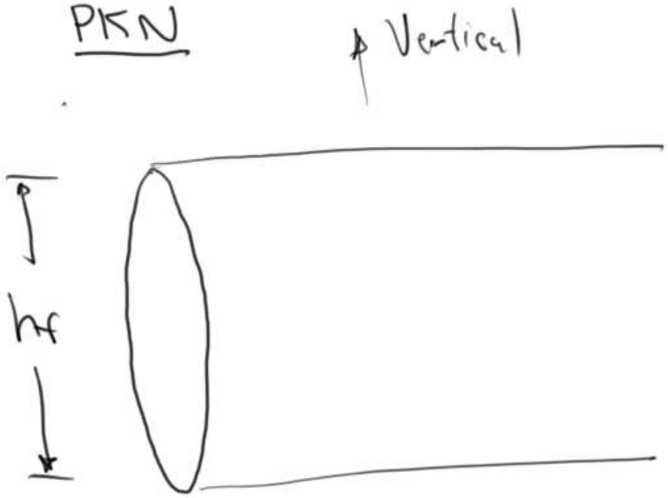
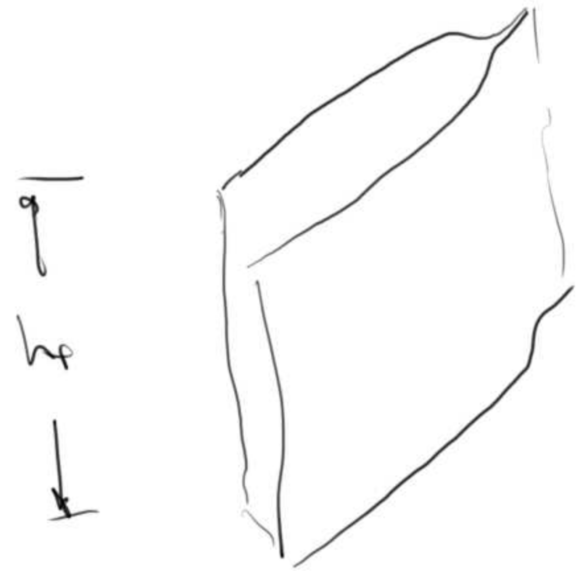
PKN



Most applicable when fractures are much longer than they are tall

KGD

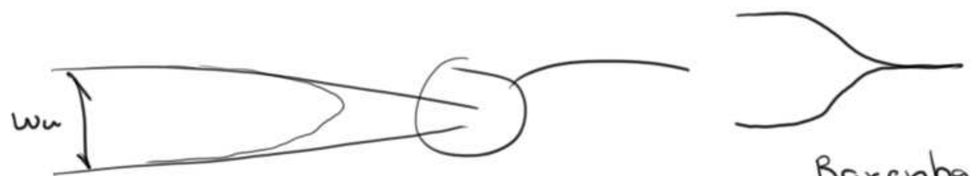


Most applicable for fractures that are much taller than they are long

KGD

Khristianovich - Gertsma - de Klerk

$$\frac{\partial p}{\partial x} = \frac{12 q \mu}{h_f w^3} \quad \text{integrate} \quad p_{net} = \frac{6 \mu q_i}{h_f} \int_0^L \frac{dx}{w^3} \qquad 1)$$



Barenblatt's tip condition

$$\int_0^L \frac{p_{net}\, dx}{\sqrt{1-(x/L)}} = 0 \qquad 2)$$

$$w_w = \frac{4}{E'} L\, p_{net} \quad \text{(Sneddon)} \qquad 3)$$

$$p_{net,w} = \left[\frac{21 \mu q_i}{64 \pi h_f L^2}\right]^{1/4} \qquad w_w = \left[\frac{84 \mu q_i L^2}{\pi E' h_f}\right]^{1/4}$$

$$L(t) = 0.38 \left[\frac{E' q_i^3}{\mu h_f}\right]^{1/6} t^{2/3} \qquad w_w = 1.48 \left[\frac{\mu q_i^3}{E' h_f^3}\right]^{1/6} t^{1/3} \leftarrow \text{no leakoff}$$

