Homework 5 Solutions

Problem 1: Kirsch Solution

Using equations of stresses around a cylindrical cavity, calculate near-wellbore effective radial σ_{rr} and hoop $\sigma_{\theta\theta}$ stresses for a vertical well 8in diameter in the directions of S_{hmin} (4500 psi – acting E-W) and S_{Hmax} (6000 psi) up to 3ft of distance. The result should be presented as plots of stresses (σ_{rr}, $\sigma_{\theta\theta}$) as a function of distance from the center of the wellbore.

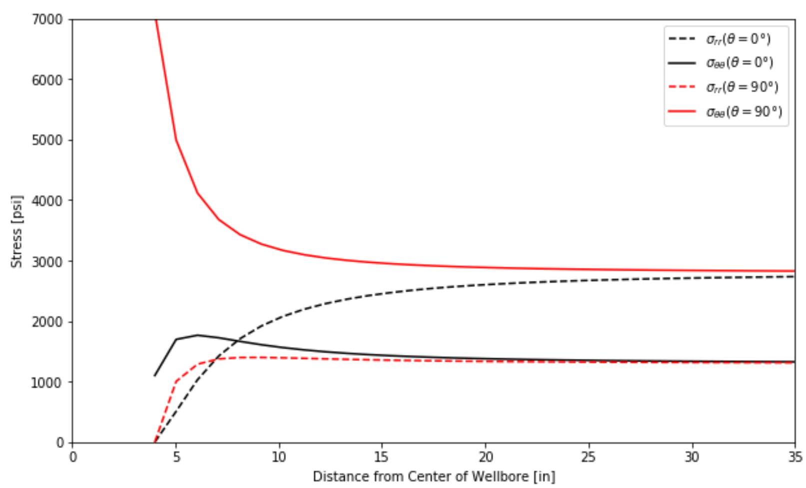
(a) $P_p = 3200psi$ **and** $P_w = 3200psi$

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

# Given parameters
Pp = 3200 # Pore pressure [psi]
Pw = 3200 # Wellbore pressure [psi]
a = 4 # Wellbore radius, a [in]
S_hmin = 4500 # Minimum total horizontal stress [psi]
S_Hmax = 6000 # Maximum total horizontal stress [psi]
sigma_hmin = S_hmin - Pp # Minimum effective horizontal stress [psi]
sigma_Hmax = S_Hmax - Pp # Maximum effective horizontal stress [psi]

# Calculate the radial stress, σrr and the tangential (hoop) stress, σϑϑ based on r and theta
r = np.linspace(4, 35, 31) # distance measured from the center of the wellbore [in]
theta = 0 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_0 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**4
sigma_thetatheta_0 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos(
theta = 90 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_90 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**
sigma_thetatheta_90 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos

# Plotting
plt.plot(r, sigma_rr_0, '--k', label=r'$\sigma_{rr} (\theta=0°)$',)
plt.plot(r, sigma_thetatheta_0, '-k', label=r'$\sigma_{\theta\theta} (\theta=0°)$',)
plt.plot(r, sigma_rr_90, '--r', label=r'$\sigma_{rr} (\theta=90°)$',)
plt.plot(r, sigma_thetatheta_90, '-r', label=r'$\sigma_{\theta\theta} (\theta=90°)$',)
# Plot labels
plt.xlabel('Distance from Center of Wellbore [in]')
plt.ylabel('Stress [psi]')
plt.legend()
# Axis range
plt.xlim([0, 35])
plt.ylim([0, 7000])
# Change plot size
fig = plt.gcf()
fig.set_size_inches(10, 6)
```



(b) $P_p = 3200psi$ **and** $P_w = 4000psi$

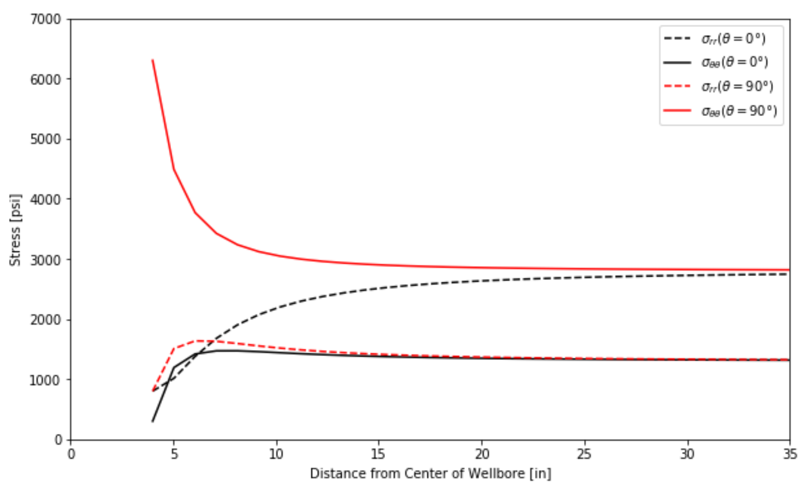
In [2]:

```
import numpy as np
import matplotlib.pyplot as plt

# Given parameters
Pp = 3200 # Pore pressure [psi]
Pw = 4000 # Wellbore pressure [psi]
a = 4 # Wellbore radius, a [in]
S_hmin = 4500 # Minimum total horizontal stress [psi]
S_Hmax = 6000 # Maximum total horizontal stress [psi]
sigma_hmin = S_hmin - Pp # Minimum effective horizontal stress [psi]
sigma_Hmax = S_Hmax - Pp # Maximum effective horizontal stress [psi]

# Calculate the radial stress, σrr and the tangential (hoop) stress, σϑϑ based on r and theta
r = np.linspace(4, 35, 31) # distance measured from the center of the wellbore [in]
theta = 0 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_0 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**4
sigma_thetatheta_0 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos(
theta = 90 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_90 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**
sigma_thetatheta_90 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos

# Plotting
plt.plot(r, sigma_rr_0, '--k', label=r'$\sigma_{rr} (\theta=0°)$',)
plt.plot(r, sigma_thetatheta_0, '-k', label=r'$\sigma_{\theta\theta} (\theta=0°)$',)
plt.plot(r, sigma_rr_90, '--r', label=r'$\sigma_{rr} (\theta=90°)$',)
plt.plot(r, sigma_thetatheta_90, '-r', label=r'$\sigma_{\theta\theta} (\theta=90°)$',)
# Plot labels
plt.xlabel('Distance from Center of Wellbore [in]')
plt.ylabel('Stress [psi]')
plt.legend()
# Axis range
plt.xlim([0, 35])
plt.ylim([0, 7000])
# Change plot size
fig = plt.gcf()
fig.set_size_inches(10, 6)
```



Problem 2: Effect of Overpressure

Consider the problem solved in class (Wellbore: vertical; Site: onshore, 7000 ft of depth, $S_{hmin} = 4300psi$, $S_{Hmax} = 6300psi$; Rock properties: $UCS = 3,500psi$, $\mu = 0.6$, $T_s = 800psi$).

(a) Calculate wellbore pressure and corresponding mud weight for (i) $w_{BO} = 70°$, (ii) $w_{BO} = 0°(P_{Wshear})$, and (iii) for inducing tensile fractures (P_b) for $\lambda_p = 0.52$ and $\lambda_p = 0.60$. Compare with $\lambda_p = 0.44$ solved in class. How does the drilling mud window change with overpressure?

We can calculate the wellbore pressure for a predetermined breakout angle:

$$P_{wBO} = P_p + \frac{(\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(\pi - w_{BO}) - UCS}{1+q}$$

Wellbore breakouts occur when the stress anisotropy σ_1/σ_3 surpasses the shear strength limit of the wellbore rock. Maximum anisotropy is found at $\theta = \pi/2$ and $3\pi/2$, so the lower limit of wellbore pressure P_{Wshear} is

$$P_{Wshear} = P_p + \frac{3\sigma_{hmin} - \sigma_{Hmax} - UCS}{1+q}$$

Wellbore tensile (or open mode) fractures occur when the minimum principal stress σ_3 on the wellbore wall goes below the limit for tensile stress: the tensile strength. The minimum hoop stress is located on the wall of the wellbore $(r = a)$ and at $\theta = 0$ and π, so the upper limit of wellbore pressure P_b to prevent wellbore tensile (or open mode) fractures from forming is:

$$P_b = P_p + 3\sigma_{hmin} - \sigma_{Hmax} + T_s + \sigma^{\Delta T}$$

The mud window gets smaller as over pressure increases. The wellbore becomes unstable when P_b is smaller than P_{wBO}.

In [3]:
```python
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ2Summary = pd.read_excel(excel_file, sheet_name=2)
DataQ2Summary.head(5)
```

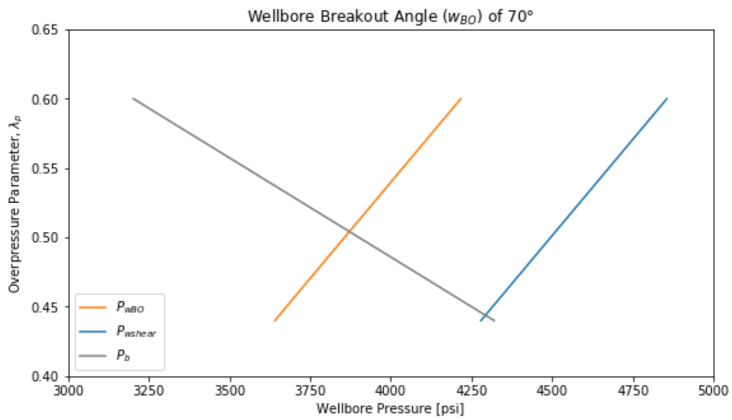
Out[3]:

	λp	wBO [°]	PwBO [psi]	Pwshear [psi]	Pb [psi]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]	-	wBO [°].1	PwBO [psi].1	Pwshear [psi].1	Pb [psi].1	PwBO [ppg].1	Pwshear [ppg].1	[ppg
0	0.44	70	3640.291458	4279.195484	4320	10.000801	11.756032	11.868132	-	0	4279.195484	4279.195484	3531.868132	11.756032	11.756032	9.7029
1	0.52	70	3928.409081	4567.313107	3761	10.792333	12.547563	10.332418	-	0	4567.313107	4567.313107	2970.332418	12.547563	12.547563	8.1602
2	0.60	70	4216.526704	4855.430730	3202	11.583865	13.339095	8.796703	-	0	4855.430730	4855.430730	2408.796703	13.339095	13.339095	6.6175

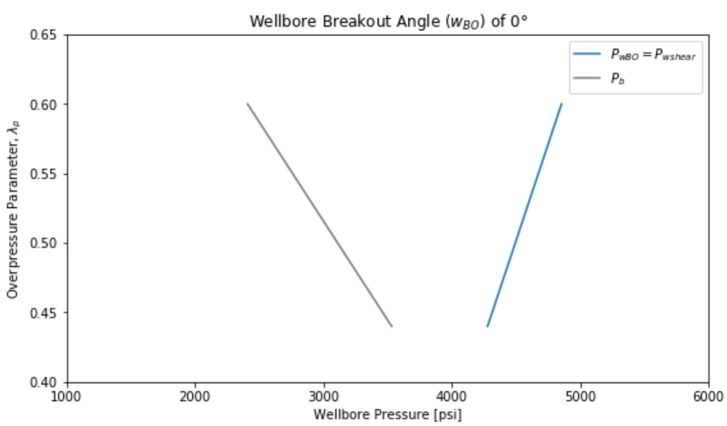
In [4]:
```python
import numpy as np
import matplotlib.pyplot as plt

lambda_p = DataQ2Summary['λp']
PwBO_70 = DataQ2Summary['PwBO [psi]']
Pwshear_70 = DataQ2Summary['Pwshear [psi]']
Pb_70 = DataQ2Summary['Pb [psi]']
PwBO_0 = DataQ2Summary['PwBO [psi].1']
Pwshear_0 = DataQ2Summary['Pwshear [psi].1']
Pb_0 = DataQ2Summary['Pb [psi].1']

