Homework 7 Solutions

Problem 1

Download the file LostHills.xls. At every depth data point along the vertical well:

In [1]:
```python
import pandas as pd

excel_file = 'HW7.xlsx'
DataQ1Summary = pd.read_excel(excel_file,sheet_name="Q1")
DataQ1Summary.head(167)
```

Out[1]:

Depth [ft]	Depth [m]	Pp [psi]	Pp [MPa]	Density [g/cc]	Density [kg/m3]	dt-comp [us/ft]	dt-shear [us/ft]	φ [-]	vp [m/s]	...	Sh [psi]	σh [psi]	Shmax [psi]	σhmax [psi]	Shmax [MPa]	σhmax [MPa]	Shmin [psi]	σhmin [psi]	Shmin [MPa]	σhmin [MPa]
1750	533	700	4.826	1.87	1870	177.3271	477.1760	0.37	1719	...	1219	519	1592	892	10.980	6.154	1376	676	9.490	4.664
1755	535	702	4.840	1.86	1860	176.3417	479.4971	0.38	1728	...	1221	519	1589	887	10.970	6.130	1376	674	9.504	4.664
1760	536	704	4.854	1.85	1850	180.1919	481.2379	0.39	1692	...	1213	509	1574	870	10.854	6.000	1364	660	9.410	4.556
1765	538	706	4.868	1.86	1860	179.5510	482.3984	0.38	1698	...	1224	518	1586	880	10.951	6.083	1376	670	9.501	4.633
1770	539	708	4.881	1.87	1870	177.1933	472.7273	0.37	1720	...	1229	521	1607	899	11.085	6.204	1387	679	9.567	4.686
1775	541	710	4.895	1.83	1830	167.9760	473.3076	0.40	1815	...	1231	521	1606	896	11.087	6.192	1391	681	9.607	4.712
1780	543	712	4.909	1.86	1860	170.4306	481.2379	0.38	1788	...	1252	540	1621	909	11.189	6.280	1410	698	9.737	4.828
1785	544	714	4.923	1.77	1770	188.3189	469.2456	0.39	1619	...	1157	443	1512	798	10.436	5.513	1301	587	8.977	4.054
1790	546	716	4.937	1.88	1880	160.5894	455.5126	0.37	1898	...	1273	557	1689	973	11.667	6.730	1452	736	10.026	5.089
1795	547	718	4.950	1.86	1860	152.6213	475.8221	0.38	1997	...	1297	579	1685	967	11.626	6.676	1469	751	10.136	5.186
1800	549	720	4.964	1.77	1770	176.1595	494.9710	0.39	1730	...	1212	492	1544	824	10.656	5.692	1354	634	9.346	4.382
1805	550	722	4.978	1.66	1660	194.9070	498.6460	0.47	1564	...	1122	400	1419	697	9.791	4.813	1243	521	8.583	3.605
1810	552	724	4.992	1.62	1620	188.1376	458.4139	0.50	1620	...	1086	362	1423	699	9.824	4.832	1221	497	8.428	3.436
1815	553	726	5.006	1.71	1710	178.8383	485.8801	0.43	1704	...	1177	451	1506	780	10.391	5.385	1316	590	9.081	4.075
1820	555	728	5.019	1.60	1600	240.9146	515.6673	0.51	1265	...	1028	300	1275	547	8.800	3.781	1117	389	7.709	2.690
1825	556	730	5.033	1.49	1490	245.7571	536.7505	0.55	1240	...	989	259	1204	474	8.309	3.276	1068	338	7.371	2.338
1830	558	732	5.047	1.42	1420	231.4573	529.0136	0.61	1317	...	975	243	1190	458	8.218	3.171	1057	325	7.300	2.253
1835	559	750	5.171	1.41	1410	256.2586	500.7737	0.62	1189	...	926	176	1144	394	7.892	2.721	996	246	6.872	1.701
1840	561	768	5.295	1.45	1450	245.5023	478.5300	0.58	1242	...	952	184	1197	429	8.260	2.965	1031	263	7.114	1.819
1845	562	787	5.426	1.61	1610	256.7654	494.0038	0.50	1187	...	1016	229	1270	483	8.758	3.332	1096	309	7.562	2.136
1850	564	805	5.550	1.58	1580	260.8673	503.0948	0.52	1168	...	1018	213	1258	453	8.678	3.128	1094	289	7.547	1.997
1855	565	823	5.674	1.52	1520	259.3640	497.6789	0.53	1175	...	1005	182	1240	417	8.551	2.877	1079	256	7.441	1.767
1860	567	841	5.798	1.55	1550	232.1263	513.1528	0.55	1313	...	1081	240	1327	486	9.160	3.362	1172	331	8.093	2.295
1865	568	863	5.950	1.52	1520	233.8973	481.0445	0.53	1303	...	1055	192	1318	455	9.096	3.146	1146	283	7.905	1.955
1870	570	861	5.936	1.48	1480	239.5141	466.1509	0.56	1273	...	1020	159	1284	423	8.860	2.924	1105	244	7.626	1.690
1875	572	859	5.923	1.75	1750	221.3747	484.7195	0.40	1377	...	1186	327	1496	637	10.328	4.405	1300	441	8.978	3.055
1880	573	857	5.909	1.60	1600	202.2344	488.0078	0.51	1507	...	1149	292	1441	584	9.949	4.040	1265	408	8.730	2.821
1885	575	855	5.895	1.54	1540	222.9948	496.1315	0.51	1367	...	1095	240	1357	502	9.372	3.477	1193	338	8.239	2.344
1890	576	853	5.881	1.61	1610	204.1745	484.9130	0.50	1493	...	1153	300	1449	596	9.998	4.117	1269	416	8.756	2.875
1895	578	852	5.874	1.63	1630	208.1765	490.5222	0.49	1464	...	1162	310	1454	602	10.039	4.165	1276	424	8.812	2.938
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2435	742	1354	9.336	1.78	1780	165.3772	381.2379	0.38	1843	...	1680	326	2203	849	15.198	5.862	1881	527	12.978	3.642
2440	744	1356	9.349	1.72	1720	159.7679	351.8375	0.42	1908	...	1627	271	2206	850	15.225	5.876	1841	485	12.709	3.360
2445	745	1358	9.363	1.78	1780	161.1219	390.3288	0.38	1892	...	1704	346	2214	856	15.277	5.914	1907	549	13.158	3.795
2450	747	1360	9.377	1.78	1780	169.0522	360.9284	0.38	1803	...	1657	297	2217	857	15.302	5.925	1859	499	12.830	3.453
2455	748	1362	9.391	1.77	1770	162.6693	355.5126	0.39	1874	...	1665	303	2246	884	15.495	6.104	1879	517	12.961	3.570
2460	750	1364	9.404	1.72	1720	176.4023	395.7447	0.42	1728	...	1646	282	2108	744	14.551	5.147	1820	456	12.562	3.158
2465	751	1366	9.418	1.88	1880	175.2418	407.7369	0.37	1739	...	1769	403	2254	888	15.547	6.129	1956	590	13.495	4.077
2470	753	1368	9.432	1.80	1800	184.3327	402.3211	0.42	1654	...	1691	323	2153	785	14.858	5.426	1861	493	12.842	3.410
2475	754	1370	9.446	1.80	1800	192.2631	397.2921	0.42	1585	...	1667	297	2126	756	14.663	5.217	1826	456	12.599	3.153
2480	756	1372	9.460	1.76	1760	186.6538	387.4275	0.40	1633	...	1650	278	2123	751	14.650	5.190	1815	443	12.525	3.065
2485	757	1374	9.473	1.65	1650	180.2708	392.2631	0.47	1691	...	1605	231	2050	676	14.142	4.669	1768	394	12.199	2.726
2490	759	1376	9.487	1.93	1930	183.7524	394.3907	0.33	1659	...	1774	398	2285	909	15.764	6.277	1959	583	13.516	4.029

