

COMP9312

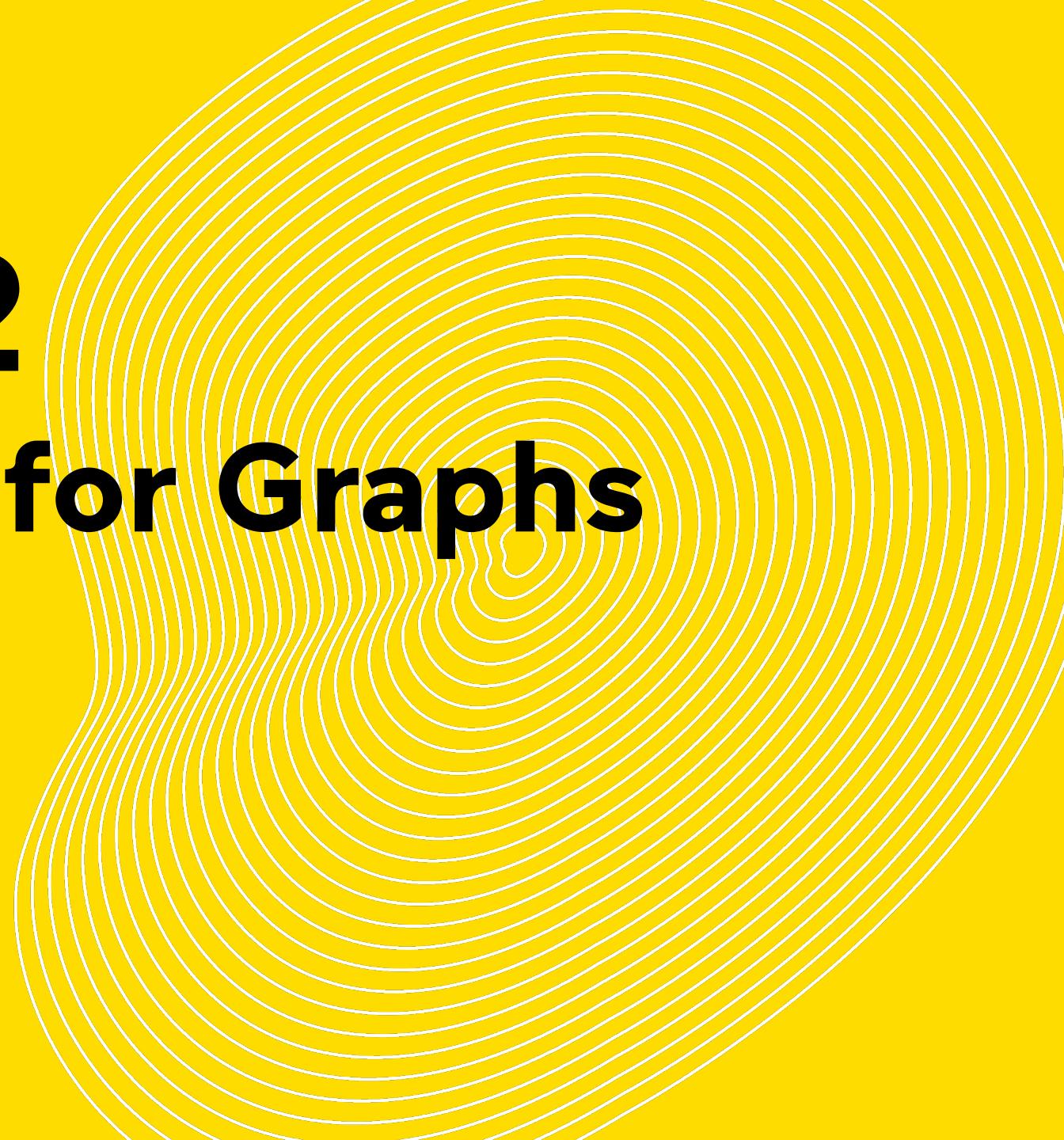
Data Analytics for Graphs

Dong Wen

2024 Term 2



UNSW
SYDNEY



Teaching Team



Dr. Dong Wen (LiC)
cse.unsw.edu.au/~dwen/

Research Interests

- **Big Data Processing**
- **Database**
- **AI4DB**

Course Info

Lectures:

4 PM – 6 PM (Monday)

week 1 – 2, 4 – 5, 7 – 10

4 PM – 6 PM (Thursday)

week 1 – 5, 7-10

Tutorials: Week 2-5, 7-10.

No tutorial in Week 1.

General consultation: start from week2. Will update later via webcms.

Course Info (cont)

LiC Email: dong.wen@unsw.edu.au or cs9312@cse.unsw.edu.au

Admin: kaiyu.chen@unsw.edu.au

For normal questions, we recommend you use the Q&A forums. You are also welcome to contact us via email if something is urgent, or book private help sessions.

EdForum: <https://edstem.org/au/courses/11987/discussion/>

See FAQ on Webcms for **the self-registration link** of EdForum.

Webcms3: <https://webcms3.cse.unsw.edu.au/COMP9312/24T2/>

Moodle: <https://moodle.telt.unsw.edu.au/course/view.php?id=84401>

Course Info (cont)

Read the Webcms course outline.

Read all previous Webcms notice after you enroll the course.

Teaching/Learning

What we do to support your learning:

- Lectures
- Reference python codes
- Tutorials (starting from Week 2): guides you through the theoretical knowledge and practical skills of the course, solutions will be released after all the tutorials are delivered.
- Private help sessions
- Assignments and projects
- Textbook & papers

Syllabus Overview

Traditional algorithms on Graphs / Graph Database (Week 1 to Week 8)

- Basic concepts of graphs
- Graph traversal – DFS, BFS, connectivity, etc.
- Path and reachability queries
- Subgraph search
- Graph Database

Deep learning methods on Graphs (Weeks 9 to 10)

- Traditional machine learning methods on graphs
- Node Embedding
- Graph neural networks

Your Background

We assume that you ...

- Have background in data structure and algorithms
- Have experience with programming languages, i.e., Python and C/C++
- Hopefully, have some knowledge of **relational databases**

You might have acquired this background in

COMP1511, COMP1531, COMP2521, COMP3311, COMP9024, COMP9311,

Textbook

Lecture notes are sufficient, but the following materials are good:

Reference Books:

- [Introduction to algorithms](#) by Thomas H. Cormen, et al
- [Cohesive subgraph computation over large sparse graphs: algorithms, data structures, and programming techniques](#) by Lijun Chang and Lu Qin.
- [Graph Representation Learning](#) by William L. Hamilton
- [Networks, Crowds, and Markets: Reasoning About a Highly Connected World](#) by David Easley and Jon Kleinberg
- [Network Science](#) by Albert-László Barabási

Assessment Structure

ass1 = mark for assignment 1 (out of **15**)
ass2 = mark for assignment 2 (out of **10**)
project = mark for project (out of **25**)
exam = mark for final exam (out of **50**)
final_mark = *ass1* + *ass2* + *project* + *exam*
grade = HD|DN|CR|PS if *final_mark* >= 50
= FL if *mark* < 50

No double pass is applied.

Email to Special Consideration **>24 hours** before deadline for extension.

Assignments

Two assignments are **done individually**.

- Released via Webcms
- Submitted via **Moodle**
- Plagiarism-checked (copying solutions -> 0 mark for the assignment)

Assignments (25%):

- Ass 1: Graph Traversal; Reachability/Path Queries; Cohesive Subgraphs (15%) (**week 2-4**)
- Ass 2: More Graph Algorithms; Graph Neural Networks (10%) (**week 9-10**)

Project

- Released via Webcms
- **Done individually**
- Submitted via Moodle

Projects (25%)

- Designing an algorithm to process graph problems (**week 5-8**)

Exam and Marks

Exam: 50%

- Graph Algorithms + Graph Neural Network
- Exam is around 3 hours.
- All answers are submitted via Moodle
- If you are ill on the day of the exam, **do not attend** the exam.
- Sample questions will be available in Week 10

Final mark = Ass1 + Ass2 + Project + Exam

Pass the course: Final mark ≥ 50

Supplementary Exam Policy

Everyone gets **exactly one chance** to pass the Exam.

If you attempt the Exam

- We assume you are fit/healthy enough to take it.
- No 2nd chance exams, even with a medical certificate.

Special consideration:

- If you are ill on the day of the exam, **do not attend** the exam.
- All special consideration requests must be submitted to UNSW.
- All special consideration requests must document how you were effect.

Beyond the course

Research Degrees: (<https://research.unsw.edu.au/higher-degree-research-programs>)

- PhD (3 -- 3.5 years)
- Master of Philosophy (1.5 -- 2 years)

Feel free to contact me if you are interested in obtaining a research degree.

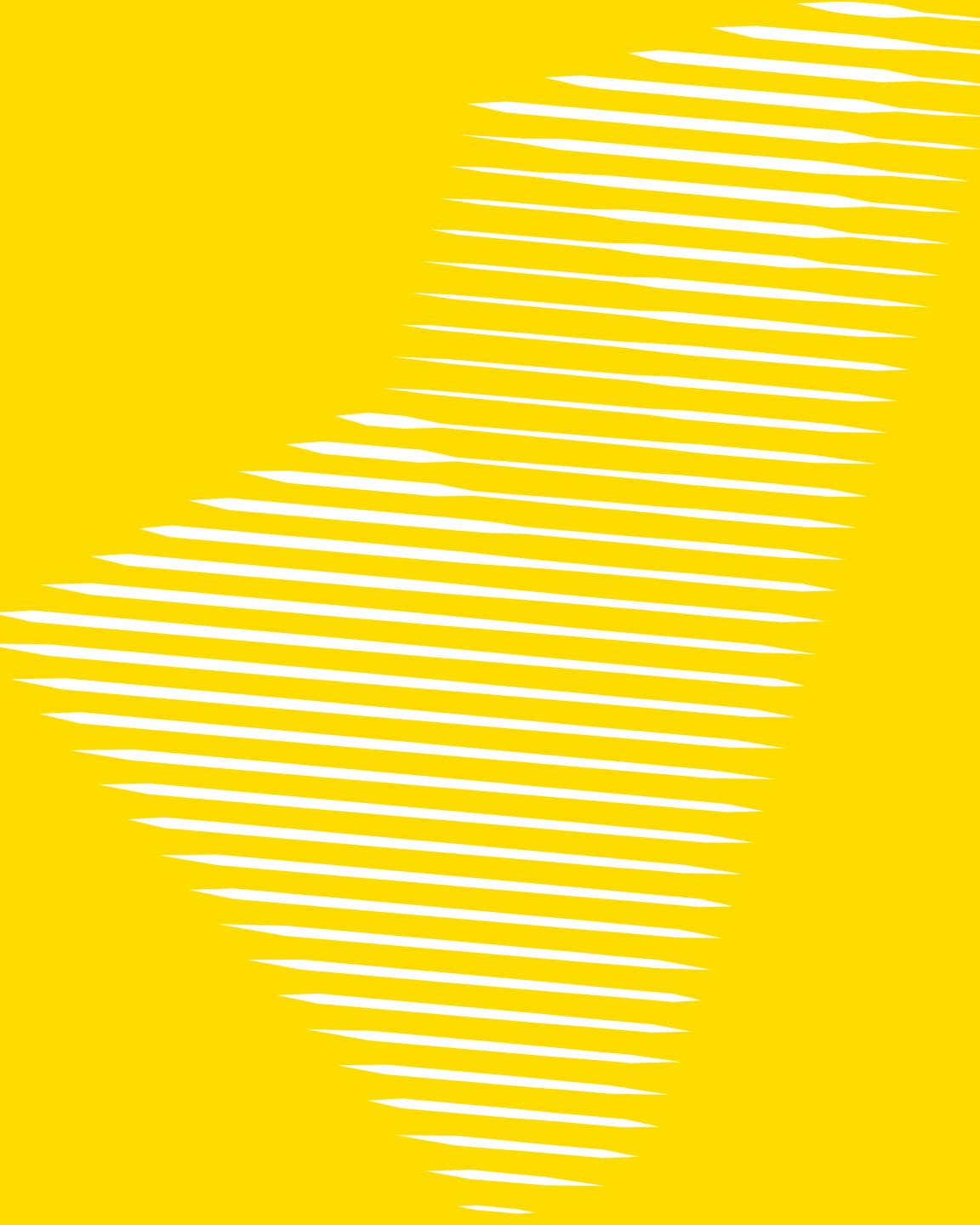
Big Data Processing; AI4DB; LLM for Data Science

Requirement:

GPA>80 with education from world top 400 universities. GPA > 75 if from world top 100 universities.
Local students are particularly welcome to apply.

Data and Knowledge Research Group in CSE:

<https://unswdb.github.io/>

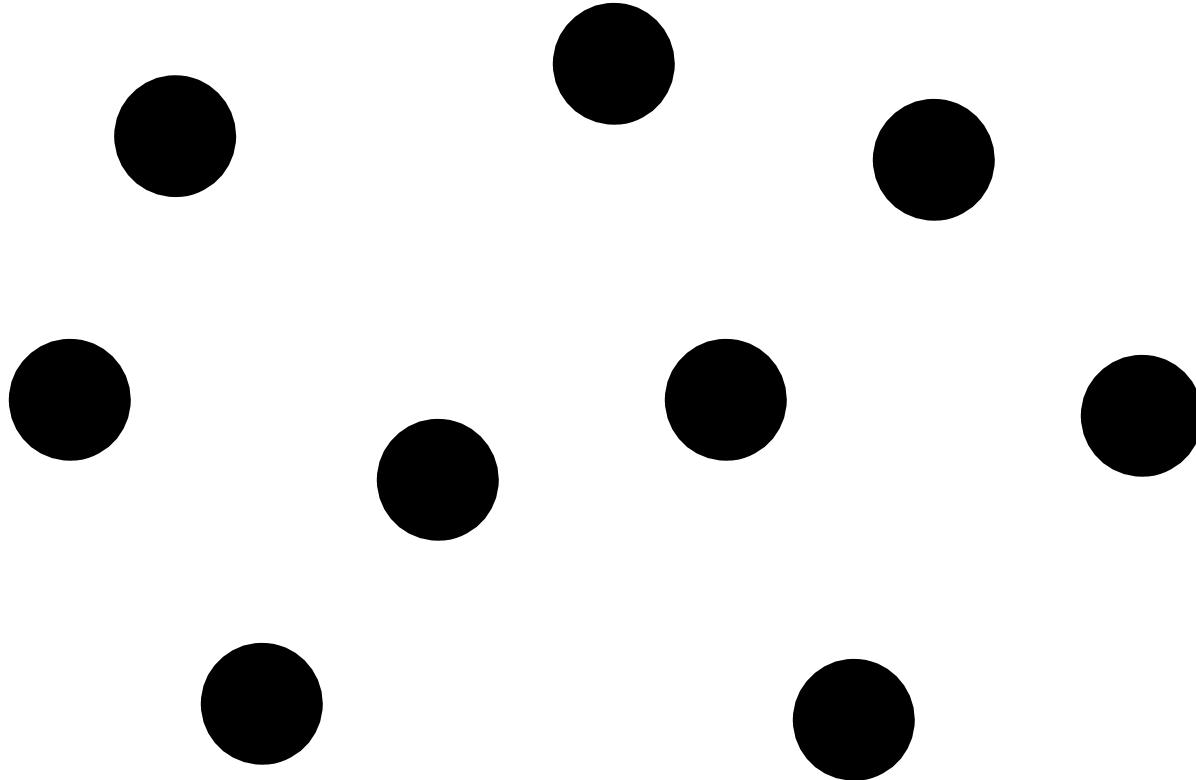


Outline

- **Basic Concepts**
 - What are networks/graphs?
 - What will we learn?
- **Characterization of graphs**
 - How to characterize graphs?
- **Data structure to represent a graph**
- **Analysing of graphs**
 - How to analyse graphs data?
- **Applications of graphs**
 - Applications of graphs analysis

