31-08-23
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CIT - 121

Done

Discrete Mathematics (8th)

↳ Discrete Mathematics and It's Application by Kenneth H. Rosen

→ Continuous number fraction based number.

① What is Discrete mathematics ?

→ Discrete mathematics is the study of discrete objects.

Discrete Vs Continuous

Discrete	Continuous
1. Discrete → can't be broken down into fractions or decimals.	1. Continuous → can be broken down into fractions or decimals.
2. Number of students, purchasing tickets, text messages etc.	2. Time, Temperature, weight, distance etc.
3. Discrete elements are ~~con~~ countable.	3. Continuous elements are measurable.

→ Why do you need to study discrete mathematics?

1. It is a very good for improving problem-solving capabilities.

2. Logical reasoning.

2. Mathematical proofs - correct.

3. complexity of algorithms →
   Execution time: Best, worst, average.

② Probability

③ Graph & Tree → Shortest path
→ computer Networks
→ Advance data structure

④ Sorting program → Algorithm
→ Counting techniques
→ Mathmatical proofs

→ It has applications to

1. Compilers.
2. Software engineering
3. Architecture
4. data bases
5. Algorithms
6. data structure and
7. Operating systems.

Course plan —

1. Logic and proofs
2. sets, functions, sequences and sums
3. Algorithms, the integers and Matrices
4. Induction and Recursion.
5. Counting
6. Discrete probability
7. Advance counting techniques
8. Relations
9. Graph
10. Trees
11. Boolean Algebra
12. Modeling computation.

1.1 Propositional Logic (valid logical arguments)

→ what is proposition?

↳ A proposition is a declarative statement that is either true(T) or False(F) but not both. [Note: True = T = 1 OR False = F = 0]

Not proposition (Not valid logical arguments)

1. What time is it? → Question
2. Read this carefully. → Instruction/comand/imperative sentence (Do your home work)
3. $x+1=2$ → Non constant value / value is not defined.
4. Bangladesh and India → Not statements
5. Shahin is the best lecturer → opinion
6. He is a college student → Person is not defined.

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Propositional variables / statement variables // sentential variables :

- Today is Friday. ⟶ p = Today is Friday
- It is raining ⟶ q = It is raining.

The truth value of the propositional variable can be True or false.

Compound proposition ——

- Compound proposition is a proposition formed by combining two or more simple proposition.
- The logical operators that are used to form compound proposition is called connectives.

Symbol	Math name	English name
$\neg$	Negation	NOT
$\vee$	Disjunction	OR
$\wedge$	Conjunction	AND
$\oplus$	EXOR	"OR ... but not both"
$\rightarrow$	Implication	"If ... then"
$\Leftrightarrow$	Equivalence / Biconditional	"if & only if"

③ Truth Tables and Logical Connectives (NOT, OR, AND)

¬ (NOT), ∧ (AND), ∨ (OR), ⊕ (X-OR), → (Implication)

↔ (Bi Conditional)

→ NOT ($\neg P$, $\bar{P}$, $\sim P$, $-P$, P', NP, $!P$)

P = Today is friday. → convert Negation

→ use not

Today is not friday = ¬P

It is not the case that today is Friday.

n → variable number.

আছি : 2

Truth Table

Input (P)	Output (¬P)
F	T
T	F

1 → 0

0 → 1

Example of Negation : (Find Negation)

① 6 is negative. 6 is non-negative / It is not the case that

② 2+1 = 3 ; 2+1 ≠ 3

③ There is no pollution in New Jersey

④ The summer is

⑤ Today is

AND / Conjunction ($\wedge$)

→ Today is Friday and It's raining. Logical expre: $P \wedge q$

P $\wedge$ q

Truth table : All of the input is true and then output is true.

Input P	q	output ($P \wedge q$)
F	F	F
F	T	F
T	F	F
T	T	T

OR / Disjunction ($\vee$)

Today is Friday or It is raining.

