Approxim a and b, discuss more in assume, compare to calibrated values

Concrete issues/assume

Why

Nucleus Ltd has completed a pilot operation of a nuclear waste repository in Northland. The repository is an underground concrete cylinder filled with clay and the waste. For five years, Plutonium-238 (Pu-238) from a nuclear reactor in Taiwan was added at six-monthly intervals. During that time and for an additional five years, the temperature was monitored every six months.

Nucleus Ltd is now seeking a resource consent to extend the pilot operation for thirty years. The application is being handled by the Northland regional council. The stakeholders are:

- Forest and Bird, an environmental group who fear that radioactive material could leak into the surrounding groundwater
- Shane Jones, Minister for Regional Development, who is interested in allowing nuclear generation in Northland as an additional form of carbon-neutral electricity production.
- Whangarei District council and Ngā Puhi Iwi do not yet have a position but will balance economic and environmental considerations.
- Local residents may be anxious about the presence of the repository in their community and will be interested in their personal safety and the local environment.

Possible outcomes of the resource consent process are:

- The pilot operation is extended for thirty years as Nucleus Ltd has requested
- The pilot operation is terminated and Nucleus Ltd must remove the waste from the repository
- The consequences of adding an additional 2kg of nuclear waste or removing 20% of the existing waste will also be explored.

How

A computer model will be used to explore how energy generated by the decaying Pu-238 impacts the structural integrity of the concrete cylinder. Modelling the environmental and thermal impacts on the surrounding groundwater and rocks, and the economic impact on the local area, is beyond the scope of the study.

With knowledge of when and how much nuclear waste was emplaced, the energy radiated by the waste can be calculated. From this, it is intended to predict the temperature of the repository over time.

To answer the central question of how the concrete is affected, the temperature at which thermal stresses in the concrete will cause it to fracture will be calculated. It can then be determined whether the repository is likely to exceed that temperature over the next thirty years, and thus whether concrete fracture is possible.

Given

Raw data from Repository site

Data is available for temperature and mass emplacement during the pilot operation. The temperature measurements were inferred, and affected by surrounding groundwater, so have a degree of volatility.