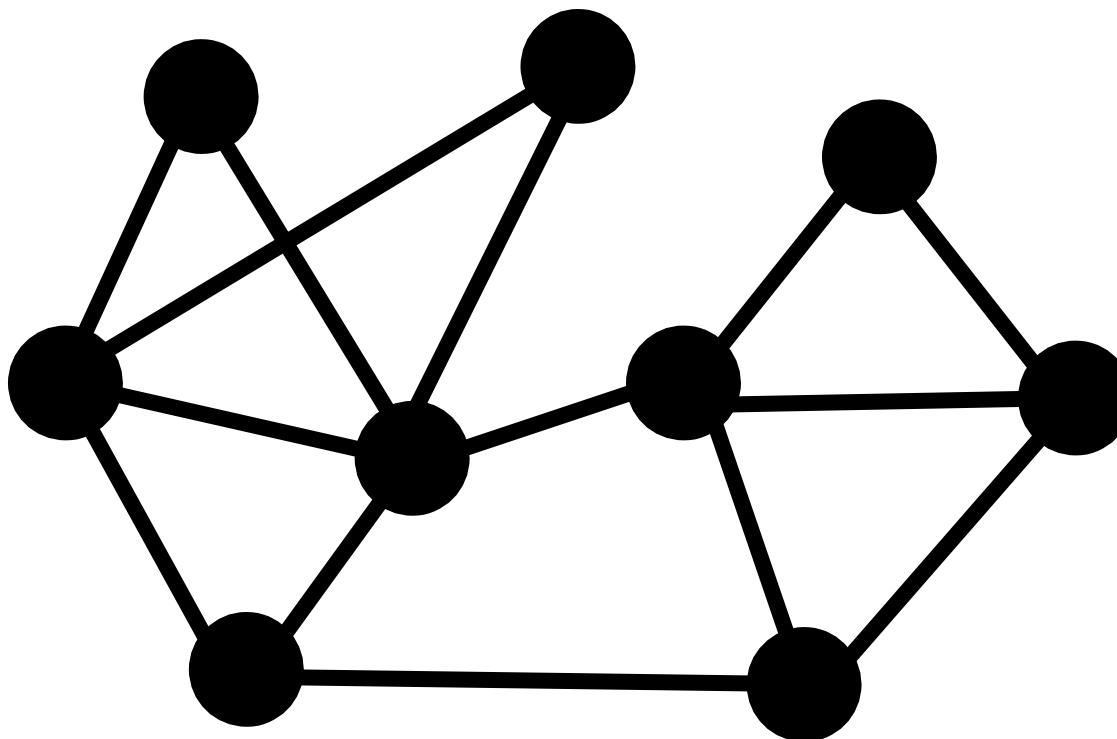
Why Graphs?

**Graph is a general language for
describing and analyzing entities
with relations/interactions.**



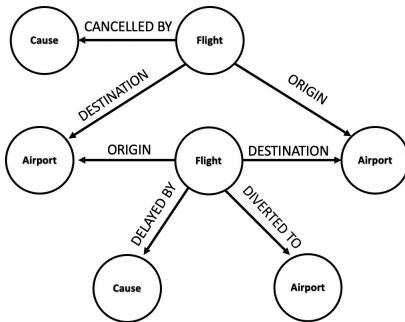
users, machines, webpages, road
intersections, and any entities...

Network / Graph



Sometimes the distinction between networks & graphs is blurred

Many Types of Data are Graphs (1)



Event Graphs

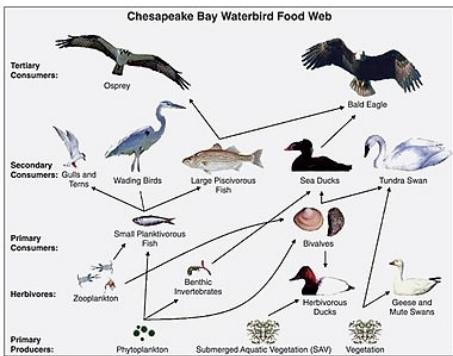


Image credit: [Wikipedia](#)

Food Webs



Image credit: [SalientNetworks](#)

Computer Networks

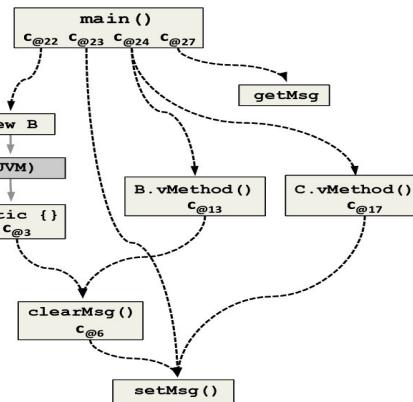
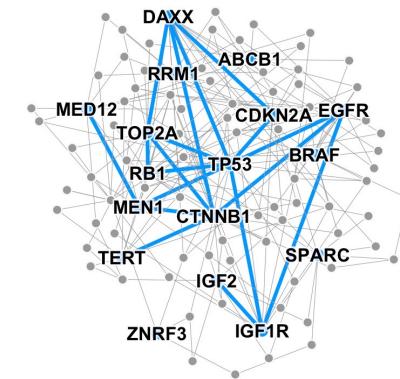


Image credit: [ResearchGate](#)

Code Graphs



Disease Pathways

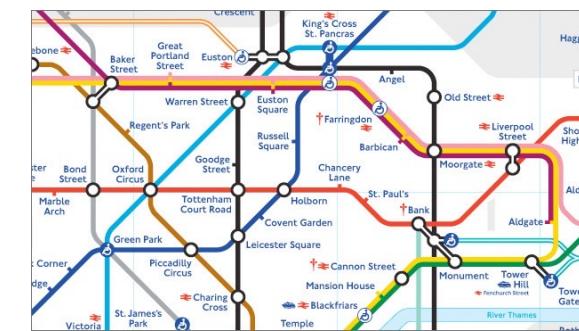


Image credit: visitlondon.com

Underground Networks

Many Types of Data are Graphs (2)

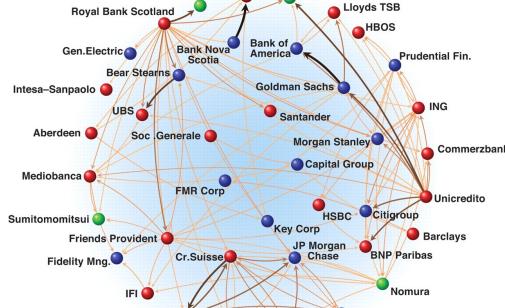


Image credit: [Science](#)
Economic Networks

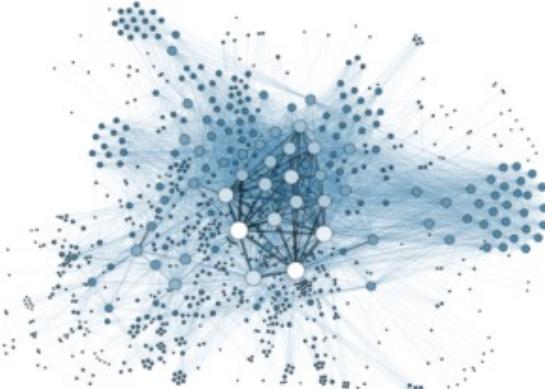
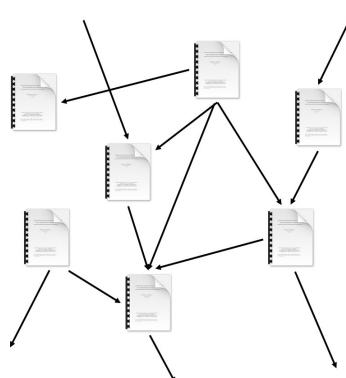


Image credit: [Lumen Learning](#)
Communication Networks



Citation Networks

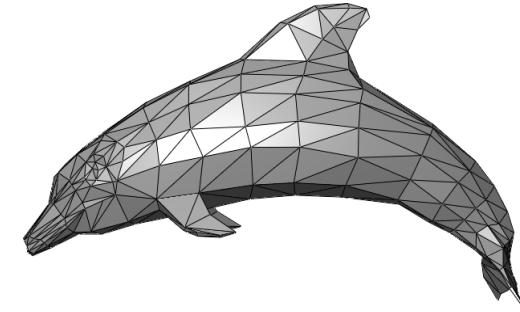


Image credit: [Wikipedia](#)

3D Shapes



Image credit: [Missoula Current News](#)

Internet

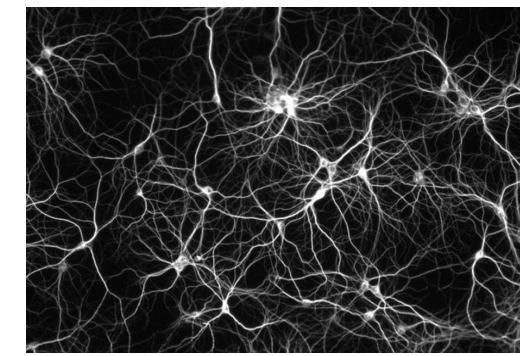


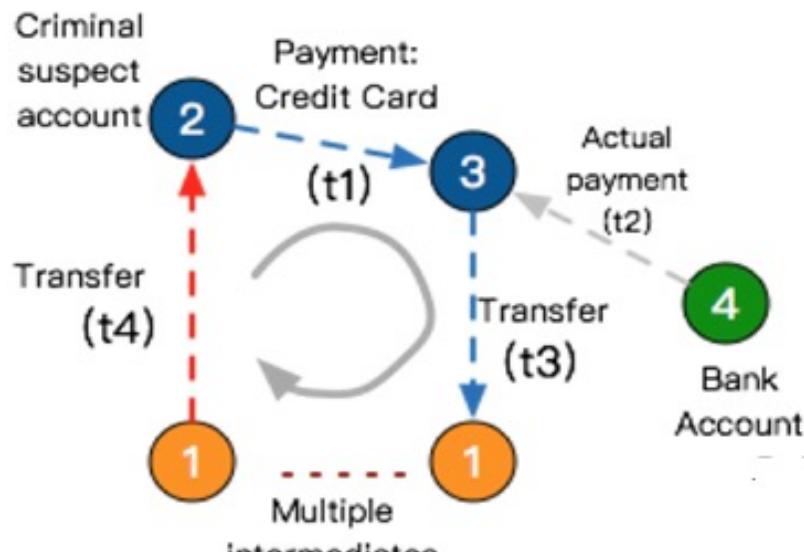
Image credit: [The Conversation](#)

Networks of Neurons

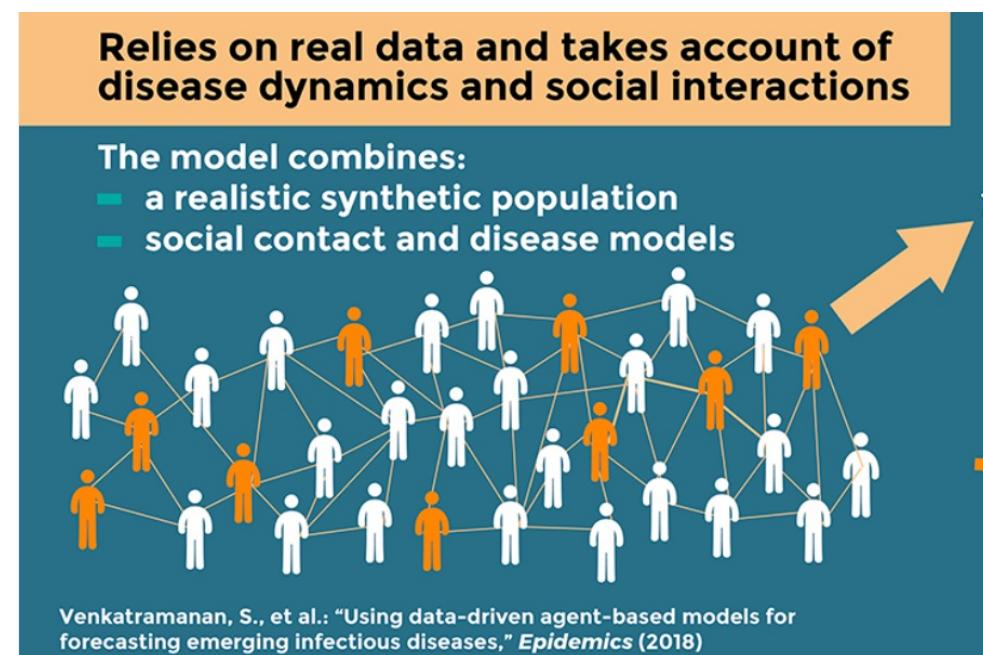
Why we study networks/graphs ?

Networks / graphs are everywhere, and we live in a **highly-connected world**.

In many applications, we need analyze **in the context of networks**, not just individuals.



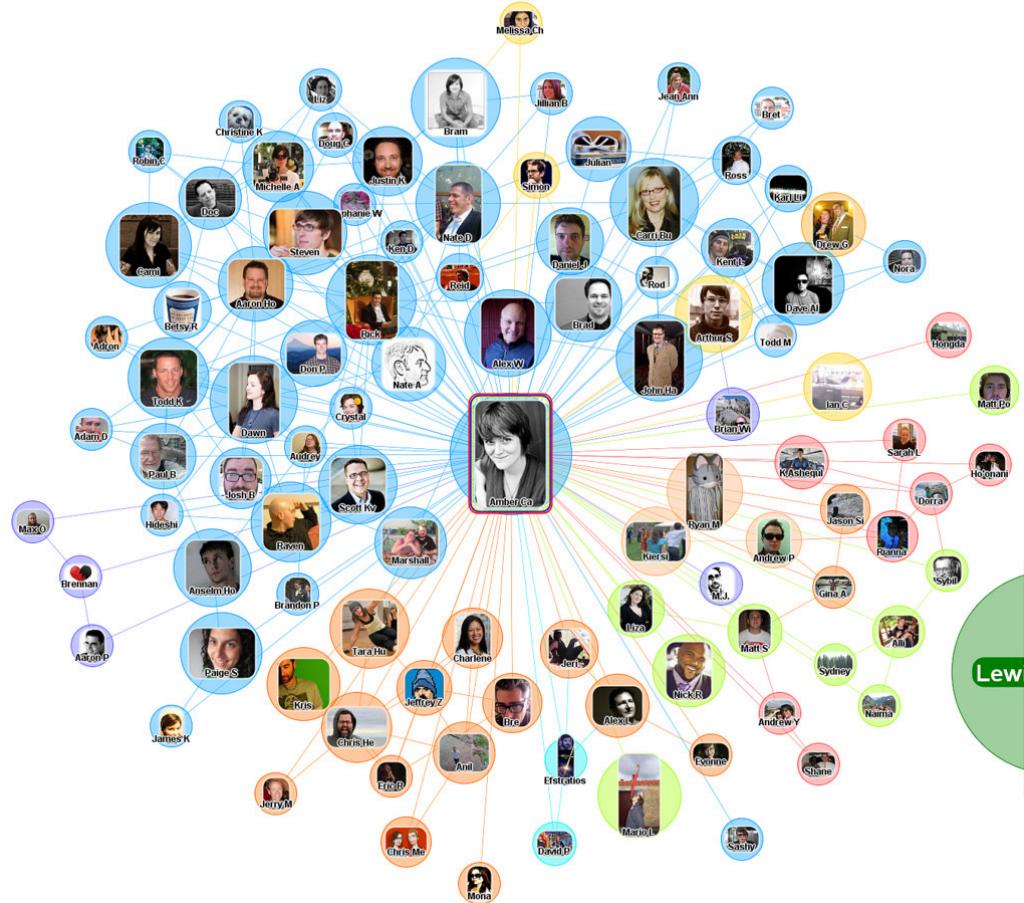
Money laundering detection



Social Networks – Facebook ego-network

- Find your communities
- Know how to approach a person via Facebook.

e.g., how to increase the chance of getting your friend invitation accepted?



