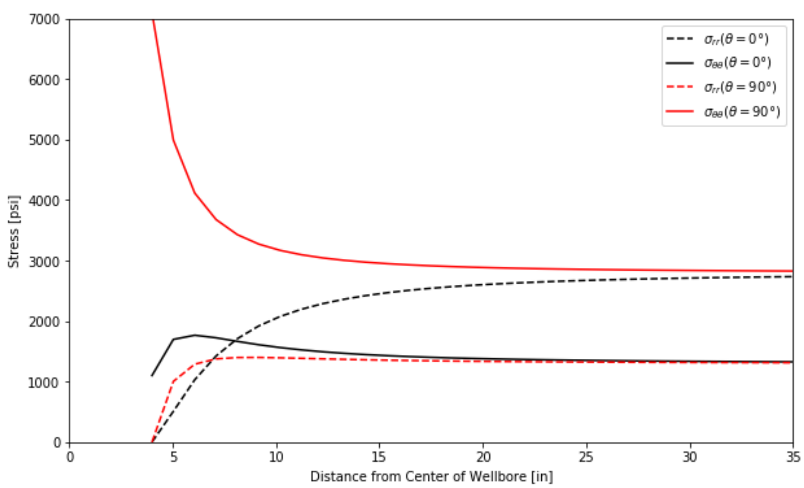
Homework 5 Solutions

Problem 1: Kirsch Solution

Using equations of stresses around a cylindrical cavity, calculate near-wellbore effective radial σ_{rr} and hoop $\sigma_{\theta\theta}$ stresses for a vertical well 8in diameter in the directions of S_{hmin} (4500 psi – acting E-W) and S_{Hmax} (6000 psi) up to 3ft of distance. The result should be presented as plots of stresses (σ_{rr}, $\sigma_{\theta\theta}$) as a function of distance from the center of the wellbore.

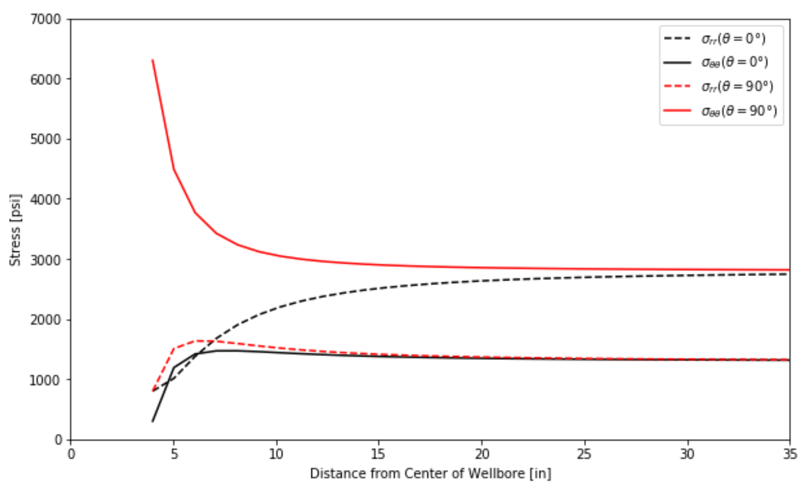
(a) $P_p = 3200psi$ **and** $P_w = 3200psi$

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

# Given parameters
Pp = 3200 # Pore pressure [psi]
Pw = 3200 # Wellbore pressure [psi]
a = 4 # Wellbore radius, a [in]
S_hmin = 4500 # Minimum total horizontal stress [psi]
S_Hmax = 6000 # Maximum total horizontal stress [psi]
sigma_hmin = S_hmin - Pp # Minimum effective horizontal stress [psi]
sigma_Hmax = S_Hmax - Pp # Maximum effective horizontal stress [psi]

# Calculate the radial stress, σrr and the tangential (hoop) stress, σϑϑ based on r and theta
r = np.linspace(4, 35, 31) # distance measured from the center of the wellbore [in]
theta = 0 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_0 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**4
sigma_thetatheta_0 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos(
theta = 90 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_90 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**
sigma_thetatheta_90 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos

# Plotting
plt.plot(r, sigma_rr_0, '--k', label=r'$\sigma_{rr} (\theta=0°)$',)
plt.plot(r, sigma_thetatheta_0, '-k', label=r'$\sigma_{\theta\theta} (\theta=0°)$',)
plt.plot(r, sigma_rr_90, '--r', label=r'$\sigma_{rr} (\theta=90°)$',)
plt.plot(r, sigma_thetatheta_90, '-r', label=r'$\sigma_{\theta\theta} (\theta=90°)$',)
# Plot labels
plt.xlabel('Distance from Center of Wellbore [in]')
plt.ylabel('Stress [psi]')
plt.legend()
# Axis range
plt.xlim([0, 35])
plt.ylim([0, 7000])
# Change plot size
fig = plt.gcf()
fig.set_size_inches(10, 6)
```



(b) $P_p = 3200psi$ **and** $P_w = 4000psi$

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt

# Given parameters
Pp = 3200 # Pore pressure [psi]
Pw = 4000 # Wellbore pressure [psi]
a = 4 # Wellbore radius, a [in]
S_hmin = 4500 # Minimum total horizontal stress [psi]
S_Hmax = 6000 # Maximum total horizontal stress [psi]
sigma_hmin = S_hmin - Pp # Minimum effective horizontal stress [psi]
sigma_Hmax = S_Hmax - Pp # Maximum effective horizontal stress [psi]

# Calculate the radial stress, σrr and the tangential (hoop) stress, σϑϑ based on r and theta
r = np.linspace(4, 35, 31) # distance measured from the center of the wellbore [in]
theta = 0 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_0 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**4
sigma_thetatheta_0 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos(
theta = 90 *(np.pi/180) # ϑ is the angle between the direction of SHmax and the point at which stress is being considered [°]
sigma_rr_90 =(((sigma_Hmax+sigma_hmin)/2)*(1-((a**2)/(r**2))))+(((sigma_Hmax-sigma_hmin)/2)*(1-(4*((a**2)/(r**2)))+(3*((a**4)/(r**
sigma_thetatheta_90 =(((sigma_Hmax+sigma_hmin)/2)*(1+((a**2)/(r**2))))-(((sigma_Hmax-sigma_hmin)/2)*(1+(3*((a**4)/(r**4))))*np.cos

# Plotting
plt.plot(r, sigma_rr_0, '--k', label=r'$\sigma_{rr} (\theta=0°)$',)
plt.plot(r, sigma_thetatheta_0, '-k', label=r'$\sigma_{\theta\theta} (\theta=0°)$',)
plt.plot(r, sigma_rr_90, '--r', label=r'$\sigma_{rr} (\theta=90°)$',)
plt.plot(r, sigma_thetatheta_90, '-r', label=r'$\sigma_{\theta\theta} (\theta=90°)$',)
# Plot labels
plt.xlabel('Distance from Center of Wellbore [in]')
plt.ylabel('Stress [psi]')
plt.legend()
# Axis range
plt.xlim([0, 35])
plt.ylim([0, 7000])
# Change plot size
fig = plt.gcf()
fig.set_size_inches(10, 6)
```



Problem 2: Effect of Overpressure

Consider the problem solved in class (Wellbore: vertical; Site: onshore, 7000 ft of depth, $S_{hmin} = 4300psi$, $S_{Hmax} = 6300psi$; Rock properties: $UCS = 3,500psi$, $\mu = 0.6$, $T_s = 800psi$).

(a) Calculate wellbore pressure and corresponding mud weight for (i) $w_{BO} = 70^{\circ}$, (ii) $w_{BO} = 0^{\circ}(P_{Wshear})$, and (iii) for inducing tensile fractures (P_b) for $\lambda_p = 0.52$ and $\lambda_p = 0.60$. Compare with $\lambda_p = 0.44$ solved in class. How does the drilling mud window change with overpressure?