# Change plot size
fig = plt.gcf()
fig.set_size_inches(20, 5)
# Plot data
plt.subplot(1, 2, 1)
plt.plot(PwBO_70,lambda_p,'tab:orange', label='$P_{wBO}$')
plt.plot(Pwshear_70,lambda_p,'tab:blue', label='$P_{wshear}$')
plt.plot(Pb_70,lambda_p,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [psi]')
plt.ylabel('Overpressure Parameter, $\lambda_p$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 70°')
plt.legend()
# Axis range
plt.xlim([3000, 5000])
plt.ylim([0.4, 0.65])
plt.show()
```



In [5]:
```python
# Change plot size
fig = plt.gcf()
fig.set_size_inches(9, 5)
# Plot data
plt.plot(PwBO_0,lambda_p,'tab:blue', label='$P_{wBO}=P_{wshear}$')
plt.plot(Pb_0,lambda_p,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [psi]')
plt.ylabel('Overpressure Parameter, $\lambda_p$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 0°')
plt.legend()
# Axis range
plt.xlim([1000, 6000])
plt.ylim([0.4, 0.65])
plt.show()
```

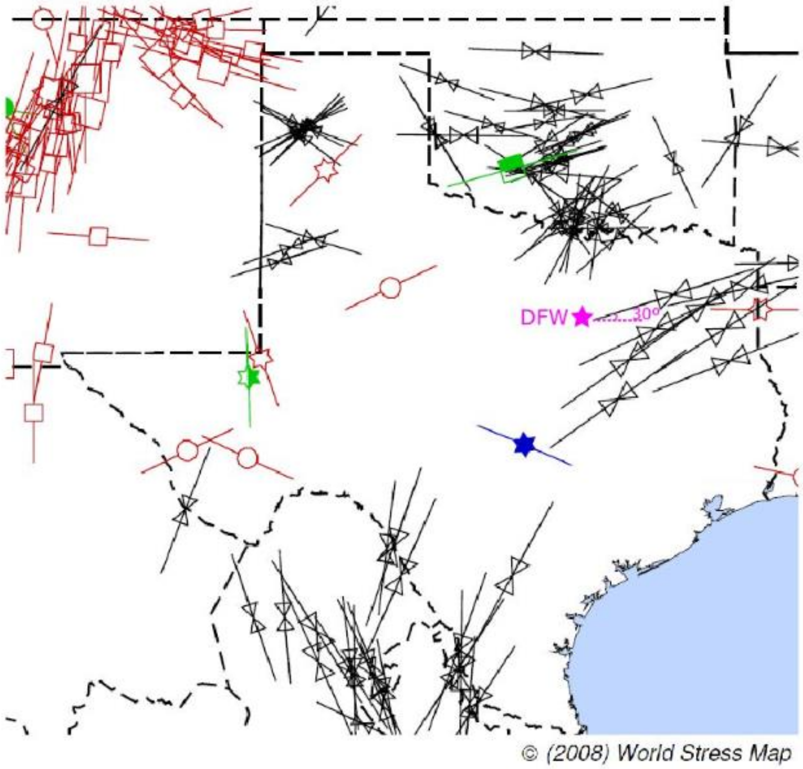


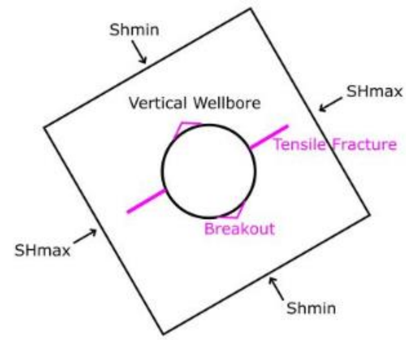
(b) Assume horizontal stress directions near Dallas-Forth Worth region. What would the azimuth of breakouts and drilling induced fractures be?
http://dc-app3-14.gfz-potsdam.de/pub/stress_data/stress_data_frame.html

The maximum stress seems to be roughly at 70 deg EW in the Dallas Fort Worth area. That means the azimuth of the breakouts and drilling induced fractures will be 070 and 160 respectively.

Tensile fractures will occur in the azimuth of S_{Hmax}. For Dallas-Forth Worth region, the azimuth of S_{Hmax} is E30°N or 060°.