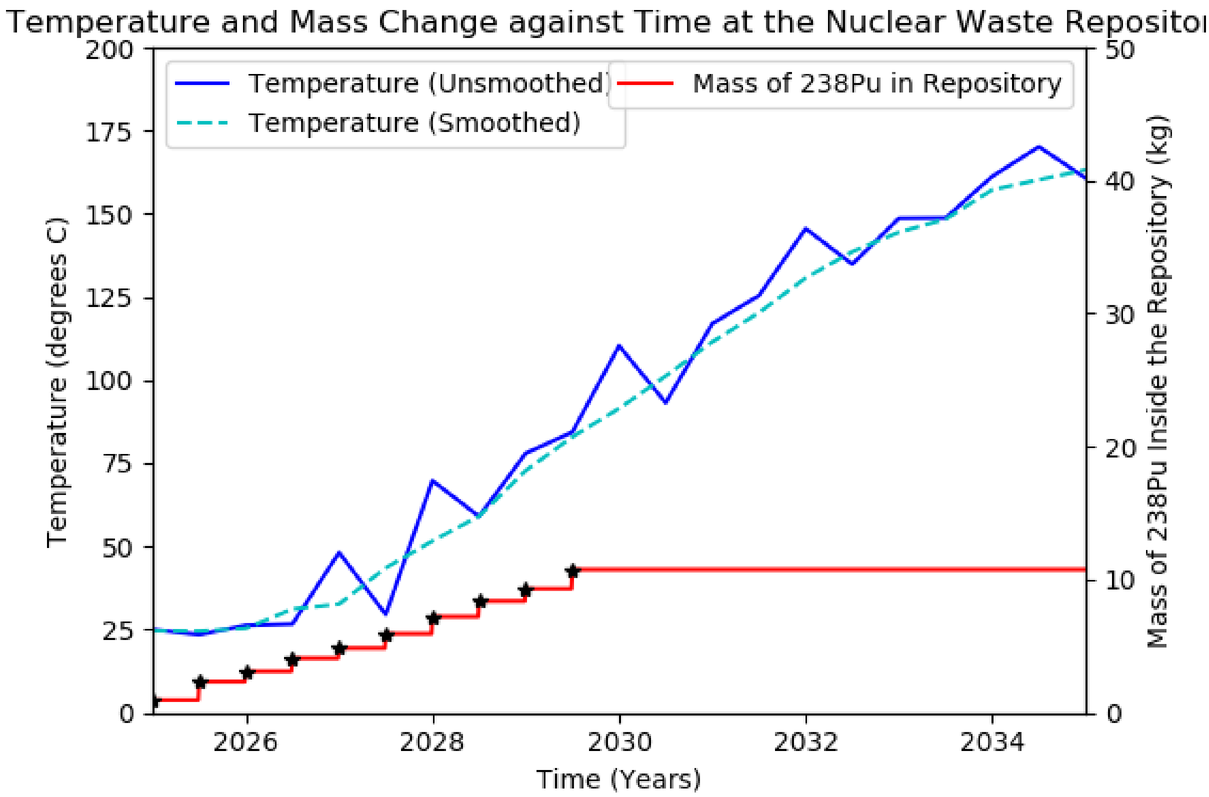

Figure 1: Data collected showing nuclear waste added to repository and temperatures recorded, between 2025 and 2035.

From Figure 1 it is clear that temperatures increased dramatically over ten years- by about 150°C- and the increase persisted even after waste was no longer being added.

Heat generation and concrete strength

Pu-238, like all nuclear waste has an exponential decay curve. From Miotla (2008), it initially gives off 570 W/kg and has a half-life of 87.7 years. It's decay product, Uranium-234 has a half-life of 246,000 years and any heat given off by that decay can be safely ignored.

Jaeger and Zimmerman (2007) give a formula for thermal stress in underground cylinders subject to heating. Kodur (2014) gives important attributes of concrete such as strength, bulk modulus and elastic modulus. Further information about concrete properties was found in Cruz and Gillen (1980), Arjun (2019), Choi et al (2009) and Guo and Xudong (2011).

<u>References</u>

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Assume

Heat Transfer

The relevant physics is considerations of conservation of energy and heat transfers. It is assumed that heat transfer will occur by conduction. The only source of heat will be the decaying radioactive waste. The repository will lose heat energy according to Newton's Law of Cooling.

We will be using a Lumped Parameter Model (LPM), and therefore it is necessary to assume that the temperature is constant throughout both the clay and concrete of the repository.

Heat transfer by convection is ignored, despite the presence of liquid groundwater surrounding the repository. The repository will be dry because the clay is moisture absorbing.

Concrete Strength

It is assumed that the concrete cylinder fractures due to hoop stresses. The formula given by Jaeger and Zimmerman (2007) for the hoop stress in underground heated tunnels and boreholes was used to calculate the thermally-induced stresses. Other possible modes of failure are ignored since this form of fracture is the most likely to occur. This formula relies on the concrete being isotropic.

It is also assumed that the properties of the concrete will not change, for example by absorbing or losing water. All stress in the concrete will be due to thermal stresses.

Domain and Initial Conditions

Using an LPM requires treating the entire repository as a single control volume. This is reasonable given that the waste and clay is spread evenly. The time domain of this problem is from 2025 to at least 2065, when the resource consent Nucleus Ltd is seeking would end.

The initial temperature, and ambient temperature of the surrounding groundwater and rocks, will be assumed to be 25°C, the first temperature recorded, and a reasonable temperature for just under the Earth's surface.