We can calculate the wellbore pressure for a predetermined breakout angle:

$$P_{wBO} = P_p + \frac{(\sigma_{Hmax} + \sigma_{hmin}) - 2(\sigma_{Hmax} - \sigma_{hmin})\cos(\pi - w_{BO}) - UCS}{1+q}$$

Wellbore breakouts occur when the stress anisotropy σ_1/σ_3 surpasses the shear strength limit of the wellbore rock. Maximum anisotropy is found at $\theta = \pi/2$ and $3\pi/2$, so the lower limit of wellbore pressure P_{Wshear} is

$$P_{Wshear} = P_p + \frac{3\sigma_{hmin} - \sigma_{Hmax} - UCS}{1+q}$$

Wellbore tensile (or open mode) fractures occur when the minimum principal stress σ_3 on the wellbore wall goes below the limit for tensile stress: the tensile strength. The minimum hoop stress is located on the wall of the wellbore $(r = a)$ and at $\theta = 0$ and π, so the upper limit of wellbore pressure P_b to prevent wellbore tensile (or open mode) fractures from forming is:

$$P_b = P_p + 3\sigma_{hmin} - \sigma_{Hmax} + T_s + \sigma^{\Delta T}$$

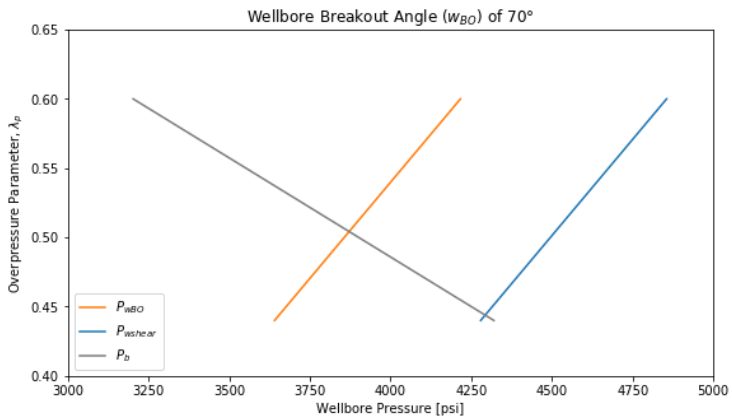
The mud window gets smaller as over pressure increases. The wellbore becomes unstable when P_b is smaller than P_{wBO}.

In [3]:
```
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ2Summary = pd.read_excel(excel_file, sheet_name=2)
DataQ2Summary.head(5)
```

Out[3]:

	λp	wBO [°]	PwBO [psi]	Pwshear [psi]	Pb [psi]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]	-	wBO [°].1	PwBO [psi].1	Pwshear [psi].1	Pb [psi].1	PwBO [ppg].1	Pwshear [ppg].1	Pb [ppg].1
0	0.44	70	3640	4279	4320	10.0	11.8	11.9	-	0	4279	4279	4320	11.8	11.8	11.9
1	0.52	70	3928	4567	3761	10.8	12.5	10.3	-	0	4567	4567	3761	12.5	12.5	10.3
2	0.60	70	4217	4855	3202	11.6	13.3	8.8	-	0	4855	4855	3202	13.3	13.3	8.8

In [4]:
```
import numpy as np
import matplotlib.pyplot as plt

lambda_p = DataQ2Summary['λp']
PwBO_70 = DataQ2Summary['PwBO [psi]']
Pwshear_70 = DataQ2Summary['Pwshear [psi]']
Pb_70 = DataQ2Summary['Pb [psi]']
PwBO_0 = DataQ2Summary['PwBO [psi].1']
Pwshear_0 = DataQ2Summary['Pwshear [psi].1']
Pb_0 = DataQ2Summary['Pb [psi].1']

# Change plot size
fig = plt.gcf()
fig.set_size_inches(20, 5)
# Plot data
plt.subplot(1, 2, 1)
plt.plot(PwBO_70,lambda_p,'tab:orange', label='$P_{wBO}$')
plt.plot(Pwshear_70,lambda_p,'tab:blue', label='$P_{wshear}$')
plt.plot(Pb_70,lambda_p,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [psi]')
plt.ylabel('Overpressure Parameter, $\lambda_p$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 70°')
plt.legend()
# Axis range
plt.xlim([3000, 5000])
plt.ylim([0.4, 0.65])
plt.show()
```