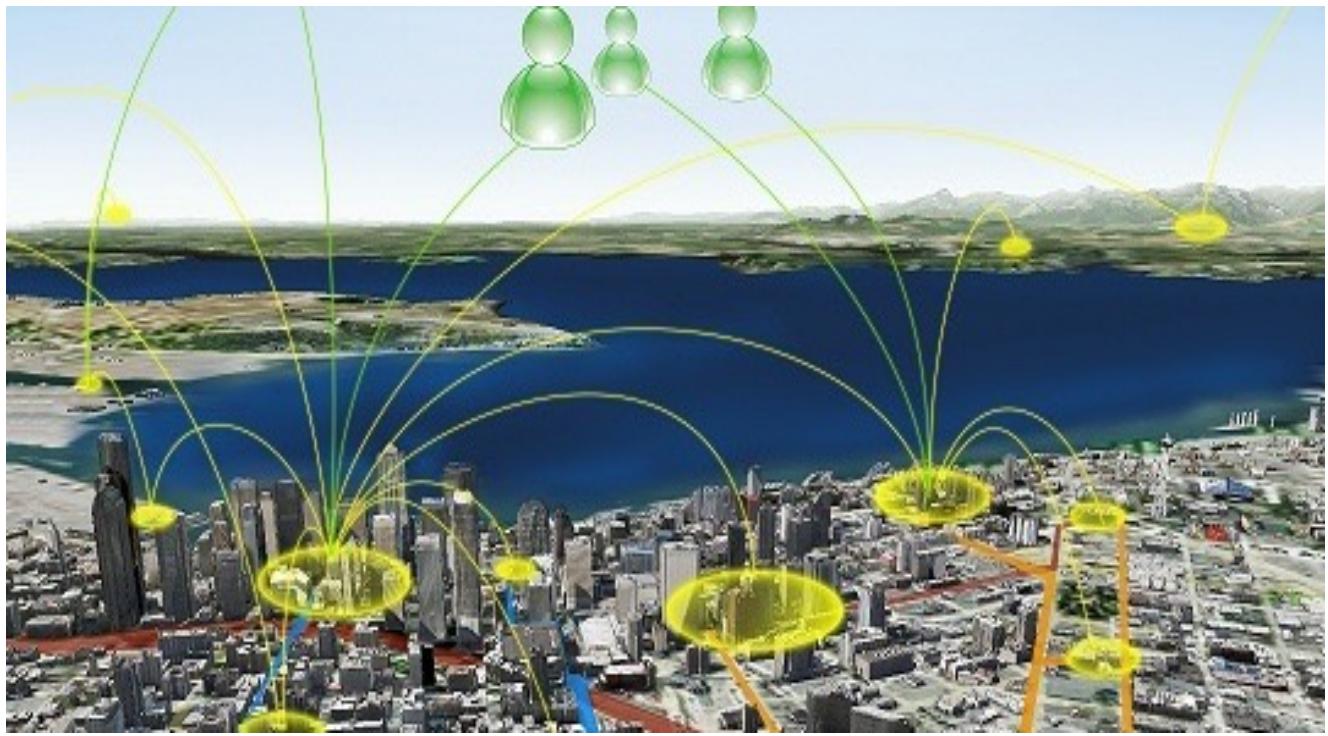
Social Networks – Facebook (location-based visualization in 2011)

Monthly active users: around **1 billion** in **2012** and **2.32 billion** (2017) - now around **2.5 billion (2020)**



Facebook social graph [Backstrom-Boldi-Rosa-Ugander-Vigna, 2011]

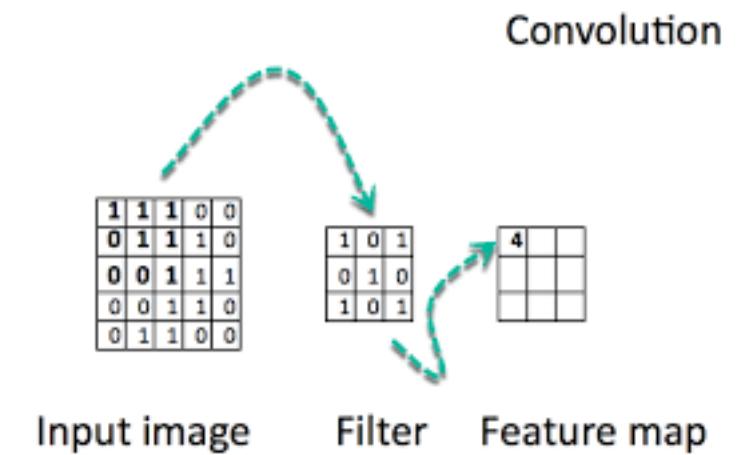
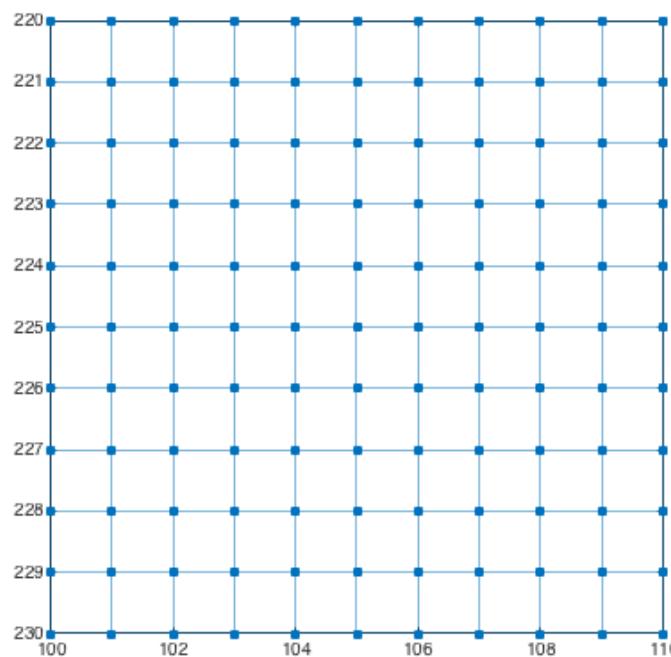
Social Networks – Location Based Social Network (LBSN)



<https://www.microsoft.com/en-us/research/project/location-based-social-networks>

Other types of graphs

- Tree
- String
- Image



Find More Real Graph Data

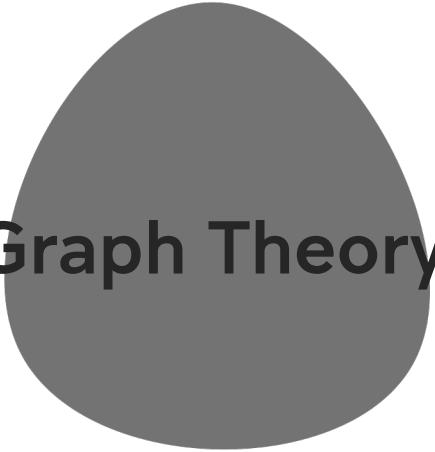
An example of [DBLP](#).

A graph dataset example of Stanford Large Network Dataset Collection ([SNAP](#)).



What we learn

We study ...



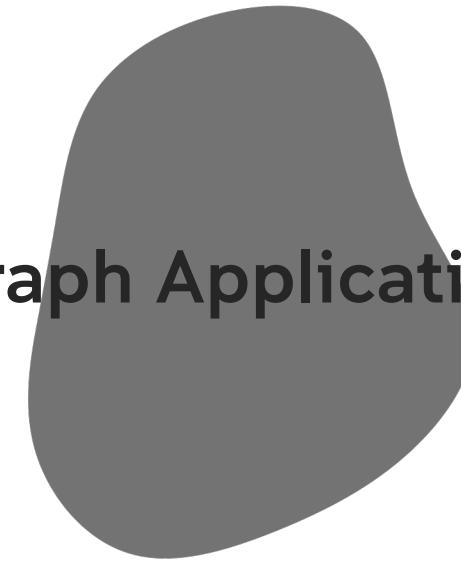
Graph Theory

Mathematical proofs



Big Graph Algorithms

Efficiency and scalability



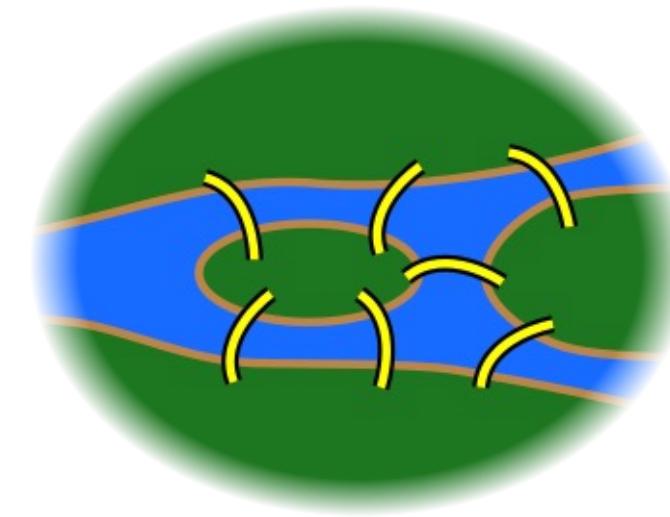
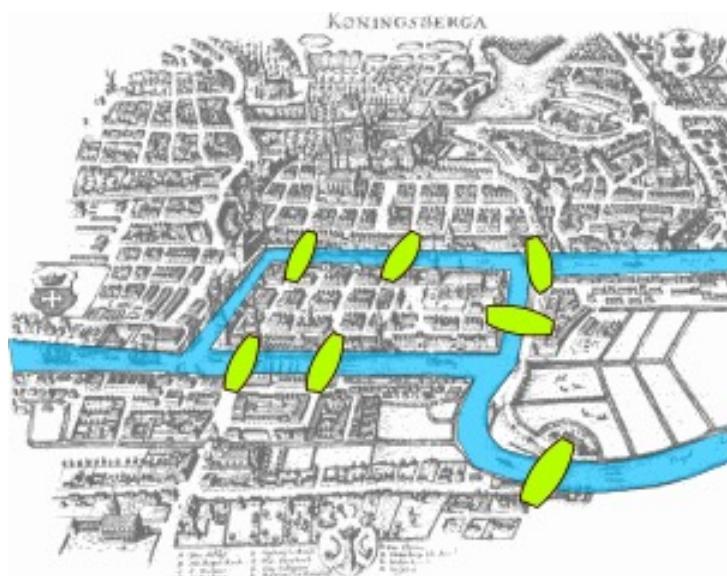
Graph Applications

Interdisciplinary tasks

Graph Theory

Leonhard Euler in 1735:

- find a roundtrip through the city that would cross each bridge once and only once
- earliest (published) work on networks/graphs, laid the foundations of graph theory



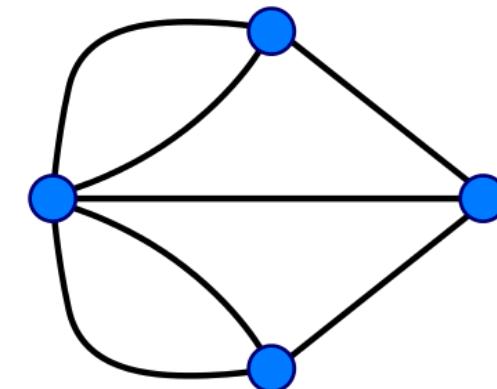
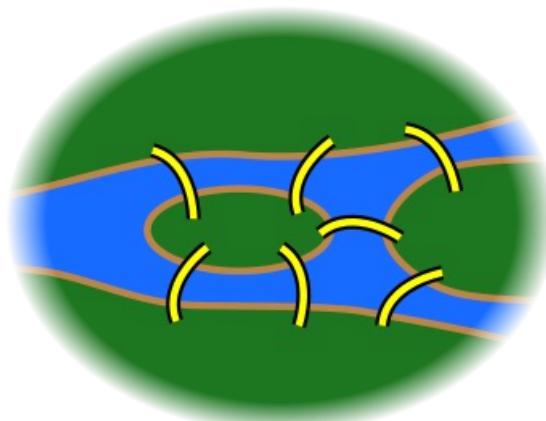
Graph Theory (cont)

Abstraction (Problem):

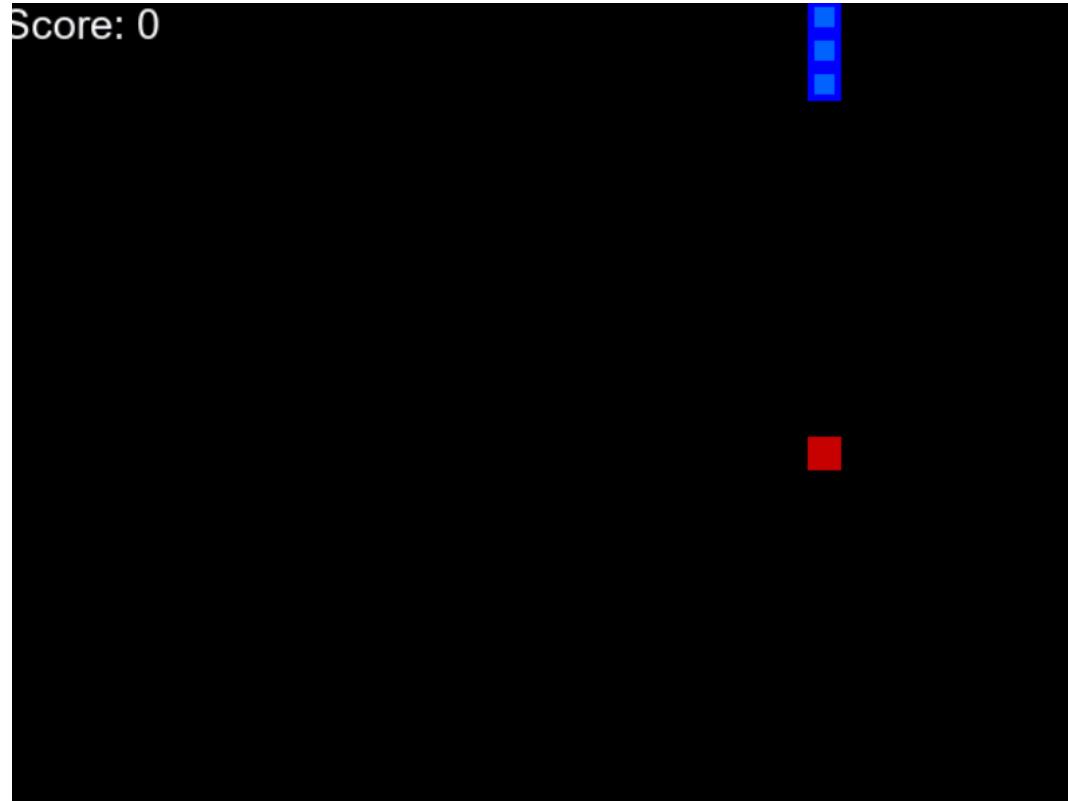
- replaced each land mass with an abstract "vertex" or node, and each bridge with an abstract connection, an "edge"
- Proved that this problem has no valid tour.