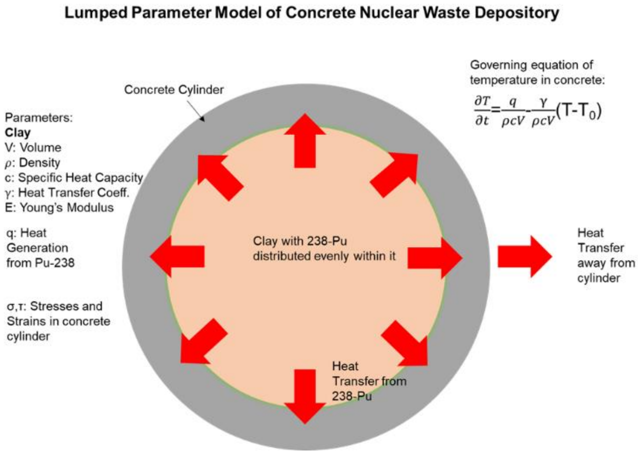

Figure 2: Graphical representation of the problem and key parameters.

Formulate

Repository Temperature

The LPM equation is:

$$\frac{dT}{dt} = aq - b(T - T_{0)}$$

Where T is the temperature, t is time, q is the rate of heat generation, T_0 is the initial temperature, and a and b are constants that relate to the rate of heat loss and absorbance by the repository.

$$q = \sum_{i=1}^{n} q_i e^{-(\frac{\ln(2)(t-t_i)}{t_h})}$$

Where q_i is the initial heat generation rate for the mass emplaced at t_i and t_h is the half-life of Pu-238.

The ODE is solved using the Improved Euler's Method with a time step of 0.05 years. The functions for this and for calculating the heat generation term q are found within the file LPM_functions.py.

An analytical solution can be obtained for the simple scenario of constant heat generation. In that scenario $q = q_0$ and:

$$T = \frac{-aq_0}{b}\left(1 - e^{-bt}\right) + T_0$$

Concrete

The thermal stress in the concrete is, as given by Jaeger and Zimmerman:

$$\sigma = 3\beta K \frac{(1-2\nu)}{(1-\nu)} \Delta T$$

$$K = \frac{E}{3(1-2\nu)}$$

Where σ is the thermal stress, β is the coefficient of thermal expansion, K is the bulk modulus, ν is Poisson's ratio, E is Young's Modulus and ΔT the change in temperature.

In the file Temp_Strength.py, the compressive strength of concrete and the thermal stress in the concrete was plotted for a range of temperatures. The point at which the strength equals the stress, 215°C, is the fracture temperature. Figure 3 shows the output of this file.

