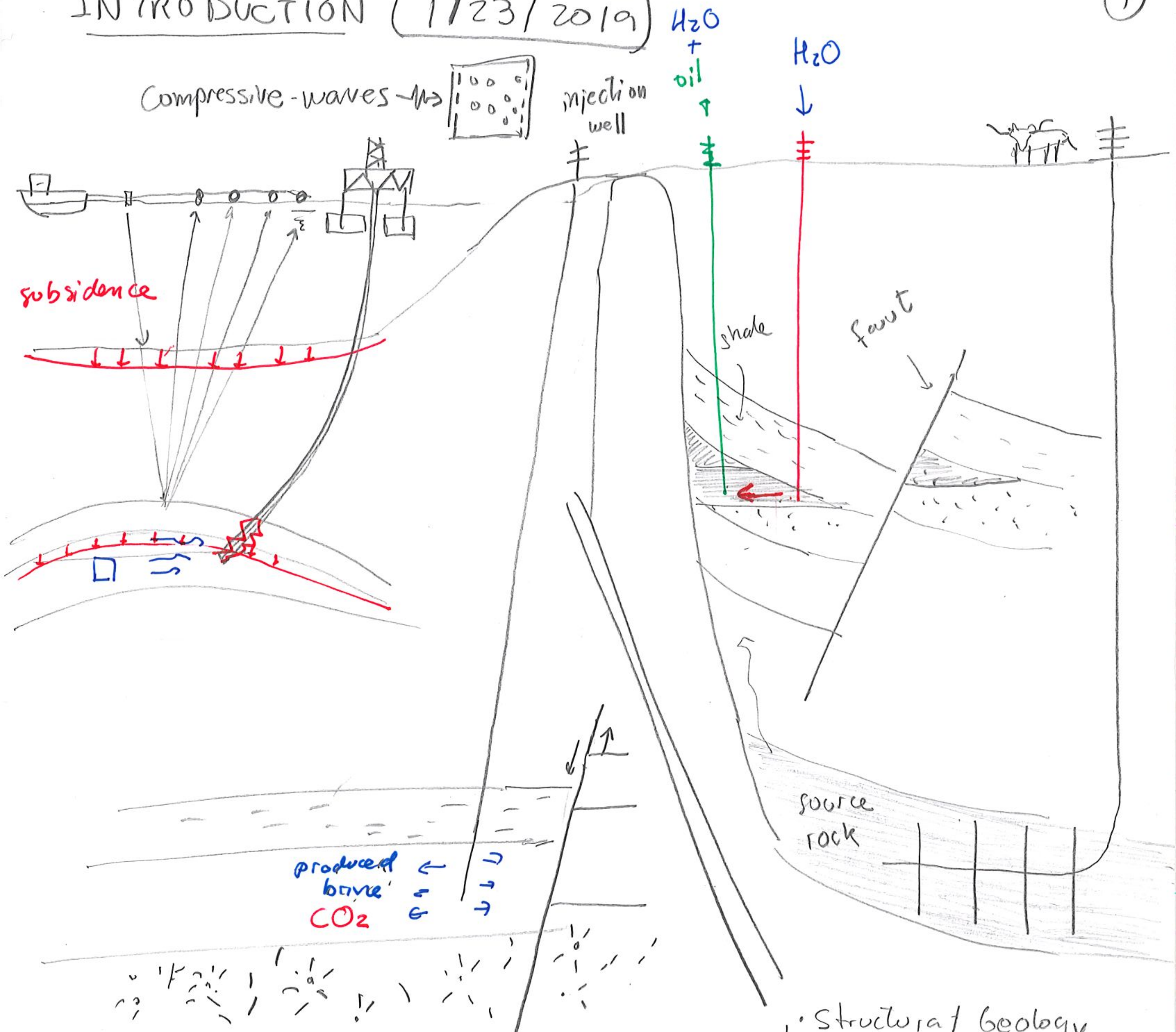
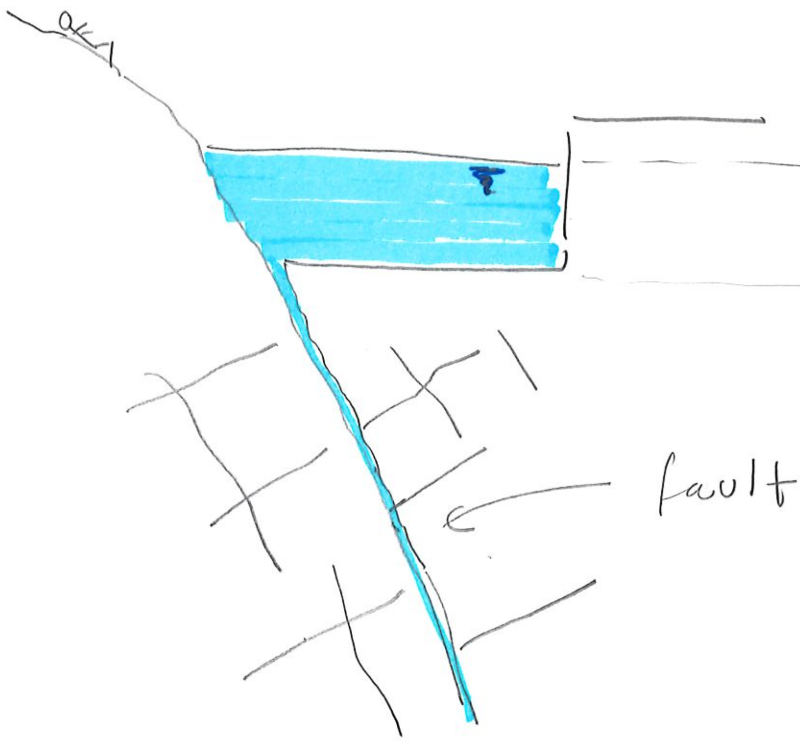
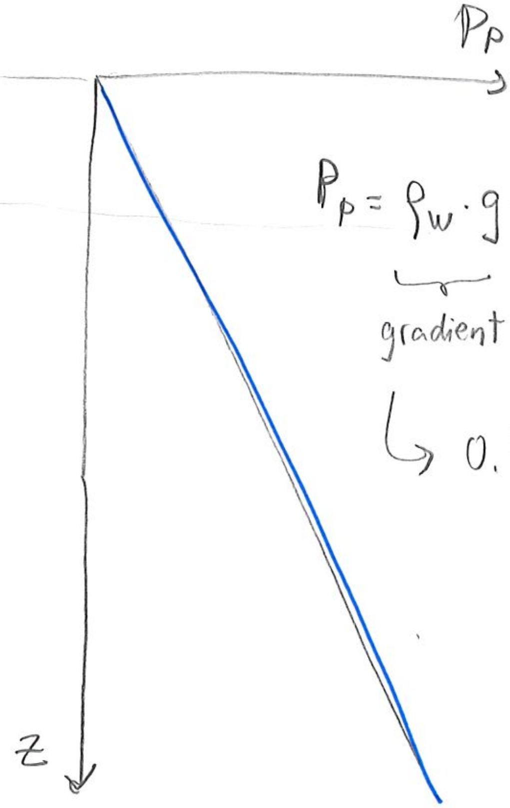
INTRODUCTION (1/23/2019)



- Exploration
  - Structural Geology
  - Seismic surveys
- Drilling & Completions
  - wellbore stability
  - fracturing
- Production
  - compaction
    ↳ rock compressibility
  - sand production
- Waste disposal
  - brine and CO_2
- well abandonment

Pore pressure (1/25/2019)





$$P_p = \underbrace{\rho_w \cdot g}_{\text{gradient}} \cdot \underbrace{z}_{\text{depth}}$$

$\hookrightarrow 0.44 \ \frac{psi}{ft}$

$$\rho_w \cdot g \cong 1000 \ \frac{kg}{m^3} \cdot 10 \ \frac{m}{s^2}$$

$$= 10^4 \ \frac{N}{m^2} \cdot \frac{1}{m}$$

$$= 10^4 \ Pa \ \frac{1}{10^{-3} \ km}$$

$$= 10 \ 10^6 Pa \ \frac{1}{km}$$

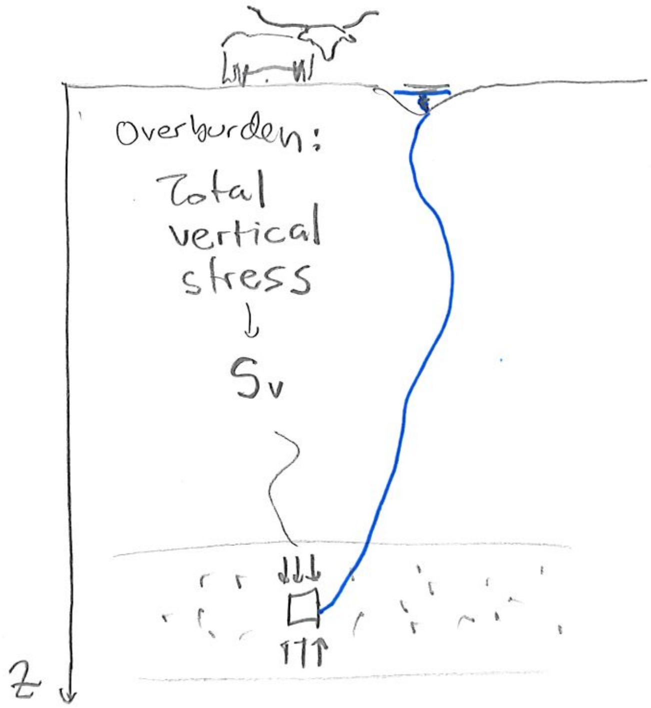
$$\boxed{\rho_w g \cong 10 \ \frac{MPa}{km}}$$