Euler Tour (Solution):

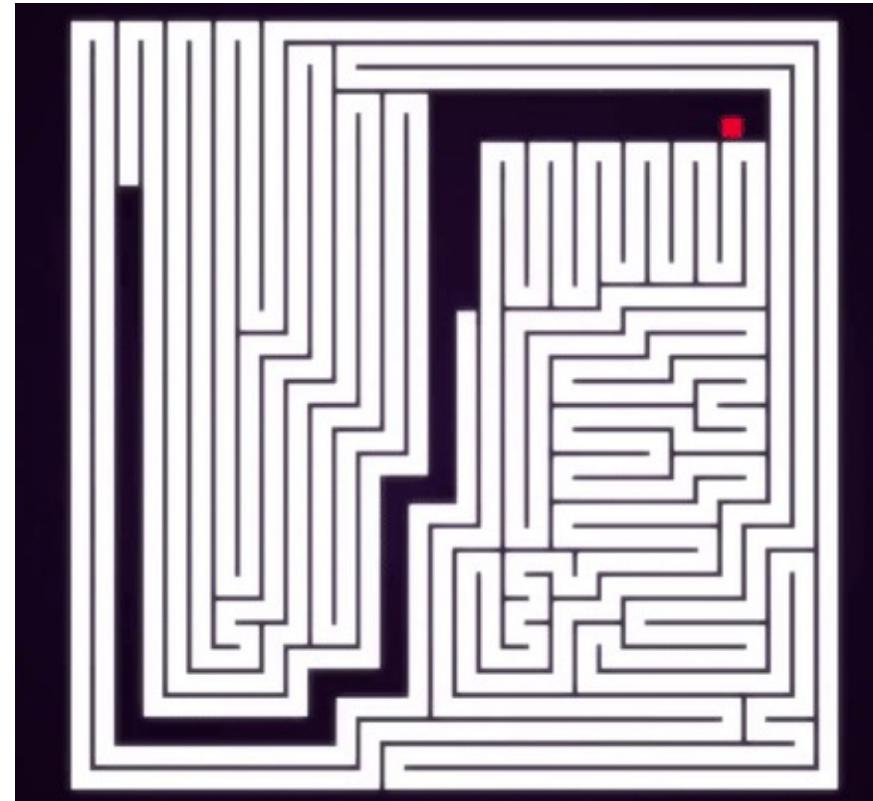
- a necessary and sufficient condition for the walk of the desired form: connected and all vertices with even degrees.



Snake



Does an optimal plan exist?



https://en.wikipedia.org/wiki/Hamiltonian_path

RESEARCH ON GRAPH:

REAL DATA

- Money Transactions
- Protein Interactions
- Internet
- System
- Medical Records
- Phone Calls
- Road Map
- Social Network
- ...

GRAPH MODEL

Traditional Graph

Streaming Graph

Heterogeneous Graph

Knowledge Graph

Attributed Graph

Temporal Graph

Evolving Graph

Uncertain Graph

GRAPH PROBLEM

Link Analysis (relevance)

SimRank

Link Prediction

Pagerank

Clustering

Degree/Neighbors (structural)

Cluster Coefficient

Triangle

Vertex Cover

Distance/Path (reachability)

Shortest Path

Spanning Tree

Reachability

Steiner Tree

K-Hop

Centrality

KNN

Keyword Search

Subgraph Detection

Cohesive Subgraph

Graph Similarity

Subgraph Enumeration

Propagation

Influence Maximization

Information Cascade

Label Propagation

Node mapping (Embedding)

Node2Vec

AI4DB

APPLICATION

Recommendation

Anomaly Detection

Visualization

Cybersecurity

Fraud Detection

Marketing

Legal Reasoning

Promotion

Traffic Monitoring

Virus Control

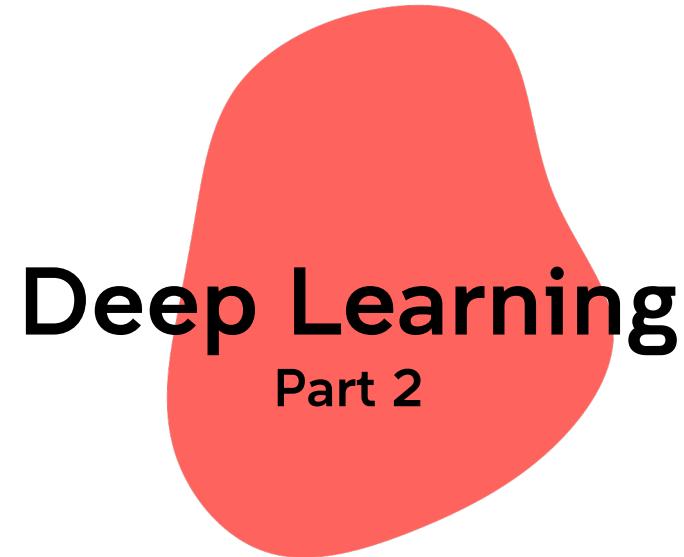
Graph Analytics System

Graph Database

We study ...



- Big data processing
- Time complexity
- External memory
- Distributed systems
- ...



- Graph neural networks
- Node classification
- AI
- ...

A Database Example

`users` Table

user_id	username	email
1	alice	alice@example.com
2	bob	bob@example.com
3	charlie	charlie@example.com
4	dave	dave@example.com
5	eve	eve@example.com
6	frank	frank@example.com

Assume rows are stored continuously:

- Query the email given a user id
- Query all friend ids given a user id
- Query the shortest path between two user_ids

`friendships` Table

friendship_id	user_id	friend_id
1	1	2
2	1	3
3	2	3
4	2	4
5	3	5
6	4	5
7	5	6
8	6	1
9	6	3

Pre-requisites

- Sorting Algorithm
- Binary Search
- Balanced Binary Search Tree (AVL-tree & red-black tree)
- Hash Set/Map
- Heap/Priority Queue
- ...

Warm Up

Given a graph with n vertices and m edges:

- How to compute the shortest distance between two query vertices?
- What is the time complexity of your solution?
- Can you further improve your solution?



Programming Languages

C/C++

Efficiency

Java

Distributed Applications
Android, Web ...

Javascript

Phone App, Web ...

...

Python

Graph Neural Networks
Plugins ...

We mainly use **Python** in this course ...

Let's see some simple codes :)

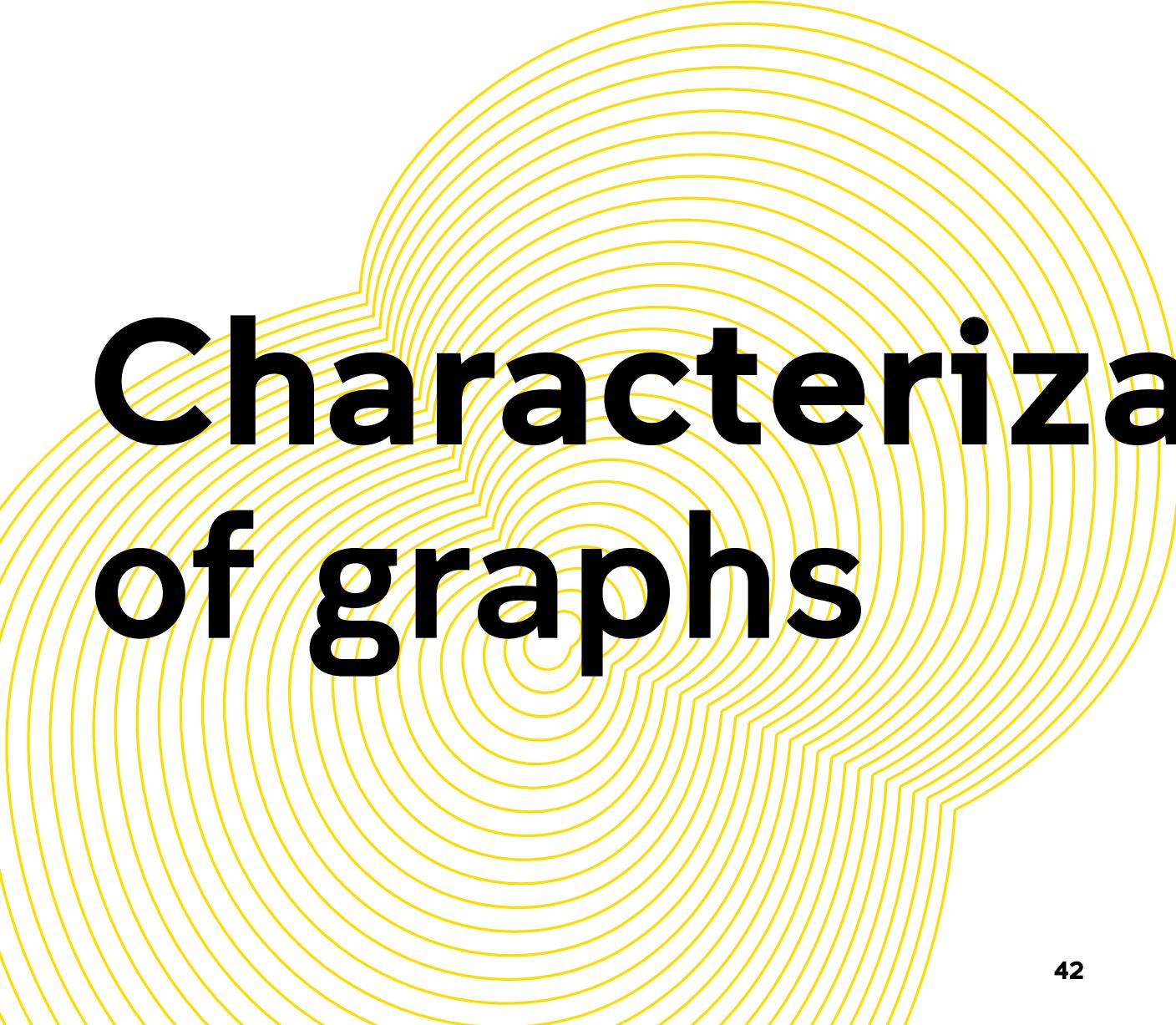
Python

```
a_set = {"aaa", "bbb", "c"}  
a_tuple = ("aaa",1)
```

List vs Array

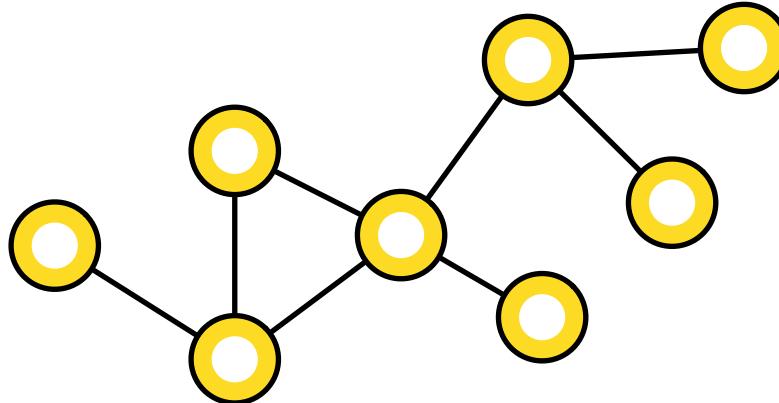
```
a_list = [1,2,3,4,5]  
b_list = [1,2,"abc",4,5]
```

```
import numpy as np  
arr = np.array([1, 2, 3, 4, 5])
```



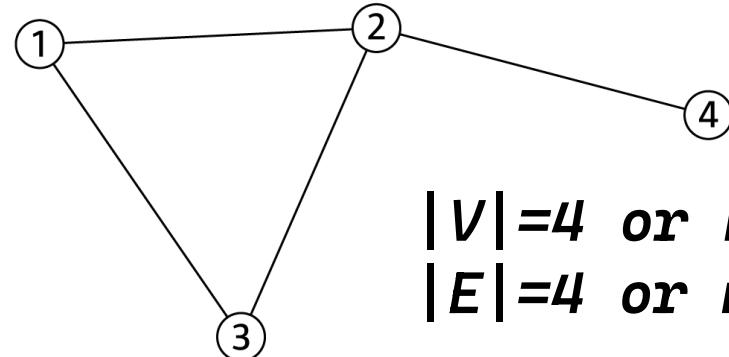
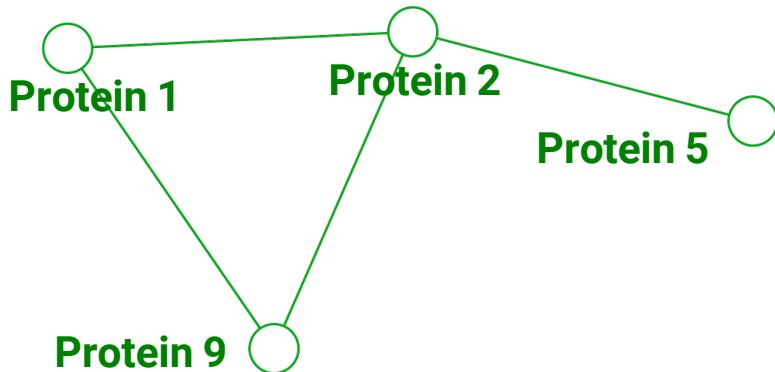
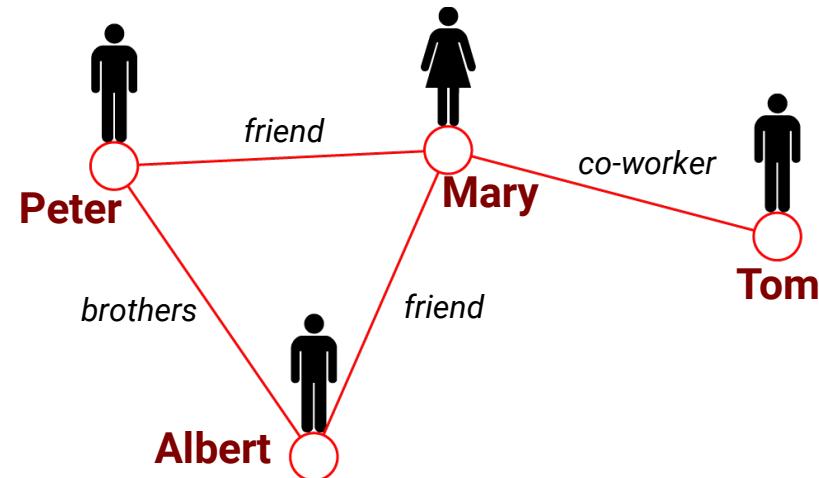
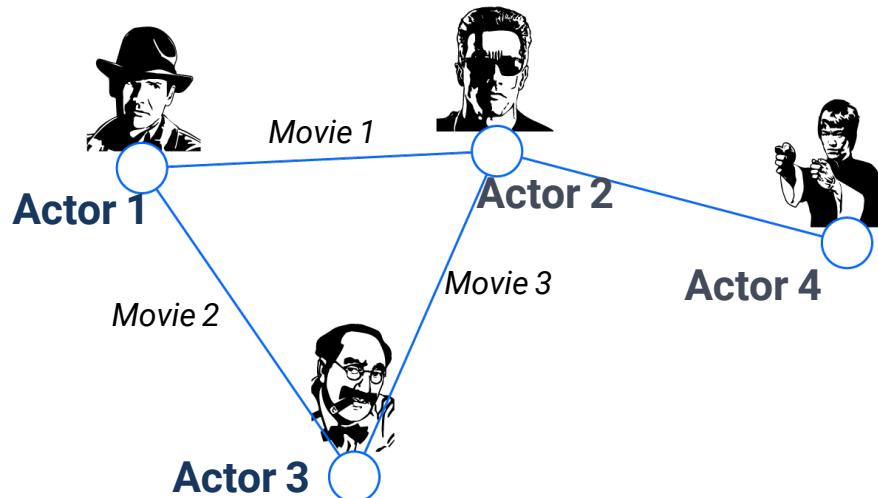
Characterization of graphs

Subgraph / component



- **Objects:** nodes, vertices $V, n=|V|$
- **Interactions:** links, edges $E, m=|E|$
- **Systems:** networks, graphs $G(V, E)$

Graphs: A Common Language

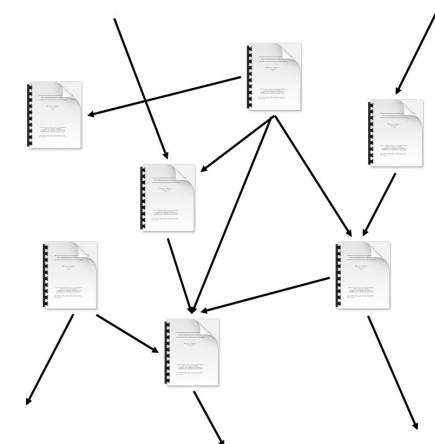


$$|V|=4 \text{ or } n=4$$
$$|E|=4 \text{ or } m=4$$

Choosing a Proper Representation

- If you connect individuals that work with each other, you will explore a **professional network**.
- If you connect those that have a sexual relationship, you will be exploring **sexual networks**.
- If you connect scientific papers that cite each other, you will be studying the **citation network**.
- **If you connect all papers with the same word in the title, what will you be exploring?**

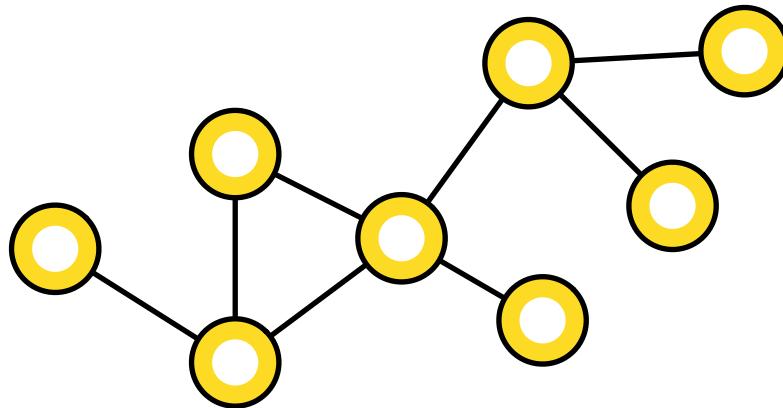
It is a graph/network, nevertheless.