2495	760	1378	9.501	1.88	1880	170.7930	385.2676	0.37	1785	...	1775	397	2310	932	15.933	6.432	1977	599	13.639	4.138
2500	762	1380	9.515	1.90	1900	196.1315	406.1896	0.35	1554	...	1741	361	2205	825	15.214	5.699	1903	523	13.130	3.615
2505	764	1382	9.529	1.85	1850	186.0735	400.5803	0.39	1638	...	1737	355	2212	830	15.264	5.735	1909	527	13.175	3.646
2510	765	1384	9.542	1.90	1900	173.5010	392.6499	0.35	1757	...	1800	416	2321	937	16.013	6.471	1997	613	13.782	4.240
2515	767	1386	9.556	1.90	1900	164.7969	368.6654	0.35	1850	...	1796	410	2384	998	16.453	6.897	2016	630	13.920	4.364
2520	768	1388	9.570	1.91	1910	156.0928	385.4932	0.35	1953	...	1856	468	2421	1033	16.707	7.137	2083	695	14.377	4.807
2525	770	1390	9.584	1.88	1880	185.2998	394.9710	0.37	1645	...	1762	372	2257	867	15.577	5.993	1940	550	13.393	3.809
2530	771	1392	9.598	1.83	1830	165.7640	393.8104	0.40	1839	...	1787	395	2297	905	15.850	6.252	1987	595	13.712	4.114
2535	773	1394	9.611	1.89	1890	170.4062	399.6132	0.36	1789	...	1828	434	2337	943	16.127	6.516	2026	632	13.982	4.371
2540	774	1396	9.625	1.88	1880	170.4062	399.4197	0.37	1789	...	1824	428	2330	934	16.076	6.451	2021	625	13.943	4.318
2545	776	1398	9.639	1.93	1930	163.0561	381.0445	0.33	1869	...	1860	462	2431	1033	16.777	7.138	2082	684	14.369	4.730
2550	777	1400	9.653	1.96	1960	168.6654	359.9613	0.32	1807	...	1828	428	2448	1048	16.891	7.238	2051	651	14.150	4.497
2555	779	1402	9.666	1.84	1840	170.8897	334.6228	0.39	1784	...	1706	304	2344	942	16.175	6.509	1913	511	13.199	3.533
2560	780	1404	9.680	1.76	1760	158.4139	382.7853	0.40	1924	...	1764	360	2287	883	15.779	6.099	1972	568	13.605	3.925
2565	782	1406	9.694	1.94	1940	166.7311	397.2921	0.33	1828	...	1891	485	2423	1017	16.719	7.025	2100	694	14.494	4.800
2570	783	1408	9.708	1.95	1950	174.6615	402.7079	0.32	1745	...	1883	475	2396	988	16.530	6.822	2080	672	14.351	4.643
2575	785	1410	9.722	1.91	1910	173.6944	413.5396	0.35	1755	...	1876	466	2359	949	16.282	6.560	2066	656	14.260	4.538
2580	786	1412	9.735	1.95	1950	165.1837	368.6654	0.32	1845	...	1870	458	2473	1061	17.060	7.325	2095	683	14.457	4.722

a) Compute total vertical stress as a function of depth (you may assume homogeneous rock above 1750 ft), and compute overpressure parameter.

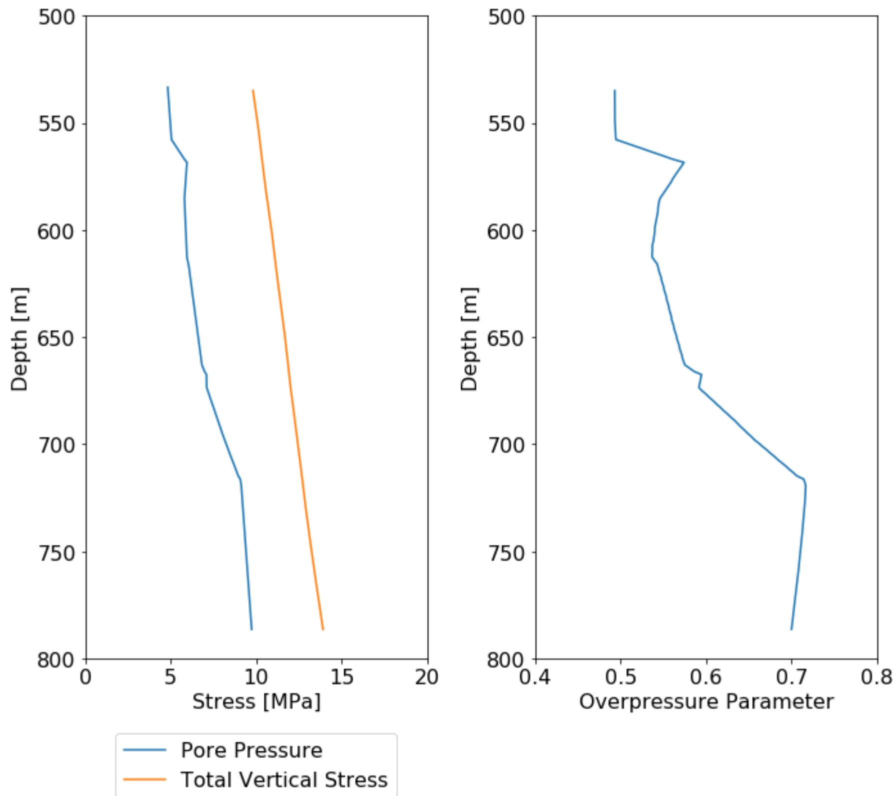
Total vertical stress can be found through the integration of bulk rock densities from the surface to the depth of interest, z

$$S_v = \int_0^z \rho g z \, \mathrm{d}z$$

Summing the overburden stress of each depth interval below the depth of interest will give the total vertical stress at that depth

In [4]:
```
import numpy as np
import matplotlib.pyplot as plt

Depth = DataQ1Summary['Depth [ft]'] * 0.3048 # [ft] -> [m]
Depth1 = DataQ1Summary['Depth [ft]'] # [ft]
Pp = DataQ1Summary['Pp [psi]'] * 0.00689476 # [psi] -> [MPa]
rho = DataQ1Summary['Density [g/cc]'] * 1000 # [g/cc] -> [kg/m3]
rho1 = DataQ1Summary['Density [g/cc]']
gravity = 9.81 # [m/s2]

# We are told the rock above 1750 ft is homogeneous, so we can calculate the overburden up till 1750 ft.
dZ = np.diff(np.linspace(0,Depth.iloc[0]))
offsetSv = np.sum(dZ*gravity*rho.iloc[0])
deltaZ = Depth.diff(1)
Sv = offsetSv + np.cumsum(gravity*rho*deltaZ) # Total Vertical Stress [Pa]
Sv = Sv*(1e-6) # [Pa] -> [MPa]
Sv1 = Sv*145.038 # [MPa] -> [psi]

# Calculate Overpressure Parameter [-]
lambdaP = Pp/Sv

# Figure size
fig = plt.figure(figsize=(10,9))

# Plot Sv and Pp
ax = fig.add_subplot(121)
ax.plot(Pp,Depth,label = 'Pore Pressure')
ax.plot(Sv,Depth,label = 'Total Vertical Stress')
# Plot labels
ax.set_xlabel("Stress [MPa]")
ax.set_ylabel("Depth [m]")
ax.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
# Axis range
plt.xlim([0, 20])
plt.ylim([800, 500])
```

```python
# Plot Overpressure
ax2 = fig.add_subplot(122)
ax2.plot(lambdaP,Depth)
# Plot labels
ax2.set_xlabel("Overpressure Parameter")
ax2.set_ylabel("Depth [m]")
# Axis range
plt.xlim([0.4, 0.8])
plt.ylim([800, 500])

# Set font size
plt.rcParams.update({'font.size': 16})
# Layout
plt.tight_layout()
```



b) Compute dynamic Poisson's ratio and dynamic Young's modulus from compressive and shear slowness (be careful with unit conversion).

