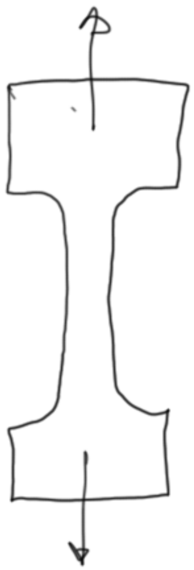
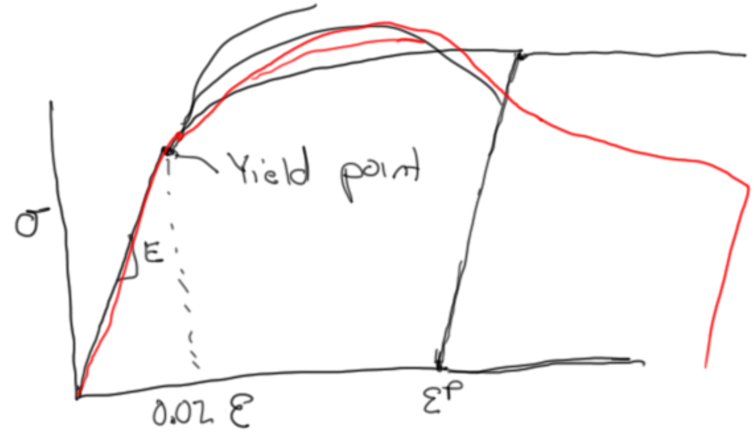
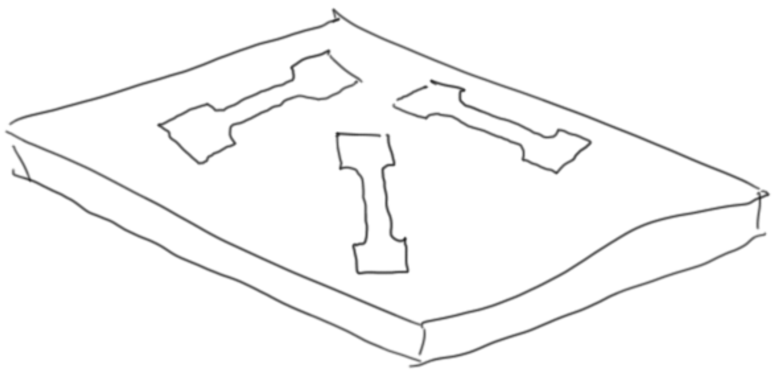
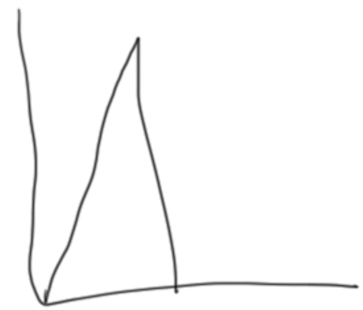
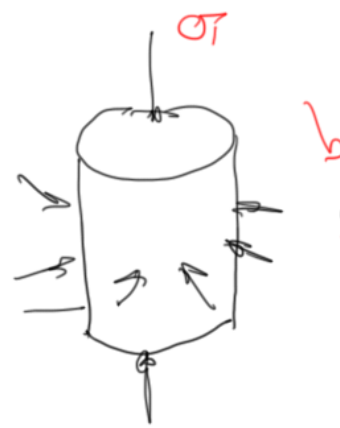