How do you define a graph?

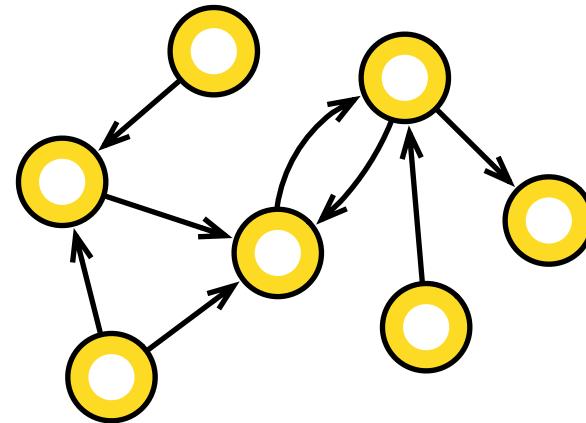
- How to build a graph:
 - What are vertices?
 - What are edges?
- Choice of the proper network representation is important:
 - The way you assign links will determine the nature of the question you can study
- Consider an email scenario:
 - User (email address, ...)
 - Email (from, to, cc, email content)
 - ...

Directed vs Undirected



An undirected graph

- Collaborations
- Friendship on Facebook

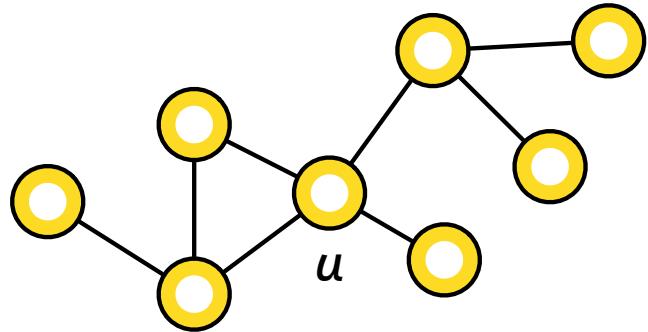


A directed graph

- Phone calls
- Following on Twitter

Node degree

Undirected



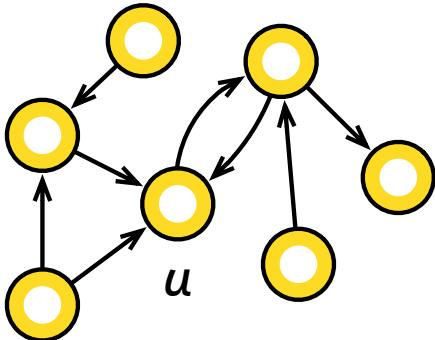
For undirected graphs:

Vertex degree: the number of edges adjacent to the vertex

$$\deg(u) = 4$$

$$\text{Average degree: } \deg_{avg} = 2m/n$$

Directed



For directed graphs:

In-degree (#in-neighbors) and out-degree (#out-neighbors)

$$\deg_{in}(u) = 3, \quad \deg_{out}(u) = 2$$

Average out-degree equals to average in-degree

Characterization

- Degree: how many friends do I have?
- Path: how far am I from another vertex?
- Connectivity: can I reach other vertices?
- Density: how dense are they? (Number of edges / Number of possible edges)
- ...

Bipartite Graph

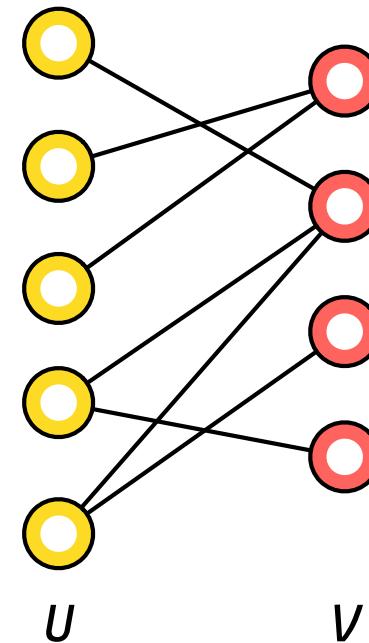
Bipartite graph is a graph whose vertices can be divided into two disjoint sets U and V such that every link connects a vertex in U to one in V ; that is, U and V are independent sets

Examples:

- Authors-to-Papers (they authored)
- Actors-to-Movies (they appeared in)
- Users-to-Movies (they rated)
- Recipes-to-Ingredients (they contain)

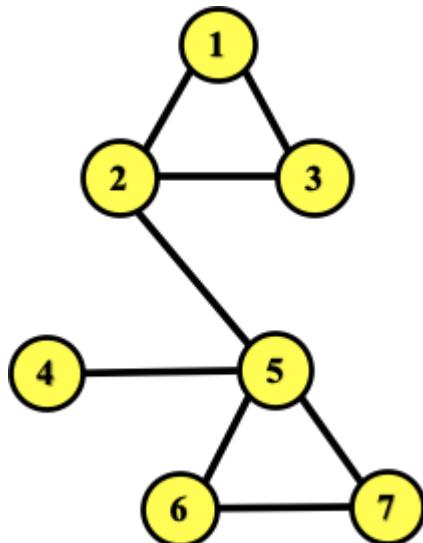
“Folded” networks:

- Author collaboration networks
- Movie co-rating networks

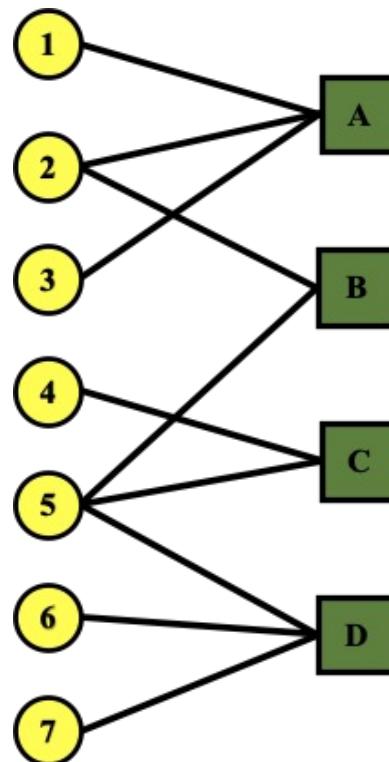


Folded/Projected Bipartite Graphs

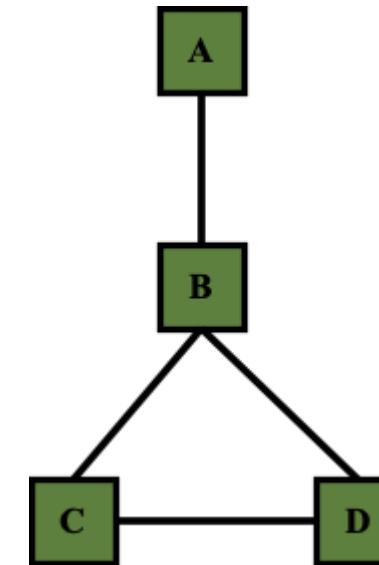
Projection U



U V



Projection V



Node and edge attributes

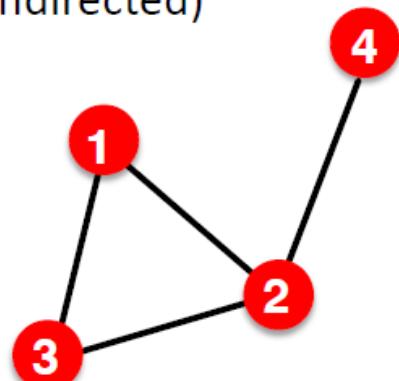
Possible options:

- Weight (e.g., frequency of communication)
- Ranking (best friend, second best friend...)
- Type (friend, relative, co-worker)
- Properties depending on the structure of the rest of the graph: Number of common friends

More Types of Graphs

- **Unweighted**

(undirected)

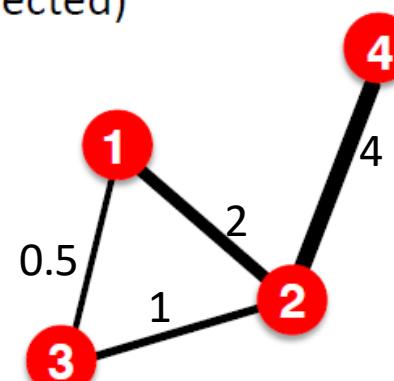


$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Examples: Friendship, Hyperlink

- **Weighted**

(undirected)

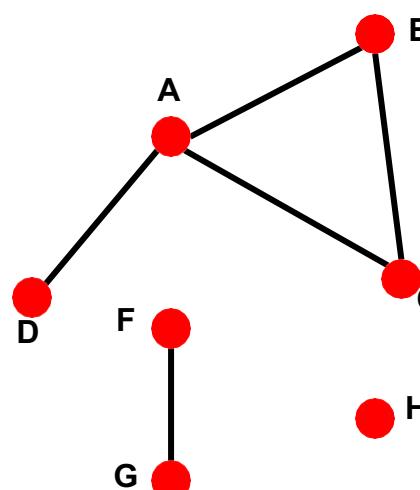
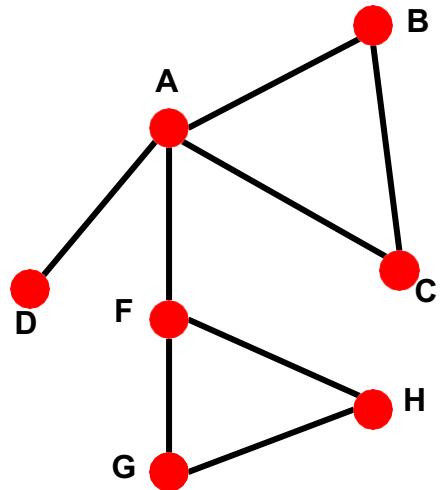


$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

Examples: Collaboration, Internet, Roads

Connectivity of Undirected Graphs

- **Connected (undirected) graph:**
 - Any two vertices can be joined by a path
- A disconnected graph is made up by two or more connected components



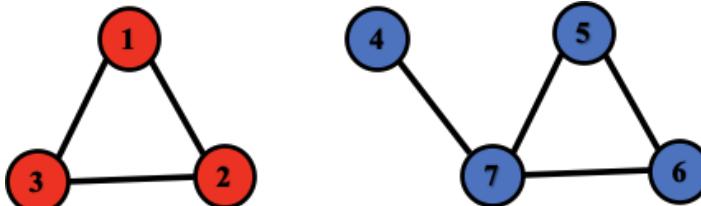
Largest Component:
Giant Component

Isolated node (node H)

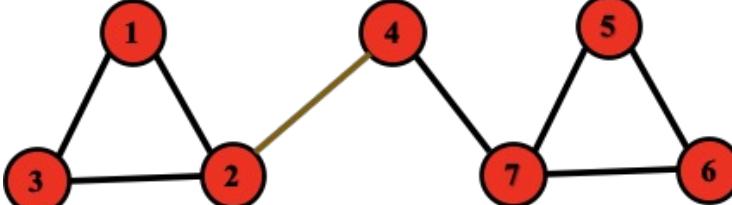
Connectivity: Example

The adjacency matrix of a network with several components can be written in a block-diagonal form, so that nonzero elements are confined to squares, with all other elements being zero:

Disconnected



Connected

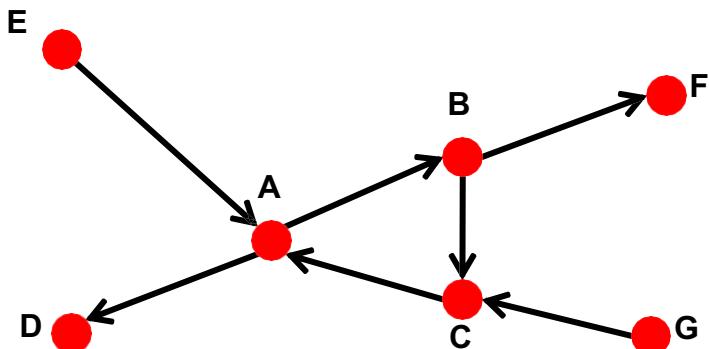


$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Connectivity of Directed Graphs

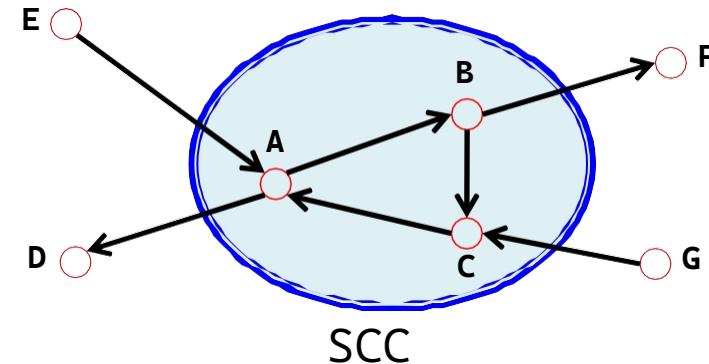
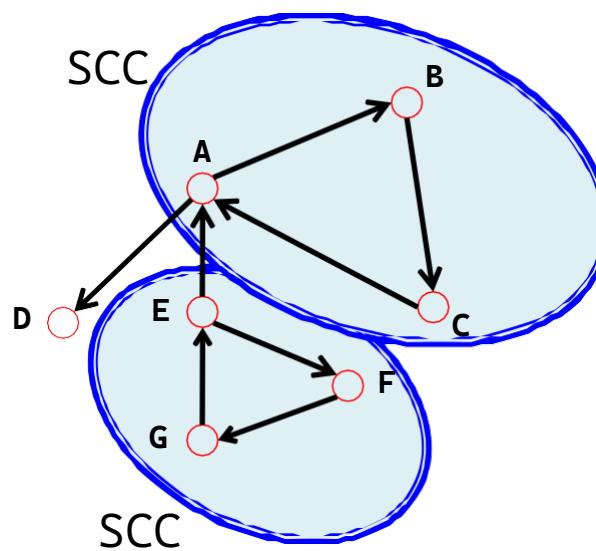
- Strongly connected directed graph
 - has a path from each vertex to every other vertex and vice versa (e.g., A-B path and B-A path)
- Weakly connected directed graph
 - is connected if we disregard the edge directions