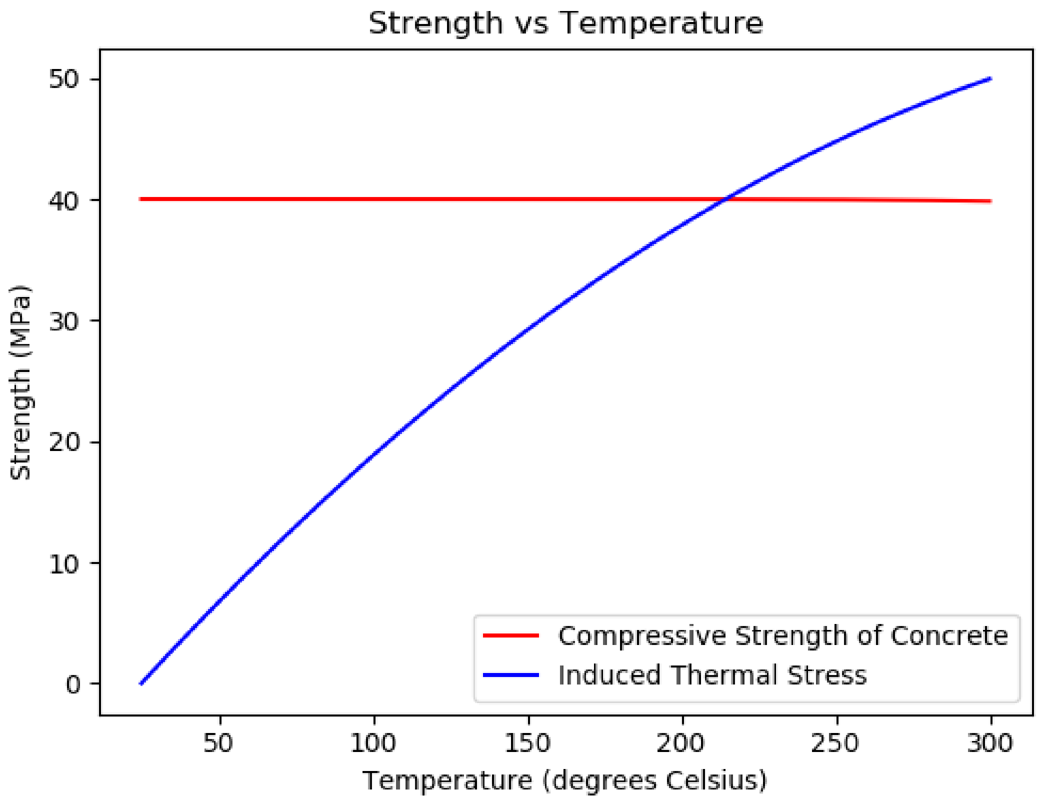

Figure 3: Compressive strength of concrete and thermal stresses in the concrete at different temperatures.

Working

Unit tests were constructed for the functions in LPM_functions.py by comparing their outputs to hand-calculated solutions. All functions passed.

Benchmarking was performed to ensure that the functions for solving differential equations were functioning correctly. The numerical solutions aligned closely with the expected answers, as can be seen in Figure 4.

