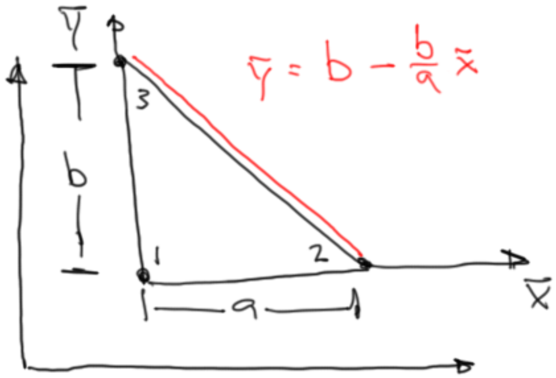


$$-\left(a_{11}\frac{\partial^2 u}{\partial x^2} + a_{22}\frac{\partial^2 u}{\partial y^2}\right) - f = 0$$

$$K_{ij} = \int_\Omega B^T C \; B \, dx\,dy$$

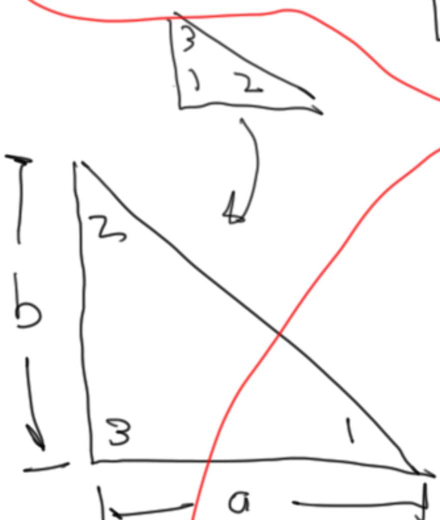
where

$$B = \begin{bmatrix} N_{1,x} & N_{2,x} & \cdots & N_{N,x} \\ N_{1,y} & N_{2,y} & \cdots & N_{N,y} \\ N_1 & N_2 & \cdots & N_n \end{bmatrix}$$

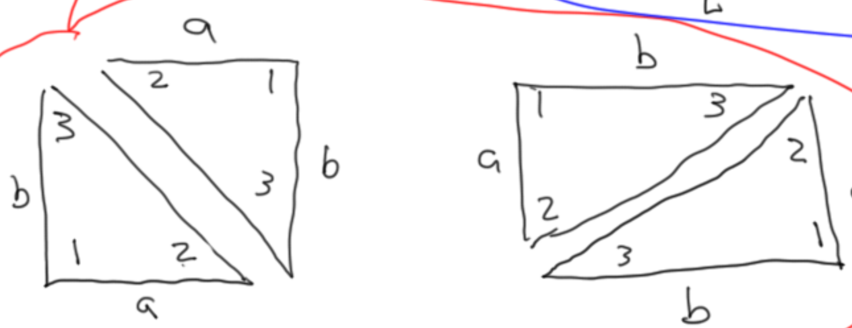
$$C = \begin{bmatrix} a_{11} & \cancel{a_{12}}^{\,0} & 0 \\ \cancel{a_{21}}^{\,0} & a_{22} & 0 \\ 0 & 0 & \cancel{a_{00}}^{\,0} \end{bmatrix}$$