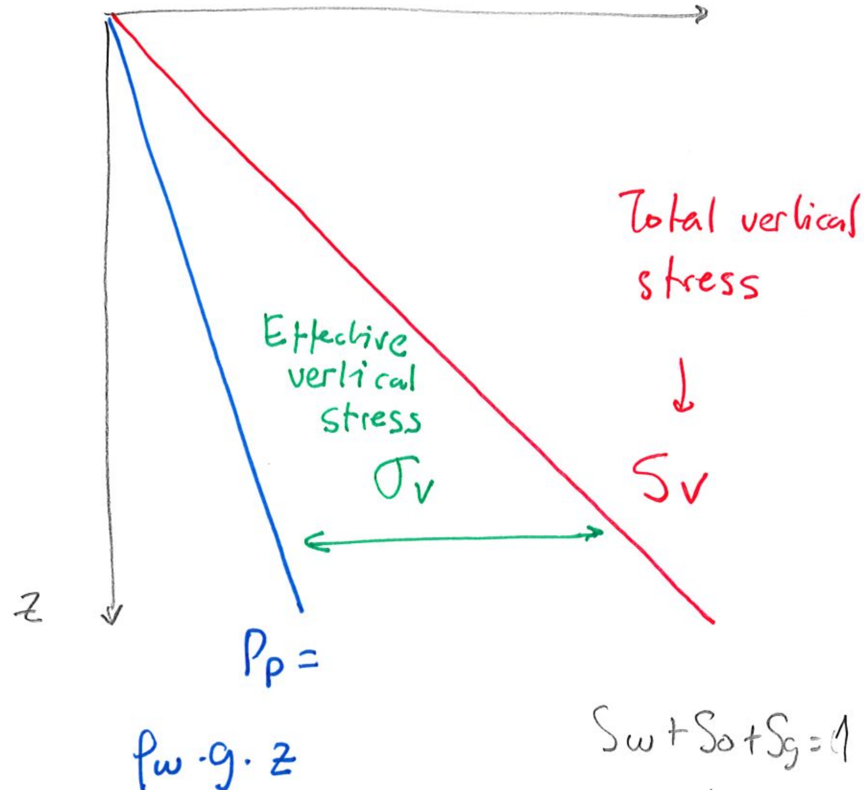
$$\rho_w = 62.4 \ \frac{lb}{ft^3}$$

$$\rho_w \cdot g = 62.4 \ \frac{\vec{lb}}{ft^3}$$

$$\rho_w \cdot g = 62.4 \ \frac{1}{(12in)^2} \ \frac{\vec{lb}}{ft}$$

$$\boxed{\rho_w \cdot g = 0.433 \ \frac{psi}{ft}}$$

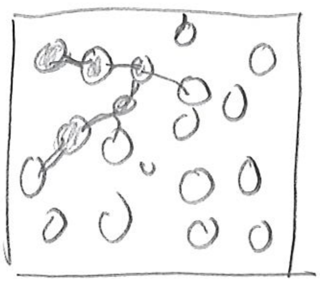




$$S_w + S_o + S_g = 1$$

$$\rho_F = \rho_w \cdot S_w + \rho_o \cdot S_o + \rho_g \cdot S_g$$

$$S_v = \rho_{bulk} \cdot g \cdot z$$



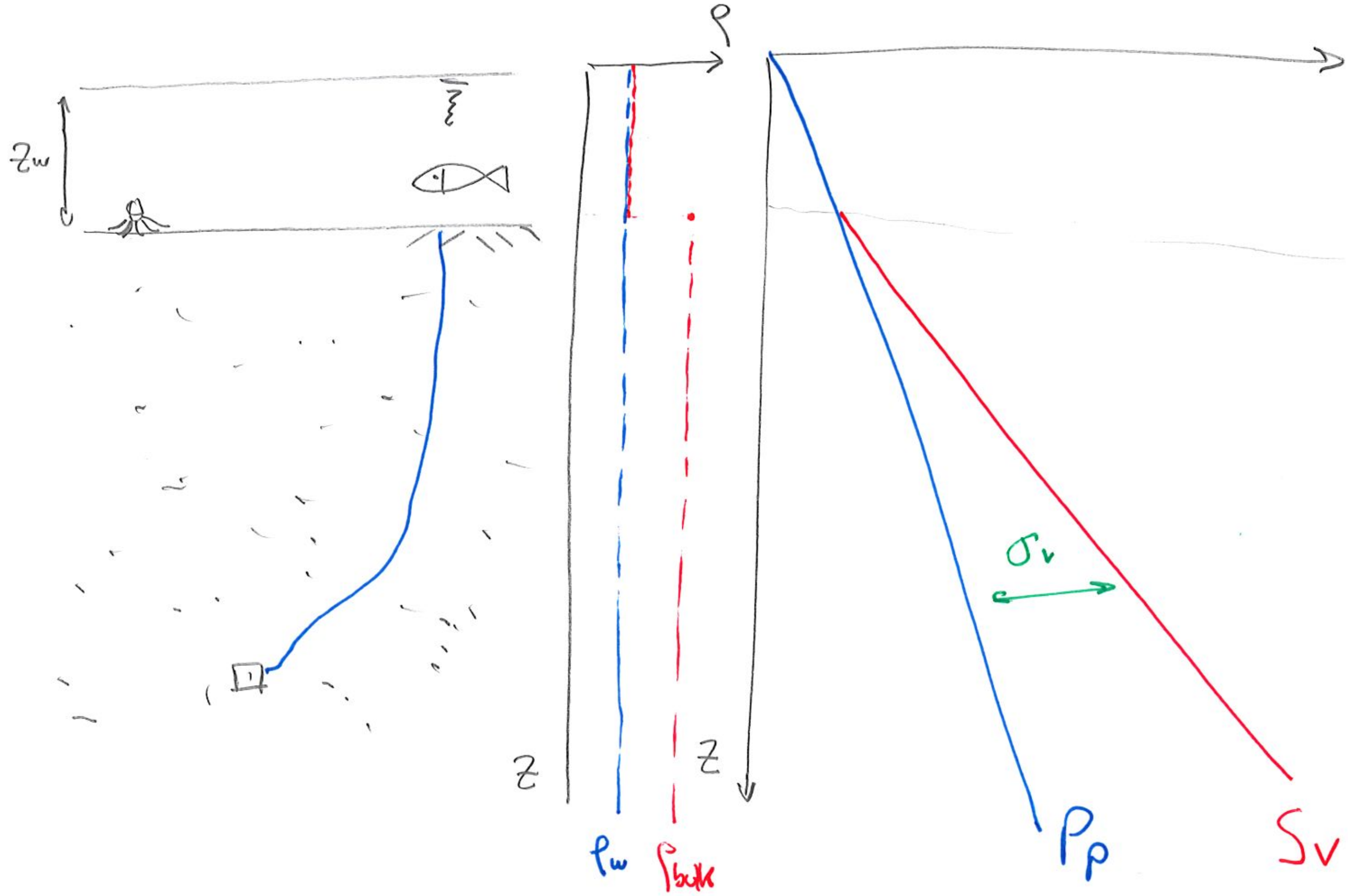
$$\rho_{bulk} = \rho_m (1-\varnothing) + \rho_F (\varnothing)$$

$$\rho_m = C_{m1} \rho_{m1} + C_{m2} \rho_{m2} + \dots + \rho_{mn} C_{mn}$$

$$\sum C_{mn} = 1$$

Sandstone
$$\left| \begin{array}{l} \varnothing = 0.20 \\ \rho_{quartz} = 2650 \ kg/m^3 \\ \rho_{brine} = 1000 \ kg/m^3 \end{array} \right\} \quad \rho_{bulk} = 2320 \ \frac{kg}{m^3}$$

$$\boxed{\frac{dS_v}{dz} = \rho_{bulk} \cdot g \cong 23 \ \frac{MPa}{km}}$$



2019/1/28

$$S_v = \underbrace{\rho_w g \cdot z_w} + \rho_{bulk} g (z - z_w)$$

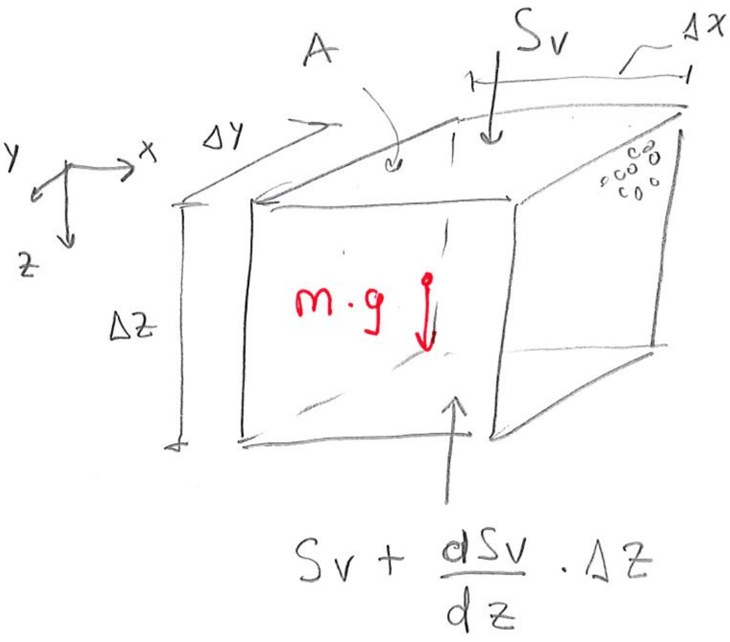
Pore pressure gradient (hydrostatic): $0.44 \frac{psi}{ft} = 10 \frac{MPa}{km}$

Total vertical stress gradient : $1 \frac{psi}{ft} = 23 \frac{MPa}{km}$

lithostatic

$\underbrace{\hphantom{1 \frac{psi}{ft}}}_{1000 \frac{kg}{m^3}}$ $\underbrace{\hphantom{23 \frac{MPa}{km}}}_{2320 \frac{kg}{m^3}}$

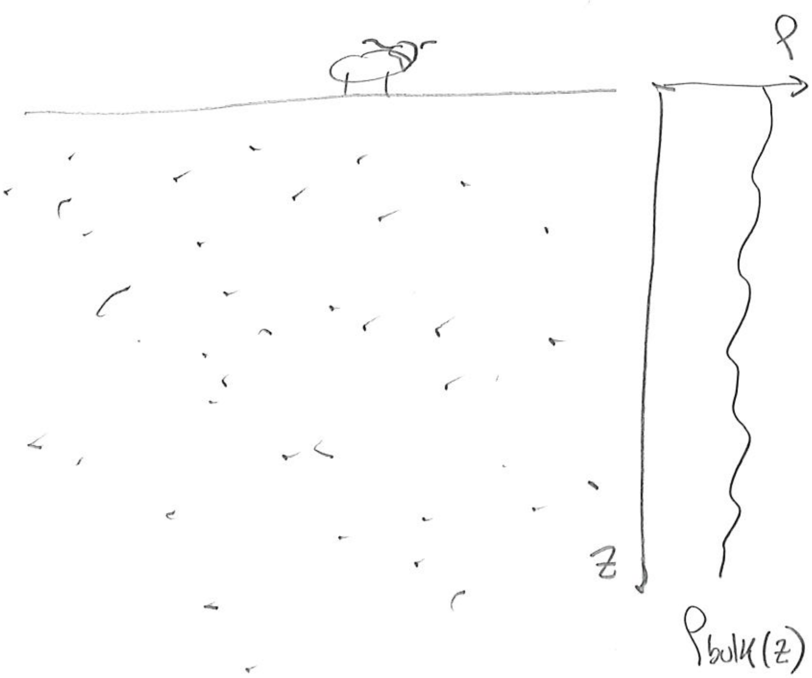
General solution for vertical stress



$$\Sigma F_z = S_v \cdot A + m \cdot g - \left(S_v + \frac{dS_v}{dz}\cdot \Delta z\right) A$$

$$= \underbrace{\cancel{S_v\,(\Delta x\,\Delta y)}} + \rho_{bulk}\,(\Delta x\,\Delta y\,\Delta z)\,g - \left(\cancel{S_v} + \frac{dS_v}{dz}\,\Delta z\right)\Delta x\,\Delta y = 0$$

