# WP2

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Before we begin, we need to recall the information we derived last time with respect to Depth E.

In [2]:
```
import numpy as np

depthSpacing_Metric = 25 #meters
depthSpacing_Field = depthSpacing_Metric*3.28084 #ft

#Instead of estimating actual values along the axes of the plot,
#I estimated the spacing between the lines on the plot and converted
#that into values based on the known spacing.

pressureSpacing = (12000-6000)/5 #psi

stressSpacing = pressureSpacing #psi

PPRS_3_A_Field = 6000 + 1.25*pressureSpacing #psi
PPRS_3_B_Field = 6000 + 1.75*pressureSpacing #psi
distanceAB_Field = 8*depthSpacing_Field #meters to feet

porePressureGradientAB_Field = (PPRS_3_B_Field-PPRS_3_A_Field)/distanceAB_Field

#Pore Pressure values at depth E
Pp_E = PPRS_3_A_Field+8.2*depthSpacing_Field*porePressureGradientAB_Field

#Total stress values at depth E
Sv_E = 6000 + 3*stressSpacing
Shmax_E = 6000 + 4.5*stressSpacing
Shmin_E = 6000 + 2.8*stressSpacing
```

In [3]:
```
def stressTensor(Sv,Shmax,Shmin):
    sValues = [Sv,Shmax,Shmin]
    sValues = sorted(sValues,reverse=True)
    return np.array([[sValues[0],0,0],
                     [0,sValues[1],0],
                     [0,0,sValues[2]]])

#Principal stress tensor for Depth E
Sp = stressTensor(Sv_E,Shmax_E,Shmin_E)
print("Principal stress tensor for Depth E [psi]:")
print()
print(Sp)
```

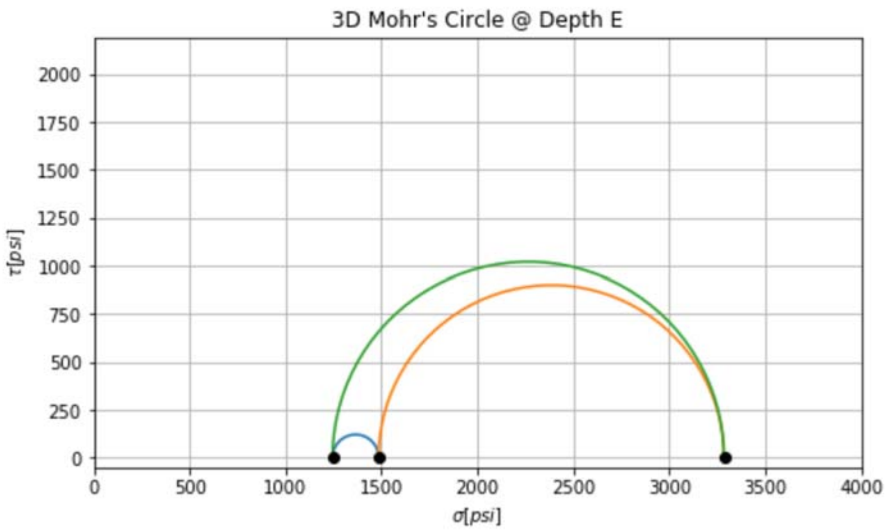
```
Principal stress tensor for Depth E [psi]:

[[11400.     0.     0.]
 [    0.  9600.     0.]
 [    0.     0.  9360.]]
```

In [4]:
```
def classifyRegime(Sv,Shmax,Shmin):
    if(Sv>Shmax and Shmax >Shmmin):
        return "Normal"
    elif(Shmax>Sv and Sv>Shmin):
        return "Strike-Slip"
    elif(Shmax>Shmin and Shmin>Sv):
        return "Reverse"
    else:
        print("Error: Potential issue with stresses provided")
        return "Bad Input"

Regime = classifyRegime(Sv_E,Shmax_E,Shmin_E)
print("Faulting Regime at Depth E: ")
print(Regime)
```

```
Faulting Regime at Depth E:
Strike-Slip
```