P $\vee$ q

P	q	$P \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

ExOR ($\oplus$) X-OR

OR "Students who have taken calculus OR Computer science can take this class".

"students who have taken calculus or computer science, but not both, can take the class."

P	q	P ∨ q
F	F	F
F	T	T
T	F	T
T	T	T

P	q	P ⊕ q
F	F	F
F	T	T
T	F	T
T	T	F

Example:

a) Coffee or tea comes with dinner. (XOR)

b) Experience with C++ or Java is required (OR)

c) Lunch includes soup or salad. (XOR)

d)

~~বিজোড়~~ ~~সংখ্যক~~ ~~ইনপুট~~ T হলে output T হবে

Conditional Statement (Implication) ($\rightarrow$)

"If p, then q" $\boxed{p \rightarrow q}$

p: Today is holiday. q: The store is closed.

If ... then

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

T F $\rightarrow$ F

a) If $1+1=2$ then $2+2=5$
T T = F

b) $1+1=3$ then $2+2=4$ T

① if p, then q
② if p, q
③ p is sufficient for q
④ q if p
⑤ q whenever p
⑥ q is necessary for p
⑦ q follows from p
⑧ A is a sufficient condition for q is p.
⑨ q when p ⑪ p only if q
⑩ q unless ¬p

Then we can write $p \rightarrow q$

p is called the hypothesis / antecedent / premise

q is called the conclusion / consequence.

tutorial point

Converse, Inverse, Contrapositive

→ Conditional → new conditional

Converse | Inverse | Contrapositive

$\boxed{p \to q}$ — Converse: $q \to p$

Inverse: $\boxed{\neg p \to \neg q}$

Contrapositive ↓ Conver + Inverse $\boxed{\neg q \to \neg p}$

p : I am in Sylhet

q : I am in Bangladesh.

If I am in Sylhet then I am in Bangladesh.

$p \longrightarrow q$

Converse: $\boxed{q \to p}$

If I am in Bangladesh then I am in Sylhet.

Inverse: $\neg p \longrightarrow \neg q$

Contrapositive: $\neg q \longrightarrow \neg p$

N.T. $p \longrightarrow q \;=\; \neg q \longrightarrow \neg p$

p	¬p	q	¬q	q→p	¬p → ¬q	¬q → ¬p
F	T	F	T	T	T	T
F	T	T	F	T	F	T
T	F	F	T	F	T	F
T	F	T	F	T	T	T

→ Converse, Inverse, Contrapositive :-

a) If it snows today, I will ski tomorrow. $(p \to q)$

p q

just write down that sentence (C, I, C)

b) I come to class whenever there is going to be a quiz.

q p

$p \to q$

Converse :

Inverse :

Contrapositive :

bi-implication

BICONDITIONAL STATEMENT $\rightleftarrows$

→ You can take the flight if and only if you buy a ticket.
p q

$$p \leftrightarrow q$$

(i) p is necessary and sufficient for q.

(ii) p iff q

(iii) If p then q and conversely

(iv) p exactly when q

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

same হলে T

Note that,
$p \leftrightarrow q$
$p \to q \wedge q \to p$

Examples:

That it is below freezing (p) is necessary and sufficient for it to be snowing (q).

F	F	→ T
T	F	→ F ✓
F	T	→ F
T	T	→ T

Determine whether these biconditionals are true or false

a) $2+2=4$ if only if $1+1=2$

Ans: True

b) $1+1=2$ if and only if $2+3=4$

Ans: False

c) $1+3=2$ if and only if monkeys can fly.