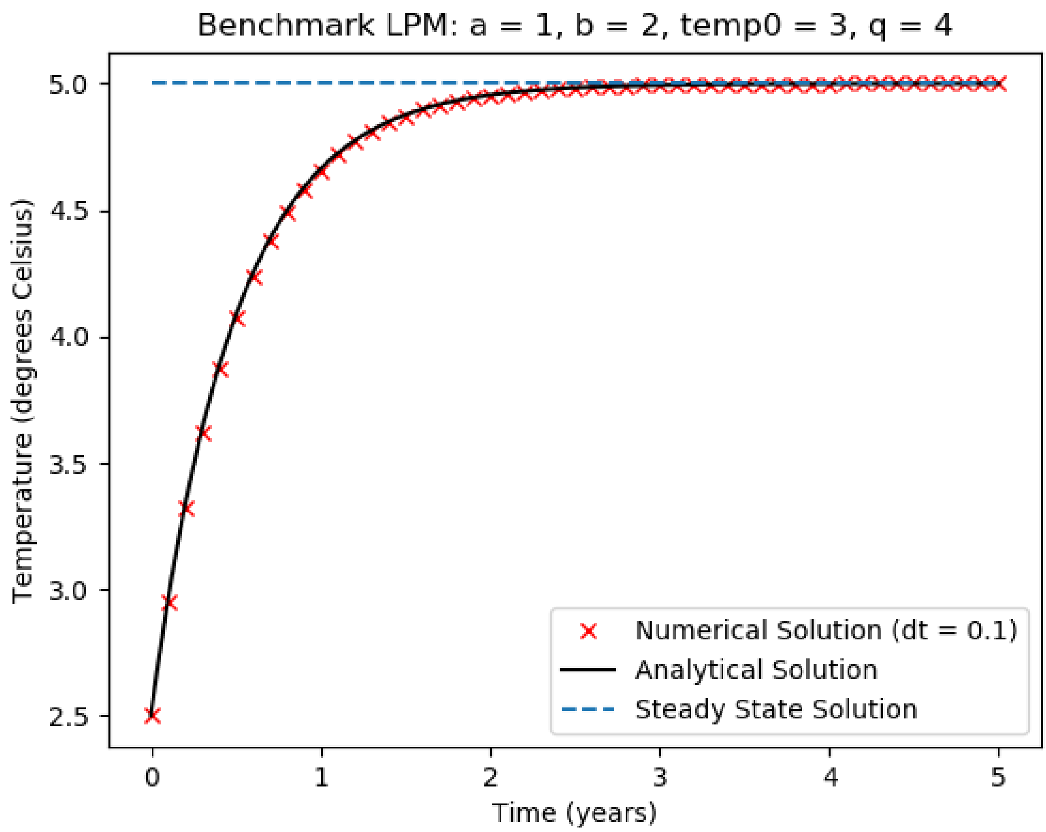

Figure 4: Comparison of numerical and analytical solutions for a scenario with constant heat generation.

Suitable

Calibration was done using gradient descent on the parameter space, and the result is displayed in Figure 5. The parameter values in the best fit model are $a = 6.00 \times 10^{-3}$ and $b = 0.200$.

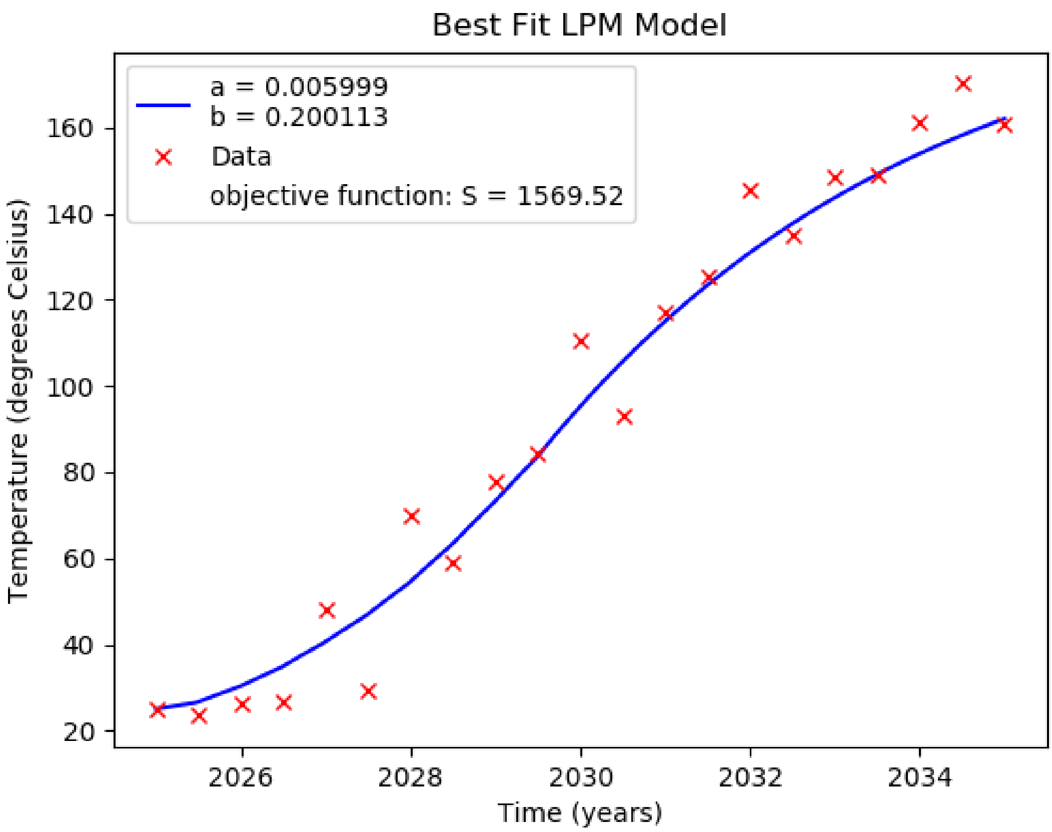

Figure 5: Calibration of model parameters against temperature data, using gradient descent over the parameter space.