$$\frac{dS_v}{dz}\,\cancel{\Delta z\,\Delta x\,\Delta y} = \rho_{bulk}\cdot g\,\cancel{\Delta x\,\Delta y\,\Delta z}$$



$$\boxed{\frac{dS_v}{dz} = \rho_{bulk}(z)\cdot g}$$

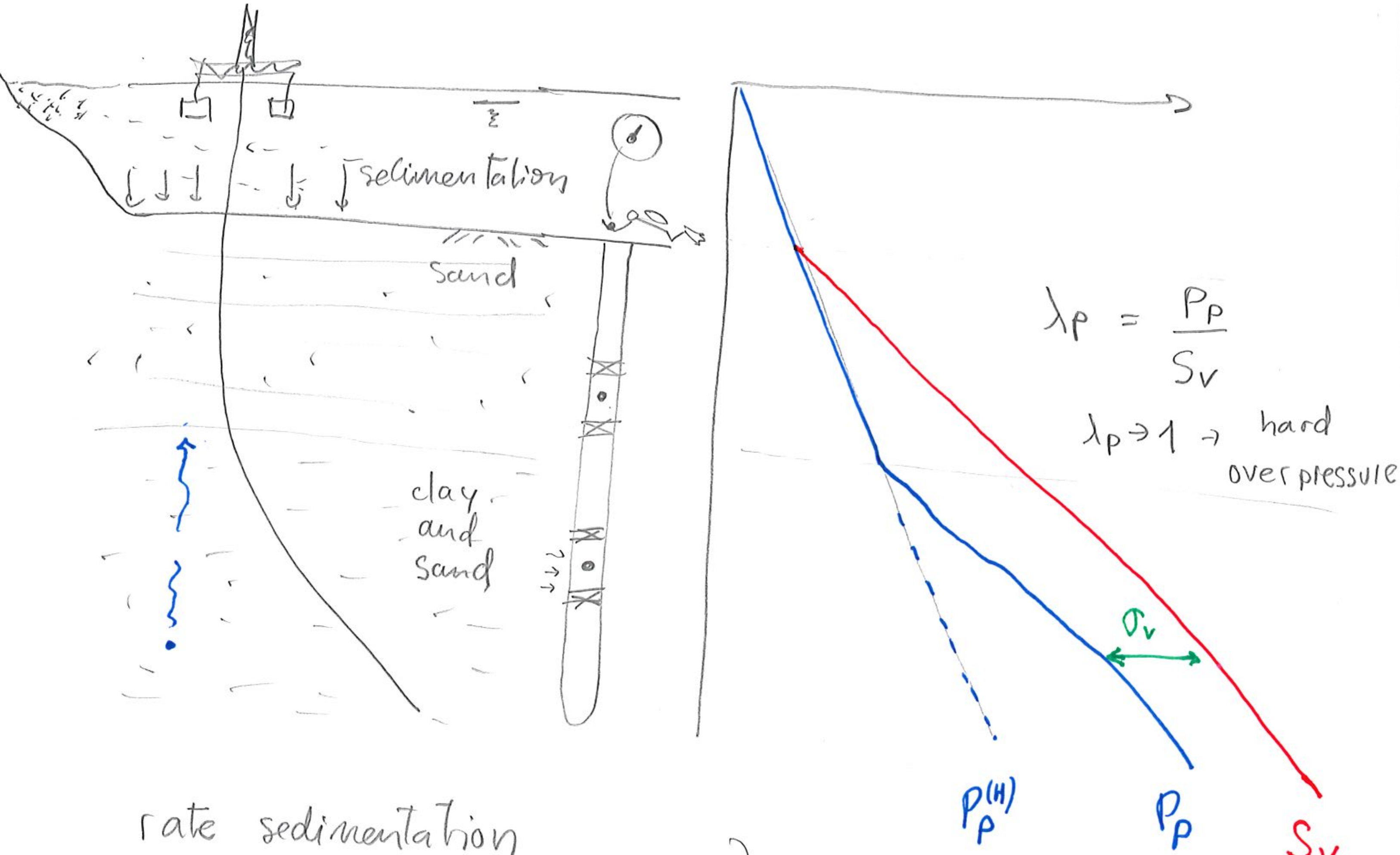
$$\int_{S_v(z=0)}^{S_v(z)} dS_v = \int_0^z \rho_{bulk}(z)\,g\,dz$$

$$S_v(z=0) = 0$$

$$\boxed{S_v(z) = \int_0^z \rho_{bulk}(z)\cdot g\cdot dz}$$

density log

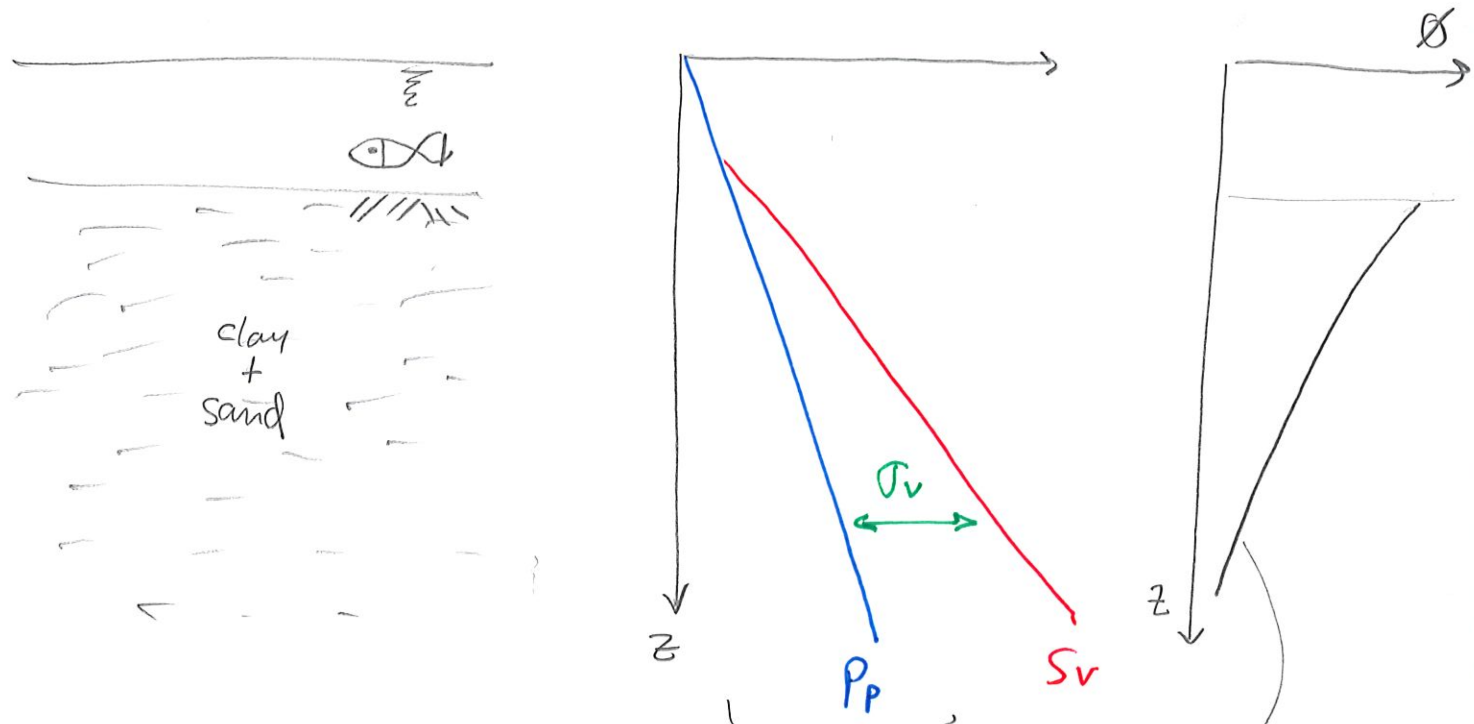
Non-hydrostatic pore pressure



rate sedimentation
$>$
rate of pore pressure diffusion $\Rightarrow$ disequilibrium compaction