Shear fractures (wellbore breakouts) will occur in the azimuth of S_{hmin}. For Dallas-Forth Worth region, the azimuth of S_{hmin} is N30°W or 150°.





Problem 3: Effect of Stress Anisotropy (Differential Stress)

Consider the following problem, Wellbore: vertical; Site: onshore, 2 km of depth, $\lambda_p = 0.44$, $\sigma_{hmin} = 0.4\sigma_V$; Rock properties: $UCS = 7MPa$, $q = 3.9$, $T_s = 2MPa$. Calculate wellbore pressure and corresponding mud weight for (i) $w_{BO} = 45°$, (ii) $w_{BO} = 0°$, and (iii) for inducing tensile fractures for

(a) $\sigma_{Hmax} = 0.6\sigma_V$

(b) $\sigma_{Hmax} = 0.8\sigma_V$

(c) $\sigma_{Hmax} = 1.0\sigma_V$

In [6]:
```
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ3Summary = pd.read_excel(excel_file, sheet_name=4)
DataQ3Summary.head(3)
```

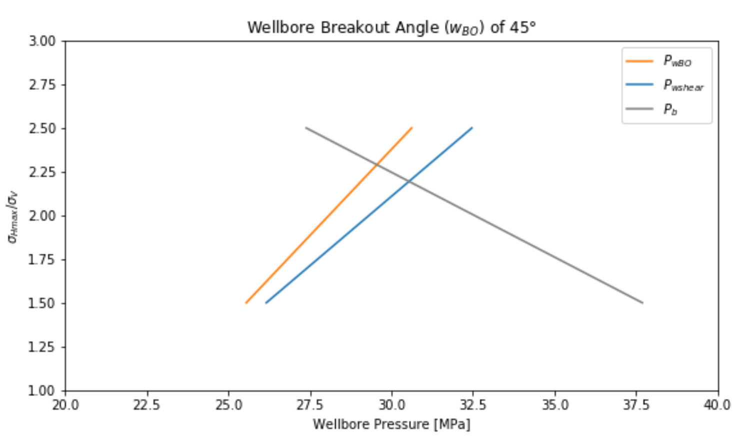
Out[6]:

	σHmax/ σV	σHmax/ σhmin	wBO [°]	PwBO [MPa]	Pwshear [MPa]	Pb [MPa]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]	-	wBO [°].1	PwBO [MPa].1	Pwshear [MPa].1	Pb [MPa].1	PwBO [ppg].1	Pwshear [ppg].1	
0	0.6	1.5	45	25.550308	26.165617	37.696	10.681567	10.938803	15.759197	-	0	26.165617	26.165617	51.455197	10.938803	10.938803	21.
1	0.8	2.0	45	28.086195	29.316813	32.544	11.741720	12.256193	13.605351	-	0	29.316813	29.316813	44.149351	12.256193	12.256193	18.
2	1.0	2.5	45	30.622081	32.468008	27.392	12.801873	13.573582	11.451505	-	0	32.468008	32.468008	36.843505	13.573582	13.573582	15.

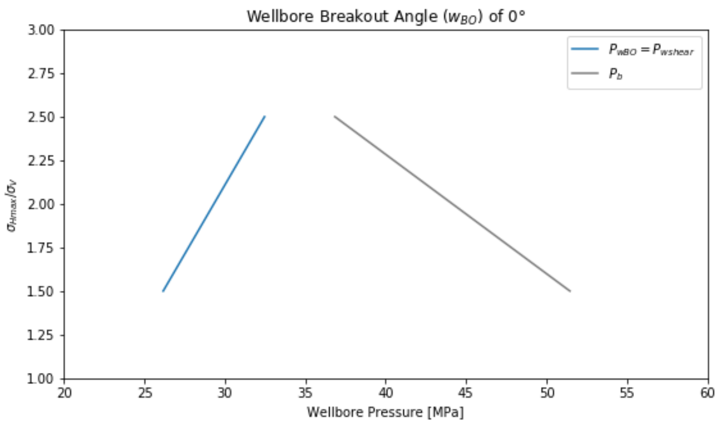
In [7]:
```
import numpy as np
import matplotlib.pyplot as plt