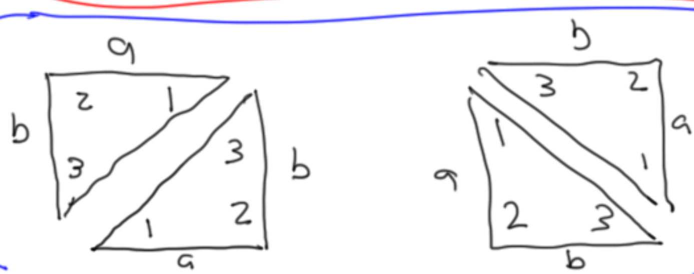
$$a_{11} = a_{22} = k_e$$

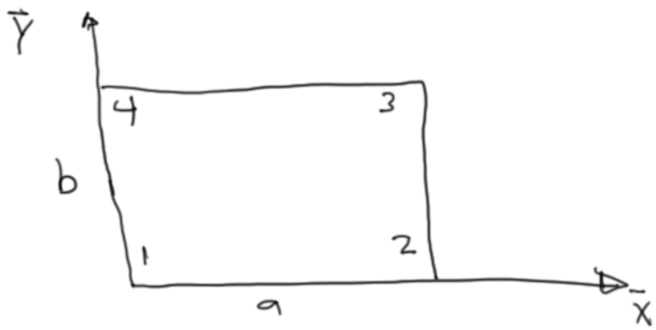
$$[k^e] = \frac{k_e}{2ba}\begin{bmatrix} b^2+a^2 & -b^2 & -a^2 \\ -b^2 & b^2 & 0 \\ -a^2 & 0 & a^2 \end{bmatrix} \qquad \{f\} = \frac{f_c\, ab}{6}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$



$$[k^e] = \frac{k_e}{2ab}\begin{bmatrix} b^2 & 0 & -b^2 \\ 0 & a^2 & -a^2 \\ -b^2 & -a & a^2+b^2 \end{bmatrix} \qquad \{f\} = \frac{f_c\, ab}{6}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$







$$X = \{1,\ x,\ y,\ xy\}$$

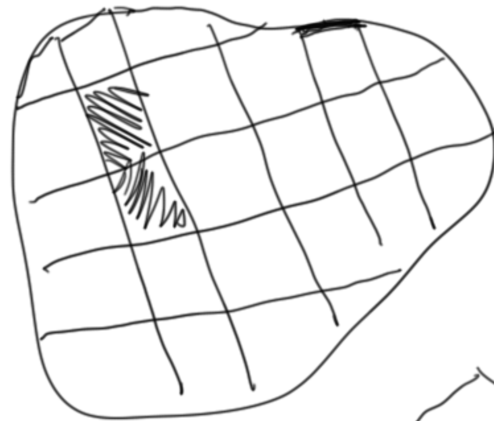
$$[k] = \frac{a_{11} b}{6a} \begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{a_{22} a}{6b} \begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

$$a_{11} = a_{22} = k_e$$

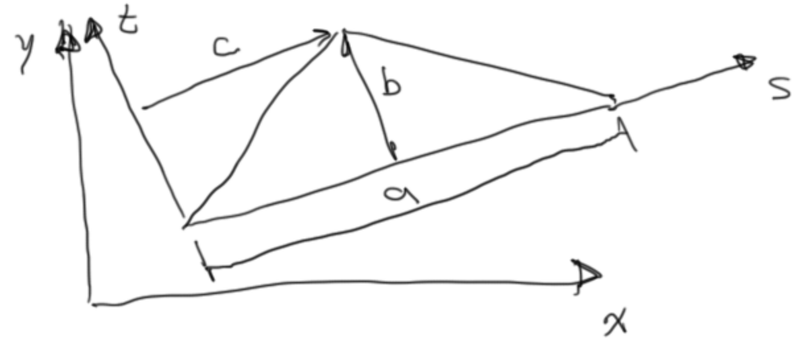
$$[k^e] = \frac{k_e}{6ab} \begin{bmatrix} 2(a^2+b^2) & a^2-2b^2 & -(a^2+b^2) & b^2-2a^2 \\ a^2-2b^2 & 2(a^2+b^2) & b^2-2a^2 & -(a^2+b^2) \\ -(a^2+b^2) & b^2-2a^2 & 2(a^2+b^2) & a^2-2b \\ b^2-2a & -(a+b^2) & a^2-2b^2 & 2(a^2+b^2) \end{bmatrix}$$