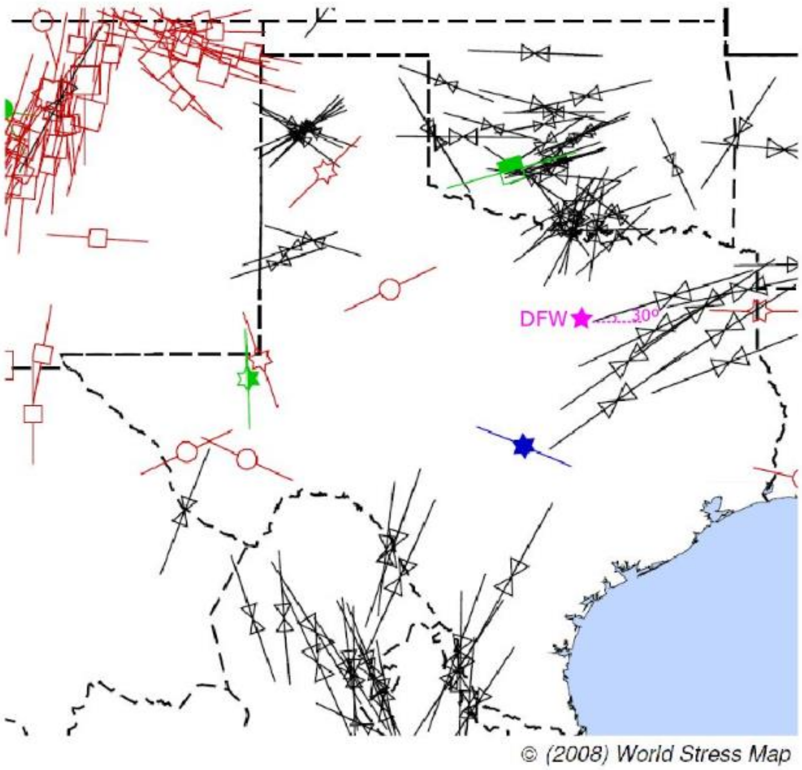
In [5]:

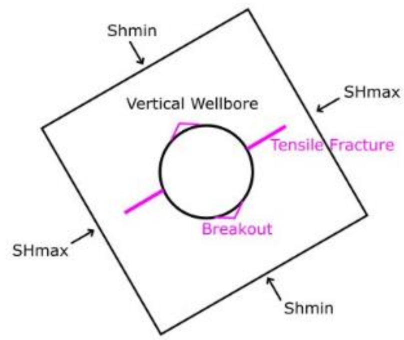
```
# Change plot size
fig = plt.gcf()
fig.set_size_inches(9, 5)
# Plot data
plt.plot(PwBO_0,lambda_p,'tab:blue', label='$P_{wBO}=P_{wshear}$')
plt.plot(Pb_0,lambda_p,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [psi]')
plt.ylabel('Overpressure Parameter, $\lambda_p$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 0°')
plt.legend()
# Axis range
plt.xlim([1000, 6000])
plt.ylim([0.4, 0.65])
plt.show()
```

(b) Assume horizontal stress directions near Dallas-Forth Worth region. What would the azimuth of breakouts and drilling induced fractures be?
http://dc-app3-14.gfz-potsdam.de/pub/stress_data/stress_data_frame.html

Tensile fractures will occur in the azimuth of S_{Hmax}. For Dallas-Forth Worth region, the azimuth of S_{Hmax} is E30°N or 060°.

Shear fractures (wellbore breakouts) will occur in the azimuth of S_{hmin}. For Dallas-Forth Worth region, the azimuth of S_{hmin} is N30°W or 150°.