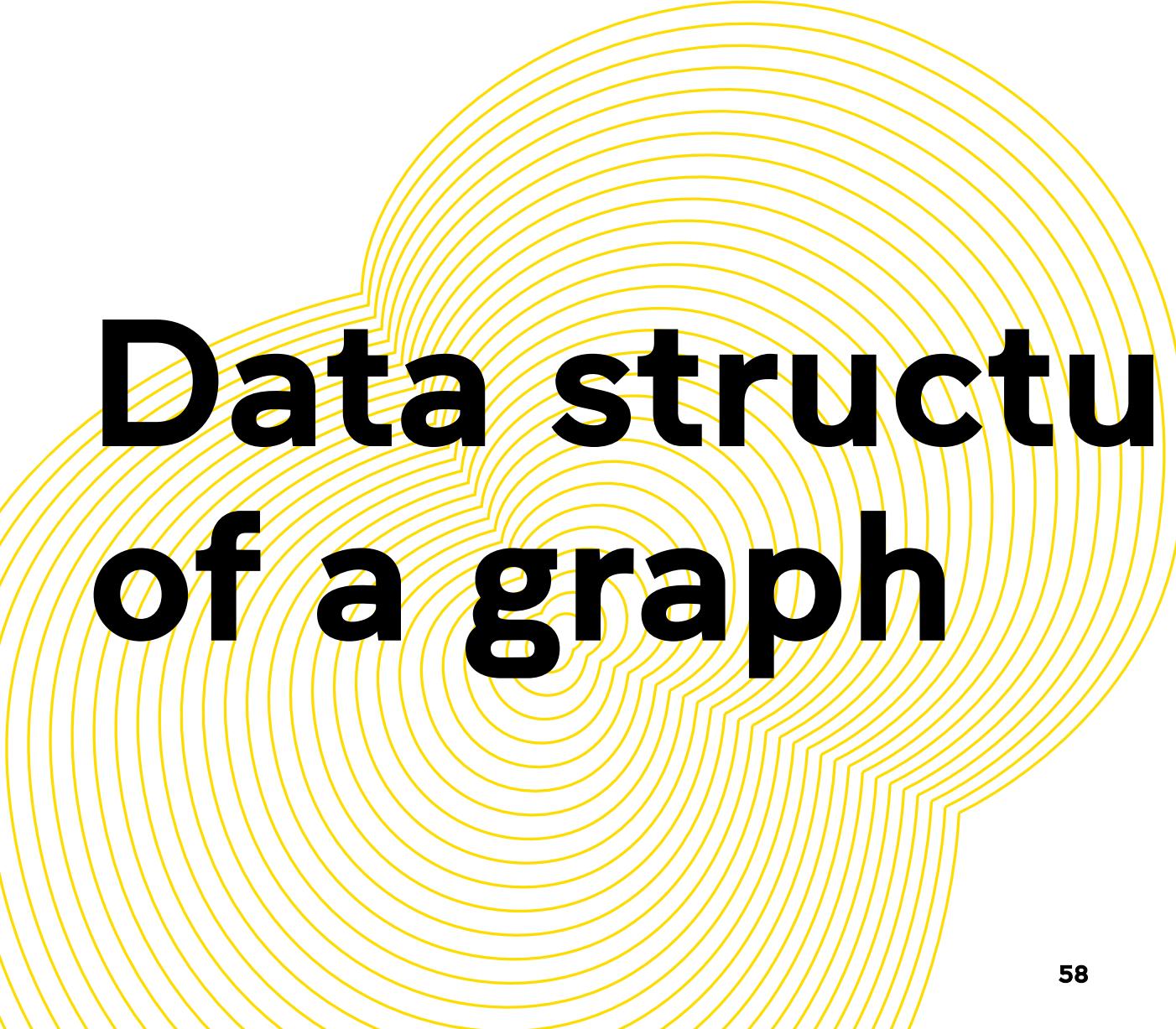
Graph on the left is connected but not strongly connected (e.g., there is no way to get from F to G by following the edge directions).

Connectivity of Directed Graphs

Strongly connected components (SCCs) can be identified, but not every vertex is part of a nontrivial strongly connected component.

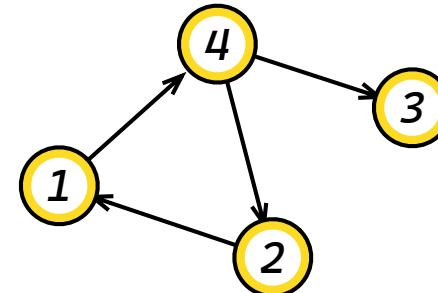
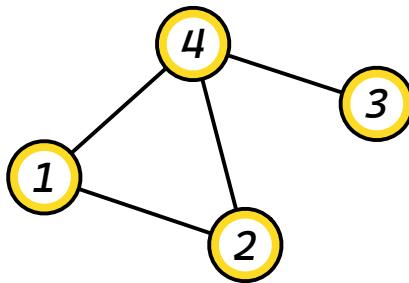


Maximal VS Maximum

A large, stylized graphic of yellow concentric circles, resembling a brain or a network, occupies the left side of the slide. It has a irregular, organic shape with some gaps and wavy lines.

Data structure of a graph

Adjacency Matrix



$A_{ij} = 1$ if there is a link from vertex i to vertex j

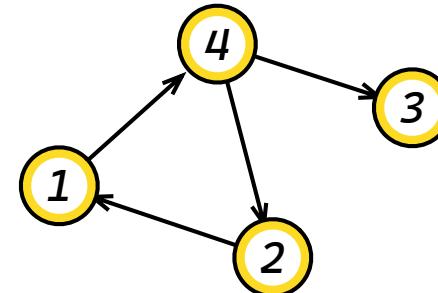
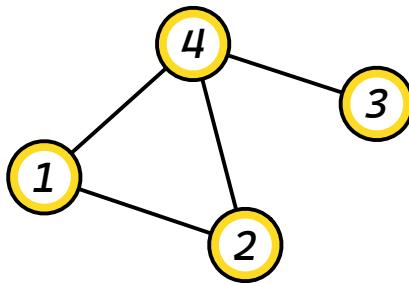
$A_{ij} = 0$ Otherwise

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Note that for a directed graph (right) the matrix is not symmetric.

Adjacency Matrix (cont)



$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} A_{ij} &= A_{ji} \\ A_{ii} &= 0 \end{aligned}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned} A_{ij} &\neq A_{ji} \\ A_{ii} &= 0 \end{aligned}$$

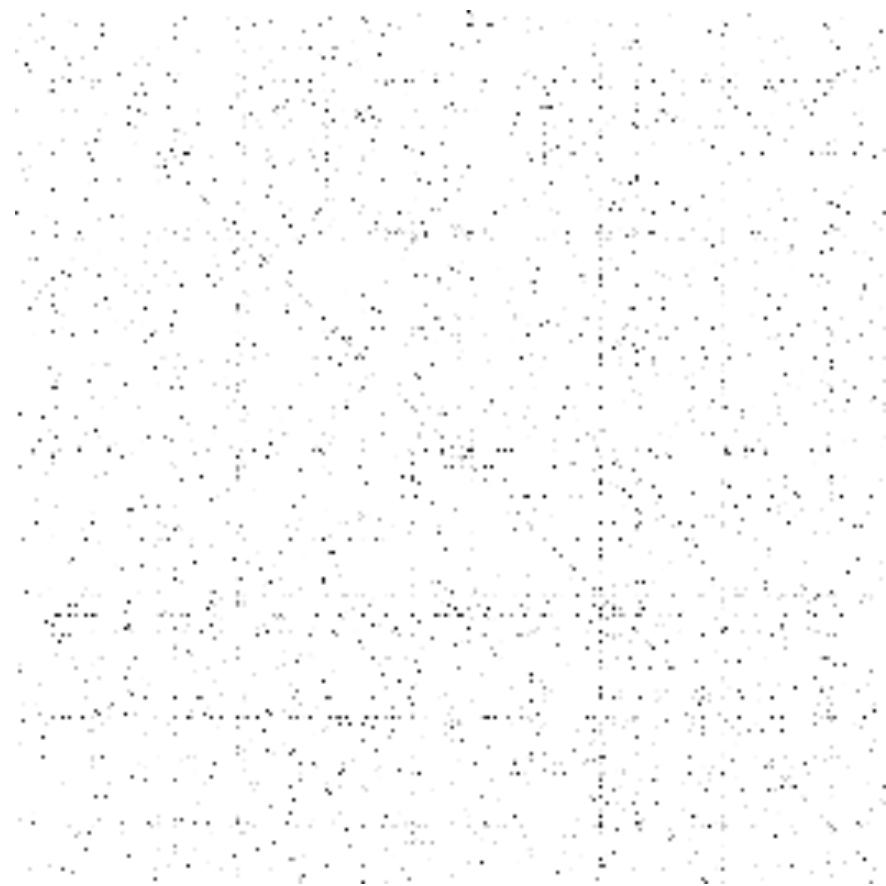
Most real-world networks are sparse

$$E \ll E_{max} \text{ or } deg_{avg} \ll |V|-1$$

NETWORK	NODES	LINKS	DIRECTED/ UNDIRECTED	N	L	$\langle k \rangle$
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.33
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email Addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Paper	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

Consequence: Adjacency matrix is filled with zeros!

Real Adjacency Matrices are sparse



$O(n^2)$ space

Scanning neighbors is inefficient

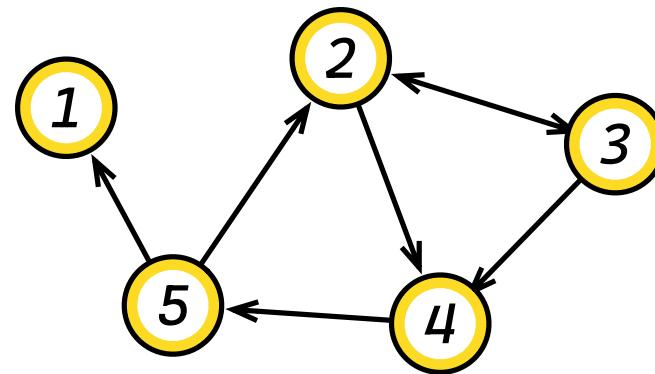
Implement adjacency matrix in Python

Explore real graphs:

<http://snap.stanford.edu/data/index.html>

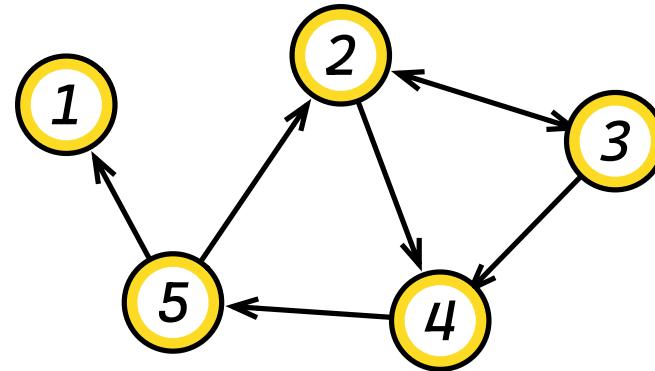
Adjacency list (Array)

- Easier to work with if network is
 - Large
 - Sparse
- Allows us to quickly retrieve all neighbors of a given vertex
 - 1:
 - 2: 3, 4
 - 3: 2, 4
 - 4: 5
 - 5: 1, 2



Adjacency list (cont)

- Easier to work with if network is
 - Large
 - Sparse
- Allows us to quickly retrieve all neighbors of a given vertex
 - 1:
 - 2: 3, 4
 - 3: 2, 4
 - 4: 5
 - 5: 1, 2



Implement adjacency list in Python

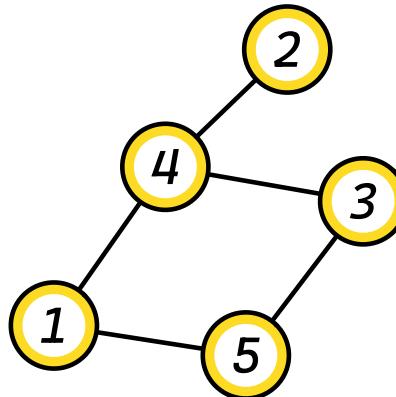
Space-efficient for sparse graphs

OK for graph updates

Efficient enough?

Quick quiz

- Compute the adjacency matrix and the adjacency list of the right graph
- Degree of each vertex
- Density of the graph

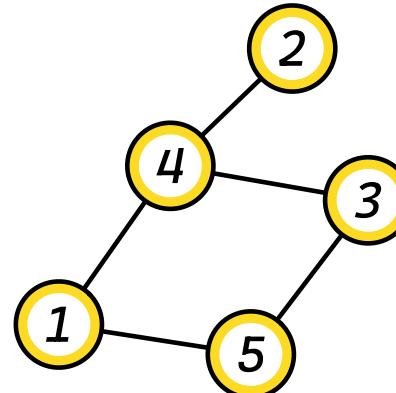


Quick quiz

- Compute the adjacency matrix and the adjacency list of the right graph
 - 1: [0, 0, 0, 1, 1]
 - 2: [0, 0, 0, 1, 0]
 - 3: [0, 0, 0, 1, 1]
 - 4: [1, 1, 1, 0, 0]
 - 5: [1, 0, 1, 0, 0]
 - 1: 4, 5
 - 2: 4
 - 3: 4, 5
 - 4: 1, 2, 3
 - 5: 1, 3
- Degree of each vertex
 - 1: 2
 - 2: 1
 - 3: 2
 - 4: 3
 - 5: 2
- Density of the graph
 $5/10 = 0.5$

Present time complexity:

- Input data size (n, m)
- Query related values
- Output data size



Complexity difference between the adjacency matrix and the adjacency list:

Storage space: ?

Query whether an edge (u, v) exists : ?

Get all the neighbors of a vertex v: ?

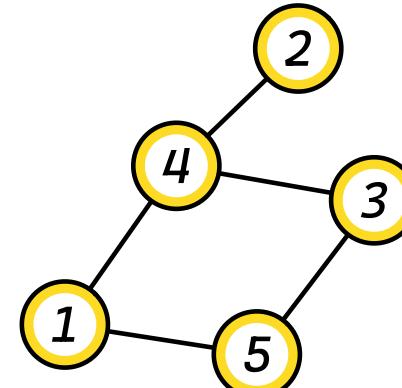
Quick quiz

- Compute the adjacency matrix and the adjacency list of the right graph
 - 1: [0, 0, 0, 1, 1]
 - 2: [0, 0, 0, 1, 0]
 - 3: [0, 0, 0, 1, 1]
 - 4: [1, 1, 1, 0, 0]
 - 5: [1, 0, 1, 0, 0]
 - 1: 4, 5
 - 2: 4
 - 3: 4, 5
 - 4: 1, 2, 3
 - 5: 1, 3

Storage space: $O(|V|^2)$ v.s. $O(|V| + |E|)$

Query whether an edge (u, v) exists : $O(1)$ v.s. $O(\deg(u)) \rightarrow O(\min(\deg(u), \deg(v))) \rightarrow$ Or better?