Ans: False.

d) $0>1$ if and only if $2>1$

Ans: False

Precedence of Logical Operator

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$\neg \quad \wedge \quad \vee \quad \rightarrow \quad \leftrightarrow$

→ Truth table construction of compound propositio[n]

(1) $(P \vee \neg q) \rightarrow (P \wedge q)$

output ↓

P	q	¬q	P ∨ ¬q	P ∧ q	(P∨¬q)→(P∧q)
F	F	T	T	F	F
F	T	F	F	F	T
T	F	T	T	F	F
T	T	F	T	T	T

(2) $(P \leftrightarrow q) \oplus (P \leftrightarrow \neg q)$

P	q	¬q	P ↔ q	P ↔ ¬q	(P↔q) ⊕ (P↔¬q)
F	F	T	T	F	T
F	T	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	T

Bi-conditional এটি False হলে False.

X-OR

③ $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow p)$

English Sentence → preposition

Let p and q be the proposition

p: It is below   but → And → $\wedge$

q: It is snowing

a) It is $\underbrace{\text{below freezing}}_{p}$ $\underbrace{\text{and}}_{\wedge}$ $\underbrace{\text{snowing}}_{q}$.

b) It is below $\underbrace{\text{freezing}}_{p}$ $\underbrace{\text{but}}_{\wedge}$ $\underbrace{\text{not snowing}}_{\neg q}$.

c) It is either snowing or freezing but not both.

$p \oplus q$

d) If it is below freezing, It is also snowing.

$p \rightarrow$

e) It is either $\underbrace{\text{below freezing}}_{p}$ $\underbrace{\text{or}}_{\vee}$ it is $\underbrace{\text{snowing}}_{q}$, $\underbrace{\text{but}}_{\wedge}$ it is $\underbrace{\text{not snowing}}_{\neg q}$. if it is below $\underbrace{\text{freezing}}_{p}$

1.2 Logic and Proofs $\boxed{4/5}$

→ Propositional Equivalences:

$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$

i) Tautology
ii) Contradiction
iii) Contingency

(i) $p \vee \neg p$ (ii) $q \wedge \neg q$ (iii) $p \to q$

$\boxed{T\ F \to TF}$ $\to F$

p	q	¬p	¬q	
F	F	T		
F	T	T		
T	F	F		
T	T	F		

Tautology: কোনো উক্তি compound এর অবস্থাগুলো True হলেই তাকে Tautology. (Always True)

Contradiction: Always False

Contingency: Mixed of Tautolog and Contradiction

T F → F ∧

আমোকানো একটি False হলেই False

□ Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology using truth table

r	p	q	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	F	T	T	F	T
F	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T
T	F	T	T	T	F	T	T
T	T	F	F	F	T	F	T
T	T	T	T	T	T	T	T

There, P, q, r → then it will be tautology.

□ Determine $(\neg p \wedge (p \to q)) \to p$ is a tautology.

□ ¬

Logical Equivalence using truth table.

Ex: Show that $p \to q$ and $\neg p \vee q$ are logically equivalent.

$\neg p \Rightarrow F$

p	q	$\neg p$	$p \to q$	$\neg p \vee q$	① ↔ ②
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	F	
T	T	F	T	T	

$p \to q$ and $\neg p \vee q$ are logically equivalent

② $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$

p	q	r	1: $p \to r$	2: $q \to r$	$1 \vee 2$	$p \wedge q$	$(p \wedge q) \to r$
F	F	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	F	T
T	F	F	F	T	T	F	T
T	F	T	T	T	T	F	T
T	T	F	T	F	T	T	F
T	T	T	T	T	T	T	T

Exercises:

① $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$

② $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$

Logic Laws

① Double Negation law

$\neg(\neg p) \equiv p$

② Idempotent law

$p \vee p \equiv p$

$p \wedge p \equiv p$

③ Identity law

$p \wedge T \equiv p$

$p \vee F \equiv p$

$(p \wedge T) \wedge (p \vee F)$

$\equiv p \wedge p$

$\equiv p$

④ Domination law

$p \vee T \equiv \cancel{\phi}\, T$

$p \wedge F \equiv F$

$(p \vee T) \wedge (p \wedge F)$

$\equiv T \wedge F$

$\equiv F$

can't understand

⑤ Commutative law

$P \vee q \equiv q \vee P$

$P \wedge q \equiv q \wedge P$

⑥ Associative law

$(P \vee q) \vee r \equiv P \vee (q \vee r)$

$(P \wedge q) \wedge r \equiv P \wedge (q \wedge r)$

⑦ Inverse law

$P \vee \neg P \equiv T$

F T
T F

$P \wedge \neg P \equiv F$

F T

$A\bar{B}\,B\bar{C} + B\bar{C}C\bar{D} + \bar{A}DA\bar{B} + ACD\bar{D}$

$= A\bar{C}\,(\bar{B} \wedge B)$

$= A\bar{C}\,0 + B\bar{D}\,0 + \bar{B}D\,0 + ACF$

$= 0$

⑧ Absorption law

$P \vee (P \wedge q) \equiv P$

$P \wedge (P \vee q) \equiv P$

⑨ Distributive law

$P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$

$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$

⑩ Conditional law

$P \rightarrow q \equiv \neg P \vee q$

⑪ Biconditional law

$P \leftrightarrow q \equiv (P \rightarrow q) \wedge (q \rightarrow P)$

Logic Law (De Morgan's Law)

De morgan's Law :

(i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$
(ii) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Similar as the three variable

→ De Morgan's law to find the negation

a) Jon is $\underset{p}{\text{rich}}$ $\underset{\wedge}{\text{and}}$ $\underset{q}{\text{happy}}$. $\neg(p \wedge q) = \neg p \vee \neg q$

b) Carlos will $\underset{p}{\text{bicycle}}$ $\underset{\vee}{\text{or}}$ run $\underset{q}{\text{tommarrow}}$. $\neg(p \vee q)$

→ Proving tautology without using truth table :

a) $\underset{p}{(p \wedge q)} \rightarrow \underset{q}{p}$ $\quad (p \rightarrow q \equiv \neg p \vee q)$

DML

$\equiv \neg(p \wedge q) \vee p$

$\equiv \neg p \vee \neg q \vee p$

$\equiv (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$ ← Tautology

$= T / 1$

b) $(p \wedge q) \rightarrow (p \vee q)$

$= \neg(p \wedge q) \vee (p \vee q)$

$= \neg p \vee \neg q \vee p \vee q$

$= (\neg p \vee p) \vee (\neg q \vee q)$

$= T \vee T$

$= T$

Logical equivalent without TT

HW

i) $\neg(p \to q) \equiv p \wedge \neg q$

ii) $\neg(p \vee (\neg p \vee q)) \equiv \neg p \wedge \neg q$

→ Logical equivl.

L.H.S

$\neg(p \to q)$

$= \neg(\neg p \vee q)$

$= \neg\neg p \wedge \neg q$

$= p \wedge \neg q$

$=$ R.H.S

(ii) $\neg(\underset{p}{p} \vee \underset{q}{(\neg p \vee q)})$

$= \neg p \wedge \neg(\neg p \vee q)$

$= \neg p \wedge \neg\neg p \wedge \neg q$

$= \underset{T}{\neg p} \wedge \underset{F}{p} \wedge \neg q$

$= 0$

1.3 Predicates & quantifiers

variable/subject predicate

i) x is greater than 3. ← এটা proposition নয় কারণ x এর value define করা নাই

$P(x) = x$ is greater than 3

$x = 2$ $P(2) = 2$ is greater than 3 (False)

$x = 4$ $P(4) = 4$ is gre — — 3 (True)

Types of predicates

→ Unary binary Ternary
1 2 3

Examples

Let $P(x)$ denote the statement $x > 3$. What are the truth values of $P(4)$ and $P(2)$?

$P(x) = x > 3$
$P(4) = 4 > 3 \longrightarrow$ True
$P(2) = 2 > 3 \rightarrow$ False

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Quantifiers:

i) Universal ($\forall$)

ii) Existential ($\exists$)

→ "x can speak English".