Problem 3: Effect of Stress Anisotropy (Differential Stress)

Consider the following problem, Wellbore: vertical; Site: onshore, 2 km of depth, $\lambda_p = 0.44$, $\sigma_{hmin} = 0.4\sigma_V$; Rock properties: $UCS = 7MPa$, $q = 3.9$, $T_s = 2MPa$. Calculate wellbore pressure and corresponding mud weight for (i) $w_{BO} = 45°$, (ii) $w_{BO} = 0°$, and (iii) for inducing tensile fractures for

(a) $\sigma_{Hmax} = 0.6\sigma_V$

(b) $\sigma_{Hmax} = 0.8\sigma_V$

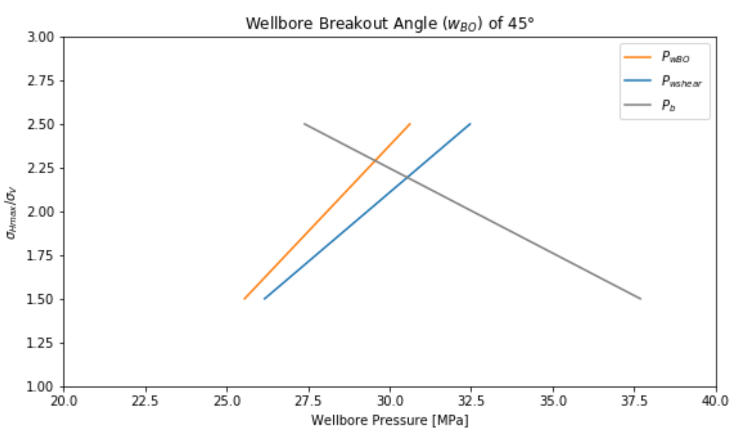
(c) $\sigma_{Hmax} = 1.0\sigma_V$

In [6]:
```
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ3Summary = pd.read_excel(excel_file, sheet_name=4)
DataQ3Summary.head(3)
```

Out[6]:

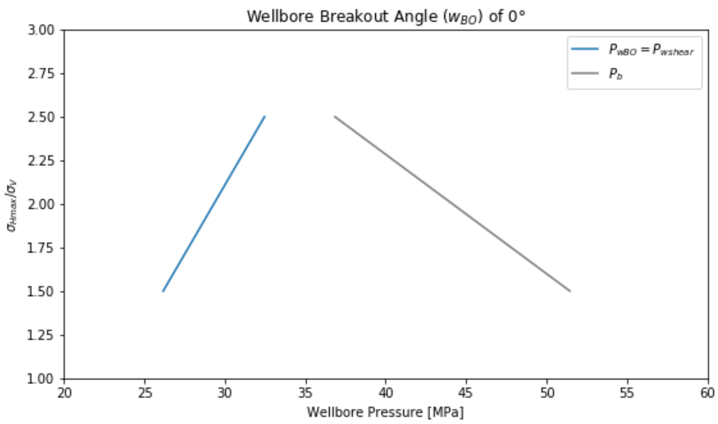
	σHmax/σV	σHmax/σhmin	wBO [°]	PwBO [MPa]	Pwshear [MPa]	Pb [MPa]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]	-	wBO [°].1	PwBO [MPa].1	Pwshear [MPa].1	Pb [MPa].1	PwBO [ppg].1	Pwshear [ppg].1	Pb [ppg].1
0	0.6	1.5	45	25.6	26.2	37.7	10.7	11.0	15.8	-	0	26.2	26.2	37.7	11.0	11.0	15.8
1	0.8	2.0	45	28.1	29.3	32.5	11.7	12.2	13.6	-	0	29.3	29.3	32.5	12.2	12.2	13.6
2	1.0	2.5	45	30.6	32.5	27.4	12.8	13.6	11.5	-	0	32.5	32.5	27.4	13.6	13.6	11.5