KGD including Carter leakoff

The volume 2-wing KGD fracture

$$V_f = \frac{\pi}{2} h_f L w_w$$

$$L = \frac{q_i w_w}{64 C_L^2 h_f}\left(e^{S^2} \operatorname{erfc}(S) + \frac{2}{\pi} S - 1 \right)$$

where

$$S = \frac{8 C_L \sqrt{\pi t}}{\pi w_w}$$

KGD for radial geometry

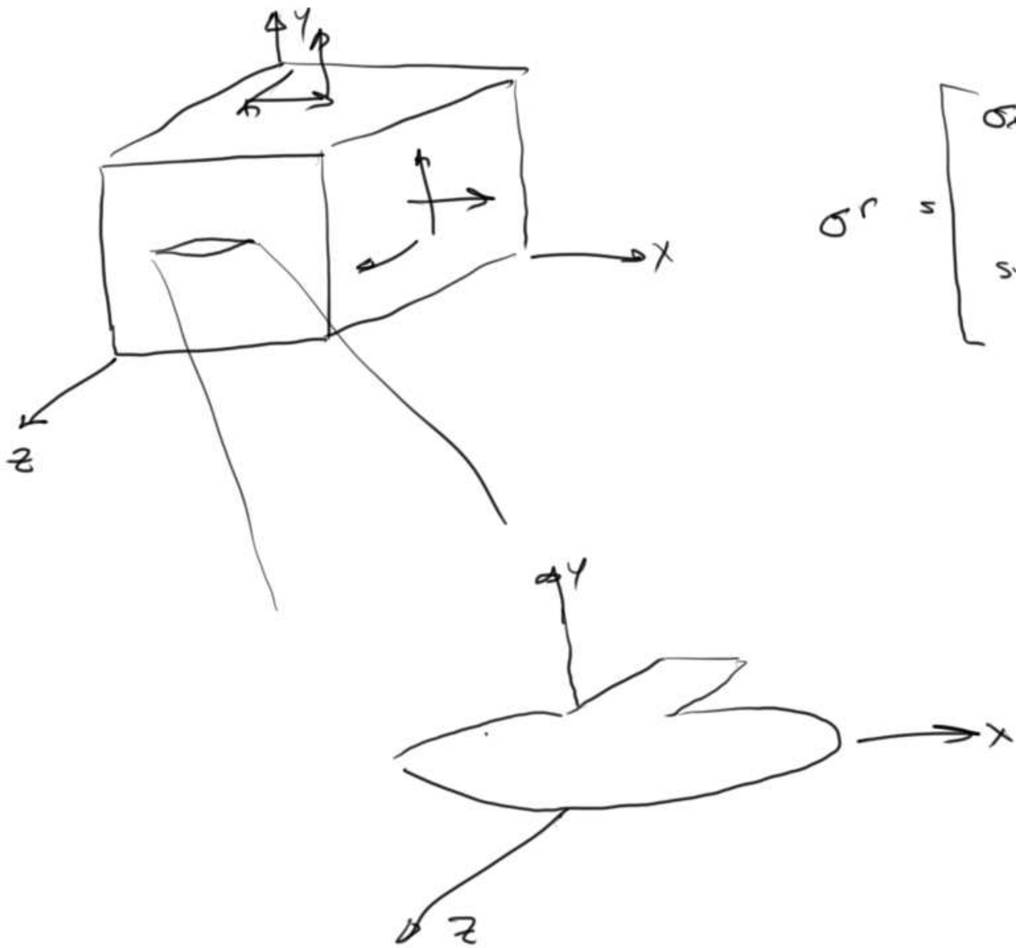
$$S = \frac{15 C_L \sqrt{\pi t}}{4 w_w + 15 S_p}$$