Get all the neighbors of a vertex v : $O(|V|)$ v.s. $O(\deg(v))$



Compressed Sparse Row (CSR)

$$A = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \left[\begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{matrix} \right] \end{matrix}$$

Adjacency matrix

Edge ID: $0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$
 $\text{edge-array} = [0, 1, 1, 2, 0, 2, 3, 1, 3]$

$\text{vertex-array} = [0, 2, 4, 7, 9]$
Vertex ID: $0 \ 1 \ 2 \ 3$



Compressed Sparse Row (CSR)

Implement CSR in Python

$$A = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

Adjacency matrix

Edge ID: 0 1 2 3 4 5 6 7 8
edge-array = [0, 1, 1, 2, 0, 2, 3, 1, 3]
vertex-array = [0, 2, 4, 7, 9]
Vertex ID: 0 1 2 3

Space-efficient for sparse graphs

Efficient for static graphs

Hard to update

Adjacency list VS CSR

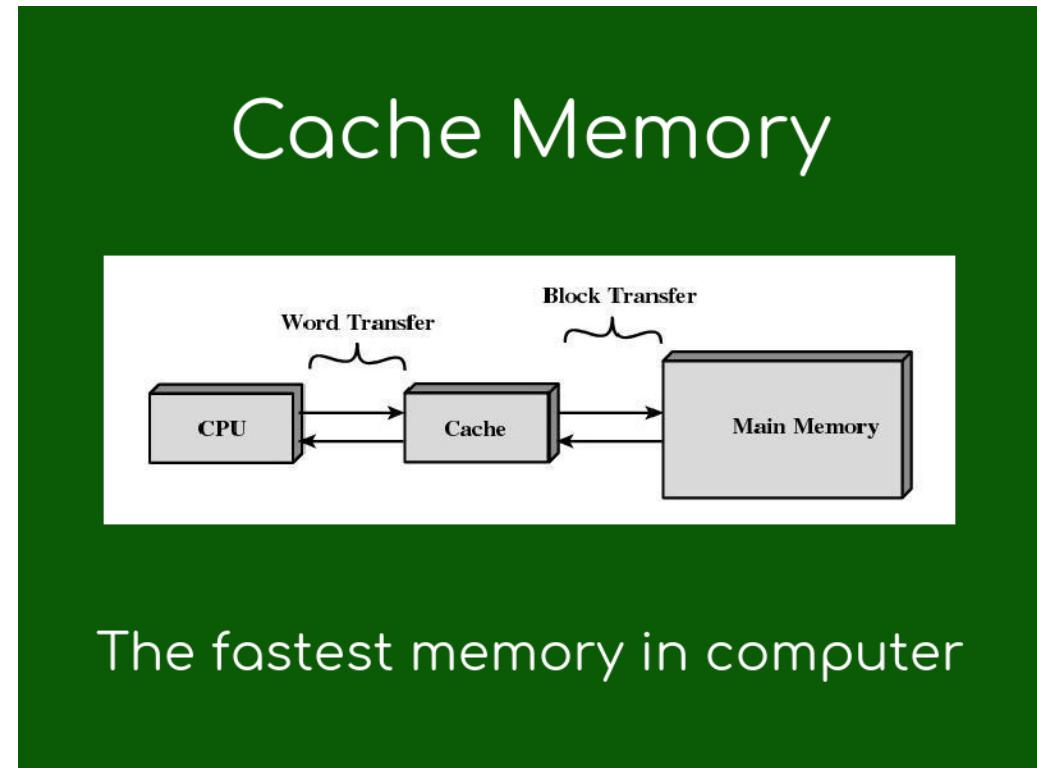
Which one is better?

Adjacency list VS CSR

Which one is better?

How does CPU read data?

Handling graph updates using adjacency list or CSR?



A quick summary for graph storage

Adjacency matrix: **best for updating**, **worst for neighbor scanning**, **worst for big graphs**

Adjacency list: **good for neighbor scanning**, **ok for updating**

CSR: **best for neighbor scanning**, **worst for updating**

Other structure to store neighbors of each vertex:

- Hash Map: **hard to choose bucket number**
 - large bucket number: **bad for neighbor scanning**, **good for updating**, **bad for big graphs**
 - small bucket number: **good for neighbor scanning**, **bad for updating**, **good for big graphs**
- Binary search tree: **ok for updating**

How about these structures?

Adjacent sorted list

Scan neighbors

Adjacent linked list

Check edge existence



Adjacent Set (Hash Table)

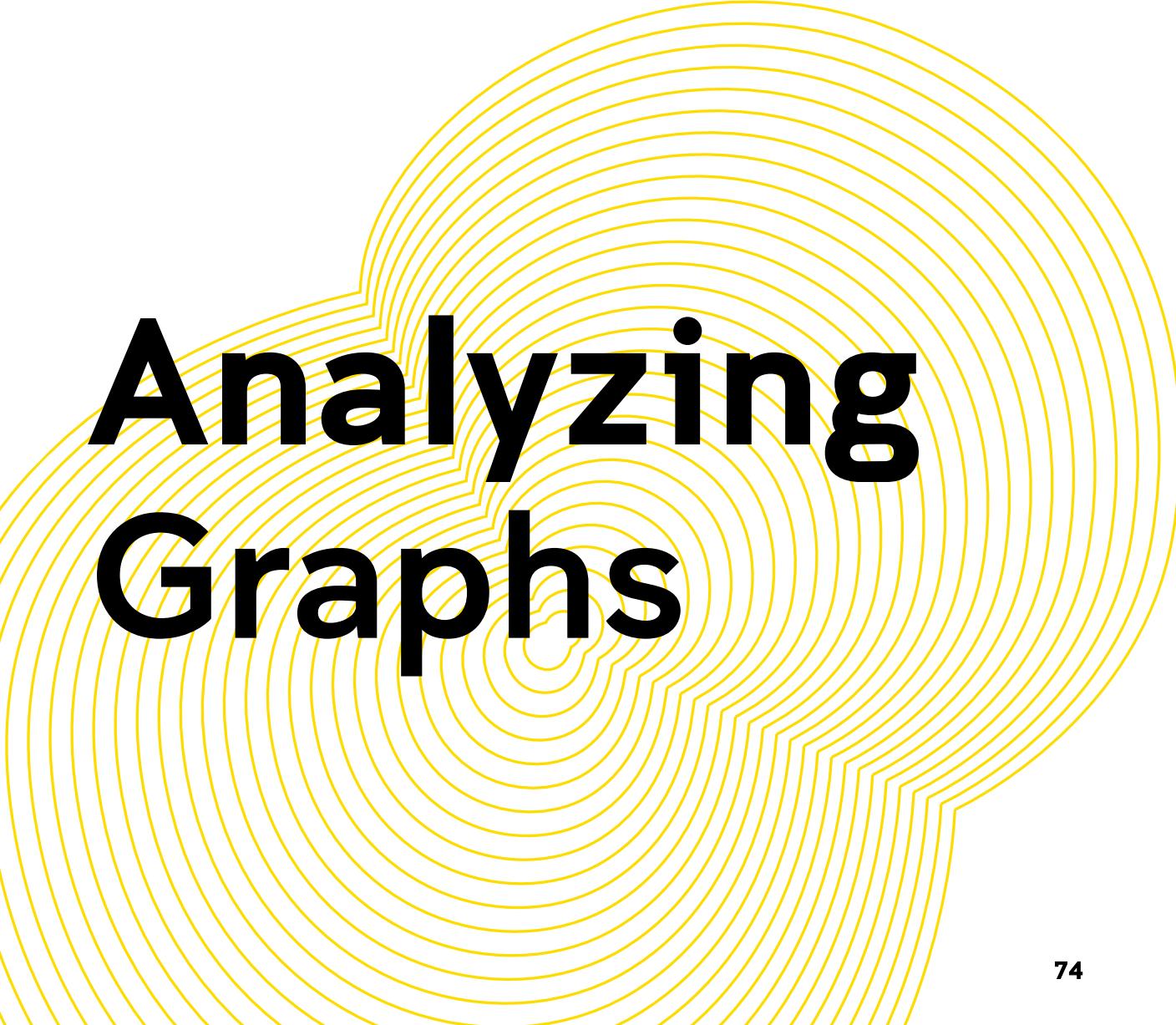
Insert edge

Adjacent BST

Delete edge

...

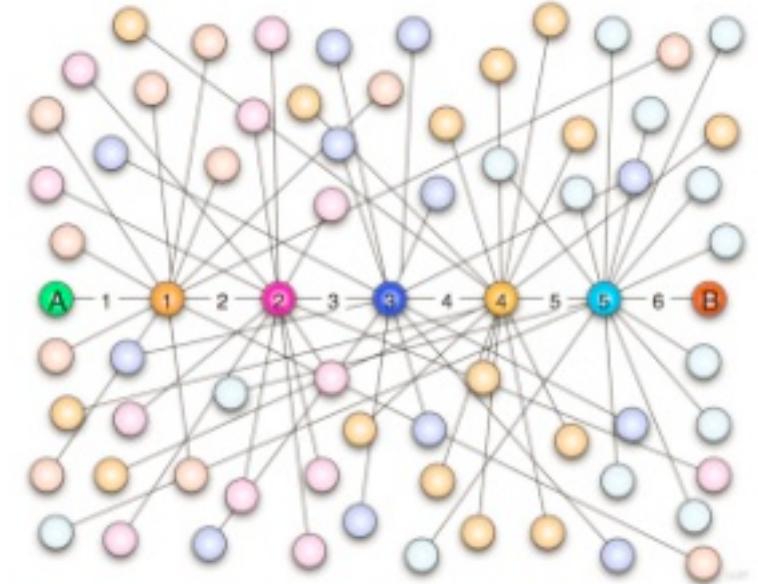
...

A large graphic of yellow concentric circles, resembling a sunburst or a series of ripples, occupies the left side of the slide. It starts from the bottom left and curves upwards towards the top left corner.

Analyzing Graphs

Searching

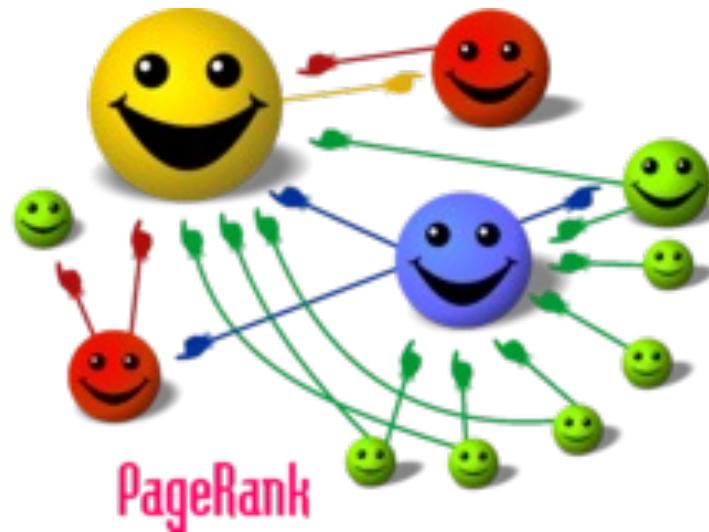
- Find the (shortest) path between two people on Facebook.
- Six degree of separation
 - : only six hops separate any two people in the world
- Can you find **a path (visible by public)** between you and Mark Zuckerberg with at least 3 hops in Facebook ?



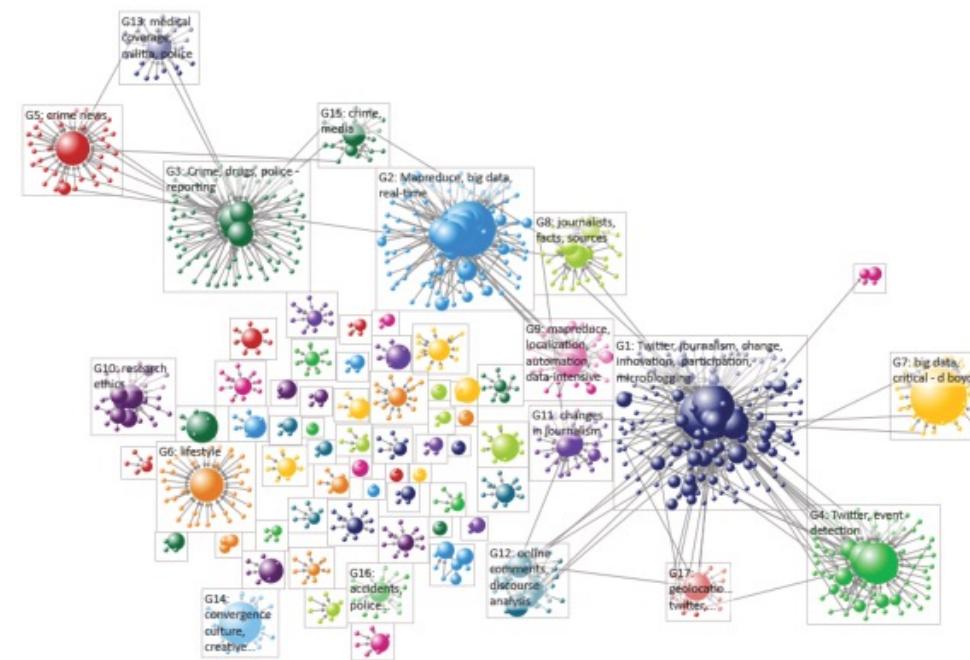
Ranking

Importance of the vertices.

E.g., which is the most influential paper in a co-citation network



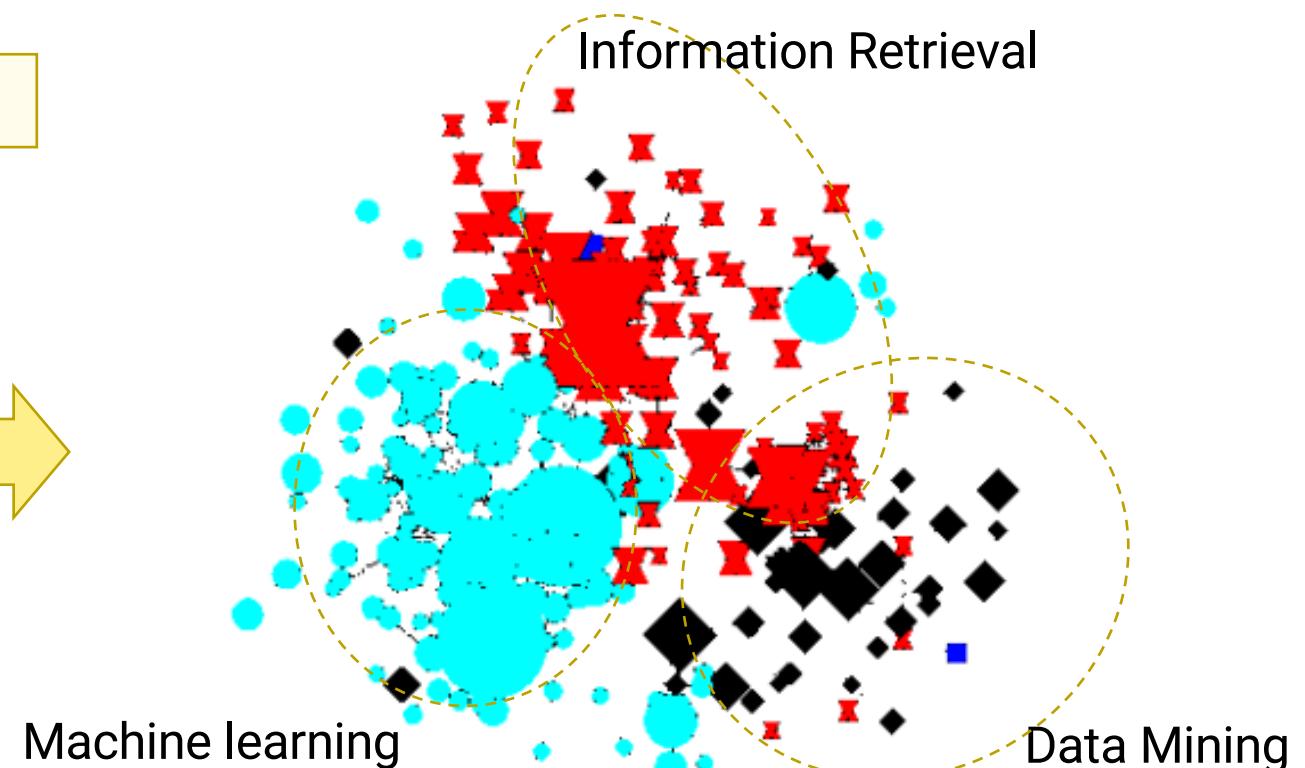
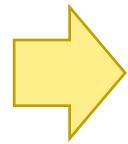
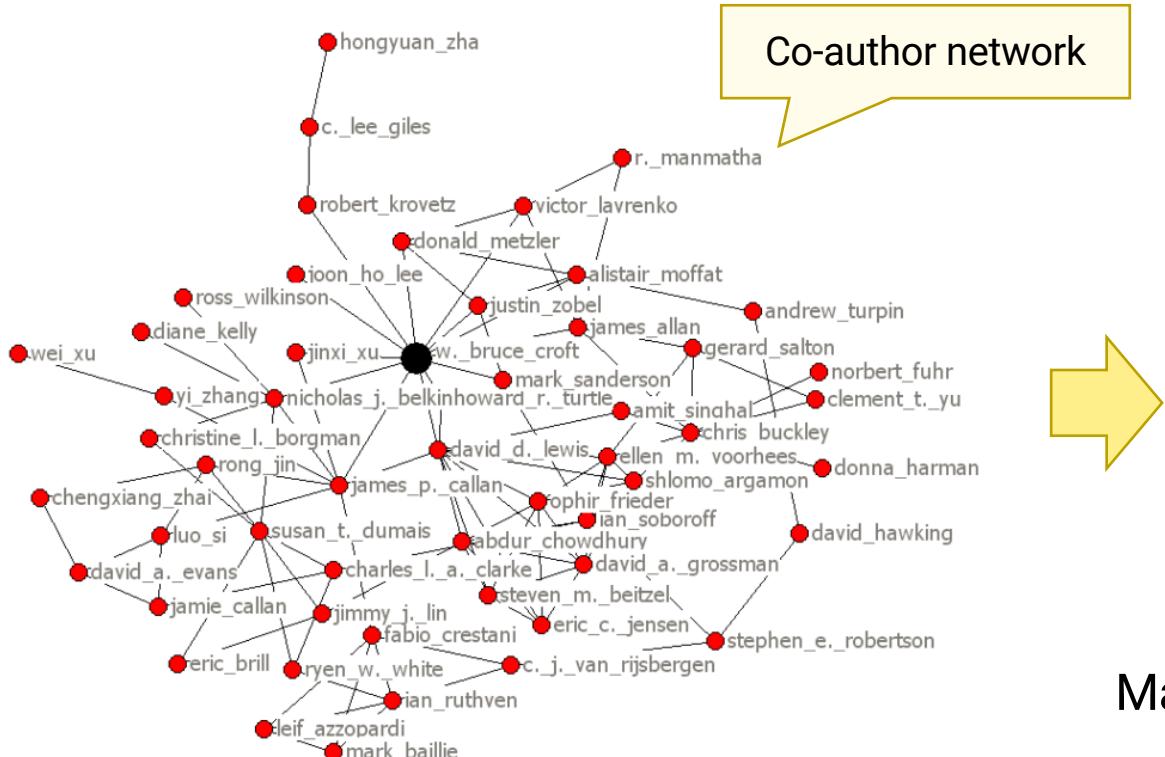
<https://en.wikipedia.org/wiki/PageRank>
Back Mirror : Nosedive (2016)