In [7]:
```
import numpy as np
import matplotlib.pyplot as plt

σHmaxσhminratio = DataQ3Summary['σHmax/σhmin']
PwBO_45 = DataQ3Summary['PwBO [MPa]']
Pwshear_45 = DataQ3Summary['Pwshear [MPa]']
Pb_45 = DataQ3Summary['Pb [MPa]']
PwBO_0 = DataQ3Summary['PwBO [MPa].1']
Pwshear_0 = DataQ3Summary['Pwshear [MPa].1']
Pb_0 = DataQ3Summary['Pb [MPa].1']

# Change plot size
fig = plt.gcf()
fig.set_size_inches(20, 5)
# Plot data
plt.subplot(1, 2, 1)
plt.plot(PwBO_45,σHmaxσhminratio,'tab:orange', label='$P_{wBO}$')
plt.plot(Pwshear_45,σHmaxσhminratio,'tab:blue', label='$P_{wshear}$')
plt.plot(Pb_45,σHmaxσhminratio,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [MPa]')
plt.ylabel('$\sigma_{Hmax}/\sigma_V$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 45°')
plt.legend()
# Axis range
plt.xlim([20, 40])
plt.ylim([1, 3])
plt.show()
```



In [8]:

```python
# Change plot size
fig = plt.gcf()
fig.set_size_inches(9, 5)
# Plot data
plt.plot(PwBO_0,σHmaxσhminratio,'tab:blue', label='$P_{wBO}=P_{wshear}$')
plt.plot(Pb_0,σHmaxσhminratio,'tab:gray', label='$P_b$')
# Plot labels
plt.xlabel('Wellbore Pressure [MPa]')
plt.ylabel('$\sigma_{Hmax}/\sigma_V$')
plt.title('Wellbore Breakout Angle ($w_{BO}$) of 0°')
plt.legend()
# Axis range
plt.xlim([20, 60])
plt.ylim([1, 3])
plt.show()
```



(d) How does the drilling mud window change with σHmax/σHmin?

As $\sigma_{Hmax}/\sigma_{hmin}$ increases, the mud window becomes narrower. The wellbore becomes unstable when P_b is smaller than P_{wBO}. More stress anisotropy creates a less stable wellbore.

Problem 4: Offshore

Consider the same formation as above but in offshore conditions, Wellbore: vertical; Site: offshore, 2 km of total depth, 500 m of water, hydrostatic pore pressure, $\sigma_{hmin} = 0.4\sigma_V$, $\sigma_{Hmax} = 0.8\sigma_V$; Rock properties: $UCS = 7MPa$, $q = 3.9$, $T_s = 2MPa$. Calculate wellbore pressure and corresponding mud weight for (i) $w_{BO} = 45°$, (ii) $w_{BO} = 0°$, and (iii) for inducing tensile fractures.

In [9]:

```python
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ3Summary = pd.read_excel(excel_file, sheet_name=6)
DataQ3Summary.head(3)
```

Out[9]:

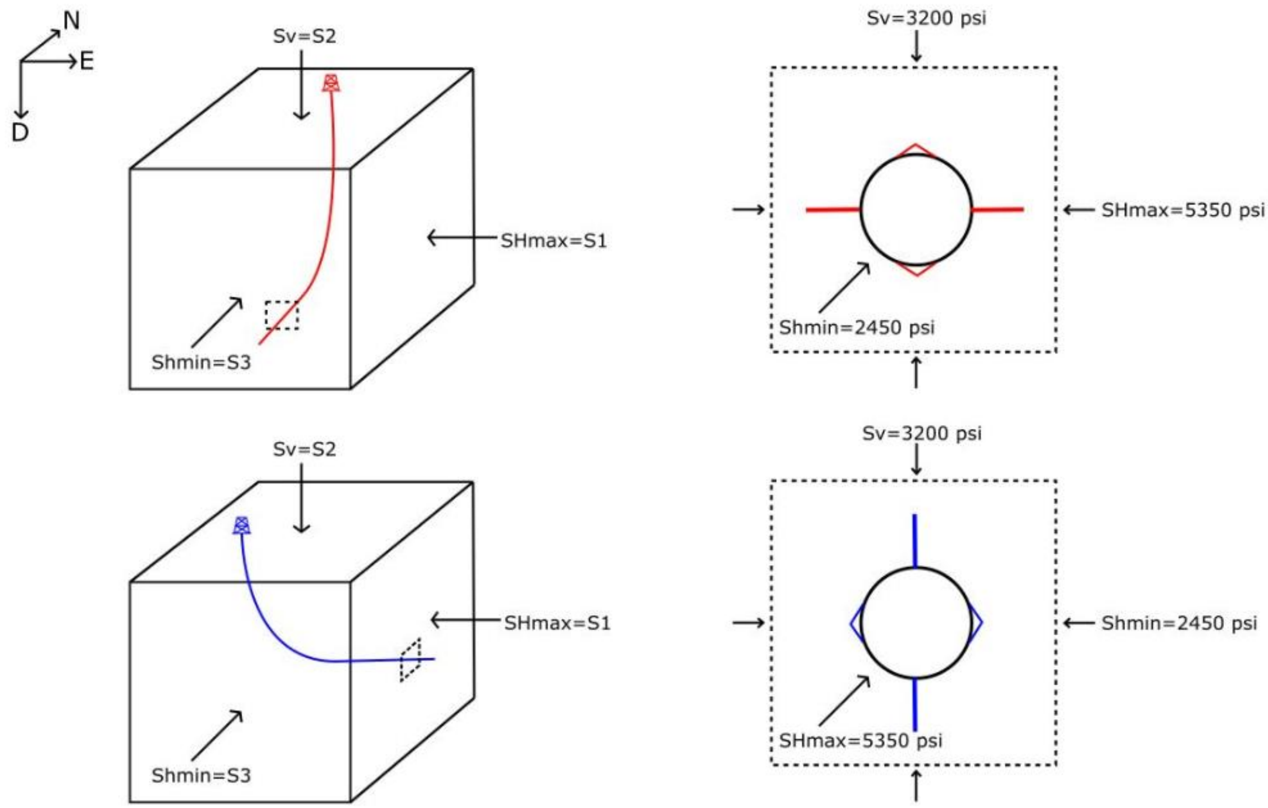
	wBO [°]	PwBO [MPa]	Pwshear [MPa]	Pb [MPa]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]
0	0	26.6	26.6	29.9	11.1	11.1	12.5
1	45	25.7	26.6	29.9	10.7	11.1	12.5

Problem 5: Horizontal Wells

Evaluate wellbore stability for horizontal wells that you will need to exploit in a gas reservoir subjected to a strike-slip stress environment.

(a) Draw cross-sections of wellbores drilled parallel to S_{hmin} and S_{Hmax}, identify involved stresses, and clearly mark expected positions of tensile fractures and wellbore breakouts.

In a strike-slip fault we have $S_{hmin} < S_v < S_{Hmax}$



(b) The horizontal wells lie at about 8000ft depth where it is estimated that $S_hmin = 50MPa$, $S_Hmax = 70MPa$ and $\lambda_p = 0.6$. The unconfined compressive strength of the rock is 8500psi, $\mu = 1.0$, and $T_s = 0psi$ is a good estimate for tensile strength, given the large density of natural fractures. Determine the mechanical stability limits on wellbore pressure for both horizontal well directions considered.

(c) Determine mud density window appropriate for these wells (keep in mind potential lost circulation).

In [10]:
```
import pandas as pd

excel_file = 'HW5.xlsx'
DataQ3Summary = pd.read_excel(excel_file, sheet_name=8)
DataQ3Summary.head(3)
```

Out[10]:

	Horizontal Well Direction	σmax	σmin	wBO [°]	PwBO [psi]	Pwshear [psi]	Pb [psi]	PwBO [ppg]	Pwshear [ppg]	Pb [ppg]
0	Horizontal in direction of SHmax	3200	2450	45	4538	4602	8950	10.9	11.1	21.5
1	Horizontal in direction of Shmin	5350	3200	45	5253	5437	9050	12.6	13.1	21.8

(d) Which one appears to have a wider mud window? Justify

The lateral drilled parallel to maximum horizontal stress has a wider mud window because the difference between the inplane max and min stress $(\sigma_{Hmax} - \sigma_{hmin})$ is smaller