No leakoff

$$w_w = 2.17 \left[\frac{\mu^2 q_i^3}{E'^2} \right]^{1/9} t^{1/9} \qquad R = 0.52 \left[\frac{E' q_i^3}{\mu} \right]^{1/9} t^{4/9}$$

Including leakoff

$$w_w = 2.56 \left[\frac{\mu q_i R}{E'} \right]^{1/4} \qquad R = \left[q_i \frac{(4 w_w + 15 S_p)}{30 \pi^2 C_L^2} \left(e^{S^2} \operatorname{erfc}(S) + \frac{2}{\sqrt{\pi}} S - 1 \right) \right]^{1/2}$$



$$\sigma^r = \begin{bmatrix} \sigma_{xx}^r & \sigma_{xy}^r & \sigma_{xz}^r \\ & \sigma_{yy}^r & \sigma_{yz}^r \\ \text{symm.} & & \sigma_{zz} \end{bmatrix}$$

$$\bar{\sigma}^c = \begin{bmatrix} \sigma_{xx}^c & \sigma_{xy}^c & \sigma_{xz} \\ & \sigma_{yy}^c & \sigma_{yz} \\ \text{symm} & & \sigma_{zz} \end{bmatrix}$$

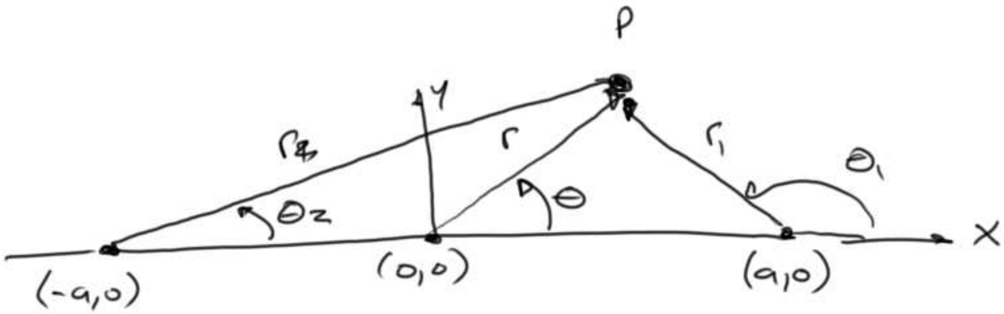
Driving stresses

$$\Delta\sigma_{I} = \left(\sigma_{yy}^r - \sigma_{yy}^c\right)$$

$$\Delta\sigma_{II} = \left(\sigma_{xy}^r - \sigma_{xy}^c\right)$$

$$\Delta\sigma_{III} = \left(\sigma_{xz}^r - \sigma_{xz}^c\right)$$

Follow <u>Pollard et al 1987</u> "tri-polar coord. sys."



$$R = \sqrt{r_1 r_2}$$

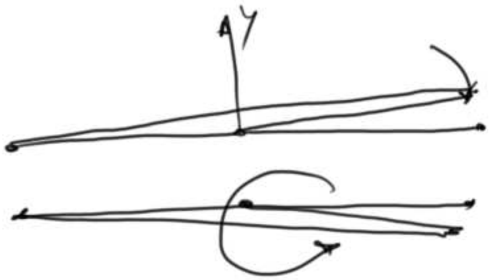
$$\Gamma = \left(\frac{\theta_1 + \theta_2}{2}\right)$$

$$u_x = \frac{1}{2\mu}\left\{\Delta\sigma_{II}\left\{2(1-\nu)\left(R\Gamma - r\sin\theta\right) + r\sin\theta\left[rR^{-1}\cos(\theta-\Gamma)-1\right]\right\}\right.$$
$$\left. + \Delta\sigma_{I}\left\{(1-2\nu)\left(R\cos\Gamma - r\cos\theta\right) - r\sin\theta \cdot \left[rR^{-1}\sin(\theta-\Gamma)\right]\right\}\right.$$