In [6]:
```
import matplotlib.pyplot as plt
%matplotlib inline

#Get effective stress, difference between total stress and pore
#pressure at each depth
sigV=Sv_E-Pp_E
sigHmax=Shmax_E-Pp_E
sigHmin=Shmin_E-Pp_E
#Determine sigma 1,2,3 where sig1>sig2>sig3. Here I used
# a = sig3, b = sig2, c = sig1.
arg = np.array([sigV,sigHmax,sigHmin])
a = np.min(arg)
c = np.max(arg)
arg = arg[np.where(arg!=a)]
arg = arg[np.where(arg!=c)]
b = arg[0]

#Copying and pasting the code for plotting Mohr's circle will get messy really quic
kly, so let us just
#make a function for convenience

def plotMohr3D(a,b,c):

    fig = plt.figure(figsize=(7,7))
    ax = fig.add_subplot(111)
    circle1X=[]
    circle1Y=[]
    circle2X=[]
    circle2Y=[]
    circle3X=[]
    circle3Y=[]

    for i in np.linspace(0,np.pi):
        circle1X.append((b-a)/2*np.cos(i) + (a+(b-a)/2))
        circle2X.append((c-b)/2*np.cos(i) + (b+(c-b)/2))
        circle3X.append((c-a)/2*np.cos(i) + (a+(c-a)/2))
        circle1Y.append((b-a)/2*np.sin(i) )
        circle2Y.append((c-b)/2*np.sin(i) )
        circle3Y.append((c-a)/2*np.sin(i) )
    ax.plot(circle1X,circle1Y)
    ax.plot(circle2X,circle2Y)
    ax.plot(circle3X,circle3Y)
    ax.plot([a,b,c],[0,0,0],'ko')
    ax.grid()
    ax.set_xlabel(r'$\sigma [psi]$')
    ax.set_ylabel(r'$\tau [psi]$')
    ax.set_title("3D Mohr's Circle @ Depth E")
    plt.axis('square')
    plt.tight_layout()
    plt.xlim(0,4000)

    return ax

plotMohr3D(a,b,c)
```

```
Out[6]: <matplotlib.axes._subplots.AxesSubplot at 0x25e50272fd0>
```



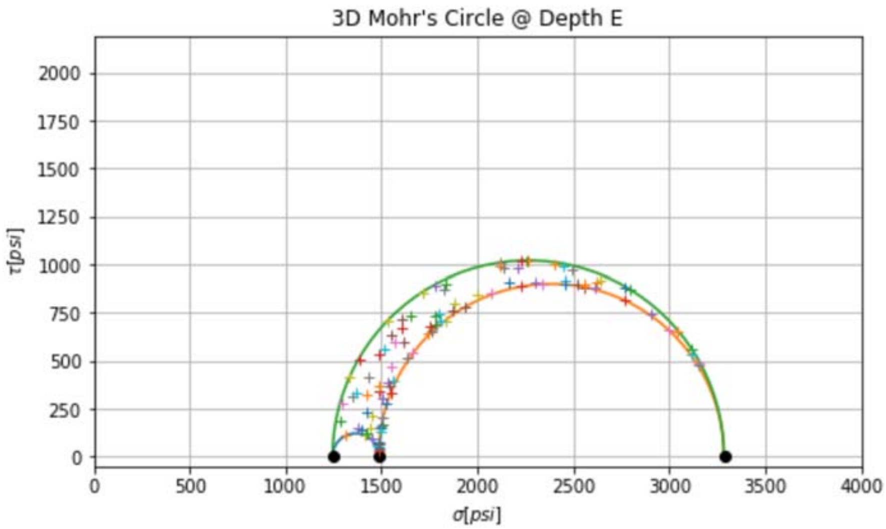

Part 1, Input the principal stress tensor at Depth E in the principal directions coordinate system and calculate the tensor in the geographical coordinate system.

In [9]:
```
#This task requires a change of basis from the principal coordinate system
#to the geographical system. This is a common task, so let us make a function
def rotationPG(alpha,beta,gamma,units = "degrees"):

    if units.lower().strip().find("deg")!=-1:
        alpha = np.deg2rad(alpha);
        beta = np.deg2rad(beta);
        gamma = np.deg2rad(gamma);

    #Easier than calling cos and sin function over and over
    cA = np.cos(alpha)
    sA = np.sin(alpha)
    cB = np.cos(beta)
    sB = np.sin(beta)
    cG = np.cos(gamma)
    sG = np.sin(gamma)

    return np.round(np.array([[cA*cB, sA*cB, -sB],
                    [(cA*sB*sG - sA*cG),(sA*sB*sG+cA*cG), cB*sG ],
                    [(cA*sB*cG + sA*sG),(sA*sB*cG-cA*sG), cB*cG ]]),3)

#We are told that the azimuth of Shmax is 90 deg
azimuthShmax = 90
#We found earlier that this is a Strike-Slip Regime, so alpha beta and
#gamma will be
alpha=azimuthShmax
beta=0
gamma=90

Rot_PG = rotationPG(alpha,beta,gamma)

#Shift Sp to Sg
Sg = np.dot(np.dot(Rot_PG.T,Sp),Rot_PG)
print("Total Stress Tensor in Geographical Coordinate System [psi]")
print()
print(Sg)
```

```
Total Stress Tensor in Geographical Coordinate System [psi]

[[ 9360.     0.     0.]
 [    0. 11400.     0.]
 [    0.     0.  9600.]]
```

