Project #4: Stresses around wellbores and fractures

For this project you will have to download and install the software FreeFEM++ (http://www.freefem.org/). FreeFEM++ help will be given in class and by the TA. You will also need to run Matlab, Python or equivalent software.

1) Wellbore problem

Consider a 2D problem of a circular cavity subjected to far field effective stresses σ_{xx}= 12 MPa and σ_{yy}= 3 MPa. The diameter of the cavity is 0.2 m. Rock properties: E = 10 GPa, ν = 0.20, unconfined compression strength UCS = 30 MPa, tensile strength T_S = 2 MPa.

a. Using Kirsch equations compute (and plot) σ_{rr}, $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ for a domain x = [-1m, +1m], and y = [-1m, +1m]. You may define a polar grid for (r,θ). How far does the presence of the wellbore influence stresses?
b. Using Kirsch equations compute (and plot) stresses in a line {x = [0.1m, 1m], y=0m} and {x = 0m, y=[0.1m, 1m]}.
c. Using Kirsch equations compute (and plot) σ_{rr} and $\sigma_{\theta\theta}$ for r = 0.1m. Is there any section of the rock in shear or tensile failure? Where?
d. Use FreeFEM++ to solve the same problem (σ_{xx}, σ_{yy} and σ_{xy}) assuming a domain size 2m by 2m. Compute σ_{xx} and σ_{yy} for the same lines as in point "b", and compare with Kirsch analytical solution. Repeat the process for a domain size 0.5m by 0.5m. Are there any differences? Why?
e. Plot the displacement field.
f. EXTRA: compute principal stresses within FreeFEM++ and plot σ_{rr} and $\sigma_{\theta\theta}$.

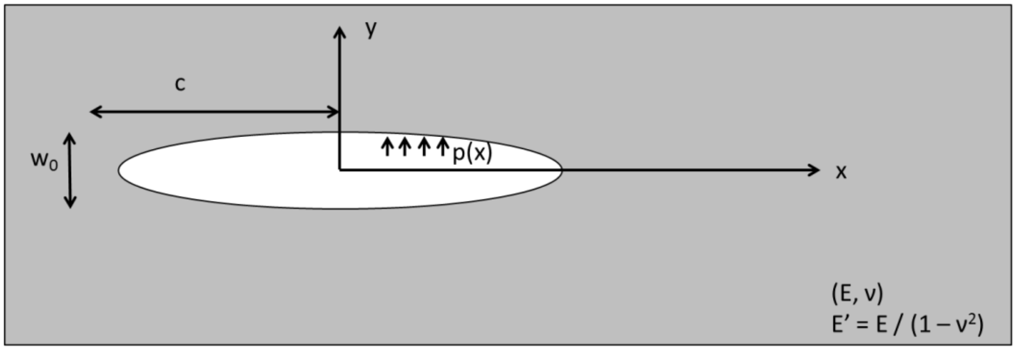
2) Fracture problem

Consider a 2D problem of an elliptical fracture (half-length c = 10m, w_0=0.1m). Set the fracture through an elliptical equation (major axis in direction y) at the center of a domain: x = [-50m, 50m] and y = [-50m, 50m]. All other boundaries will have zero displacement. Rock properties: E = 30 GPa, ν = 0.20.

a. Use FreeFEM++ to solve for σ_{xx}, σ_{yy} and σ_{xy} imposing a fracture pressure p = 10 MPa. Plot results.
b. Plot stress perpendicular to the fracture direction σ_{xx} at the middle of the fracture {x=0, y=[w_0/2, 50m]}. How far does the influence of the fracture go?
c. Plot x-displacements at the face of the fracture. Compare with analytical equation.
d. Plot σ_{xx} along line {x=0, y=[w_0/2, 50m]} and compare with analytical solution.
e. EXTRA: compare FreeFEM++ solution to analytical solution by Sneddon and Elliot, 1946.

(1) Width of an elliptical crack

Griffith crack problem



Boundary conditions

$$\begin{cases} \sigma_{yy}(x,0) = -\,p(x) & \text{for } 0 \le x \le c \\ u_y(x,0) = 0 & \text{for } x > c \end{cases}$$

- Linear elastic and homogeneous solid
- Constant pressure p_0
- No failure

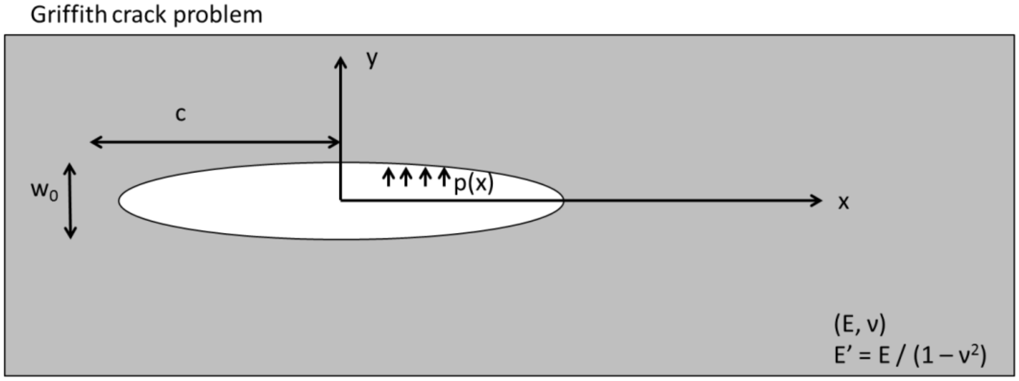
$$\begin{cases} \sigma_{yy}(x,0) = p_0[x/(x^2-c^2)^{1/2}-1] \\ u_y(x,0) = 2p_0/E' * (c^2-x^2)^{1/2} \end{cases}$$

→

- σ_{yy}: <u>tension</u> for x>c; $\sigma_{yy}=-p_0$ at 1.15c
- $w_0 = 4 \cdot c \cdot p_0 / E'$

(2) Stress Intensity Factor at Fracture Tip

Griffith crack problem



$\sigma_{yy}(x,0) = p_0[x/(x^2-c^2)^{1/2}-1]$

- $\sigma_{yy} \rightarrow \infty$ at x=c!

A more convenient amount to compare is the stress intensity factor

$$K_I = \lim_{r \to 0^+}\left[2\pi\, r^{1/2} \sigma_{yy}\left(x = c + r, y = 0\right)\right]$$

Constant pressure crack:

$$K_I = p_0\left(\pi c\right)^{1/2}$$

If $K_I > K_{IC}$ (Fracture Toughness)
→ Fracture propagates