σHmaxσhminratio = DataQ3Summary['σHmax/σhmin']
PwBO_45 = DataQ3Summary['PwBO [MPa]']
Pwshear_45 = DataQ3Summary['Pwshear [MPa]']
Pb_45 = DataQ3Summary['Pb [MPa]']
PwBO_0 = DataQ3Summary['PwBO [MPa].1']
Pwshear_0 = DataQ3Summary['Pwshear [MPa].1']
Pb_0 = DataQ3Summary['Pb [MPa].1']

# Change plot size
fig = plt.gcf()
fig.set_size_inches(20, 5)
# Plot data
plt.subplot(1, 2, 1)
plt.plot(PwBO_45,σHmaxσhminratio,'tab:orange', label='$P_{wBO}$')
plt.plot(Pwshear_45,σHmaxσhminratio,'tab:blue', label='$P_{wshear}$')
plt.plot(Pb_45,σHmaxσhminratio,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [MPa]')
plt.ylabel('$\sigma_{Hmax}/\sigma_V$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 45°')
plt.legend()
# Axis range
plt.xlim([20, 40])
plt.ylim([1, 3])
plt.show()
```



In [8]:
```
# Change plot size
fig = plt.gcf()
fig.set_size_inches(9, 5)
# Plot data
plt.plot(PwBO_0,σHmaxσhminratio,'tab:blue', label='$P_{wBO}=P_{wshear}$')
plt.plot(Pb_0,σHmaxσhminratio,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [MPa]')
plt.ylabel('$\sigma_{Hmax}/\sigma_V$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 0°')
plt.legend()
# Axis range
plt.xlim([20, 60])
plt.ylim([1, 3])
plt.show()
```



(d) How does the drilling mud window change with σHmax/σHmin?

As $\sigma_{Hmax}/\sigma_{hmin}$ increases, the mud window becomes narrower. The wellbore becomes unstable when P_b is smaller than P_{wBO}. More stress anisotropy creates a less stable wellbore.

Problem 4: Offshore

Consider the same formation as above but in offshore conditions, Wellbore: vertical; Site: offshore, 2 km of total depth, 500 m of water, hydrostatic pore pressure, $\sigma_{hmin} = 0.4\sigma_V$, $\sigma_{Hmax} = 0.8\sigma_V$; Rock properties: $UCS = 7MPa$, $q = 3.9$, $T_s = 2MPa$. Calculate wellbore pressure and corresponding mud weight for (i) $w_{BO} = 45°$, (ii) $w_{BO} = 0°$, and (iii) for inducing tensile fractures.

In [9]:
```
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ3Summary = pd.read_excel(excel_file, sheet_name=6)