Part 2, Generate 100 randomly distributed fracture orientations (strike and dip) and compute their effective normal stress and shear stress. Plot all in a σn-τ diagram together with the 3D Mohr circle(s).

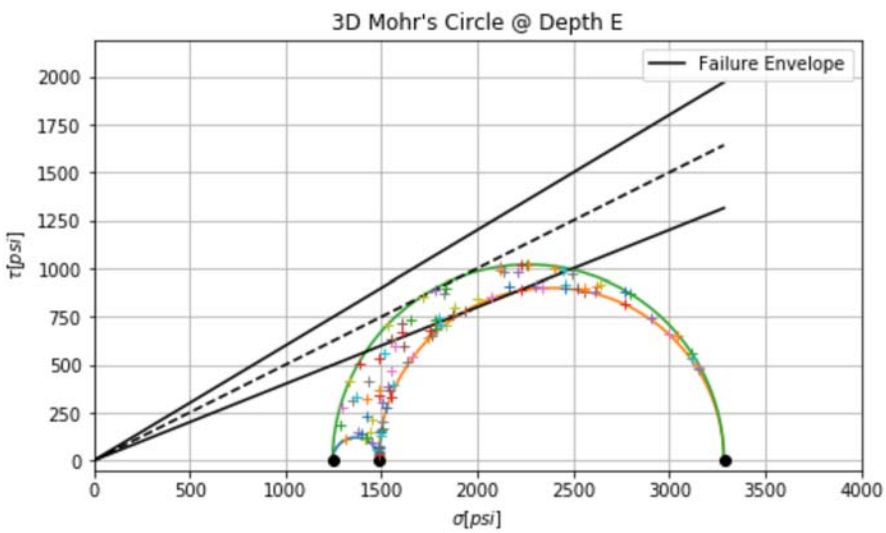
In [10]:
```
#This task will require a few helpful functions

#Step 1: generate 100 randomly distributed fracture orientations
#(strike and dip)
# Max dip angle = 90 deg
# Max stirke angle = 180 deg
numSamples = 100

strikeArray = np.random.randint(360, size= numSamples)
dipArray = np.random.randint(90, size=numSamples)

#Step 2: Compute effective normal stress and shear stress

def calcNVectors(strike,dip,units = "degrees"):

    if units.lower().strip().find("deg")!=-1:
        strike = np.deg2rad(strike)
        dip = np.deg2rad(dip)

    cS = np.cos(strike)
    sS = np.sin(strike)
    cD = np.cos(dip)
    sD = np.sin(dip)
    nDict = {}
    nDict['n'] = np.array([[-sS*sD, cS*sD, -cD]]).T
    nDict['s'] = np.array([[cS,sS,0]]).T
    nDict['d'] = np.array([[-sS*cD, cS*cD, sD]]).T

    return nDict


ax = plotMohr3D(a,b,c)


#Plot all the associated stress values on the Mohr's circle as + symbols
for i in range(0,len(dipArray)):
    nDict = calcNVectors(strikeArray[i],dipArray[i])
    t = np.dot(Sg,nDict['n'])
    Sn = np.dot(nDict['n'].T,t) #Total normal stress
    sign = Sn - Pp_E
    tau = np.sqrt(np.linalg.norm(t,2)**2 - Sn**2)
    ax.plot(sign,tau,'+')


# I am not going to lie, I thought it was really cool seeing
# all the randomly generated points land inside the 3D-Mohr's
# circle contour!
```




Part 3, Are there any fractures prone to shear slip (assume friction coefficient mu = 0.5 +/- 0.1)? What fractures are likely to be hydraulically conductive and which others are not?

