cents), nickels(5 cents) and pennies(1 cents). Prove that your algorithm yields an optimal solution. (Hints: consider how many pennies, nickels, dimes and dime plus nickels are taken by an optimal solution at most.)
   (b) For the previous problem, give a set of coin denominations for which the greedy algorithm does not yield an optimal solution. Assume that each coin's value is an integer. Your set should include a penny so that there is a solution for every value of n. (15 points)

Ungraded Problems

1. Consider a collection of n ropes which have lengths $L_1, L_2, \ldots, L_n$, respectively. Two ropes of length L and L' can be connected to form a single rope of length $L + L'$, and doing so has a cost of $L + L'$. We want to connect the ropes, two at a time, until all ropes are connected to form one long rope. Design an efficient algorithm for finding an order in which to connect all the ropes with minimum total cost. You do not need to prove that your algorithm is correct.

2. Suppose you want to drive from USC to Santa Monica. Your gas tank, when full, holds enough gas to drive p miles. Suppose there are n gas stations along the route at distances $d_1 \le d_2 \le \ldots \le d_n$ from USC. Assume that the distance between any neighboring gas stations, and the distance between USC and the first gas station, as well as the distance

between the last gas station and Santa Monica, are all at most p miles. Assume you start from USC with the tank full. Your goal is to make as few gas stops as possible along the way. Design an efficient algorithm for determining the minimum number of gas stations you must stop at to drive from USC to Santa Monica. Prove that your algorithm is correct. Analyze the time complexity of your algorithm.

3. The array A below holds a max-heap. What will be the order of elements in array A after a new entry with value 18 is inserted into this heap? Show all your work.
$A = \{15, 12, 11, 8, 7, 9, 3, 2, 4, 2, 1\}$