$P(x)$ = x can speak english.

uni, $\forall x\ P(x)$ $\exists x\ P(x)$

Universal ($\forall x$) quantifiers; for --, given any

$\forall x\ P(x)$ → For all x, $P(x)$ is true

Existential quantifiers; There is, exists, for some, for at least one (কোনো একটি True হলেই True return করবে)

Example - 1

Statement $Q(x)$ → "$x < 2$", what is the truth value of the quantification $\forall x\, Q(x)$, where the domain consits all real numbers?

→ 0, 1 Exceding less equal

English to Logical Expression:

using prediction & quantifiers

① Every student in the class has studied calcul

$\forall_x P(x)$ = calculus

② For every person x, if person x is a student in the class then x has studied calculus.

$\forall_x$ ~~person~~ $P(x)$ = person = domain.

$S(x)$ = x is a student

$c(x)$ = x has studied calculus

$\forall_x (s(x) \longrightarrow c(x))$

② Some student in this class has visited Mexico.

~~$S(x)$~~ = $\exists(x)$ = visited mexico $M(x)$

There is a person x, having the properties that x is a student in the class and x has visited Mexico.

$\exists x\,(S(x) \wedge M(x))$

4 → For every Person x, if x is a student in the class, then x has visited Mexico or x has visited Canada.

domain → Perso.

$\forall x\,(S(x) \rightarrow (M(x) \vee C(x)))$

→ Using quantifier in system specification

Every mail massage larger than one megabyte will be compressed.

$\forall x$ $S(m, y)$ = m mess larger than y megabyte

$C(m)$ = m is msg that will be compressed

$\forall x\,(S(m, 1) \longrightarrow C(m))$

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but / And $\longrightarrow \wedge$
either $\longrightarrow \vee$

$\rightarrow$ No student at your school can speak Russian or knows C++

$\exists x \neg (P(x) \vee Q(x))$

$\rightarrow$ Logical to English

a) $\forall x \, (C(x) \rightarrow F(x))$

All people of x, x is a comedian and funny. Every comedian is funny.

H.W

$\exists x \, (C(x) \longrightarrow F(x))$

$\exists x \, (C(x) \wedge F(x))$

Negation Quantified Expression

(i) $\forall x \; P(x)$

$= \neg(\forall x \; P(x))$

$= \exists x \; \neg \; P(x)$

(ii) $\exists x \; P(x)$

$= \neg(\exists x \; P(x))$

$= \forall x \; \neg \; P(x)$

[1] Every student in my class has taken a course in calculus.

$\forall x \; C(x) \rightarrow \neg(\forall x \; C(x)) \rightarrow \exists x \; \neg \; C(x)$

There is a student in my cls who hasn't taken a course

[2] There is an honest politician.

$\exists x \; H(x)$

Nested Quantifiers :

$\forall x\,(\exists y\ P(x,y))$

☐ Translate these into Eng :

$\forall x\ \forall y\big((x>0)\wedge(y<0)\rightarrow(xy<0)\big)$

Domain : real numbers

→ For every real number x and y, if x is positive and y is negative then xy is Negative.

→ Eng to logical :

The sum of two positive integers is always positive,

$\cancel{1(x)\vee 2(x)}$

$\forall x\forall y\big((x>0)\wedge(y>0)\longrightarrow(x+y)>0\big)$

→ $\forall x\ (C(x)\vee\exists y\ (C(y)\wedge F(x,y)))$

all ~~some~~ or some ~~comp~~ ~~that~~ x and y are fried

For every student x in my school, x has a computer or there is a student y that is y has a computer and x and y are friend.

Let, $L(x,y) \equiv x$ loves y

domain $\rightarrow$ all people

a) Everybody loves jerry. $\forall x \ L(x, \text{Jerry})$

b) Everybody loves somebody. $\forall x \exists y \ L(x,y)$

$\boxed{\text{gap}}$

→ Rules of Inference:

Valid Arguments in Propositional logic:

"If you have a current password (p), then ($\rightarrow$) you can log on to the network (q).