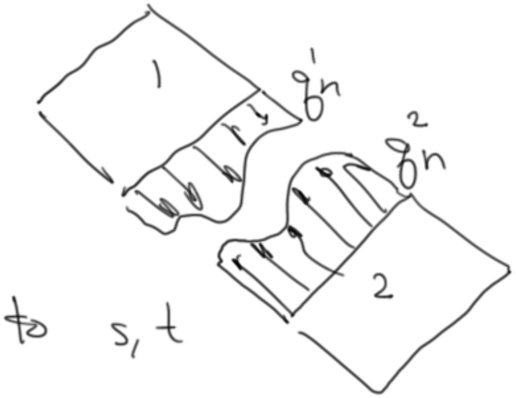
$$Q_i = \oint_{\Gamma_e} N_i \, q \, dS$$

$$\boxed{\Gamma_e \cap \Gamma}$$



Consider



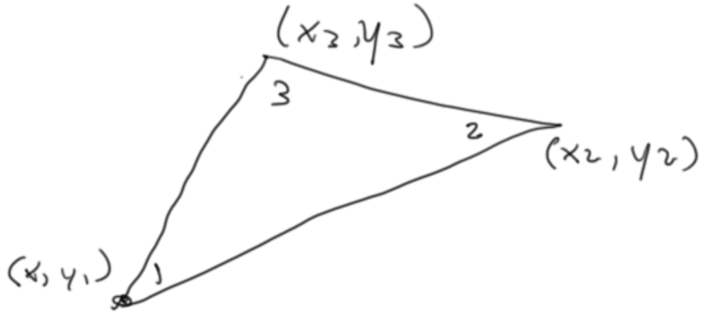


x, y are related to s, t

Solve $a_1, b_1, c_1, a_2, b_2, c_2$

$$X = a_1 + b_1 s + c_1 t$$

$$Y = a_2 + b_2 s + c_2 t$$



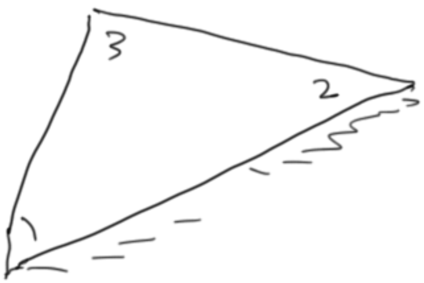
$s = 0, \; t = 0 \qquad x = x_1 \quad y = y_1$

$s = a, \; t = 0 \qquad x = x_2 \quad y = y_2$

$s = c, \; t = b \qquad x = x_3 \quad y = y_3$

$$x(s,t) = x_1 + (x_2 - x_1)\frac{s}{a} + \left[\left(\frac{c}{a} - 1\right)x_1 - \frac{c}{a}x_2 + x_3\right]\frac{t}{b}$$

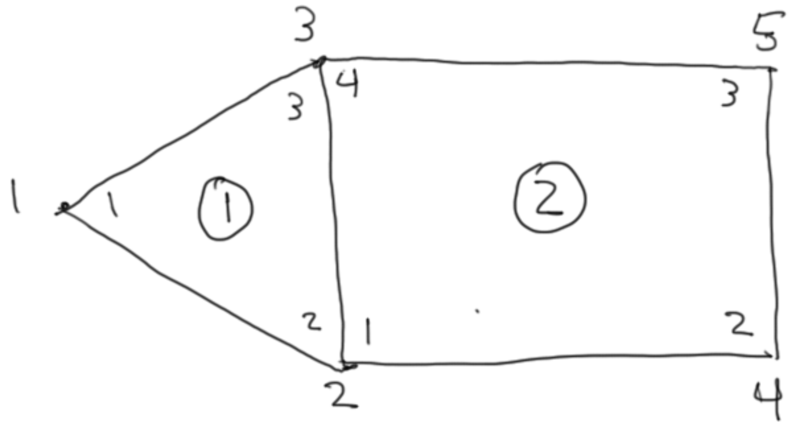
$$y(s,t) = y_1 + (y_2 - y_1)\frac{s}{a} + \left[\left(\frac{c}{a} - 1\right)y_1 - \frac{c}{a}y_2 + y_3\right]\frac{t}{b}$$



Side 1 – 2

$$N_i(s) = N_i(s, 0) = \left[\underbrace{1 - \frac{s}{a}, \ \frac{s}{a}}, \ 0\right]^T$$

$$Q_i = \int_{1-2} N_i(s)\, q_n(s)\, dS + \int_{2-3} N_i(s)\, q_n(s)\, dS + \int_{3-1} N_i(s)\, q_n(s)\, dS$$