$\to$ overpressure

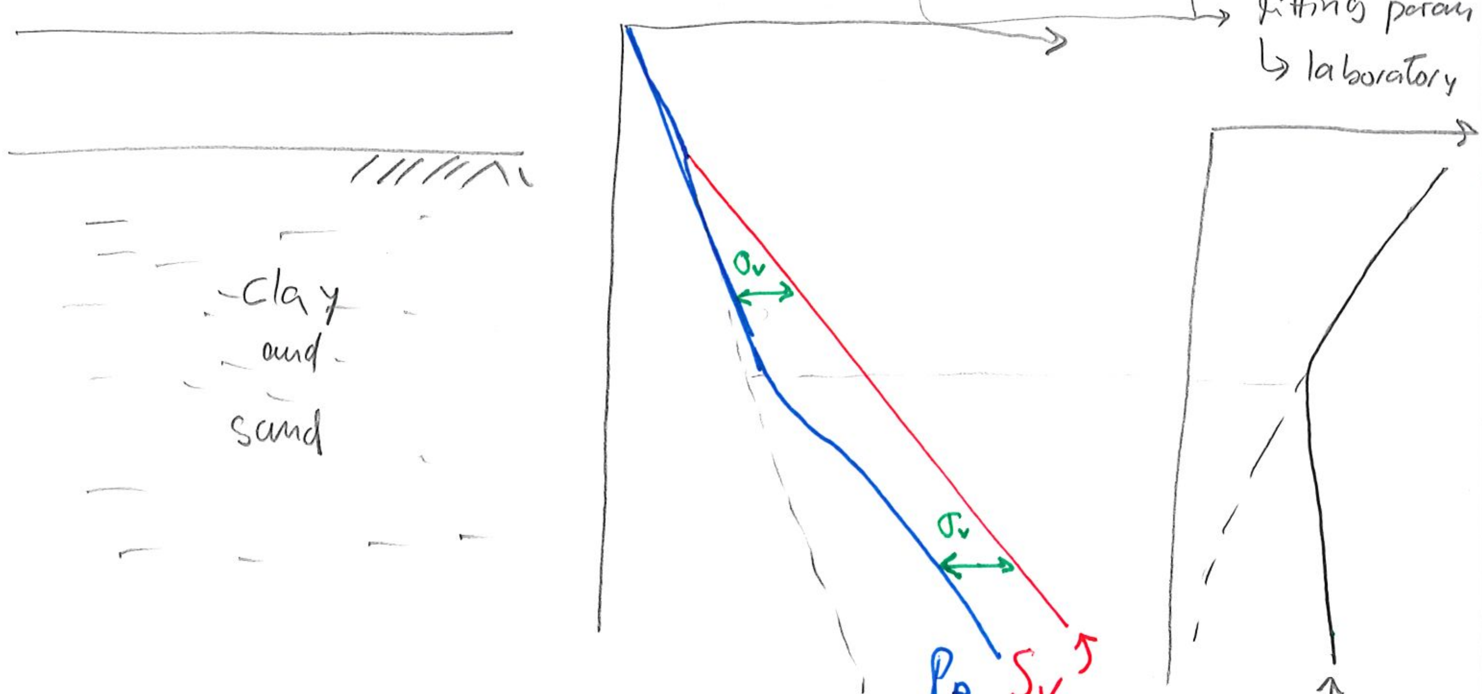
$\to$ underconsolidation



normal compaction

$$\boxed{\phi = \phi_0 \exp(-\beta \cdot \sigma_v)}$$

fitting param
↳ laboratory



disequilibrium compaction

DPHI
NPHI
RHO
DT

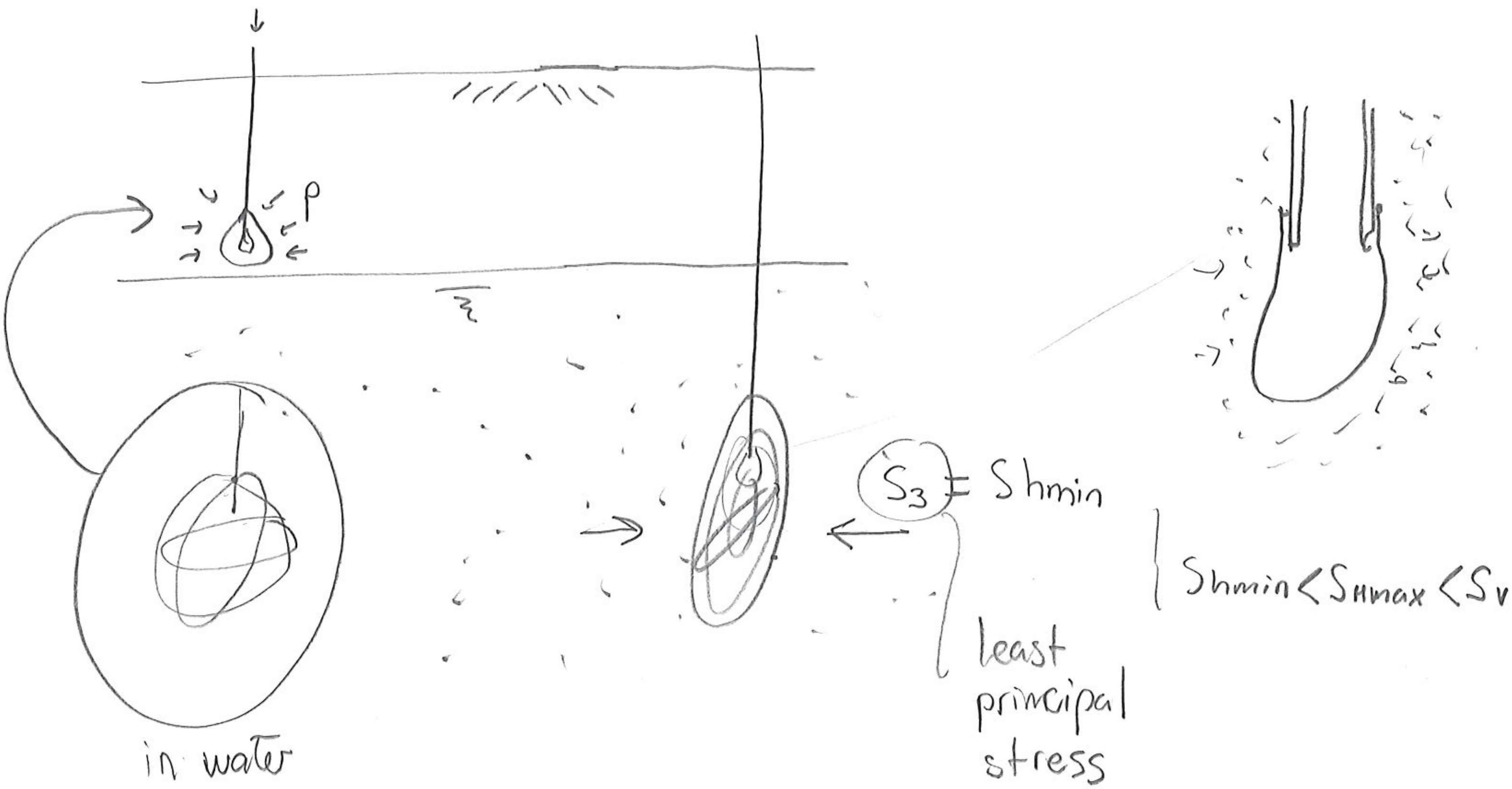
actual porosity

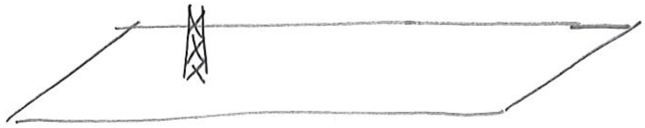
algebra ($\sigma_v = S_v - P_p$)

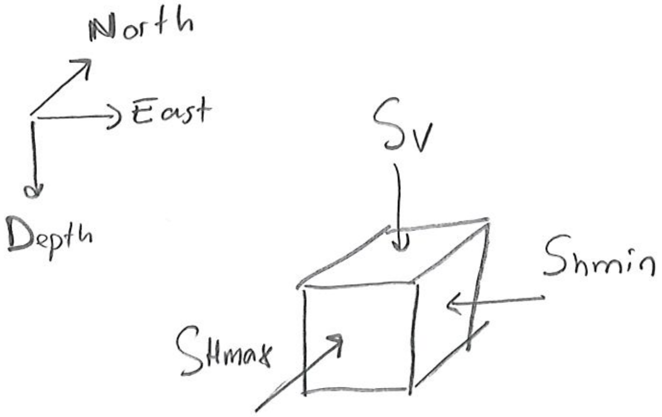
$$P_p = S_v + \underbrace{\frac{\ln(\phi/\phi_0)}{\beta}}_{<0}$$

WORKFLOW:
1) Calculate S_v
2) Determine σ_v from ϕ, ϕ_0, β
3) $P_p = S_v - \sigma_v$

Horizontal stresses







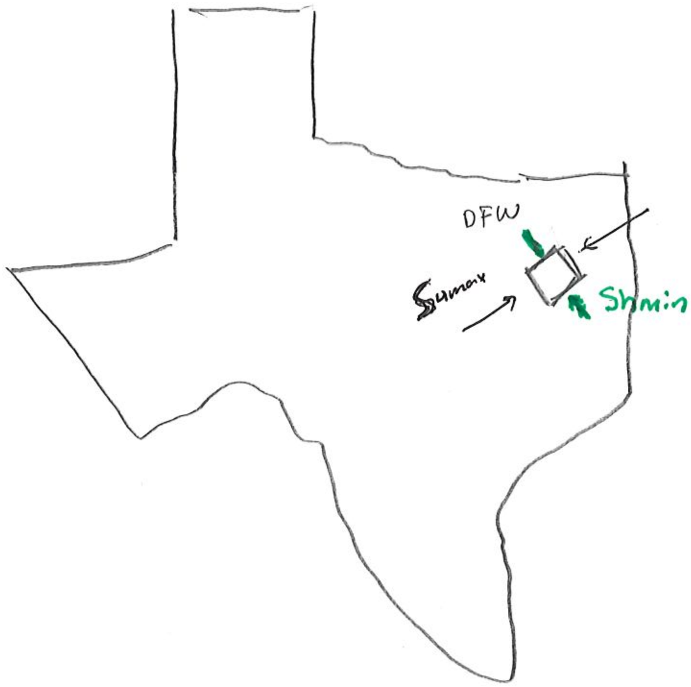
- 3 values
- 3 directions
- Full knowledge of stress state

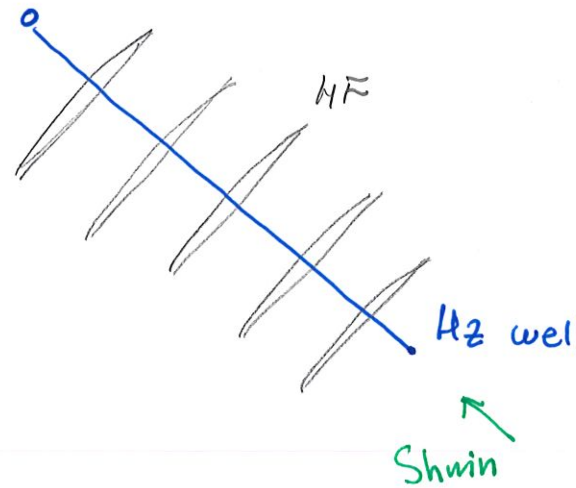
Stress regime	S_1 >	S_2 >	S_3
Normal Faulting (Extensional) • Permian, EF	Sv	SHmax	Shmin
Strike Slip • California	SHmax	Sv	Shmin
Reverse/Thrust Faulting • Argentina (Vaca Muerta) • Austalia (some depths)	SHmax	Shmin	Sv

Barnett shale

Normal Faulting

$S_v > \underbrace{S_{Hmax} > S_{hmin}}_{?}$





2/1/2019

wellbore failure

S_v | S_{Hmax} | S_{hmin}

$$\int_0^z \rho_{bulk} \cdot g \cdot dz$$

S_{hmin}

↓

if S_3

↓

conduct hydraulic fracture test

HW2

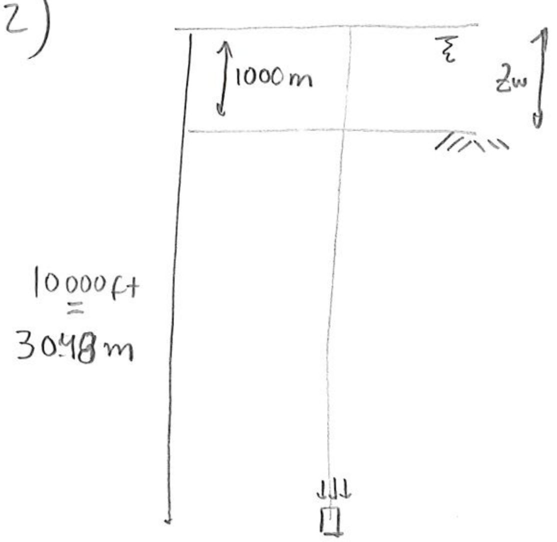
1) $1\ g/cm^3 = 1\ g/cc = 1000\ Kg/m^3$

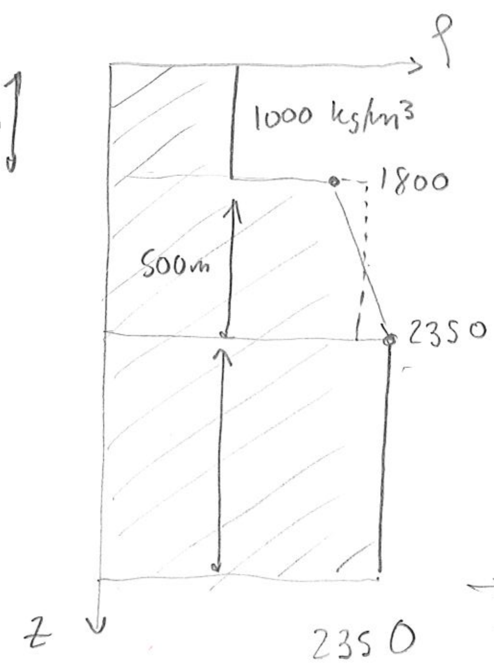
$\rho_{bulk} =$ ______

$\rho_{bulk} \cdot g = \begin{cases} \sim 26\ MPa/Km \\ \sim 1.15\ psi/ft \\ \sim 22\ ppg \end{cases}$