Therefore thakle $\therefore$, $p \therefore q$

Toutology হলে valid.

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Must be Tautology

(i) Modus ponens (Ex) If it is $\underbrace{\text{raining}}_{p}$, then I will $\xrightarrow{}$ study discrete math. ($\underbrace{\text{discrete math}}_{q}$)

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore p \end{array} \quad / \quad \begin{array}{l} p \to q \\ q \\ \hline q \end{array}$$

If it is raining

Therefore I will study discrete math

(2) Modus Tollens $\to$ If it is $\underbrace{\text{snowing}}_{p}$, then I $\xrightarrow{}$ will $\underbrace{\text{study math}}_{q}$.

I will not study discrete math

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array} \quad \left((p \to q) \wedge \neg q\right) \to \neg p$$

(3) Hypothetical syllogism

Ex: If it is $\underbrace{\text{raining}}_{p}$ then I will study dis. $\underbrace{\text{math}}_{q}$ $\quad p \to q$

If I study dis. math, I will get $\underbrace{\text{an A}}_{r}$ ~~grade~~

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

④ Disjuctive Syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Ex: I will study dis math or I will study eng Lit. $p \vee q$

I will not study dis math. $\neg p$

⑤ Addition: — OR ⑦ Conjunction:

$$\begin{array}{l} p \rightarrow T \\ \hline \therefore p \vee q \rightarrow T \end{array}$$

$$\begin{array}{l} p \rightarrow T \\ q \rightarrow T \\ \hline p \wedge q \rightarrow T \end{array}$$

⑥ Simplification

$$\begin{array}{l} \overset{T}{p} \wedge \overset{T}{q} \rightarrow T \\ \hline \therefore p \text{ or } \therefore q \end{array}$$

Ex: p: I will study English

q: I will study Math

$p \wedge q$: I will study English and math

⑧ Resolution:

$$\begin{array}{l} p \vee q \\ \neg p \vee r \end{array}$$

$$((p \vee q) \wedge (\neg p \vee r)) \rightarrow q \vee r$$

Ex: p: I will study English

q: I will " Math

r: I " " Bangla

Last (DimDum)

Example:

From the single proposition $p \wedge (p \to q)$ show that q is a conclusion.

1. $p \wedge (p \to q)$ premi.
2. p [1, simplification]
3. $p \to q$ [1, simpli]
4. q [2, 3 modus ponens]

Ex 2: 1. $T \to (M \vee E)$ S M is a conclusion

2. $S \to \neg E$
3. $T \wedge S$
4. T [3 simplification]
5. S [3 simplification]
6. $M \vee E$ [1, 4 Modus Ponens]
7. $\neg E$ [2, 5 Modus Ponens]
8. M [6, 7 disjunction]

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3. 1. p $\quad\quad$ $\neg r$ is conclusion.

2. $p \rightarrow q$

3. $q \rightarrow \neg r$

4. q [1, 2 modus ponen]

5. $\neg r$ [3, 4 $\boxed{\text{modus ponen}}$]

Ex: Show that hypotheses:

1. It is not sunny this afternoon and it is colder than Yesterday.

We will go swimming only if it is sunny.

If we don't go swimming then we will take a cannoe trip.

If we take a canones trip, then we will be home by sunset.

~~H~~ home এর জন্য conclusion বের করতে হবে।

1. $\neg p \wedge q$

2. $r \rightarrow p$

3. $\neg r \rightarrow t$

4. $t \rightarrow h$

5. $\neg p$ [1, simplification]

6. $\neg r$ [2, 5, Modus ~~pon~~ Tollens]

7. t (3, 6, Modus ponens)

8. h (4, 7, ~~mod~~ modus pon)

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