Dynamic Poisson's ratio (v) can be calculated using the below formula.

$$v = \frac{(V_p^2 - 2V_s^2)}{2(V_p^2 - V_s^2)} = \frac{[(V_p/V_s)^2 - 2]}{2[(V_p/V_s)^2 - 1]}$$

P-wave velocity (V_p) and S-wave velocity (V_s) [m/s] can be calculated from the slowness (dt) [$\mu s/ft$]

$$V = \frac{10^6}{dt} * 0.3048$$

Dynamic Young's modulus (E) can be calculated using the below formulas, where K is the bulk modulus and G is the shear modulus. Density (ρ) in [kg/m^3] and velocities in [m/s] will give you modulus in units of [Pa]

$$E = \frac{9KG}{(3K + G)}$$

$$G = \rho V_s^2$$

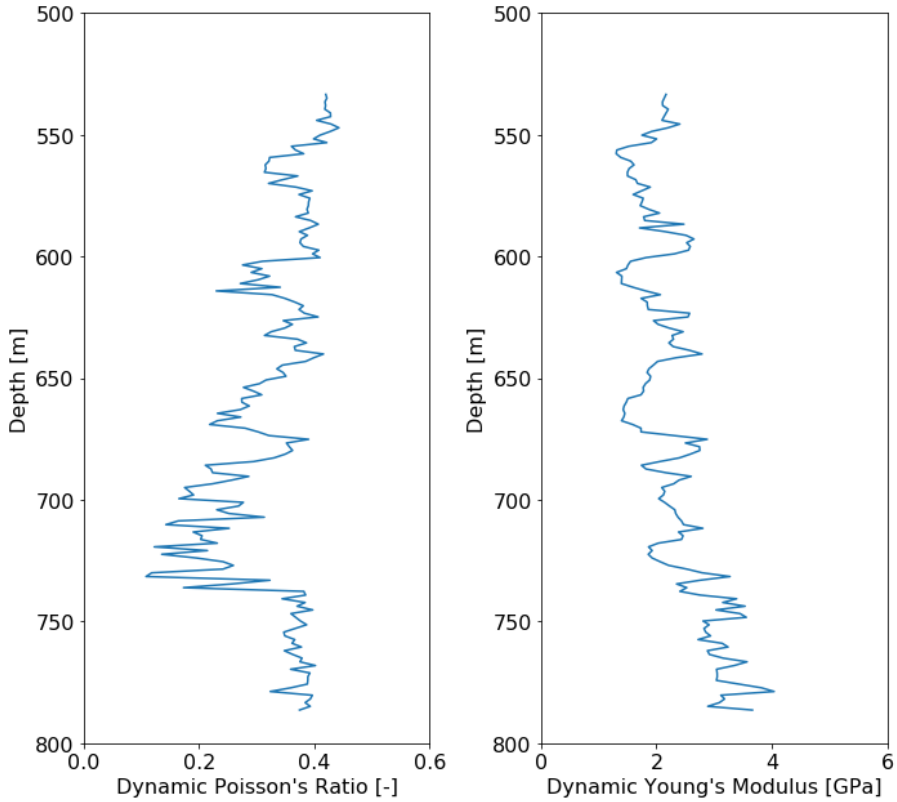
$$K = \rho(V_p^2 - \frac{4}{3}V_s^2)$$

Dynamic Young's modulus (E) can also be calculated using the below formulas

$$E = \rho V_p^2 \frac{(1+v)(1-2v)}{(1-v)}$$

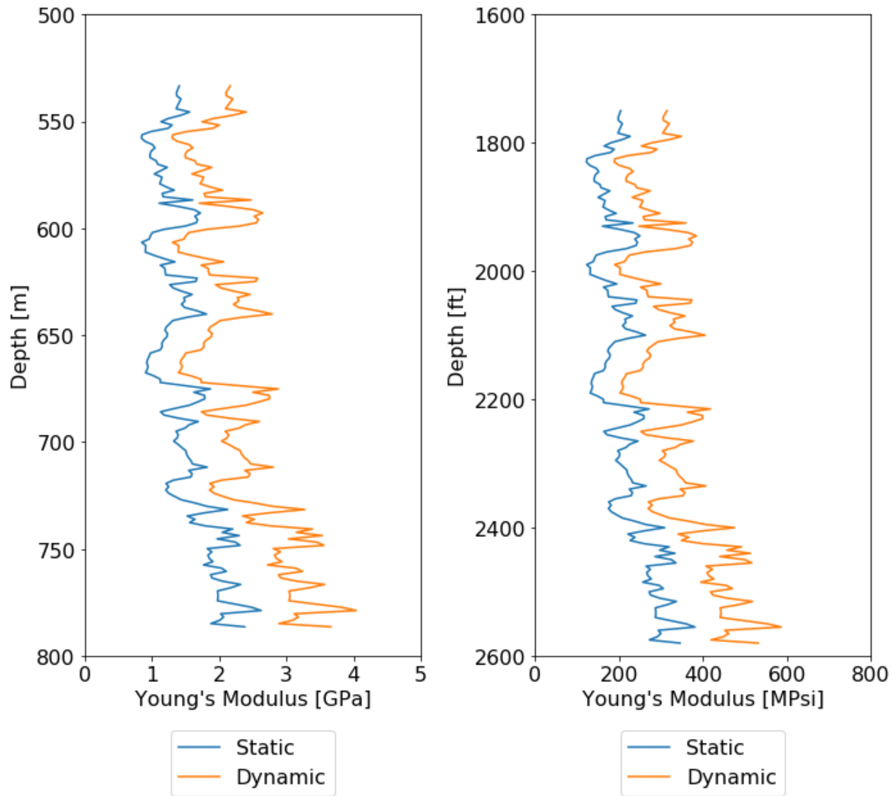
$$E = \rho V_s^2 \frac{[3(V_p/V_s)^2 - 4]}{[(V_p/V_s)^2 - 1]}$$

In [5]:
```
Vp = 1/(DataQ1Summary['dt-comp [us/ft]'] * (1e-6)/0.3048) # P-wave Slowness [us/ft] -> P-Wave Velocity [km/s]
Vs = 1/(DataQ1Summary['dt-shear [us/ft]'] * (1e-6)/0.3048) # S-wave Slowness [us/ft] -> S-Wave Velocity [km/s]

M = (Vp**2)*rho1 # Constrained Modulus []
G = (Vs**2)*rho1

v_Dyn = (Vp**2 - 2*(Vs**2))/(2*(Vp**2 - Vs**2)) # Dynamic Poisson's Ratio [-]
E_Dyn = rho*(Vs**2)*((3*(Vp**2) - 4*(Vs**2))/((Vp**2) - (Vs**2)))*1e-9 # Dynamic Young's Modulus [GPa]

# Figure size
fig = plt.figure(figsize=(10,9))

# Plot v
ax = fig.add_subplot(121)
ax.plot(v_Dyn,Depth)
# Plot labels
ax.set_xlabel("Dynamic Poisson's Ratio [-]")
ax.set_ylabel("Depth [m]")
# Axis range
plt.xlim([0, 0.6])
plt.ylim([800, 500])

# Plot E
ax1 = fig.add_subplot(122)
ax1.plot(E_Dyn,Depth)
# Plot labels
ax1.set_xlabel("Dynamic Young's Modulus [GPa]")
ax1.set_ylabel("Depth [m]")
# Axis range
plt.xlim([0, 6])
plt.ylim([800, 500])

# Set font size
plt.rcParams.update({'font.size': 16})
# Layout
plt.tight_layout()
```