The calibrated model cannot account for the initial lack of temperature increase up to 2027, but thereafter does quite a good job of capturing the rise in temperature over time and the decline in the rate of temperature change from about 2031. The differences between measured and predicted temperatures are usually within 15 °C.

Improve

Python has a built-in function curve_fit that can calibrate models using the Levenberg-Marquardt algorithm. When this function is used instead of the gradient descent functions that we wrote, the parameters in the best fit model were found to be $a = 6.10 \times 10^{-3}$ and $b = 0.194$. The objective function using these parameter values was 1411.09, lower than when using gradient descent and indicative of a better fitting model.

Use

The calibrated model will be used to consider a number of scenarios:

- The resource consent is granted, and the waste can remain in the repository for an additional 30 years
- The resource consent is not granted, and the waste must be removed in 2035.
- The resource consent is partially granted, and 20% of the waste must be removed.

- The resource consent is granted, and in addition Nucleus Ltd wins consent to store an additional 2kg of waste that is emplaced in 2035.

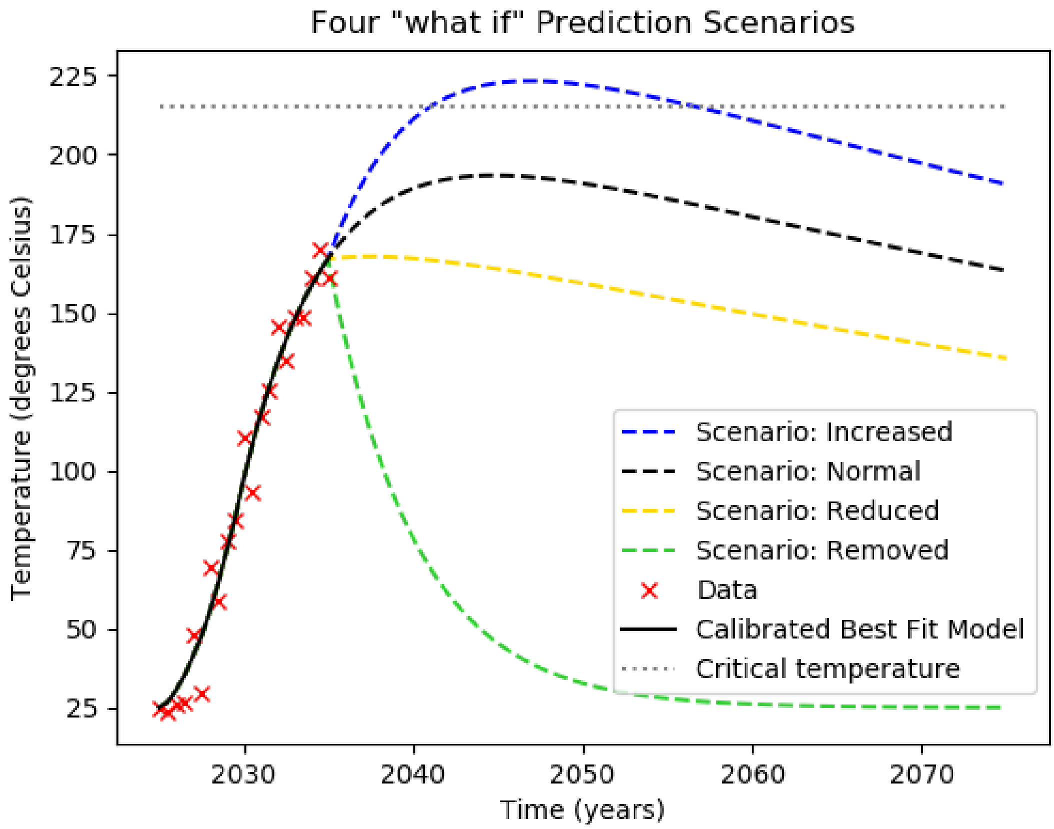

Figure 6: Predicted temperatures under the scenarios of increased waste (blue), business as usual (black), removal of some waste (yellow) and complete removal (green). The fracture temperature is indicated as a grey line.