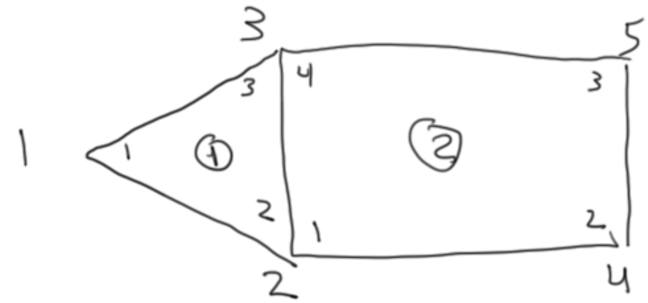


$$u_1^{e_1} = U_1 \;,\quad u_2^{e_1} = u_1^{e_2} = U_2 \;,\quad u_3^{e_1} = u_4^{e_2} = U_3 \;,$$

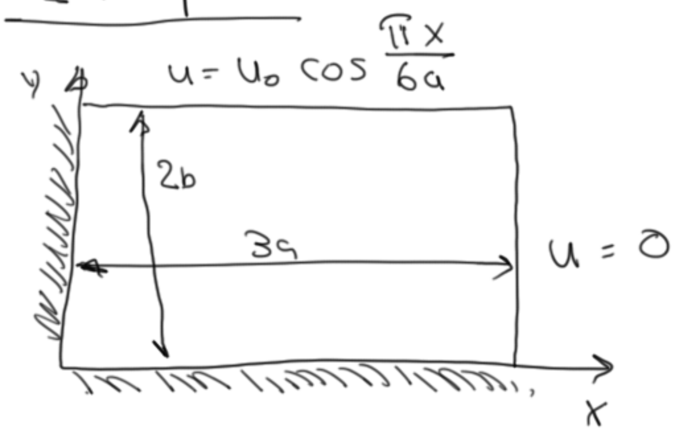
$$[k^{e_1}_{\triangle}] = \begin{bmatrix} k_{11}^{e_1} & k_{12}^{e_1} & k_{13}^{e_1} \\ k_{21}^{e_1} & k_{22}^{e_1} & k_{23}^{e_1} \\ k_{31}^{e_1} & k_{32}^{e_1} & k_{33}^{e_1} \end{bmatrix} \begin{Bmatrix} \cancel{u_1^{e_1}}\, U_1 \\ \cancel{u_2^{e_1}}\, U_2 \\ \cancel{u_3^{e_1}}\, U_3 \end{Bmatrix} \qquad [k^{e_2}_{\square}] = \begin{bmatrix} \quad \end{bmatrix} \begin{Bmatrix} \cancel{u_1^{e_2}}\, U_2 \\ \cancel{u_2^{e_2}}\, U_4 \\ \cancel{u_3^{e_2}}\, U_5 \\ \cancel{u_4^{e_2}}\, U_3 \end{Bmatrix}$$

$$[K] = \begin{bmatrix} k_{11}^{e_1} & k_{12}^{e_1} & k_{13}^{e_1} & 0 & 0 \\ k_{21}^{e_1} & k_{22}^{e_1} + k_{11}^{e_2} & k_{23}^{e_1} + k_{14}^{e_2} & K_{12}^{e_2} & k_{13}^{e_2} \\ k_{31}^{e_1} & k_{32}^{e_1} + k_{41}^{e_2} & k_{33}^{e_1} + K_{44}^{e_2} & K_{42}^{e_2} & k_{43}^{e_2} \\ 0 & k_{21}^{e_2} & k_{24}^{e_2} & k_{22}^{e_2} & k_{23}^{e_2} \\ 0 & k_{31}^{e_2} & k_{34}^{e_2} & k_{32}^{e_2} & k_{33}^{e_2} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix}$$

$$B_c = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 5 & 3 \end{bmatrix}$$



Example



$$-k \nabla^2 u = 0$$

$$-k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$$