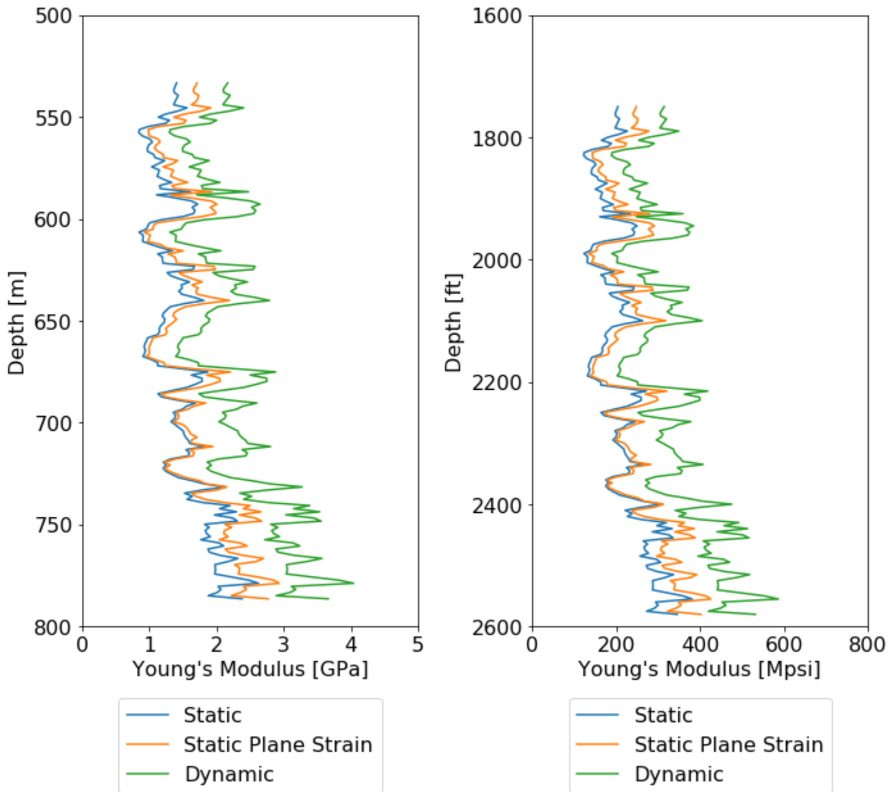
c) Compute static Young's modulus using a coefficient $E_{static} = 0.65 \times E_{dynamic}$

In [6]:
```
E_Static = E_Dyn*0.65 # [GPa]
E_Static1 = E_Static *145038 /1000 # [Mpsi]

# Figure size
fig = plt.figure(figsize=(10,9))

# Plot E in SI
ax = fig.add_subplot(121)
ax.plot(E_Static,Depth,label = "Static")
ax.plot(E_Dyn,Depth,label = "Dynamic")
# Plot labels
ax.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax.set_xlabel("Young's Modulus [GPa]")
ax.set_ylabel("Depth [m]")
# Axis range
plt.xlim([0, 5])
plt.ylim([800, 500])

# Plot E in Field
ax1 = fig.add_subplot(122)
ax1.plot(E_Static1,Depth1,label = "Static")
ax1.plot(E_Dyn1,Depth1,label = "Dynamic")
# Plot labels
ax1.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax1.set_xlabel("Young's Modulus [MPsi]")
ax1.set_ylabel("Depth [ft]")
# Axis range
plt.xlim([0, 800])
plt.ylim([2600, 1600])

# Set font size
plt.rcParams.update({'font.size': 16})
# Layout
plt.tight_layout()
```



d) Compute static plane strain modulus $E'_{static} = E_{static}/(1 - \nu^2)$ (Poisson ratio remains the same with depth).

In [7]:
```
E_Static_Plane = E_Static/(1-v_Dyn**2) # [GPa]
E_Static_Plane1 = E_Static_Plane *145038 /1000 # [Mpsi]

# Figure size
fig = plt.figure(figsize=(10,9))

# Plot in SI
ax = fig.add_subplot(121)
ax.plot(E_Static,Depth,label = "Static")
ax.plot(E_Static_Plane,Depth,label = "Static Plane Strain")
ax.plot(E_Dyn,Depth,label = "Dynamic")
# Plot labels
ax.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax.set_xlabel("Young's Modulus [GPa]")
ax.set_ylabel("Depth [m]")
# Axis range
plt.xlim([0, 5])
plt.ylim([800, 500])

# Plot in Field
ax1 = fig.add_subplot(122)
ax1.plot(E_Static1,Depth1,label = "Static")
ax1.plot(E_Static_Plane1,Depth1,label = "Static Plane Strain")
ax1.plot(E_Dyn1,Depth1,label = "Dynamic")
# Plot labels
ax1.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax1.set_xlabel("Young's Modulus [Mpsi]")
ax1.set_ylabel("Depth [ft]")
# Axis range
plt.xlim([0, 800])
plt.ylim([2600, 1600])

# Set font size
plt.rcParams.update({'font.size': 16})
# Layout
plt.tight_layout()
```



e) Compute total maximum and minimum horizontal stress assuming theory of elasticity and $\epsilon_{Hmax} = 0.0015$ and $\epsilon_{hmin} = 0$.

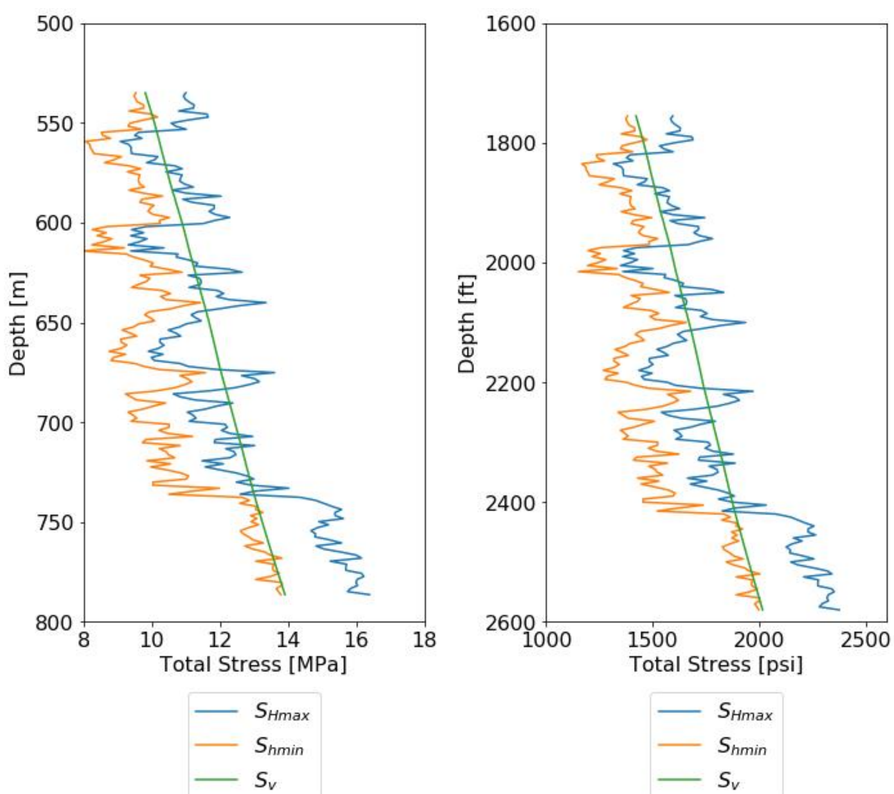
The maximum effective horizontal stress is given by

$$\sigma_{Hmax} = \frac{E}{1-\nu^2}\epsilon_{Hmax} + \frac{\nu E}{1-\nu^2}\epsilon_{Hmin} + \frac{\nu}{1-\nu}\sigma_V$$