$$u_y = \frac{1}{2\mu}\left\{\sigma_{I}\left\{2(1-\nu)\left(R\sin\Gamma - r\sin\theta\right) - r\sin\theta\left[rR^{-1}\cos(\theta-\Gamma)-1\right]\right\}\right.$$
$$\left. - \Delta\sigma_{II}\left\{(1-2\nu)\left(R\cos\Gamma - r\cos\theta\right) + r\sin\theta \cdot \left[rR^{-1}\sin(\theta-\Gamma)\right]\right\}\right\}$$

$$u_z = \frac{1}{2\mu}\left\{2\Delta\sigma_{III}\left(R\sin\Gamma - \sin\theta\right)\right\}$$

On crack surface



$r = |x|$ $\qquad \theta = 0, \pi$ $\qquad R = (a^2 - x^2)^{1/2}$

$r_1 = a - x$ $\qquad \theta_1 = -\pi, \pi$ $\qquad \Gamma = \frac{\pi}{2}, \frac{3\pi}{2}$

$r_2 = a + x$ $\qquad \theta_2 = 0, 2\pi$

$$\begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \pm \begin{Bmatrix} \Delta\sigma_{II} \\ \Delta\sigma_{I} \\ \Delta\sigma_{III} \end{Bmatrix} \frac{1-\nu}{\mu} (a^2 - x^2)^{1/2} + \begin{Bmatrix} -\Delta\sigma_{I} \\ \Delta\sigma_{II} \\ 0 \end{Bmatrix} \frac{(1-2\nu)}{2\mu} x$$

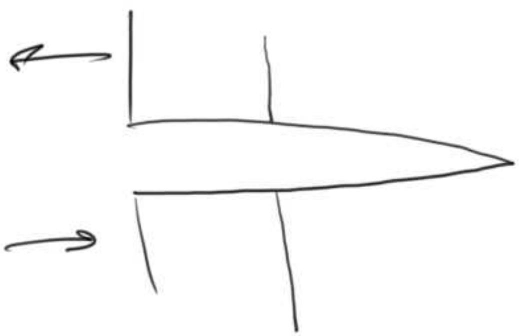
$\pm$ refers $y = 0^+$, $y = 0^-$

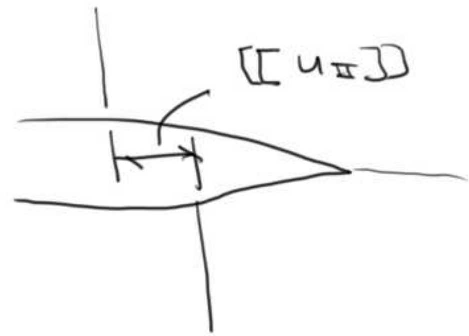
$$[[u_m]] = (u_i^+ - u_i^-)$$

$i = x, y, z$

$m = I, II, III$

$$\boxed{\begin{Bmatrix} [[u_I]] \\ [[u_{II}]] \\ [[u_{III}]] \end{Bmatrix} = \begin{Bmatrix} \Delta\sigma_{I} \\ \Delta\sigma_{II} \\ \Delta\sigma_{III} \end{Bmatrix} \frac{2(1-\nu)}{\mu} (a^2 - x^2)^{1/2}}$$





$[[u_1]]$

$$\frac{\partial}{\partial t}(\rho w) = \frac{\partial}{\partial x}\left(\frac{\rho w^3}{\partial \mu}\frac{\partial p}{\partial x}\right)$$

Rice 1969

$$K_I = \frac{\mu}{4(1-\nu)}\sqrt{\frac{2\pi}{r}}\,[[u_I]]$$

$$K_{II} = \frac{\mu}{4(1-\nu)}\sqrt{\frac{2\pi}{r}}\,[[u_{II}]]$$

$$K_I > K_{Ic}$$

$$K_{II} > K_{IIc}$$