$0.44\ \frac{psi}{ft} = 8.3\ ppg$

2)





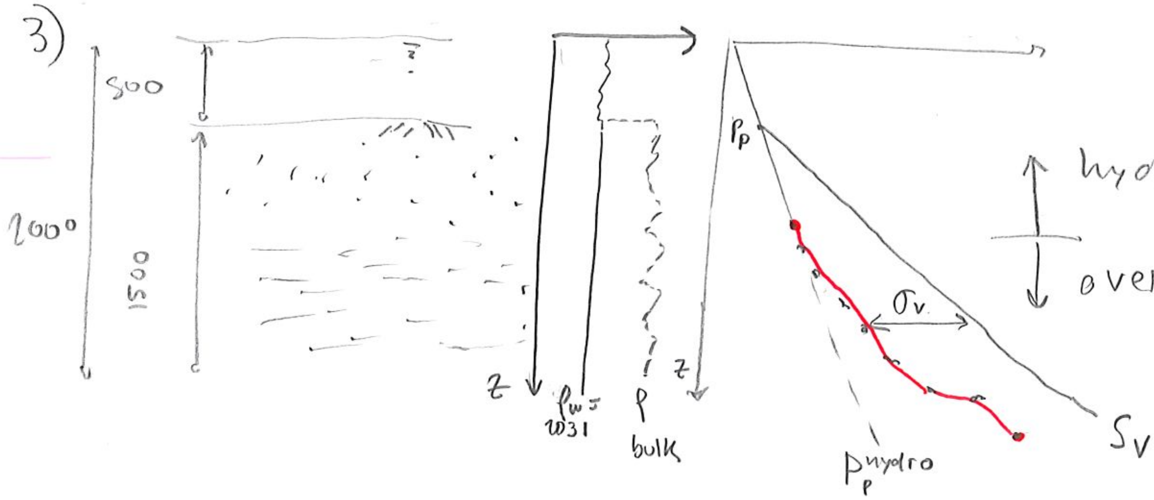
$3000\ psi = 20.6\ MPa$

$1\ MPa = 145\ psi$

$\rightarrow S_v = \int_0^z \rho_{bulk} \cdot g \cdot dz$

$S_v \cong 56\ MPa$

3)



hydrostatic

overpressure

$$\lambda_p = \frac{P_p}{S_v} \ll 1$$

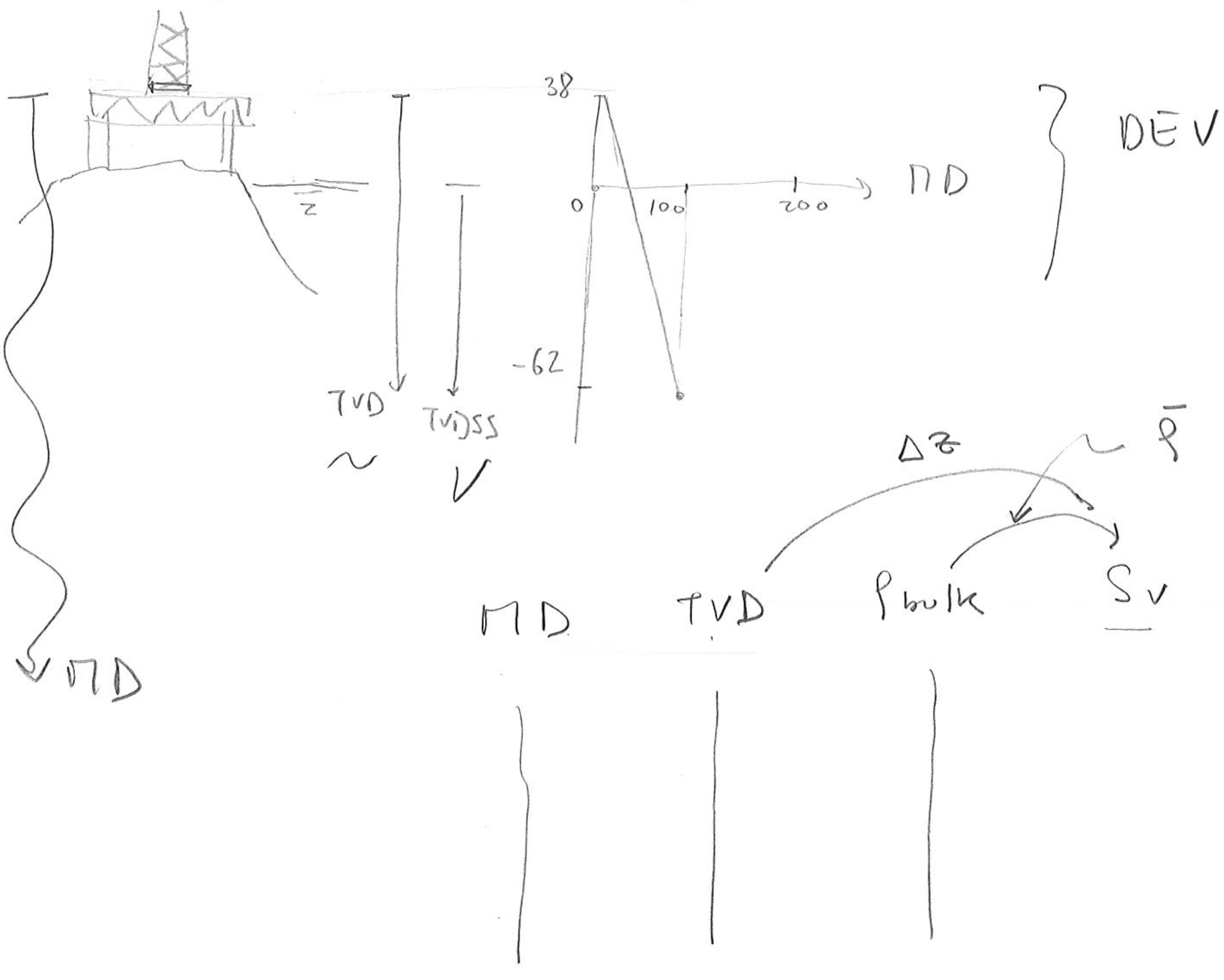
1) Hypothesis: hydrostatic

2) Shale porosity → actual pore pressure

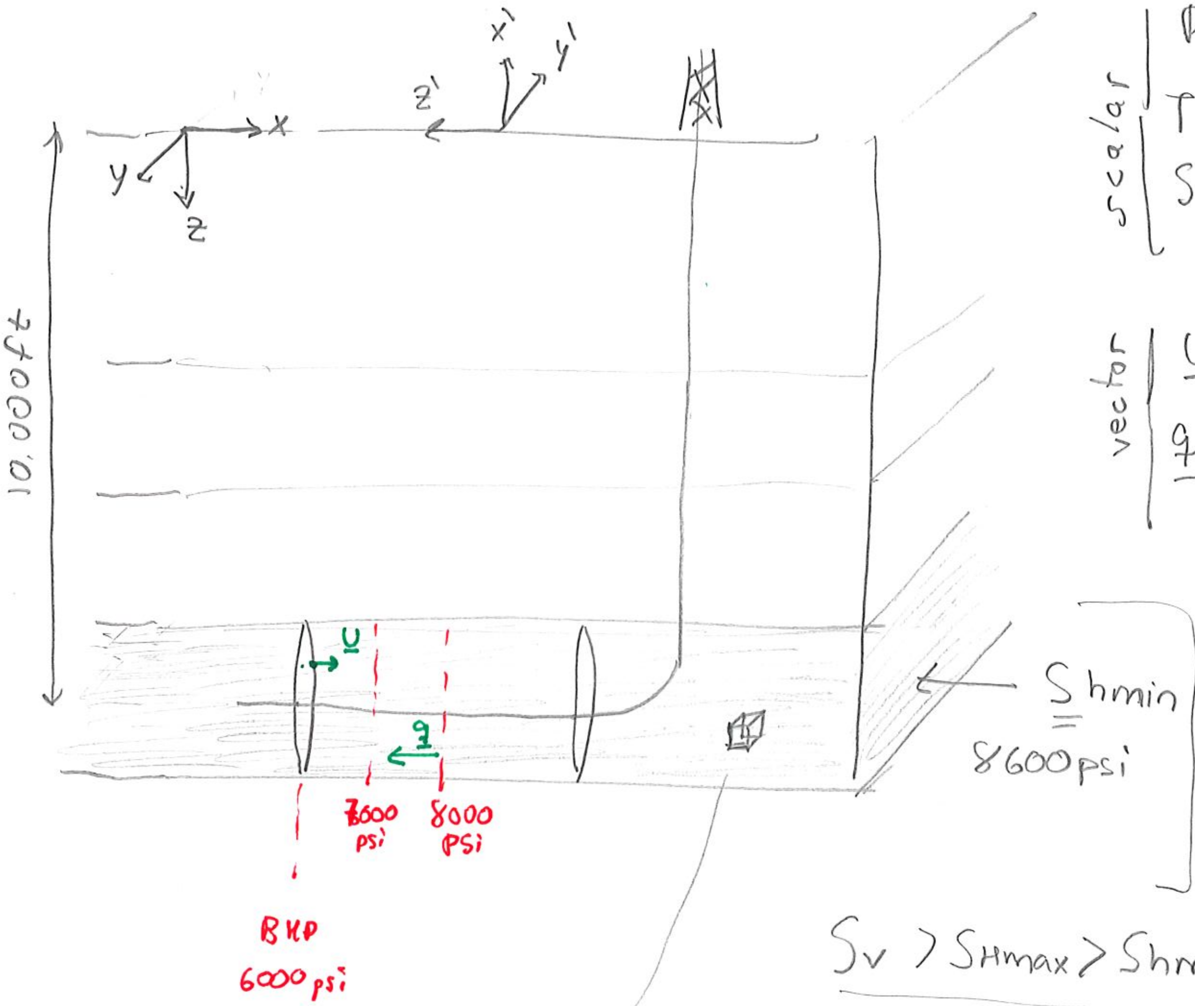
4) Plaquemines Parish, LA

.LAS | EKB (bushing) 36 ft

data 1864 - 7551 ft



$\boxed{2/4/2019}$



scalar

$P = 8000\ psi$

$T = 200°F$

$S_o = 0.7$

vector

$\underline{U} = (2mm, 0mm, 0mm)$

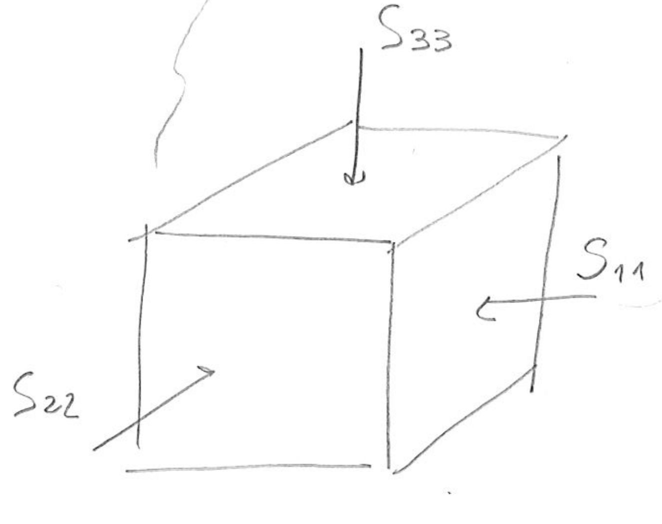
$\underline{q} = \left(1\frac{ft}{day}, 0\frac{ft}{day}, 0\frac{ft}{day}\right)$