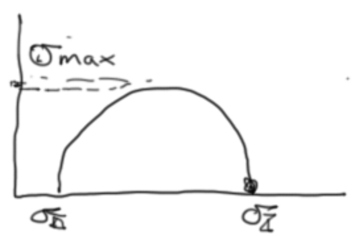


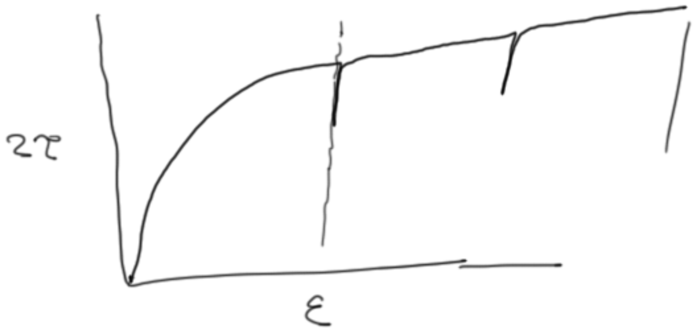




$$\sigma_{22} = \sigma_{33} = \sigma_H$$

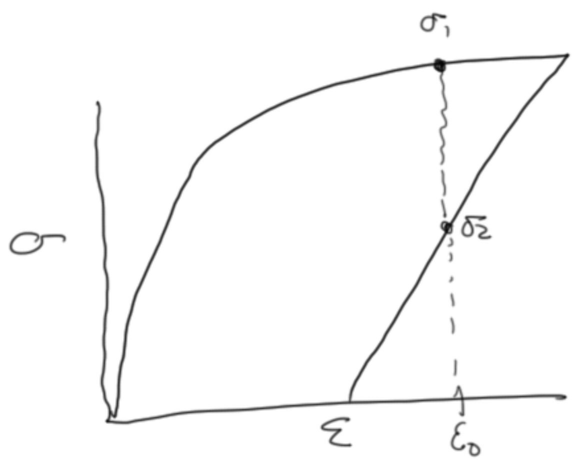
$$2\sigma_{max} = (\sigma_{11} - \sigma_{22})$$





$$\sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}}$$

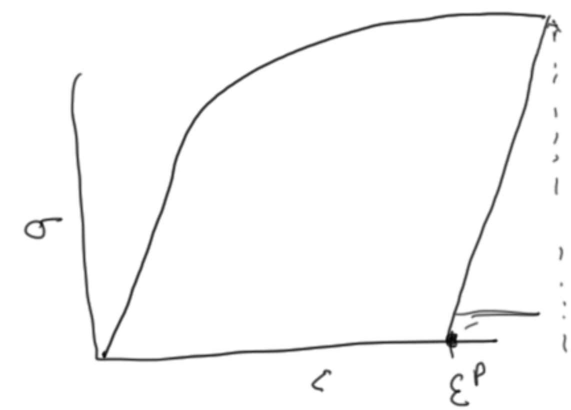
$$W(\rho\psi) = W(\varepsilon_{ij}, T)$$



$$\sigma = \sigma(\varepsilon_{ij}, T)$$
$$= \sigma(\varepsilon_{ij}, T, \vec{\xi})$$

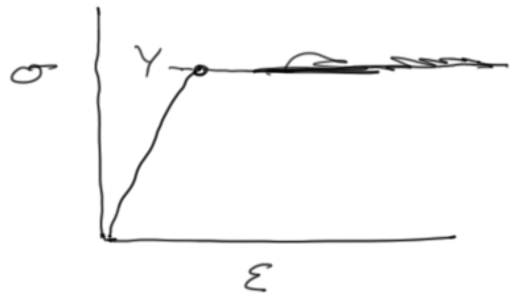
ξ may be "physical" variable

- → structure
- → physico-chemical reaction
- → phase change
- → densities of structural defects
- → <u>phenomenological</u>
  - plastic strain



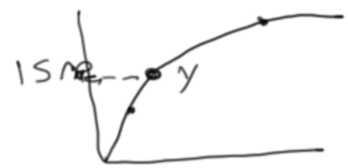
True "small" strains → $\underline{\varepsilon = \varepsilon^e + \varepsilon^p}$ $\quad \underline{\|\nabla u\| << 1}$

