Generalized Hooke's law

$$\vec{\sigma} = \boldsymbol{C}\,\vec{\varepsilon}$$

For isotropic materials

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{Bmatrix}$$

Poroelasticity

Poroelasticity Assumptions

1. There is an interconnected pore system uniformly saturated with fluid.
2. The total volume of the pore system is small compared to the volume of the rock.
3. The pore pressure, the total stress acting on the rock externally, and the stresses acting on the grains are statistically defined.

Effective stress

Terzaghi definition

$$\boldsymbol{\sigma}' = \mathbf{S} - P_p \mathbf{I}$$

"Exact" effective stress

$$\boldsymbol{\sigma}'' = \mathbf{S} - \alpha P_p \mathbf{I}$$

α is called Biot's coefficient

Biot's coefficeint

$$\alpha = 1 - \frac{K_T}{K_S}$$

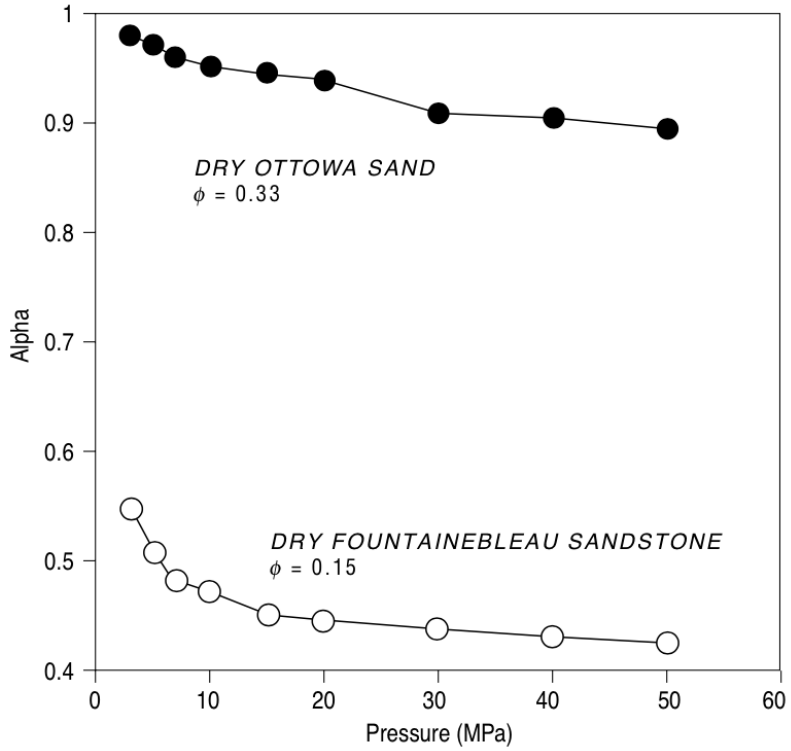
For sand

$$K_S >> K_T \qquad \alpha \approx 1$$

For rocks

$$\alpha \approx \frac{2}{3}$$

Biot's coefficient (cont.)



 Zoback, *Reservoir Geomechanics* (Fig. 3.5c, pp. 69)