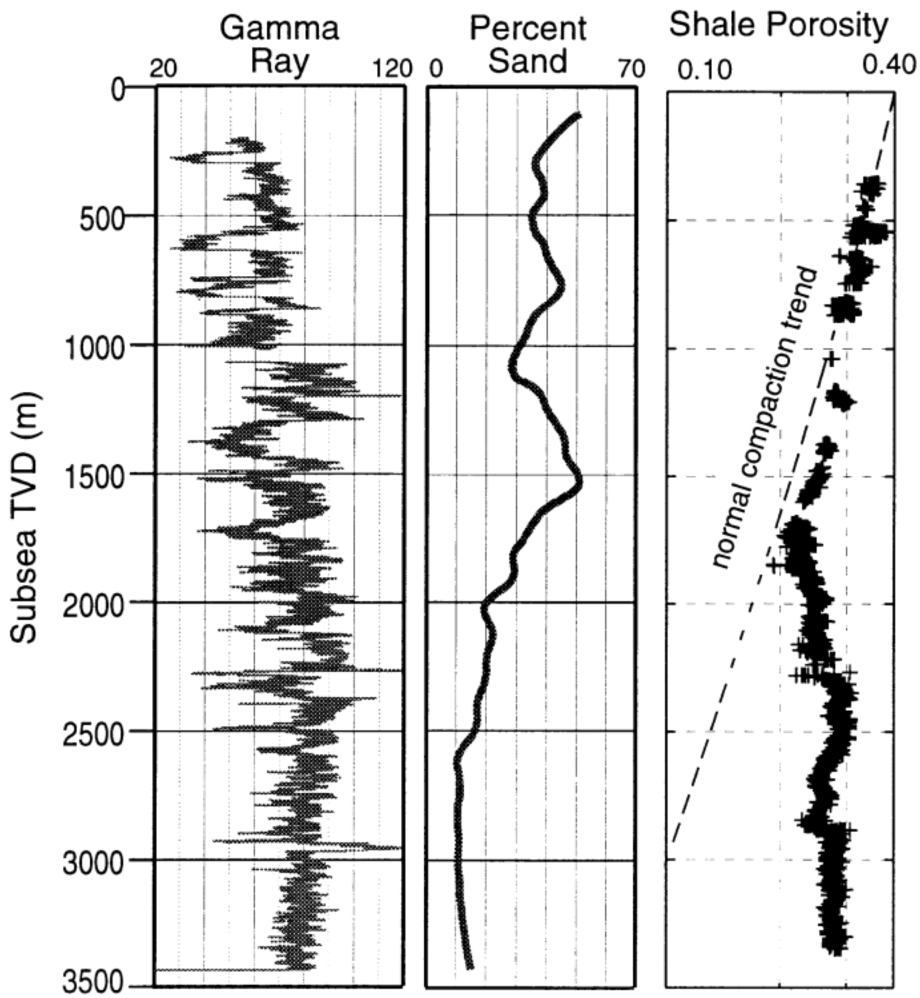
Project #7: Soft rock constitutive models

1) Compressibility of mudrocks

The following data set contains well-logging measurements of porosity of a mudrock as a function of depth (Eugene Island – offshore Louisiana):



Gordon and Flemings, 1998 – Basin Research

a) Compute and plot pore pressure assuming a hypothetical hydrostatic pore pressure gradient dPp/dz = 0.465 psi/ft.
b) Compute and plot total vertical stress assuming dSv/dz = 0.950 psi/ft and pick the seafloor from the shallowest data point in "percent sand" plot.
c) Digitize shale porosity data (at least ~20 equally spaced points) and fit an equation of porosity as a function of vertical effective stress from depth 400 m to 1800 m assuming hydrostatic pore pressure and models:

$$\phi = \phi_0 \exp(-\beta \sigma_V) \text{ (Exponential on porosity)}$$

$$e = e_0 - C_c ln\left(\frac{\sigma_V}{1MPa}\right) \text{ (Logarithmic on void ratio)}$$

Show the porosity-effective vertical stress and void ratio-effective vertical stress plots.
d) Calculate and plot actual pore pressure between the interval 1800 m to 3400 m assuming porosity is a function of vertical effective stress with the models calculated in point c.
e) Calculate and plot overpressure parameter λ_p as a function of depth.
f) Summarize all results with plots of

Left) Porosity (model and data) in log scale as a function of depth (y-axis)
Middle) S_V and actual P_P as a function of depth (y-axis)
Right) Overpressure parameter as a function of depth (y-axis)

2) Cam-clay model

Write a script that simulates a (axisymmetric) triaxial loading test for a mudrock with the following properties:

Elastic shear modulus, G = 1 MPa;
Pre-consolidation stress, p'_o = 250 [kPa]
Friction angle at critical state, ϕ_{CS} = 24°
Loading compressibility, λ = 0.25;
Unloading compressibility, κ = 0.05;
Initial void ratio, e_o = 1.15;

The initial state of stress is p'= 200 kPa; q = 0 kPa. Load the sample until the critical state.

a) Plot the stress path q VS p'. Plot the initial yield surface and the final yield surface. Is there hardening or softening?
b) Plot q as a function of ε_q. Why does it approximate an asymptotic value?
c) Plot void ratio e as a function of p' (with p' in logarithmic scale). Why is there a clear change of slope?

Incremental elastic deformations: $d\varepsilon_p^e = \frac{\kappa}{\upsilon}\frac{dp'}{p'}$; $d\varepsilon_q^e = \frac{dq}{3G}$

Incremental plastic deformation: $\begin{bmatrix} d\varepsilon_p^p \\ d\varepsilon_q^p \end{bmatrix} = \frac{\lambda-\kappa}{\upsilon p'(M^2+\eta^2)} \begin{bmatrix} M^2-\eta^2 & 2\eta \\ 2\eta & \frac{4\eta^2}{M^2-\eta^2} \end{bmatrix} \begin{bmatrix} dp' \\ dq \end{bmatrix}$

where $\upsilon = 1 + e$ is the specific volume, $\eta = q/p'$, and $de = \upsilon\, d\varepsilon_p^p$.

The incremental change of the yield surface is: $dp'_o = d\varepsilon_p^p \frac{\upsilon}{\lambda-\kappa} p'_o$