Holds always → $\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p$



"Perfectly Plastic"

$$\sigma = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



$\sigma_{11} = 10$ MPa

$\sigma_{22} = 20$ MPa

$Y = 15$ MPa

$\sigma_{11} < Y \Rightarrow \sigma_{11} = E\varepsilon_{11}$

$\sigma_{11} = Y \Rightarrow \sigma_{11} = Y$

$\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$

$$S_{ij}\Big|_{\text{Post Yield}} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\,\delta_{ij} = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \frac{Y}{3} & 0 & 0 \\ 0 & \frac{Y}{3} & 0 \\ 0 & 0 & \frac{Y}{3} \end{bmatrix} = \overbrace{\begin{bmatrix} \frac{2}{3}Y & 0 & 0 \\ 0 & -\frac{1}{3}Y & 0 \\ 0 & 0 & -\frac{1}{3}Y \end{bmatrix}}^{S_{ij}'}$$

$$\sigma_{eq} \equiv \sqrt{3J_2} = \sqrt{\frac{3}{2}S_{ij}S_{ij}} = Y$$

von Mises stress = $\bar{\sigma} = \sigma_{vm} = \sigma_{eq}$

von Mises Plastity

Assumption: Under triaxial state, the material is yielding when $\sigma_{eq} \geq Y$

$$\sigma_{eq} = \sqrt{3J_2} = \Big[\tfrac{1}{2} \big\{ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{33} - \sigma_{11})^2 + (\sigma_{22} - \sigma_{33})^2 \big\}$$