tensor

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$S_v > S_{Hmax} > S_{hmin}$

$$\underline{\underline{S}} = \begin{bmatrix} S_{hmin} & 0 & 0 \\ 0 & S_{Hmax} & 0 \\ 0 & 0 & S_v \end{bmatrix}$$



$$\underline{\underline{S}} = \underbrace{\begin{bmatrix} 8600 & 0 & 0 \\ 0 & 9000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}}_{\text{principal stresses} \equiv \text{eigen values}} psi$$

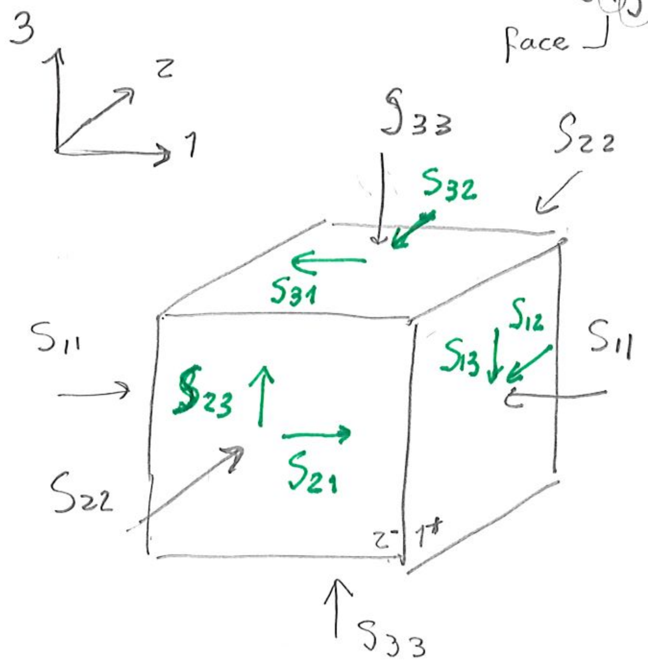
$1 \rightarrow x$

$2 \rightarrow y$

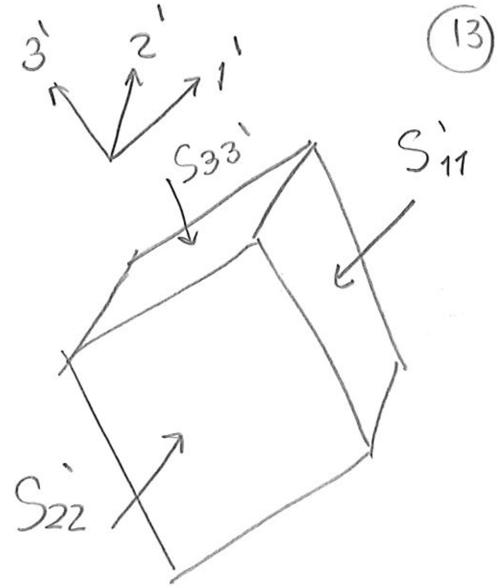
$3 \rightarrow z$

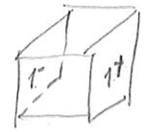
$S_{(i)(j)}$ – direction

face



principal stresses & directions $\Longrightarrow$





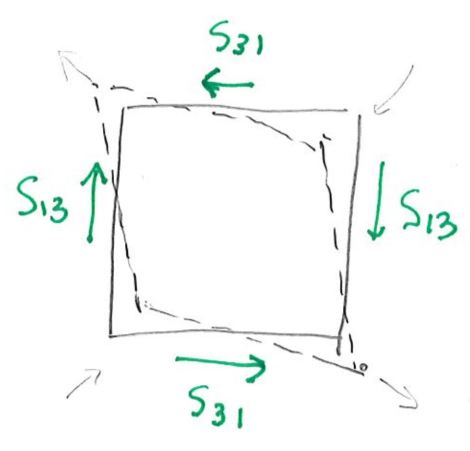
normal stress positive → compression

all shear stresses are zero

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \xrightarrow{\text{eig}} \underline{\underline{S}}' = \begin{bmatrix} S'_{11} & 0 & 0 \\ 0 & S'_{22} & 0 \\ 0 & 0 & S'_{33} \end{bmatrix}$$

symmetric

$$\underline{\underline{S}}' = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$



$S_1 \geq S_2 \geq S_3$

Mohr circle

Eigenvalues → eig(_)

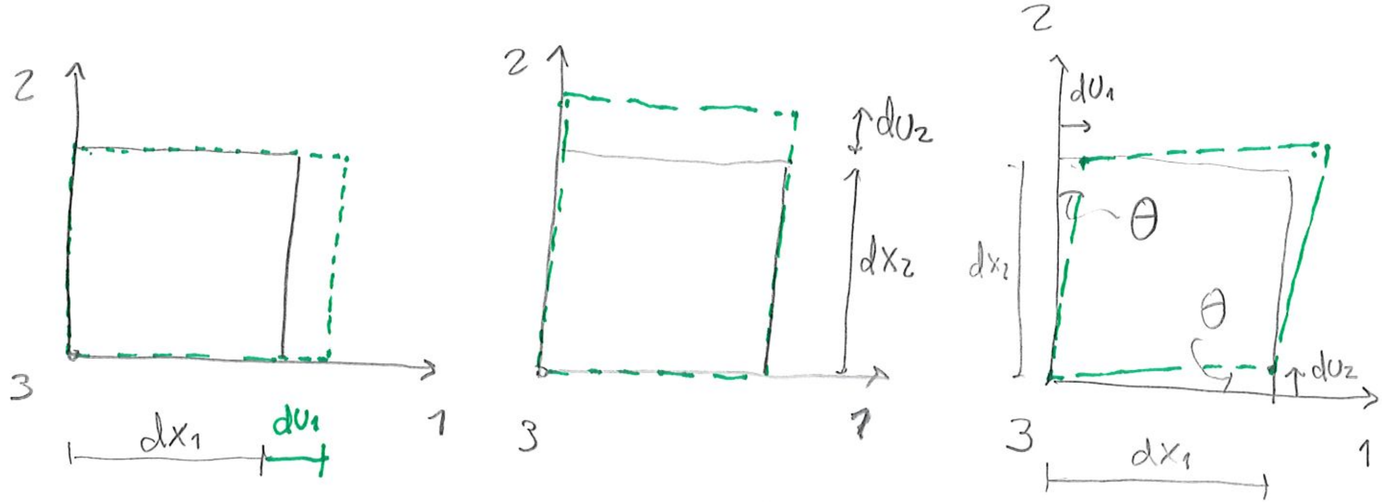
$S_{31} = S_{13}$ → angular momentum Equilibrium

$$\underline{\underline{\sigma}} = \underline{\underline{S}} - P_p \, \underline{\underline{I}}$$

$$\overbrace{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{bmatrix}}^{\text{Effective stress}} = \overbrace{\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ & S_{22} & S_{23} \\ & & S_{33} \end{bmatrix}}^{\text{Total stress}} - \overbrace{\begin{bmatrix} P_p & 0 & 0 \\ & P_p & 0 \\ & & P_p \end{bmatrix}}^{\text{Pore pressure}}$$

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \underbrace{\begin{bmatrix} S_{11}-P_p & S_{12} & S_{13} \\ S_{21} & S_{22}-P_p & S_{23} \\ S_{31} & S_{32} & S_{33}-P_p \end{bmatrix}}_{\text{Effective stress}}$$

Strains (deformation)



$$\varepsilon_{11} = \frac{dU_1}{dX_1} \quad ; \quad \varepsilon_{22} = \frac{dU_2}{dX_2} \quad ; \quad \varepsilon_{12} = \frac{1}{2}\left(\frac{dU_1}{dX_2} + \frac{dU_2}{dX_1}\right)$$

$$\underbrace{\varepsilon_{33} = \frac{dU_3}{dX_3}}_{\text{linear strains}}$$

linear strains ↳ ΔVol

$$\underbrace{\tan\theta = \frac{dU_1}{dX_2} = \frac{dU_2}{dX_1}}_{\text{shear strain}}$$

shear strain

↓

no volume change

strain tensor

↓

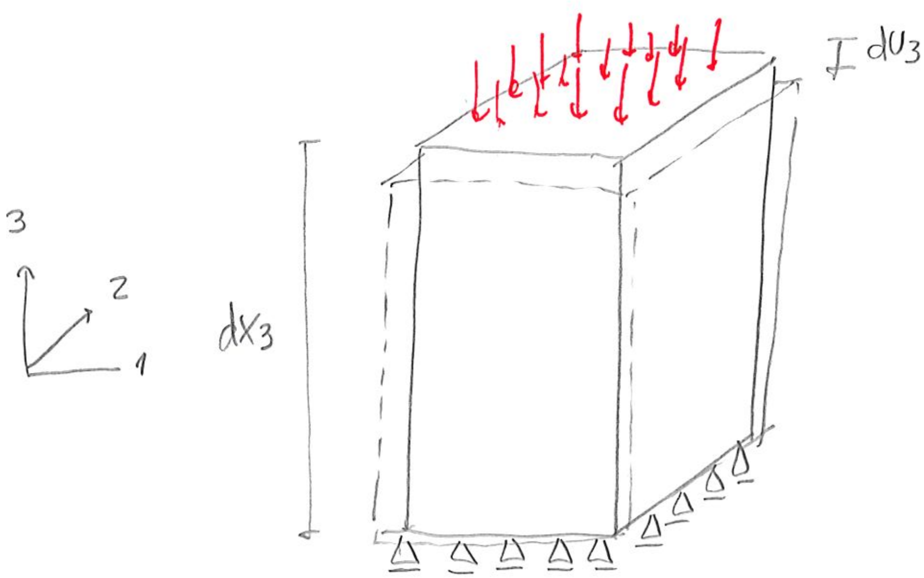
$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \frac{dU_1}{dX_1} & \frac{1}{2}\left(\frac{dU_1}{dX_2} + \frac{dU_2}{dX_1}\right) & \frac{1}{2}\left(\frac{dU_1}{dX_3} + \frac{dU_3}{dX_1}\right) \\ & \frac{dU_2}{dX_2} & \frac{1}{2}\left(\frac{dU_2}{dX_3} + \frac{dU_3}{dX_2}\right) \\ & & \frac{dU_3}{dX_3} \end{bmatrix}$$

$$\underline{\underline{\sigma}} = f(\underline{\underline{\varepsilon}})$$

$$\underline{\underline{\varepsilon}} = f(\underline{\underline{\sigma}})$$

?

