Name(last, first): _____

CS570
Analysis of Algorithms
Fall 2005
Final Exam

Name: _____

Student ID: _____

	Maximum	Received
Problem 1	20	
Problem 2	10	
Problem 3	10	
Problem 4	15	
Problem 5	15	
Problem 6	15	
Problem 7	15	

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1. 20 pts
   Define each of the following terms or symbols.

   a- NP-complete

   b- NP

   c- $\leq_P$

   d- Flow network

   e- Cut capacity

f- Residual graph

g- Linear programming

h- Dynamic programming

i- Decision problem

j- Optimization problem

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2. 10 pts
   Given a bipartite graph, define a 2-perfect matching M to be a set of edges such that each vertex on the left side has exactly 2 incident edges in M and each vertex on the right side has exactly 1 incident edge in M. Give an algorithm to determine whether there is a 2-perfect matching and to output one if it exists.

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3. 10 pts
   A world-famous musician from Los Angeles is planning a concert tour to various countries. Banks in each country offer two exchange rates for each foreign currency, one for buying and one for selling, and travelers generally lose money with each exchange of currency. The musician knows in advance exactly how much of each currency he will need and wishes to do whatever he can to minimize his losses to currency exchange.

   Suppose that he would like to carry only the currency of the country he is visiting in order to avoid confusion, and therefore plans to exchange all the currency he is holding at each border. Assuming that the exchange rates will be stable during his trip, he wishes to arrange his itinerary so as to visit each country exactly once and minimize his losses to currency exchange. Show that this problem is NP-complete.

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4. 15 pts
   Design a set of customer surveys with the following guidelines and the specific set of data provided.
   Guidelines:
   - Each customer will receive questions about a certain subset of the products he/she has purchased
   - The number of questions each customer will receive should be within the minimum and the maximum provided for that customer
   - The number of customers surveyed for each product should be between the minimum and the maximum provided for that product

Customer ID	Products purchased	Minimum # of Questions	Maximum # of Questions
1	B, C, D, F	2	4
2	A, B, D	3	3
3	B, C, E, F	2	4
4	C, E	2	2
5	A, D	2	2

Product ID	Minimum # of Customers surveyed	Maximum # of Customers surveyed
A	1	2
B	2	3
C	3	3
D	2	3
E	2	2
F	2	2

If a solution does not exist describe why. Show all your work.

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Additional space.

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5. 15 pts
We are given an undirected graph G = (V, E). The following algorithm computes a vertex cover C in the graph.

```
Approx-Vertex-Cover (G)
C = NULL
E' = E
While (E' ≠ NULL) do
    Let (u,v) be an arbitrary edge of E'
    C = C ∪ {u,v}
    Remove from E' every edge incident on either u or v
Endwhile
Return C
```

Argue that C is a vertex cover and that the algorithm is a 2-approximation to the smallest vertex cover problem.

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6. 15 pts
   Assume that you have a list of n numbers, both positive and negative. Give an efficient algorithm to find the value of the maximum contiguous sum. For example, the maximum contiguous sum in the sequence (-2, -2, 5, 7, -3, 4, -4) will be 13.

   Note: grading will be based on the efficiency of your algorithm. Details are in the table below:

Complexity of your algorithm	Partial credit
$O(n^3)$	1
$O(n^2)$	2
O(n lgn)	5
O(n)	Full credit

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7. 15 pts
   Consider a variant of the maximum flow problems in a network G = (V, E) in which there is a positive real (non-integer) lower bound l_{ij} on the flow of each arc (i,j) as well as a positive real upper bound u_{ij} on the flow. Rather than send the most amount of flow from s to t, the objective in this problem is to send the least amount of flow from s to t, while satisfying the lower and upper bound constraints and conservation of flow at all nodes other than s and t. Give a polynomial time solution to this problem.

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