The minimum effective horizontal stresses is given by

$$\sigma_{Hmin} = \frac{\nu E}{1-\nu^2}\epsilon_{Hmax} + \frac{E}{1-\nu^2}\epsilon_{Hmin} + \frac{\nu}{1-\nu}\sigma_V$$

In [8]:
```python
eHmax = 0.0015
ehmin = 0
Ep = E_Static_Plane*(1e3) # [GPa] -> [MPa]
sigV = Sv - Pp # [MPa]
sigHmax = Ep*eHmax + v_Dyn*Ep*ehmin + sigV*(v_Dyn/(1-v_Dyn)) # [MPa]
sighmin = v_Dyn*Ep*eHmax + Ep*ehmin + sigV*(v_Dyn/(1-v_Dyn)) # [MPa]
SHmax = sigHmax + Pp # [MPa]
Shmin = sighmin + Pp # [MPa]
SHmax1 = SHmax *145.038 # [psi]
Shmin1 = Shmin *145.038 # [psi]

# Figure size
fig = plt.figure(figsize=(10,9))

# Plot in SI
ax = fig.add_subplot(121)
ax.plot(SHmax,Depth,label = "$S_{Hmax}$")
ax.plot(Shmin,Depth,label = "$S_{hmin}$")
ax.plot(Sv,Depth,label = "$S_{v}$")
# Plot labels
ax.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax.set_xlabel("Total Stress [MPa]")
ax.set_ylabel("Depth [m]")
# Axis range
plt.xlim([8, 18])
plt.ylim([800, 500])

# Plot in Field
ax1 = fig.add_subplot(122)
ax1.plot(SHmax1,Depth1,label = "$S_{Hmax}$")
ax1.plot(Shmin1,Depth1,label = "$S_{hmin}$")
ax1.plot(Sv1,Depth1,label = "$S_{v}$")
# Plot labels
ax1.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax1.set_xlabel("Total Stress [psi]")
ax1.set_ylabel("Depth [ft]")
# Axis range
plt.xlim([1000, 2600])
plt.ylim([2600, 1600])

# Set font size
plt.rcParams.update({'font.size': 16})
# Layout
plt.tight_layout()
```



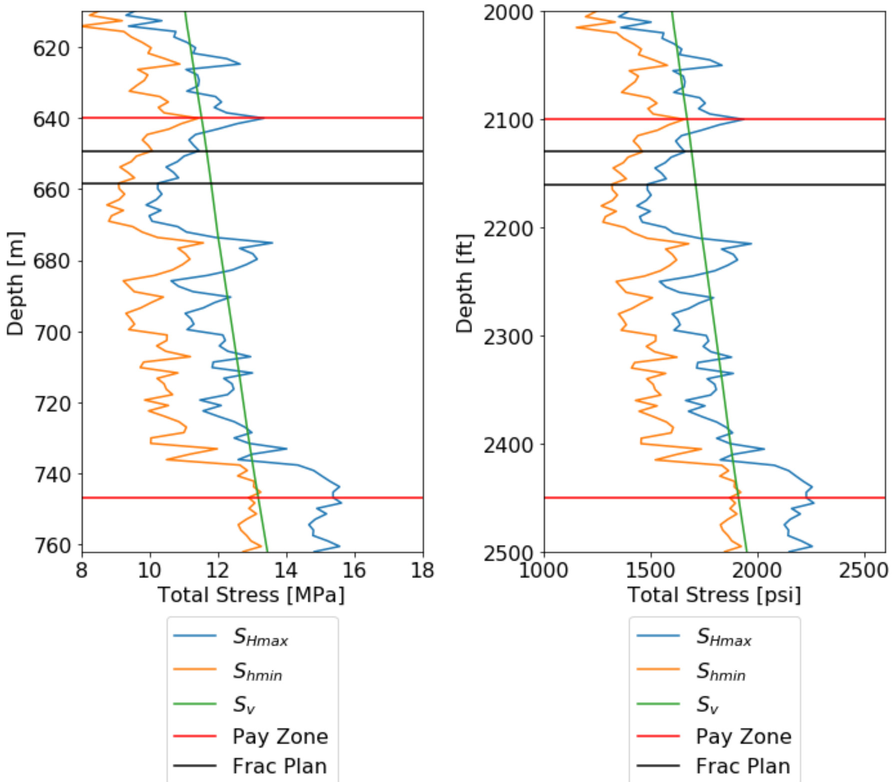
f) The pay-zone is between 2,100 ft and 2,450 ft. A hydraulic fracture is planned to be executed with a vertical well at a depth between 2,130 ft and 2,160 ft. What will be the height of this fracture? Will it reach out to the entire pay zone?

A hydraulic fracture will open perpendicular to least principal stress. Since for the pay zone of interest we have the case of $S_V > S_{Hmax} > S_{hmin}$, we can expect hydraulic fractures to open up perpendicular to S_{Hmin}.

If a fracture is initiated between 2130 ft and 2160 ft (black boundary lines), the fracture will expand up and down until it reaches the region where the horizontal stress is significantly higher (local peaks at approximately 2100 and 2210 ft), making the height of the fracture a maximum of ~110ft. A second set of perforations between 2300 and 2400 ft may be needed to extend the hydraulic fracture through the entire pay zone

In [9]:

```
#Payzone
payUpperBound1 = 2100 # [ft]
payUpperBound = 2100 * 0.3048 # [m]
payLowerBound1 = 2450 # [ft]
payLowerBound = 2450 * 0.3048 # [m]

#Fracture Plan
fracUpperBound1 = 2130 # [ft]
fracUpperBound = 2130 * 0.3048 # [m]
fracLowerBound1 = 2160 # [ft]
fracLowerBound = 2160 * 0.3048 # [m]

# Figure size
fig = plt.figure(figsize=(10,9))

# Plot in SI
ax = fig.add_subplot(121)
ax.plot(SHmax,Depth,label = "$S_{Hmax}$")
ax.plot(Shmin,Depth,label = "$S_{hmin}$")
ax.plot(Sv,Depth,label = "$S_{v}$")
ax.plot([0,18e6],[payUpperBound,payUpperBound],'r',label='Pay Zone')
ax.plot([0,18e6],[payLowerBound,payLowerBound],'r')
ax.plot([0,18e6],[fracUpperBound,fracUpperBound],'k',label='Frac Plan')
ax.plot([0,18e6],[fracLowerBound,fracLowerBound],'k')
# Plot labels
ax.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax.set_xlabel("Total Stress [MPa]")
ax.set_ylabel("Depth [m]")
# Axis range
plt.xlim([8, 18])
plt.ylim([762, 610])

# Plot in Field
ax1 = fig.add_subplot(122)
ax1.plot(SHmax1,Depth1,label = "$S_{Hmax}$")
ax1.plot(Shmin1,Depth1,label = "$S_{hmin}$")
ax1.plot(Sv1,Depth1,label = "$S_{v}$")
ax1.plot([0,2600],[payUpperBound1,payUpperBound1],'r',label='Pay Zone')
ax1.plot([0,2600],[payLowerBound1,payLowerBound1],'r')
ax1.plot([0,2600],[fracUpperBound1,fracUpperBound1],'k',label='Frac Plan')
ax1.plot([0,2600],[fracLowerBound1,fracLowerBound1],'k')
# Plot labels
ax1.legend(loc=9, bbox_to_anchor=(0.5, -0.1), ncol=1)
ax1.set_xlabel("Total Stress [psi]")
ax1.set_ylabel("Depth [ft]")
# Axis range
plt.xlim([1000, 2600])
plt.ylim([2500, 2000])

# Set font size
plt.rcParams.update({'font.size': 16})
# Layout
plt.tight_layout()
```