Figure 6 shows the predicted temperatures until 2075 under each scenario. Additionally, the maximum temperature reached in each scenario was recorded. This was 223.2°C when 2 kg was added, 193.4°C under business as usual, 167.7°C if 20% was removed and 166.7°C if all of the nuclear waste was removed.

Unknown

As previously stated, the temperature data is not exact and has an inherent uncertainty. The error or uncertainty of the data was not given, so was estimated by considering the sum of squared distances found through calibration. This gave a standard deviation of approximately 8.32°C

A posterior distribution of a and b was constructed by considering the objective function:

$$S(\theta) = \sum \frac{1}{\sigma^2} (\hat{p} - p(\theta))^2$$

Where σ^2 is the variance of the data.

The relative probability of each point in the posterior parameter space is:

$$P(\theta) = e^{-S(\theta)/2}$$

A multivariate normal distribution was then used to take a sample of 100 models from the posterior. This is plotted below:

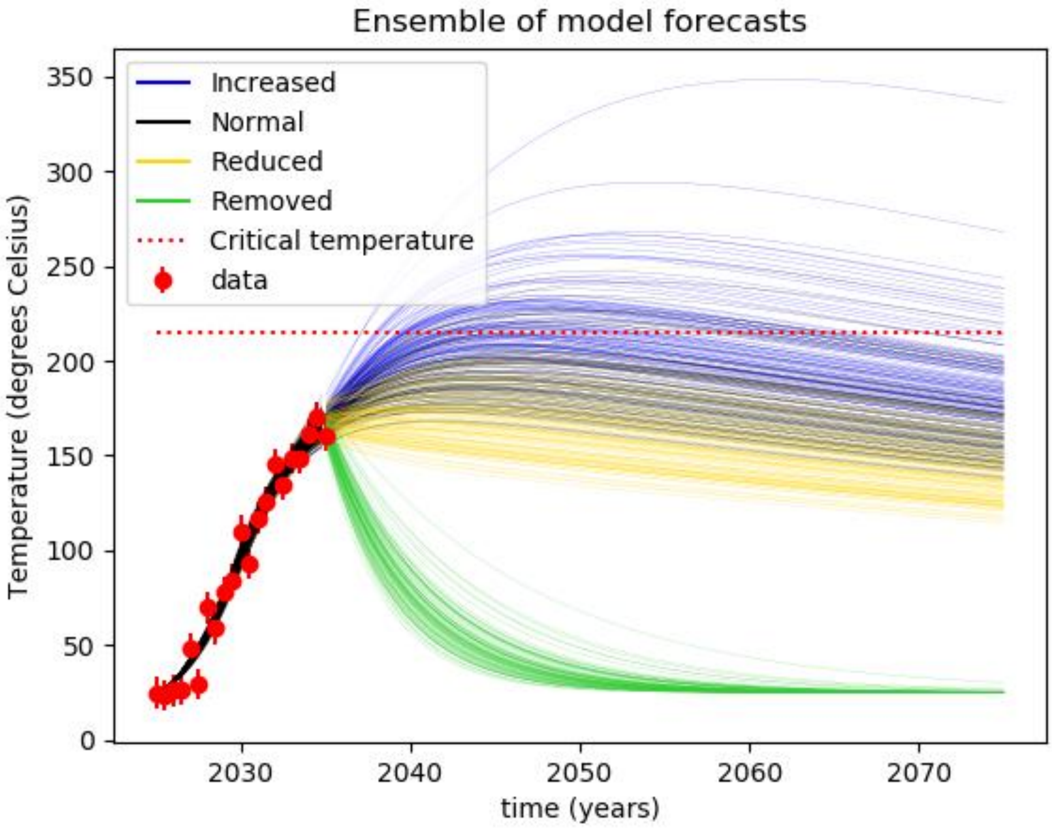

Figure 7: Forecast for the four scenarios showing 100 different possible outcomes.

