

```

n = 8
edges = [[0, 1], [0, 2], [1, 2], [1, 3], [1, 4], [4, 5], [6, 7]]

adj_list = [[] for j in range(n)]
for edge in edges:
    adj_list[edge[0]].append(edge[1])
    adj_list[edge[1]].append(edge[0])

# create CSR
offset = [0]*(n+1);
csr_edges = [];
for i in range(n):
    offset[i] = len(csr_edges)
    csr_edges.extend(adj_list[i])
offset[n] = len(csr_edges)

# BFS
# A bad implementation
def BFS(u):
    visited = [False] * n
    queue = []

    queue.append(u)
    visited[u] = True
    while queue:
        s = queue.pop(0)

        print(s)
        for i in range(offset[s], offset[s+1]):
            nbr_of_s = csr_edges[i]
            if visited[nbr_of_s]: continue
            queue.append(nbr_of_s)
            visited[nbr_of_s] = True

# BFS
from collections import deque

def BFS(u):
    visited = [False] * n
    # queue = []
    q = deque()

    q.append(u)
    visited[u] = True
    while len(q) > 0:
        s = q.popleft()
        print(s)
        for i in range(offset[s], offset[s+1]):
            nbr_of_s = csr_edges[i]
            if visited[nbr_of_s]: continue
            q.append(nbr_of_s)
            visited[nbr_of_s] = True

```

```

# DFS recursive
visited = [False] * n
def DFS_recursive(u):
    print(u)
    visited[u] = True
    for i in range(offset[u], offset[u+1]):
        nbr_of_u = csr_edges[i]
        if visited[nbr_of_u]: continue
        DFS_recursive(nbr_of_u)

# DFS iterative
def DFS_iterative(u):
    visited = [False] * n
    stack = []
    stack.append(u)

    while (len(stack)):
        s = stack.pop()
        if(visited[s]):
            continue;

        visited[s] = True
        for i in range(offset[s], offset[s+1]):
            nbr_of_s = csr_edges[i]
            if visited[nbr_of_s]: continue
            stack.append(nbr_of_s)

```

Doing the assignment:

Types of Time complexity:

Worst case | Average | Amortized

How to write pseudocode:

A safe way is to just write any programming language

add any explanation for your pseudocode

```

## connectivity / reachability for undirected graphs
def connectivity(u,v):
    visited = [False] * n
    queue = deque()

    queue.append(u)
    visited[u] = True
    while queue:
        s = queue.popleft()

```

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# dequeue.popleft() for bfs & pop() for dfs
for i in range(offset[s], offset[s+1]):
    nbr_of_s = csr_edges[i]
    if nbr_of_s == v: return True
    if visited[nbr_of_s]: continue
    queue.append(nbr_of_s)
    visited[nbr_of_s] = True

return False

# better idea? Bidirectional BFS
# Why Bidirectional BFS is more efficient?

```

Output of Connected Components:

assume we have a graph with six vertices: 0, 1, 2, 3, 4, 5

```

[
    [0, 3, 5],
    [1, 2],
    [4]
]

```

```
[0, 1, 1, 0, 2, 0]
```

linear => equivalent

naive connected component

```

def naive_cc():
    undiscovered_vertices = [i for i in range(n)]
    result = []

    while undiscovered_vertices:
        current_cc = []
        u = undiscovered_vertices.pop()
        queue = []
        queue.append(u)
        while queue:
            s = queue.pop(0)
            current_cc.append(s)
            for i in range(offset[s], offset[s+1]):
                nbr_of_s = csr_edges[i]
                if nbr_of_s in undiscovered_vertices:
                    queue.append(nbr_of_s)
                    undiscovered_vertices.remove(nbr_of_s)
            result.append(current_cc)
    return result

# suggestions:

```

optimized connected component

```

# 1. swap the visited vertex with the last unvisited
...

# 2. doubly linked list
...

# 3. lazy updates

def cc():

    result = []
    visited = [False] * n

    for i in range(n):
        if visited[i]: continue
        current_cc = []
        queue = deque()
        queue.append(i)
        visited[i] = True
        current_cc.append(i)

        while queue:
            s = queue.popleft()
            for i in range(offset[s], offset[s+1]):
                nbr_of_s = csr_edges[i]
                if visited[nbr_of_s]: continue
                queue.append(nbr_of_s)
                visited[nbr_of_s] = True
                current_cc.append(nbr_of_s)
            result.append(current_cc)

    return result

# print(cc())

### disjoint set

parent = [i for i in range(n)]
size = [1 for i in range(n)]

def find(x):
    if parent[x] != x:
        parent[x] = find(parent[x])
    return parent[x]

def union(x, y):
    x = find(x)
    y = find(y)
    if x == y:
        return
    if size[x] < size[y]:
        size[y] += size[x]
        parent[x] = y
    else:
        size[x] += size[y]
        parent[y] = x

def ds_cc():
    for edge in [[0, 1], [0, 2], [1, 2], [1, 3], [1, 4], [4, 5], [6, 7]]:
        union(edge[0], edge[1])

# ds_cc()
# print(find(1) == find(7))

```

Topological Sort:

```
def ts():
    result = []
    s = []
    in_deg = [0 for i in range(n)]

    # calculate in-degree
    for u in range(n):
        for i in range(offset[u], offset[u+1]):
            nbr = csr_edges[i]
            in_deg[nbr] = in_deg[nbr]+1

    for u in range(n):
        if in_deg[u] == 0: s.append(u)

    while s:
        u = s.pop()
        result.append(u)
        for i in range(offset[u], offset[u+1]):
            nbr = csr_edges[i]
            in_deg[nbr] = in_deg[nbr]-1
            if in_deg[nbr] == 0: s.append(nbr)

    return result
```

Queue VS Stack

what if only in-neighbors of each vertex is stored?