In [12]:
```
#Let us plot the shear failure envelope

ax = plotMohr3D(a,b,c)

#Plot shear failure envelope

sigN_maxTau = a + (c-a) #Normal stress corresponding to maximum shear stress on Moh
rs circle

mu = 0.5 #friction coefficient
err = 0.1 #uncertainty in friction coefficient value
ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu+err)],'k-',label='Failure Envelope')
ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu)],'k--')
ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu-err)],'k-')


#Plot all the associated stress values on the Mohr's circle as + symbols
for i in range(0,len(dipArray)):
    nDict = calcNVectors(strikeArray[i],dipArray[i])
    t = np.dot(Sg,nDict['n'])
    Sn = np.dot(nDict['n'].T,t) #Total normal stress
    sign = Sn - Pp_E
    tau = np.sqrt(np.linalg.norm(t,2)**2 - Sn**2)
    ax.plot(sign,tau,'+')
ax.legend()
plt.xlim(0,4000)
```

Out[12]: (0, 4000)




**Any fracture within the failure envelope (so in the plot any + sign within the solid black lines) will be prone to shear slip.**

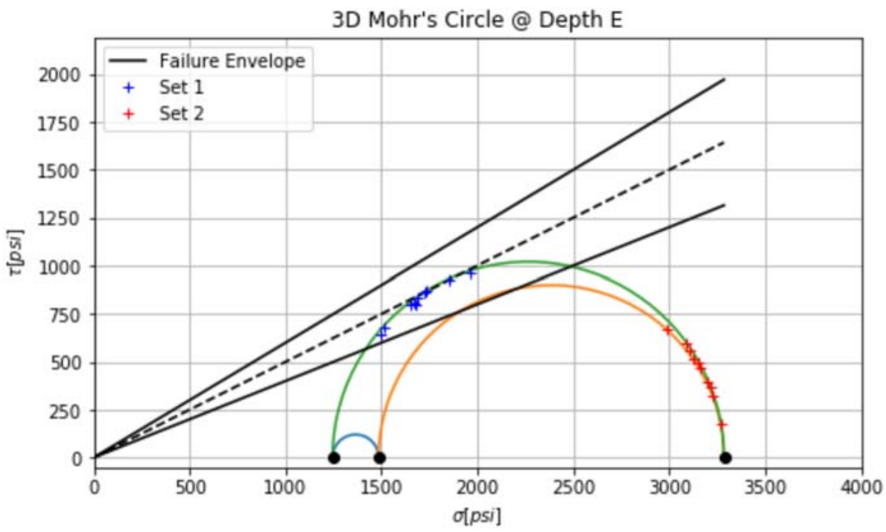
**Those fractures within the envelope (especially those close to the mean) will most likely be hydraulically conductive and those outside will probably not be.**

Part 4, Wellbore images actually show that there are two major sets of fractures:

- Set 1: strike = 60 deg +/- 5 deg, dip = 80 deg +/- 5 deg
- Set 2: strike = 10 deg +/- 5 deg, dip = 80 deg +/- 5 deg Plot results for 10 fractures for each set (errors represent one standard deviation with normal distribution). Which fracture set is more likely to slip in shear? Why?

In [14]:
```
#Bring back Mohr's circle
ax = plotMohr3D(a,b,c)

#Step 1: Sample 10 points from each normal distribution of strike and slip values

numSamples = 10
sigma = 5
set1StrikeArray = np.random.normal(60,sigma,numSamples)
set1DipArray = np.random.normal(80,sigma,numSamples)

set2StrikeArray = np.random.normal(10,sigma,numSamples)
set2DipArray = np.random.normal(80,sigma,numSamples)


ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu+err)],'k-',label='Failure Envelope')
ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu)],'k--')
ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu-err)],'k-')


#Step 2: Plot all the associated stress values on the Mohr's circle as + symbols
#(set 1 will be blue, set 2 will be red)

for i in range(0,len(set1DipArray)):
    nDict = calcNVectors(set1StrikeArray[i],set1DipArray[i])
    t = np.dot(Sg,nDict['n'])
    Sn = np.dot(nDict['n'].T,t) #Total normal stress
    sign = Sn - Pp_E
    tau = np.sqrt(np.linalg.norm(t,2)**2 - Sn**2)
    if i == 0:
        ax.plot(sign,tau,'b+',label='Set 1')
    else:
        ax.plot(sign,tau,'b+')

for i in range(0,len(set2DipArray)):
    nDict = calcNVectors(set2StrikeArray[i],set2DipArray[i])
    t = np.dot(Sg,nDict['n'])
    Sn = np.dot(nDict['n'].T,t) #Total normal stress
    sign = Sn - Pp_E
    tau = np.sqrt(np.linalg.norm(t,2)**2 - Sn**2)
    if i == 0:
        ax.plot(sign,tau,'r+',label='Set 2')
    else:
        ax.plot(sign,tau,'r+')
ax.legend()
plt.xlim(0,4000)
```