Linear Elasticity



Unconfined Loading → in one direction

$$\varepsilon_{33} = \frac{dU_3}{dX_3}$$

$$\boxed{E = \frac{\sigma_{33}}{\varepsilon_{33}}}$$ Young's Modulus

$$\sigma_{33} = E \cdot \varepsilon_{33}$$

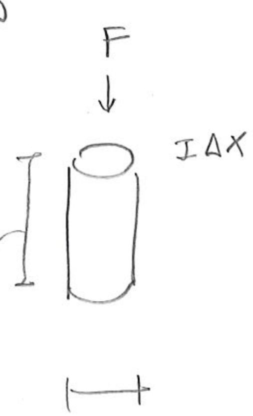
$$\varepsilon_{33} = \frac{\sigma_{33}}{E}$$

$$\boxed{\nu = -\frac{\varepsilon_{11}}{\varepsilon_{33}} = -\frac{\varepsilon_{22}}{\varepsilon_{33}}}$$

Poisson's ratio

Class Exp

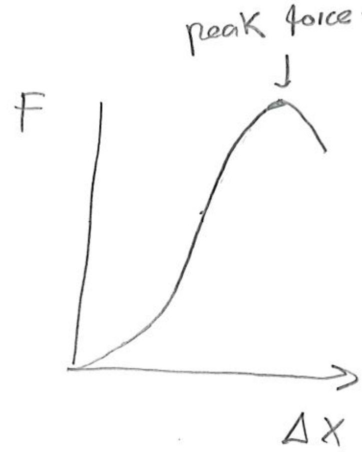
Berea Sandstone



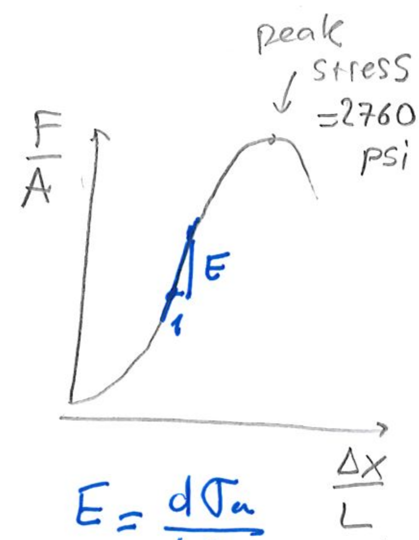
$L = 0.8$ in

$D = 0.38$ in

$A = 0.11 \text{ in}^2$



⇒



1 turn = 0.15 mm = 0.006 in

$$E = \frac{d\sigma_a}{d\varepsilon_a}$$

$E = 94 \cdot 10^3$ psi $\rightarrow$ H

$262 \cdot 10^3$ psi $\rightarrow$ K, M ✓

$8.9 \cdot 10^4$ MPa $\rightarrow$ B

$\Delta F =$ 110 lb
↓ /A
$\Delta\sigma =$ 1000psi

10 kg	0 turn
50 kg	0.75
100 kg	1.25
138 kg	broke

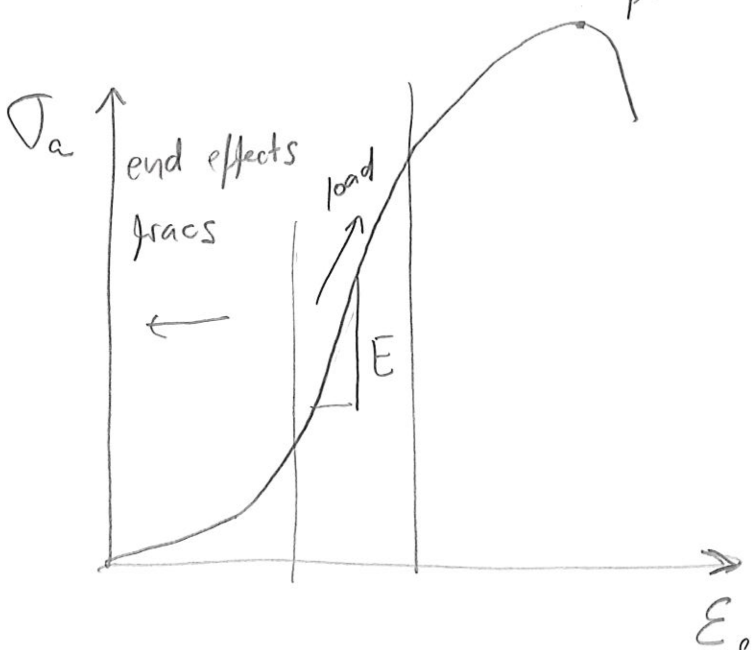
0.003/0.8

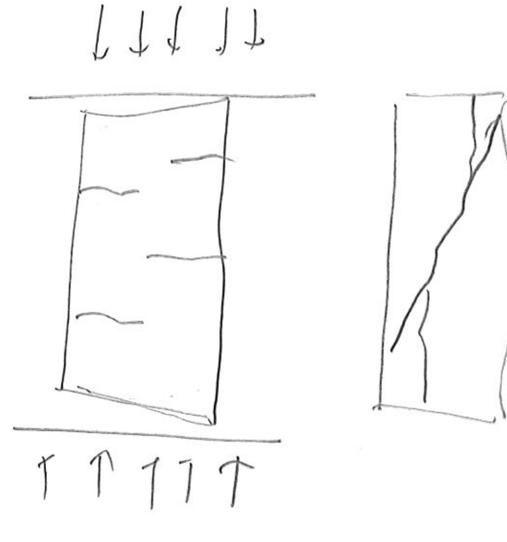
$\Delta X = 0.003$ in $\Rightarrow \varepsilon_a = 3.75 \cdot 10^{-3}$

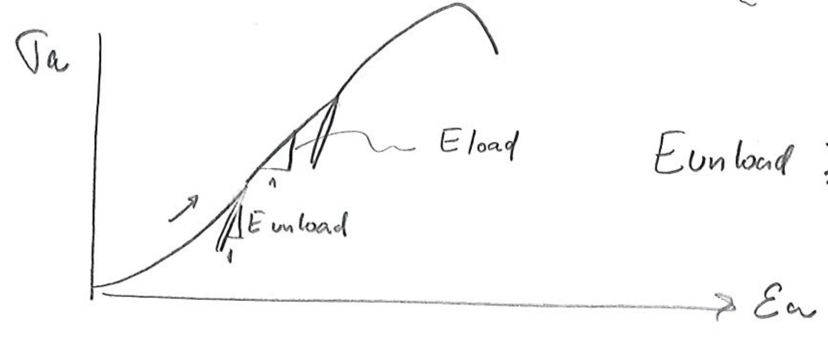
$$\boxed{E = 266 \cdot 10^3 \text{ psi}}$$

$$\boxed{E = 1.84 \cdot 10^9 \text{ Pa}}$$

peak stress + failure

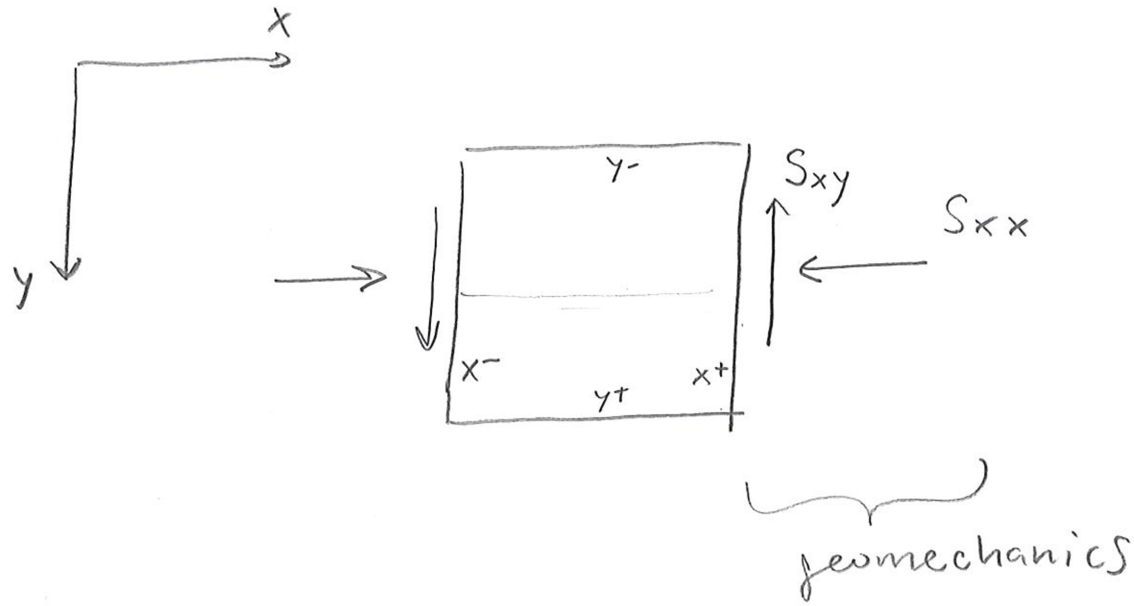






Eunload $\geq$ Eload

HW3



(2019 / 2/ 11)

$$\begin{cases} \varepsilon_{11} = \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} & \leftarrow \varepsilon_{11} = -\nu\,\varepsilon_{33} \\ \varepsilon_{22} = -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ \varepsilon_{33} = -\frac{\nu}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \end{cases}$$

$$\begin{cases} 2\cdot\varepsilon_{12} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{12} \\ 2\cdot\varepsilon_{13} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{13} \\ 2\cdot\varepsilon_{23} = \emptyset + \emptyset + \emptyset\,\sigma_{33} + \frac{1}{G}\sigma_{23} \end{cases}$$

G: shear modulus, $G = \frac{E}{2(1+\nu)}$

$$\underbrace{\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}_{6\times1}}_{\text{VOIGT NOTATION}} = \underbrace{\left[\begin{array}{ccc|ccc} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{array}\right]_{6\times6}}_{\text{compliance matrix}} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}_{6\times1}$$