<https://www.researchgate.net/>

Finding Communities

Who tend to work together

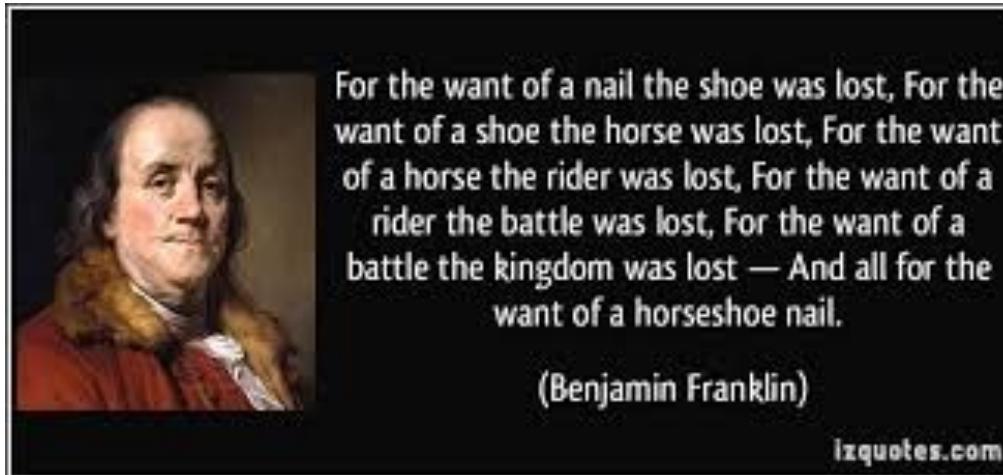


- Q.Mei, D.Cai, D.Zhang, and C.Zhai, Topic Modeling with Hitting Time, WWW 2008

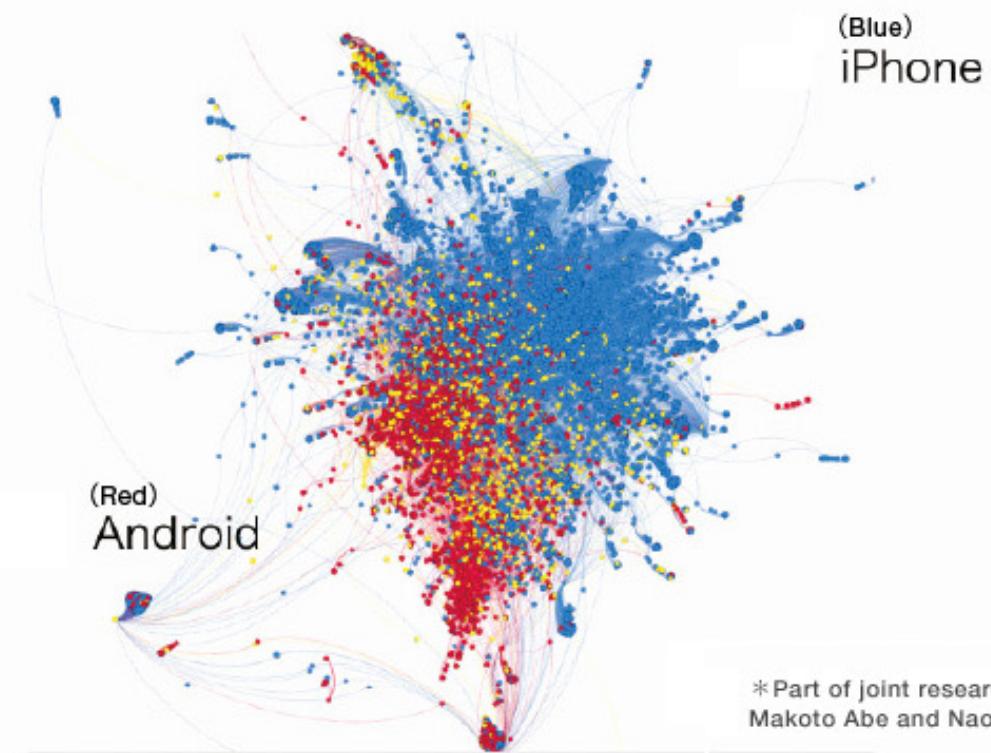
Network Dynamic Analysis

Cascade behaviour from node to node like an epidemic,

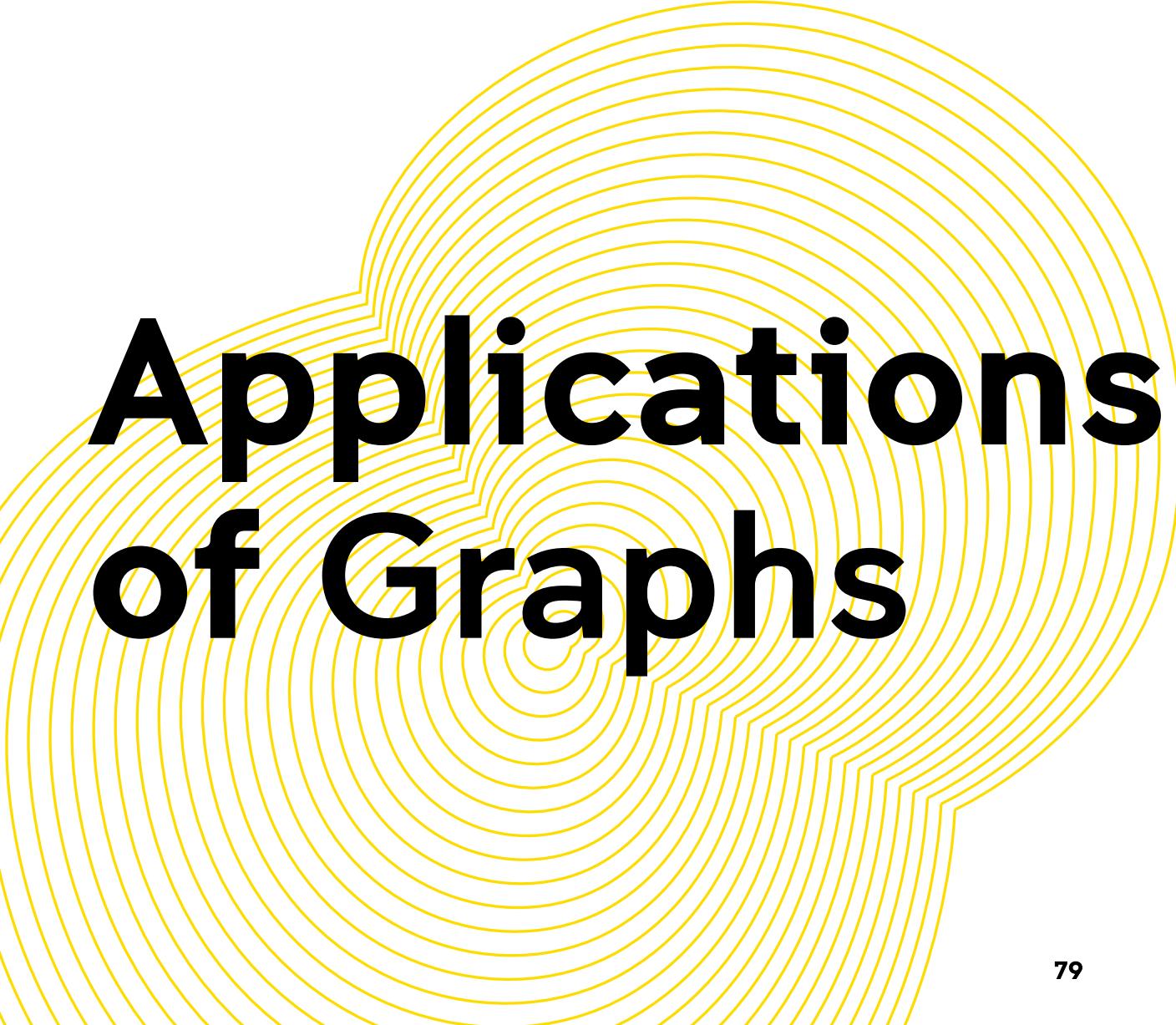
- Marketing, online advertising
- News, opinions, rumours, virus
- Adoption of innovation
- Joining a community, buying a book



Information diffusion network on Twitter

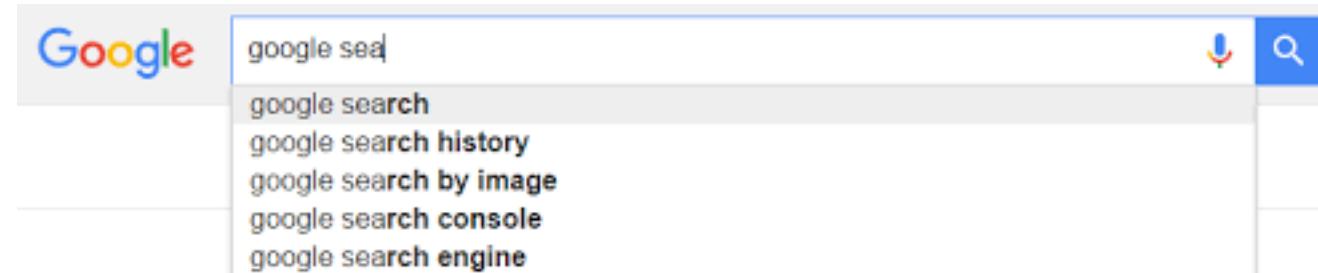


* Part of joint research with
Makoto Abe and Naoki Shinbo



Applications of Graphs

Web search



Yahoo directory

Social search

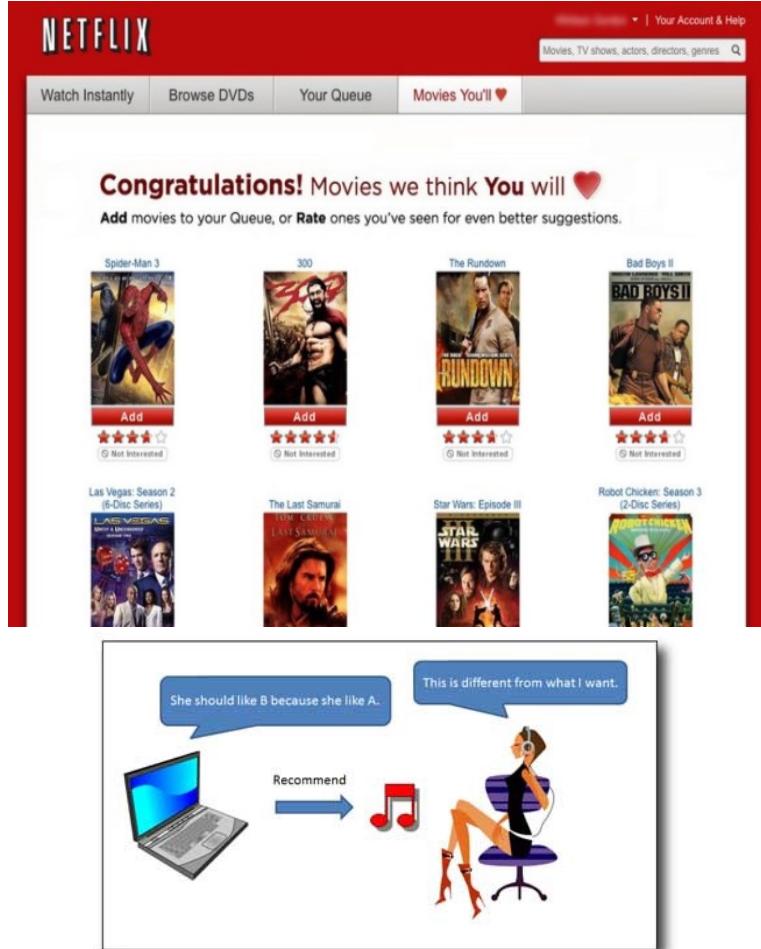
Social Search is an enhanced version of web search, also takes into account social relationships between the results and the searcher, such as work for the same companies, belong to the same social groups etc.



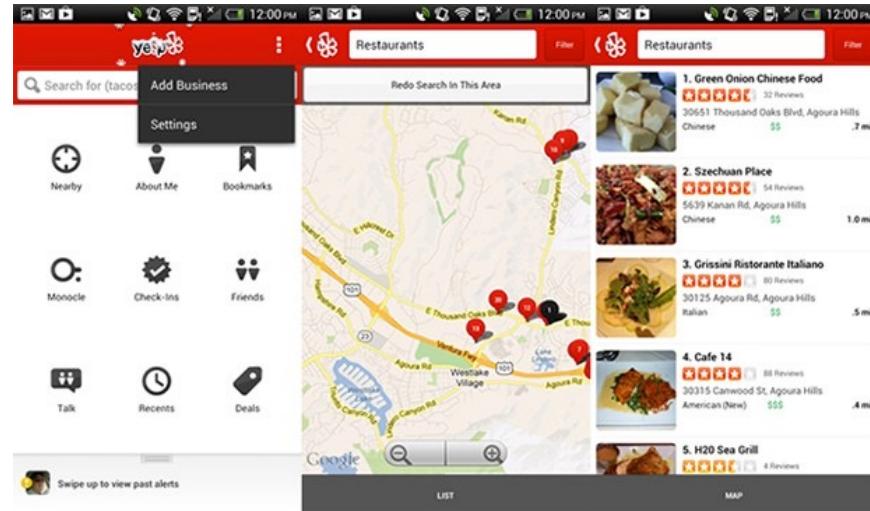
Facebook graph search



Recommendation



http://www.kis.kansai-u.ac.jp/res_music_e.html



Learning outcomes

Understand the basic graph structure and how to represent a graph

Know about characterization of graphs

Know about different types of graphs