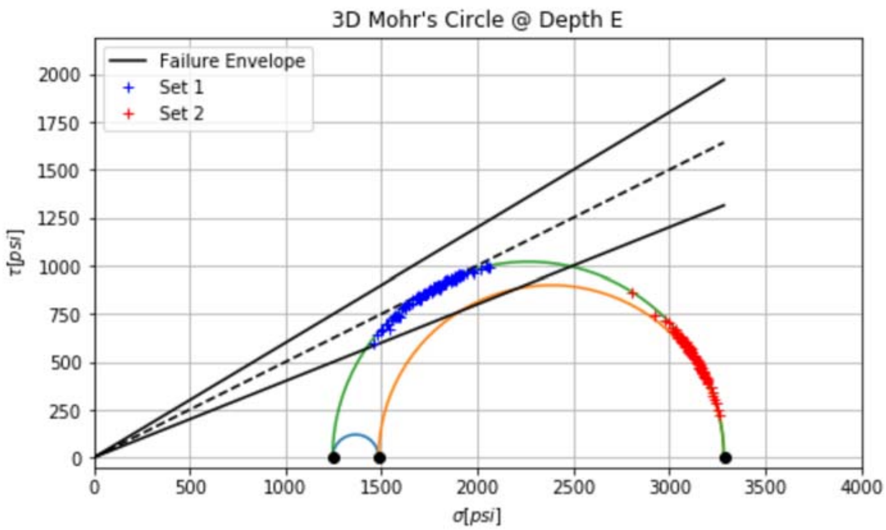
```
Out[14]: (0, 4000)
```




**Since set 1 falls under the shear failure envelope we setup earlier, it is likely to be the most hydraulically conductive and prone towards slip in shear.**

In [13]:
```
#Difficult to see at 10 samples, increasing to 100

#Bring back Mohr's circle
ax = plotMohr3D(a,b,c)

#Step 1: Sample 10 points from each normal distribution of strike and slip values

numSamples = 100
sigma = 5
set1StrikeArray = np.random.normal(60,sigma,numSamples)
set1DipArray = np.random.normal(80,sigma,numSamples)

set2StrikeArray = np.random.normal(10,sigma,numSamples)
set2DipArray = np.random.normal(80,sigma,numSamples)


ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu+err)],'k-',label='Failure Envelope')
ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu)],'k--')
ax.plot([0,sigN_maxTau ],[0,sigN_maxTau *(mu-err)],'k-')


#Step 2: Plot all the associated stress values on the Mohr's circle as + symbols
#(set 1 will be blue, set 2 will be red)

for i in range(0,len(set1DipArray)):
    nDict = calcNVectors(set1StrikeArray[i],set1DipArray[i])
    t = np.dot(Sg,nDict['n'])
    Sn = np.dot(nDict['n'].T,t) #Total normal stress
    sign = Sn - Pp_E
    tau = np.sqrt(np.linalg.norm(t,2)**2 - Sn**2)
    if i == 0:
        ax.plot(sign,tau,'b+',label='Set 1')
    else:
        ax.plot(sign,tau,'b+')

for i in range(0,len(set2DipArray)):
    nDict = calcNVectors(set2StrikeArray[i],set2DipArray[i])
    t = np.dot(Sg,nDict['n'])
    Sn = np.dot(t.T,nDict['n']) #Total normal stress
    sign = Sn - Pp_E
    tau = np.sqrt(np.linalg.norm(t,2)**2 - Sn**2)
    if i == 0:
        ax.plot(sign,tau,'r+',label='Set 2')
    else:
        ax.plot(sign,tau,'r+')
ax.legend()
plt.xlim(0,4000)
```

```
Out[13]: (0, 4000)
```


3D Mohr's Circle @ Depth E
Failure Envelope
Set 1
Set 2
2000
1750
1500
1250
1000
750
500
250
0
τ[psi]
0
500
1000
1500
2000
2500
3000
3500
4000
σ[psi]