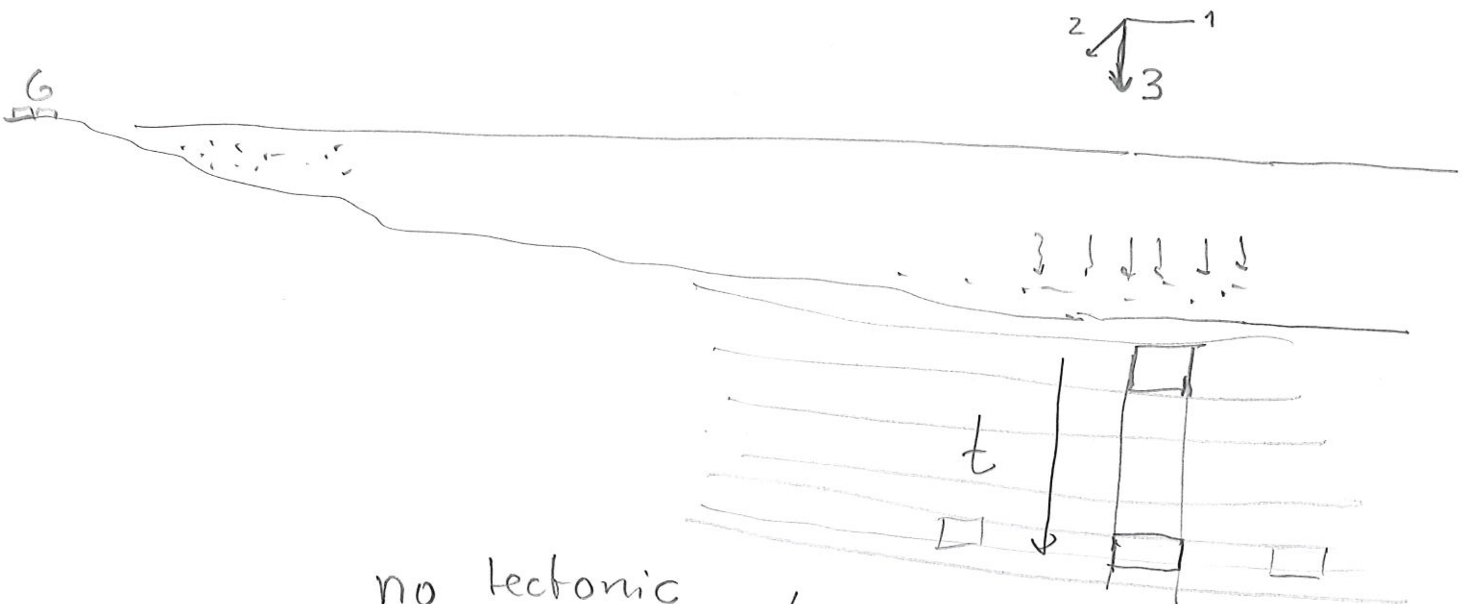
$$\underline{\underline{\varepsilon}} = \underline{\underline{\underline{\underline{D}}}} \cdot \underline{\underline{\sigma}} \implies \underset{\text{vector}}{\underline{\varepsilon}} = \underset{\text{matrix}}{\underline{D}} \cdot \underset{\text{vector}}{\underline{\sigma}}$$

$$\underline{\underline{\sigma}} = \underbrace{\underline{\underline{C}}}_{\text{stiffness matrix}} \; \underline{\underline{\varepsilon}}$$

$$\underline{\underline{\sigma}} = \underline{\underline{D}}^{-1} \; \underline{\underline{\varepsilon}}$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \hline \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \left[\begin{array}{ccc|ccc} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{array}\right] \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \hline 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{bmatrix}$$

Uniaxial-strain stress path

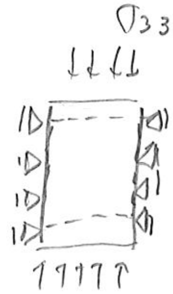


no tectonic strains

$$\left\{ \begin{array}{l} \varepsilon_{11} = \varepsilon_{22} = 0 \qquad \varepsilon_{ij} = 0 \quad i \neq j \\ \varepsilon_{33} \neq 0 \end{array} \right.$$

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu \\ \nu & 1-\nu & \nu \\ \nu & \nu & 1-\nu \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \varepsilon_{33} \end{bmatrix}$$

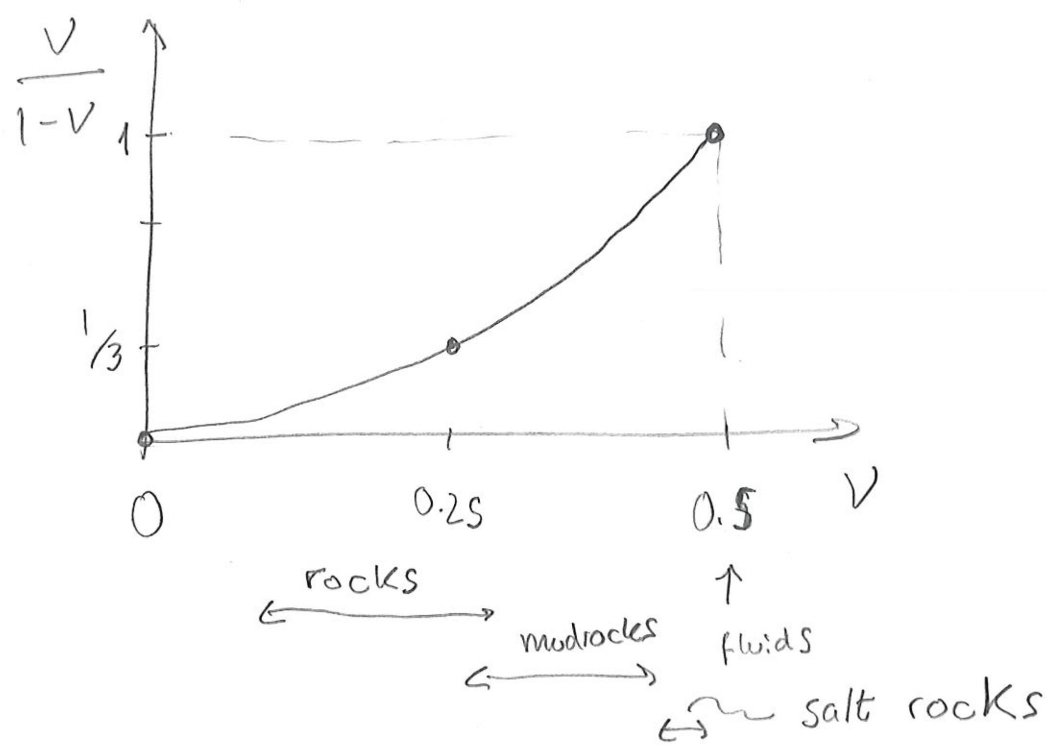
$$\begin{cases} \sigma_{33} = \overbrace{\boxed{\dfrac{(1-\nu)E}{(1+\nu)(1-2\nu)}}}^{M:\ \text{constrained modulus}} \cdot \varepsilon_{33} \\[2ex] \sigma_{11} = \sigma_{22} = \dfrac{\nu E}{(1+\nu)(1-2\nu)} \varepsilon_{33} = \dfrac{\nu \cancel{E}}{\cancel{(1+\nu)(1-2\nu)}} \dfrac{\cancel{(1+\nu)(1-2\nu)}}{(1-\nu)\cancel{E}} \sigma_{33} \end{cases}$$



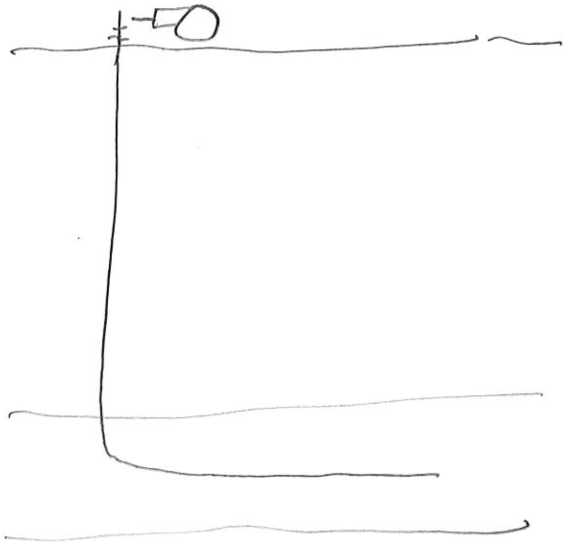
$M > E$

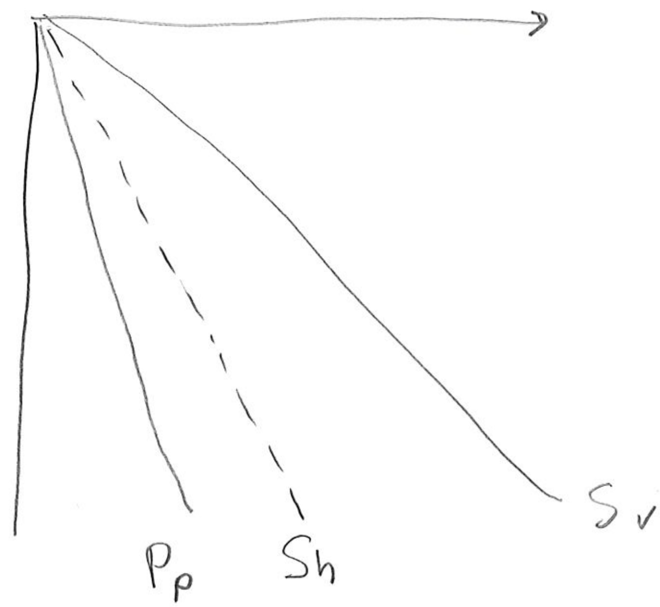
$$\boxed{\sigma_{22} = \underbrace{\frac{\nu}{1-\nu}}_{\text{lateral effective stress coefficient}} \sigma_{33}}$$

$\rightarrow$ valid for effective stresses



$S_v \geq S_{Hmax} > S_{hmin}$





$$S_{hmin} = S_{Hmax} = S_h$$

$$\underbrace{S_h}_{\text{Total}} = \underbrace{\sigma_h}_{\text{effective}} + P_p$$

$$S_h = \frac{\nu}{1-\nu}\sigma_v + P_p$$

$$S_h = \frac{\nu}{1-\nu}\left(S_v - P_p\right) + P_p$$

absolute values

$$S_h = \frac{\nu}{1-\nu} S_v + \frac{1-2\nu}{1-\nu} P_p$$

gradient

$$\underbrace{\frac{\Delta S_h}{\Delta z}}_{\text{frac gradient}} = \frac{\nu}{1-\nu} \underbrace{\frac{\Delta S_v}{\Delta z}}_{\text{lithostatic gradient}} + \frac{1-2\nu}{1-\nu} \underbrace{\frac{\Delta P_p}{\Delta z}}_{\text{pore pressure gradient}}$$