The most striking feature, seen in Figure 7, is the relatively high levels of uncertainty, particularly for the scenarios where at least some nuclear waste is retained in the repository. This would suggest that the parameter a is more uncertain than b. An examination of the posterior distribution (Figure 8), indicates that this is indeed the case, with the highest possible values of a nearly three times as large as the smallest.

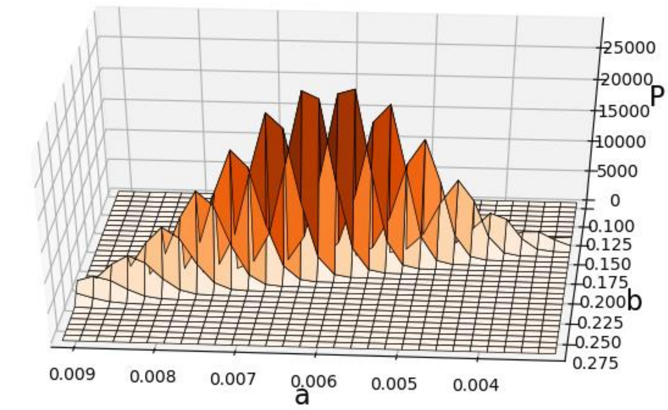

Figure 8: Posterior distribution of parameters a and b

90% Confidence Intervals for the maximum temperature reached were obtained by extracting the 5th lowest and 5th highest maximum temperature for each scenario. These intervals (in °C) were [199.1, 266.5], [173.1,226.5], [160.4,197.6] and [159.2,172.5].

Recommend

This study has modelled temperature changes in the nuclear waste repository under four different scenarios. Under two scenarios- retaining the existing waste and adding an additional 2kg, the upper bound of the confidence interval for the maximum temperature exceeded 215°C. The other scenarios had reasonably similar maximum temperatures that were well below 215°C.

The consequences of a leak of nuclear waste due to the concrete shell fracturing would be severe. The groundwater and surrounding environment would be contaminated, and a sizeable area could become uninhabitable. These fears were a contributing factor to New Zealand adopting a "Nuclear Free" policy in 1987. This policy has been popular and is embraced by many New Zealanders. For these reasons residents in the area surrounding the repository are likely to be anxious about and opposed to the continuing existence of nuclear waste in their community.

While the properties and strength of the concrete had to be estimated, the 215°C figure assumed a worst-case scenario for the compressive strength of the concrete. We can be confident that any temperatures below 215°C will not damage the repository. Economic considerations highly favour the retention of the repository. Removing the waste will cause Nucleus Ltd to lose revenue from their storage of the waste and will cause them expenses and safety issues involved with removing the waste. The repository is also a source of wealth for the region, and its complete repository would be an economic setback.

The recommendation is to remove 20% of the waste within the repository. This protects the environment and residents while maximizing the economic benefits. However, this recommendation is subject to several conditions. Monitoring of the repository must continue and should be more frequent than the six monthly measurements of the past. If the temperature continued to increase, more waste would need to be removed immediately. Nucleus Ltd should also consult and communicate with the community, in order to ensure that locals understand what is happening and are reassured about the risks of the pilot operation.

For future studies of this sort, better information on the concrete that a repository is made of would be beneficial. This would allow the temperature at which fracture will occur to be more accurately and confidently calculated, reducing the uncertainty in the model and the risk when the modelling is used to make decisions. If the fracture temperature of the concrete was actually higher than 215°C, then this information could have allowed the pilot operation to continue, but in the absence of this data it has been necessary to take a cautious approach.