Depth [m]
620
640
660
680
700
720
740
760
8
10
12
14
16
18
Total Stress [MPa]
Depth [ft]
2000
2100
2200
2300
2400
2500
1000
1500
2000
2500
Total Stress [psi]
S_{Hmax}
S_{hmin}
S_v
Pay Zone
Frac Plan

Problem 2

A single hydraulic fracture treatment will be performed in a tight sandstone. The hydraulic fracture height is expected to be $h_f = 170ft$. The tight sandstone has a plane-strain modulus E' = 8.9 MMpsi. The (two-wing) injection rate will be 50 bbl/min (constant) during 1 hour. The fracturing fluid has a viscosity 2 cP. Compute:

In [10]:
```python
import pandas as pd

excel_file = 'HW7.xlsx'
DataQ2Summary = pd.read_excel(excel_file,sheet_name="Q2")
DataQ2Summary.head(11)
```

Out[10]:

	Parameter	Symbol	Value	-	t [min]	t [sec]	xf [m]	ww,0 [mm]	Pnet [MMPa]
0	Fracture Half Length	hf [ft]	170.000000	-	0.00	0	0	0.000	0.000
1	Plane-Strain Modulus	E' [MMpsi]	8.900000	-	0.25	15	19	1.608	0.952
2	Fracturing Fluid Viscosity	mu [cP]	2.000000	-	1.00	60	58	2.122	1.256
3	One-Wing Injection Rate	i [bbl/min]	25.000000	-	2.50	150	120	2.548	1.509
4	Injection Period	te [hour]	1.000000	-	5.00	300	209	2.927	1.733
5	Fracture Half Length	hf [m]	51.816000	-	10.00	600	363	3.363	1.991
6	Plane-Strain Modulus	E' [MMPa]	61363.364000	-	20.00	1200	633	3.863	2.287
7	Fracturing Fluid Viscosity	mu [Pas]	0.002000	-	30.00	1800	875	4.189	2.480
8	One-Wing Injection Rate	i [m3/s]	0.066243	-	40.00	2400	1102	4.437	2.627
9	Injection Period	te [s]	3600.000000	-	50.00	3000	1317	4.640	2.747
10	2-Wing Fracture Volume	2Vf [m3]	477.512048	-	60.00	3600	1524	4.812	2.849

a) The expected fracture half-length x_f, fracture width at the wellbore $w_{w,0}$, and net pressure p_n as a function of time with the PKN model (no leak-off).

Fracture half-length x_f can be calculated:

$$x_f = 0.524\left(\frac{i^3 E'}{\mu h_f^4}\right)^{1/5} t^{4/5}$$

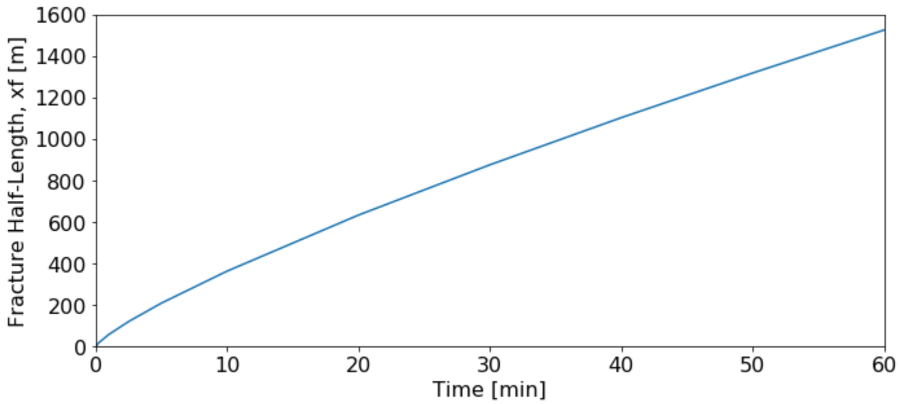
Fracture width at the wellbore $w_{w,0}$ can be calculated:

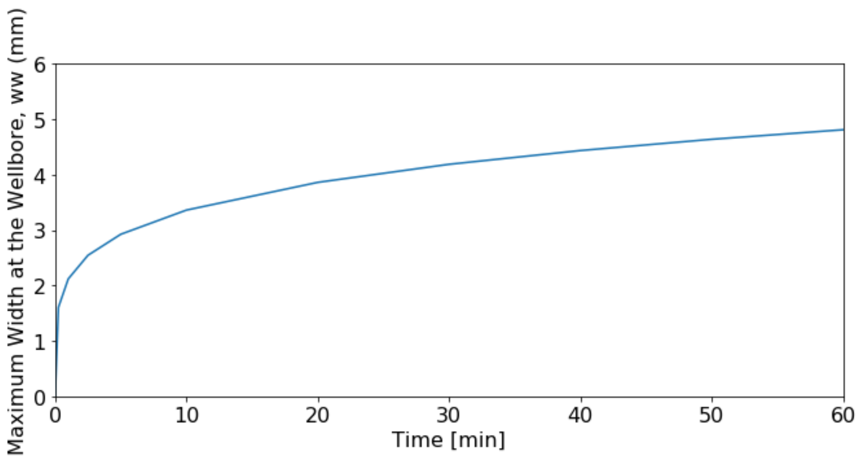
$$w_{w,0} = 3.04\left(\frac{i^2 \mu}{E' h_f}\right)^{1/5} t^{1/5}$$

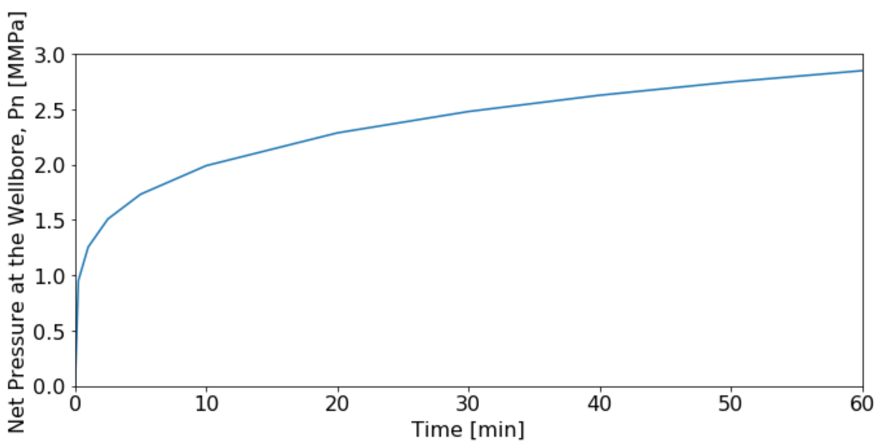
Net pressure P_n can be calculated:

$$P_n = 1.52\left(\frac{E'^4 i^2 \mu}{h_f^6}\right)^{1/5} t^{1/5}$$

In [11]:
```
import numpy as np
import matplotlib.pyplot as plt

Time = DataQ2Summary['t [min]']
xf = DataQ2Summary['xf [m]']
ww = DataQ2Summary['ww,0 [mm]']
Pnet = DataQ2Summary['Pnet [MMPa]']

# Figure size
fig = plt.figure(figsize=(10,15))

# Plot Fracture Half-Length
ax = fig.add_subplot(311)
ax.plot(Time,xf)
# Plot labels
ax.set_xlabel("Time [min]")
ax.set_ylabel("Fracture Half-Length, xf [m]")
# Axis range
plt.xlim([0, 60])
plt.ylim([0, 1600])

# Plot Maximum Width at the Wellbore
ax = fig.add_subplot(312)
ax.plot(Time,ww)
# Plot labels
ax.set_xlabel("Time [min]")
ax.set_ylabel("Maximum Width at the Wellbore, ww (mm)")
# Axis range
plt.xlim([0, 60])
plt.ylim([0, 6])

# Plot Fracture Half-Length
ax = fig.add_subplot(313)
ax.plot(Time,Pnet)
# Plot labels
ax.set_xlabel("Time [min]")
ax.set_ylabel("Net Pressure at the Wellbore, Pn [MMPa]")
# Axis range
plt.xlim([0, 60])
plt.ylim([0, 3])

# Set font size
plt.rcParams.update({'font.size': 16})
# Layout
plt.tight_layout()
```







b) The total amount of water (volume) and sand (weight) required assuming a constant volume ratio 90% water-10% sand. How many water swimming pools (100,000 L) and sand trucks (10 metric tons) are needed to complete the hydraulic fracturing job?

Fracture volume for one-wing is given by:

$$v_f = \overline{w} x_f h_f$$

Where for the PKN model

$$\overline{w} = \frac{\pi}{5} w_{w,0}$$

Two-wing fracture volume 2Vf = 477 m^3

90% water volume = 429 m^3 = 4.3 swimming pools

10% sand volume = 48 m^3 = 126.4 metric tons = 12.6 trucks

Problem 3

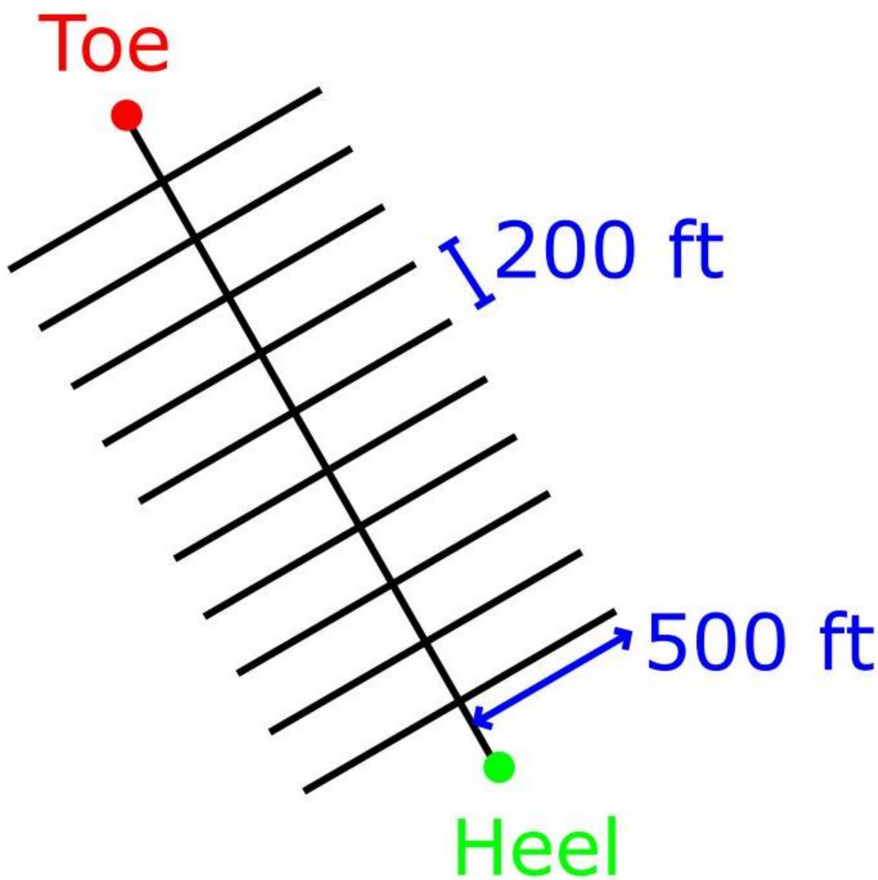
Consider the design of a completion job with horizontal wellbores and multistage hydraulic fracturing in the Barnett shale at 8,200 ft with pore pressure of 4100 psi (NF).

a) What is the direction of horizontal wellbores that maximize drainage area using multistage fractures? (You may need to check the US stress map)

Wells must be drilled in the direction of S3 to maximize drainage surface area from multistage hydraulic fracturing.

In the Barnett shale, a normal faulting stress environment prevails with SHmax striking at ~N60°E. Hence, wellbores should be drilled at an azimuth of N30°W or S30°E.

b) Sketch a horizontal well with 10 fracture stages spaced every 200ft from a top view. Fracture half-length is ~500ft.



c) Calculate a lower bound (absolute minimum) of the pressure needed to propagate hydraulic fractures using the theory of elasticity (v=0.25) and limit equilibrium of faults (q~3.5). Perforations will be done, so you may ignore the effect of near-wellbore stresses.

The minimum pressure to open a hydraulic fracture is $P = S_3(P_{net} = 0)$. Thus, the problem is asking for Shmin.

Assuming linear elasticity with perfect containment:

$$\sigma_{hmin} = \left(\frac{\nu}{1-\nu}\right)\sigma_V = 0.333 \cdot (8200psi\text{–}4100psi) = 1365psi$$

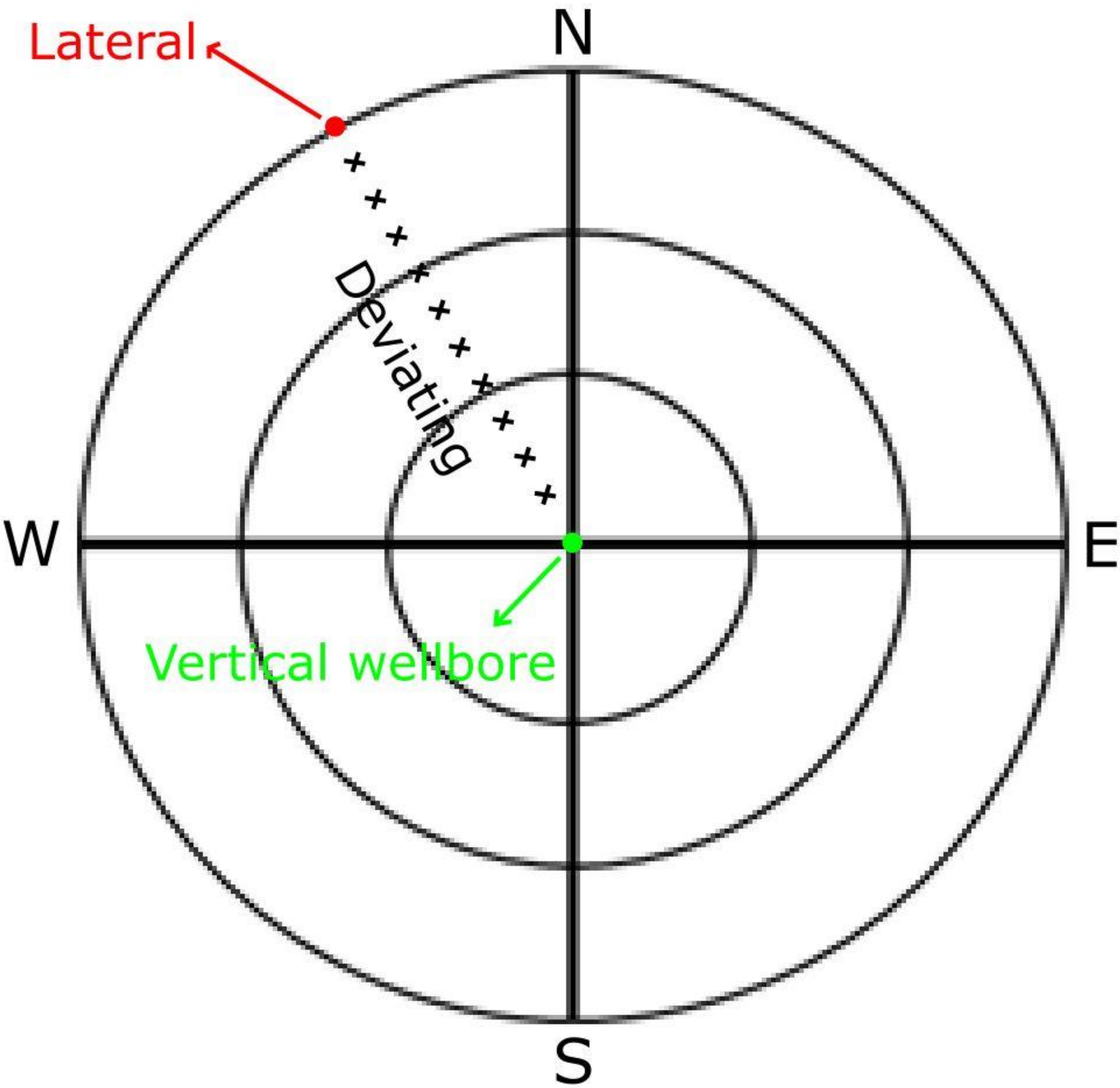
$$S_{hmin} = \sigma_{hmin} + P_p = 5465psi$$