$$+ 3\sigma_{12}^2 + 3\sigma_{13}^2 + 3\sigma_{23}^2 \Big]^{1/2}$$

$\sigma_{11} = 10$ MPa

$\sigma_{22} = \sigma_{33} = 20$ MPa

$Y = 15$

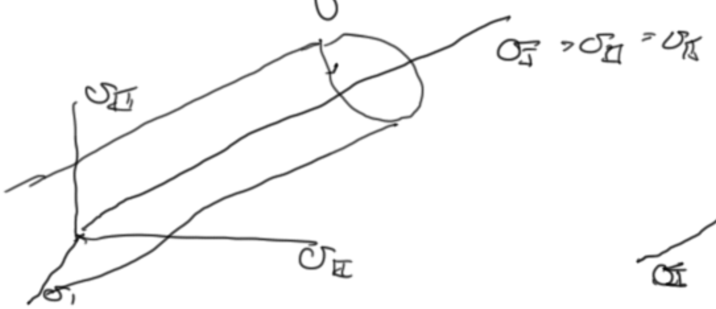
material constant

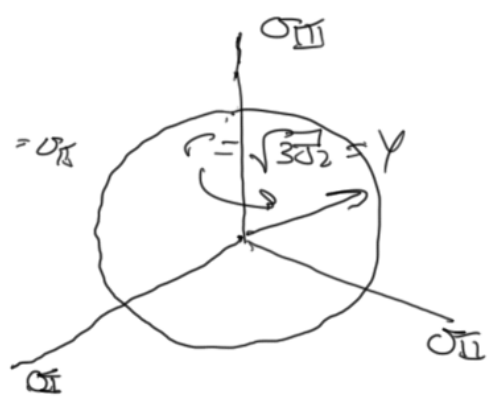
$f(\sigma_{ij}) = \sqrt{3J_2} - Y = 0$

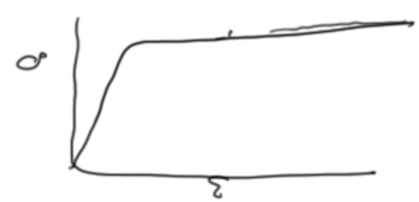
$f(\sigma_{ij}) < 0$ elastic

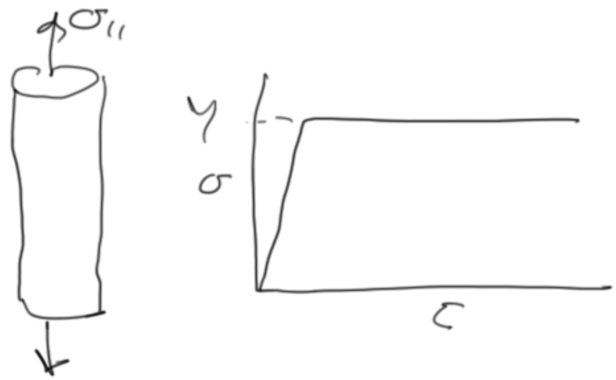
$f(\sigma_{ij}) = 0$ plastic

$\bar{\sigma}_{eq} = 10$ MPa









$$\varepsilon_{11} = 0.20\%$$

$$\varepsilon_{11}^{e} = \frac{\sigma_{11}}{E} - \frac{\nu}{E}\left(\cancelto{0}{\sigma_{22}} + \cancelto{0}{\sigma_{33}}\right) = \frac{\sigma_{11}}{E} = \frac{Y}{E}$$

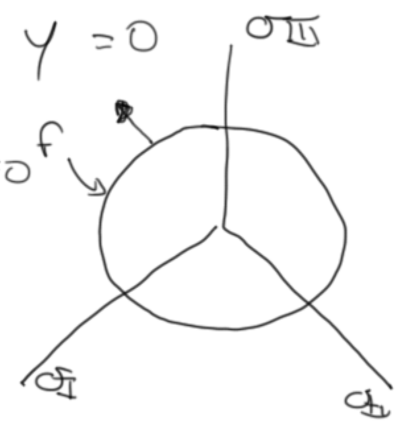
$$\varepsilon_{22} = \frac{-\nu}{E}\sigma_{11} = \frac{-\nu Y}{E}$$

$$\varepsilon_{33} = \frac{-\nu}{E}\sigma_{11} = \frac{-\nu Y}{E}$$

$$\vec{\varepsilon} = \vec{\varepsilon}^{\,e} + \vec{\varepsilon}^{\,p}$$

$$f = \sqrt{3J_2} - Y = 0$$

$$= J_2 - \frac{Y^2}{3} = 0$$



Flow rule

$$\dot{\varepsilon}^{p} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}$$

$$d\varepsilon^{p} = d\lambda\frac{\partial f}{\partial \sigma_{ij}}$$

$$\frac{\partial f}{\partial \sigma_{ij}} = \frac{\partial}{\partial \sigma_{ij}}\left(J_2 - \cancel{\frac{\sigma_y^2}{3}}^{\,0} \right) = \frac{\partial J_2}{\partial \sigma_{ij}} = S_{ij}$$

$$= \frac{\partial}{\partial \sigma_{ij}}\left(\frac{1}{2} S_{kl} S_{kl} \right)$$

$$= \frac{1}{2}\left(\frac{\partial}{\partial \sigma_{ij}}(S_{kl})\, S_{kl} + S_{kl} \frac{\partial}{\partial \sigma_{ij}}(S_{kl}) \right)$$

$$= S_{kl} \frac{\partial}{\partial \sigma_{ij}}(S_{kl})$$

$$= S_{kl} \frac{\partial}{\partial \sigma_{ij}}\left(\sigma_{kl} - \frac{1}{3}\sigma_{mm}\delta_{kl} \right)$$

$$= S_{kl}\left(\frac{\partial \sigma_{kl}}{\partial \sigma_{ij}} - \frac{1}{3}\frac{\partial \sigma_{mm}}{\partial \sigma_{ij}}\delta_{kl} \right)$$

$$= S_{kl}\left(\underbrace{\delta_{il}\delta_{kj}} - \frac{1}{3}\underbrace{\delta_{im}\delta_{jm}}_{\delta_{ij}}\delta_{kl} \right)$$

$$= S_{ki}\delta_{kj} - \frac{1}{3} S_{kk}\delta_{ij}$$

$$= S_{ji} - \frac{1}{3}\cancel{S_{kk}}^{\,0}\delta_{ij}$$

$$= S_{ji} = S_{ij}$$

$$\boxed{d\varepsilon^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}} = d\lambda\, S_{ij}}$$