DataQ3Summary.head(3)
```

Out[9]:

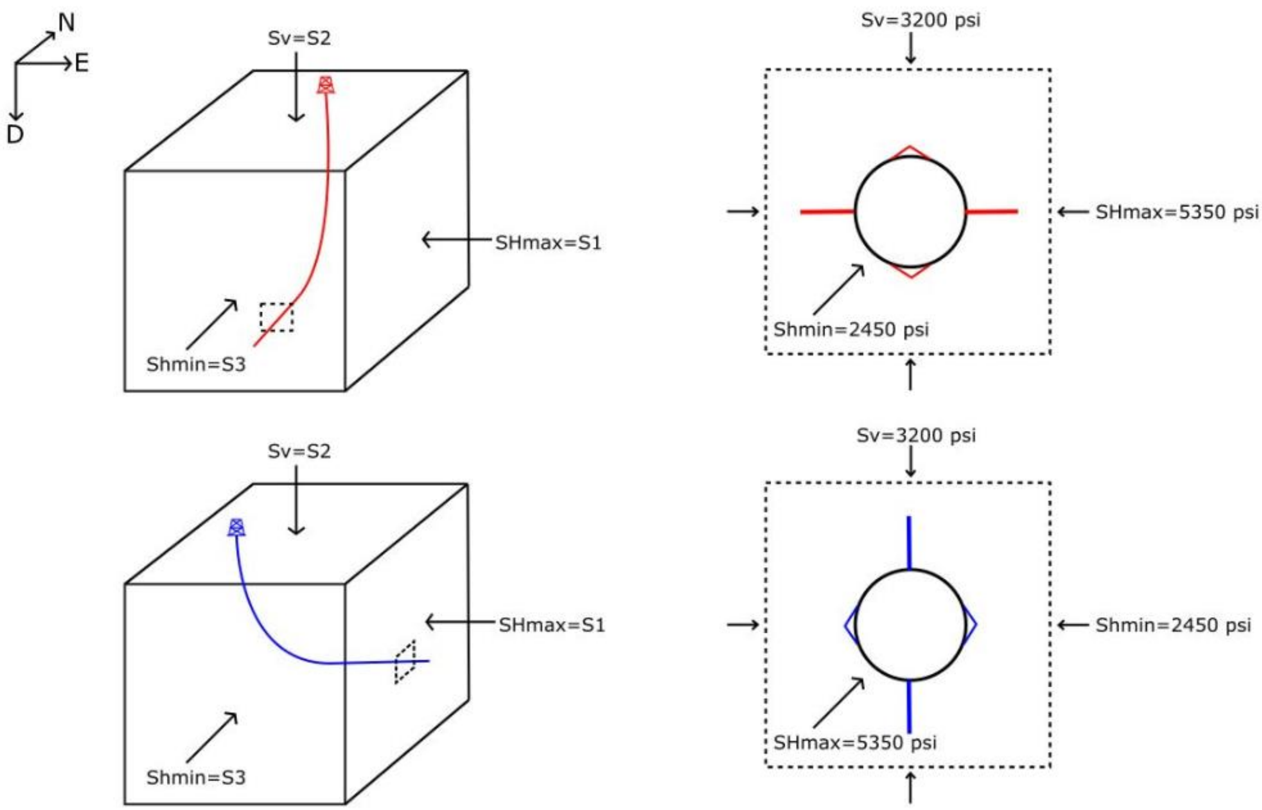
	wBO [°]	PwBO [MPa]	Pwshear [MPa]	Pb [MPa]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]
0	0	26.601874	26.601874	40.361042	11.121185	11.121185	16.873345
1	45	25.676811	26.601874	29.872527	10.734453	11.121185	12.488515

Problem 5: Horizontal Wells

Evaluate wellbore stability for horizontal wells that you will need to exploit in a gas reservoir subjected to a strike-slip stress environment.

(a) Draw cross-sections of wellbores drilled parallel to S_{hmin} and S_{Hmax}, identify involved stresses, and clearly mark expected positions of tensile fractures and wellbore breakouts.

In a strike-slip fault we have $S_{hmin} < S_v < S_{Hmax}$



(b) The horizontal wells lie at about 8000ft depth where it is estimated that $S_h min = 50MPa$, $S_H max = 70MPa$ and $\lambda_p = 0.6$. The unconfined compressive strength of the rock is 8500psi, $\mu = 1.0$, and $T_s = 0psi$ is a good estimate for tensile strength, given the large density of natural fractures. Determine the mechanical stability limits on wellbore pressure for both horizontal well directions considered.

(c) Determine mud density window appropriate for these wells (keep in mind potential lost circulation).

In [10]:
```
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ3Summary = pd.read_excel(excel_file, sheet_name=8)
DataQ3Summary.head(3)
```

Out[10]:

	Horizontal Well Direction	σmax	σmin	wBO [°]	PwBO [psi]	Pwshear [psi]	Pb [psi]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]
0	Horizontal in direction of SHmax	3200	2450	45	4537.957249	4602.297077	8950.000000	10.908551	11.063214	21.514423
1	Horizontal in direction of Shmin	5350	3200	45	5252.601910	5437.042751	9071.514423	12.626447	13.069814	21.806525

(d) Which one appears to have a wider mud window? Justify

The lateral drilled parallel to maximum horizontal stress has a wider mud window because the difference between the inplane max and min stress $(\sigma_{Hmax} - \sigma_{hmin})$ is smaller