Assuming shear equilibrium of faults:

$$\sigma_{hmin} = \left(\frac{1}{q}\right)\sigma_V = 0.286 \cdot (8200psi\text{–}4100psi) = 1171psi$$

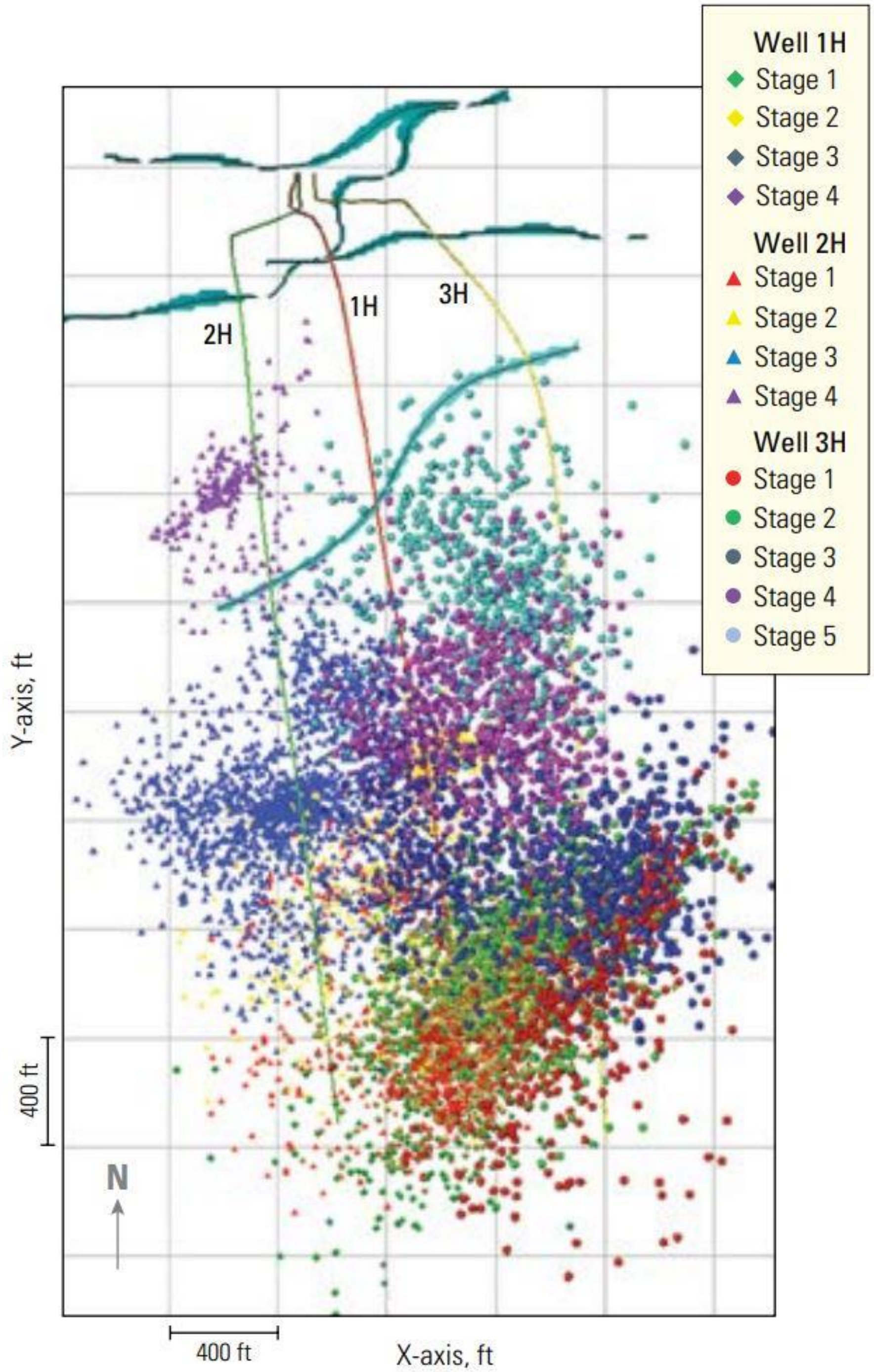
$$S_{hmin} = \sigma_{hmin} + P_p = 5271psi$$

d) Draw the "path" of wellbore orientation on a semi-sphere projection. The wellbore starts vertical on the surface and deviates close to the pay zone until it gets horizontal.



Problem 4

The following figures correspond to microseismicity images from hydraulic fracturing stimulation in the Barnett shale (Hydraulic Fracturing Insights from Microseismic Monitoring – SLB Oilfield Review https://www.slb.com/~/media/Files/resources/oilfield_review/ors16/May2016/02-microseismic.pdf).



"Stimulation process performed on the three horizontal wells using plug and perf and slickwater treatment with fault traces mapped at laterals depth interval in cyan. The zipper-style frac performed on the 1H (central well in red) and the 3H (east well in yellow) was monitored from 2H (west well in green). The four stimulation stages performed on the 2H were monitored from the 3H."

a) What is the length of the laterals? What is the distance between laterals? What is the approximate distance between stages? (axis units: feet) Note: 1H is 80 ft higher in elevation than 2H and 3H.

Length of laterals is ~3300 ft.

Distance between laterals is ~500 ft.

Distance between stages is ~400-800 ft.

b) What is the strike of the hydraulic fractures at the toe of the laterals? How does the strike interpreted from microseismicity compare to the strike obtained in problem 3?

Strike = N50°E (050°)

This is similar to the strike of 060° from problem 3.

b) What is the average fracture half-length x_f as interpreted from the microseismic cloud?

$x_f = 600ft$

d Assuming a pay zone thickness of 200 ft, what is the Stimulated Reservoir Volume $[ft^3]$?

$Area = 2000ft \cdot 2000ft = 4 \cdot 10^6 ft^2$

$SRV = Area \cdot Height = 4 \cdot 10^6 ft^2 \cdot *200ft = 8 \cdot 10^8 ft^3$