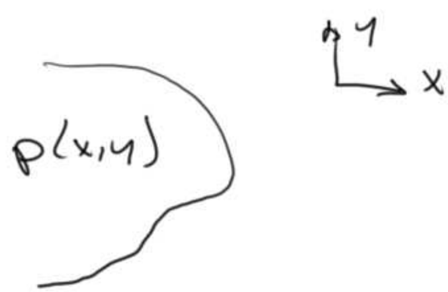
Planar 3D Models

Assumption that fracture is planar and oriented perpendicular to far-field minimum in-situ stress

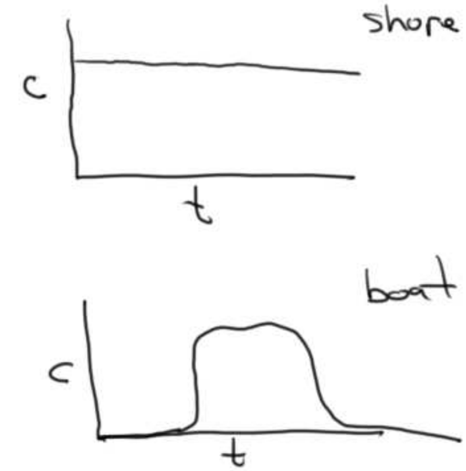
- fracture is defined by its <u>width</u> and <u>shape</u> at periphery
- width & shape vary w/ time

- width profile in a fracture of known shape & pressure distribution
- shape of fracture
- flow of fluid in a fracture of known shape & width



Width

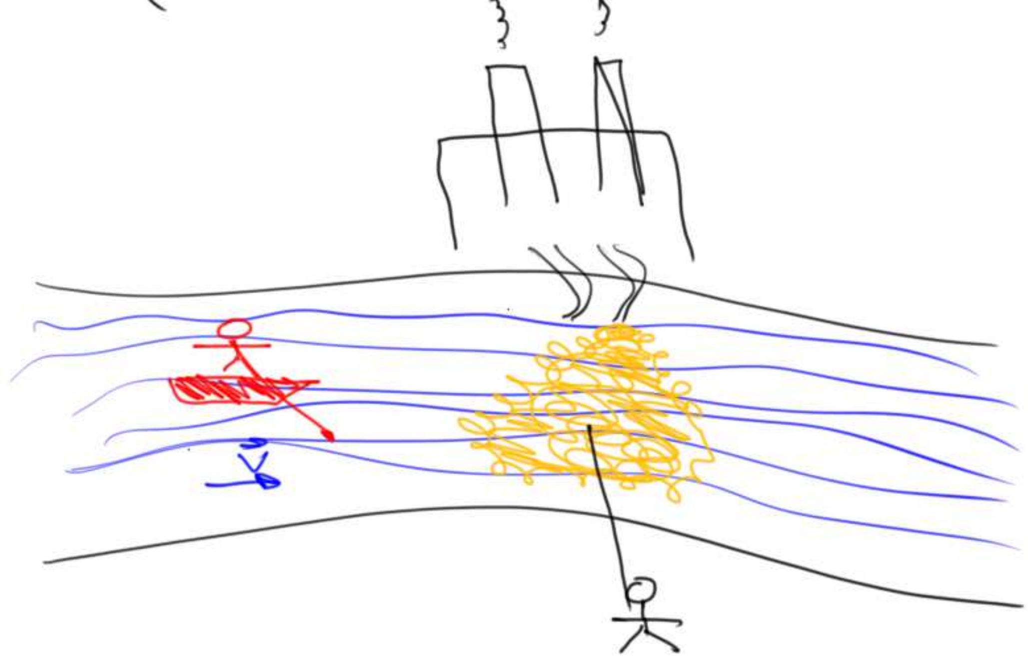
Hirthe & Lothe (1968) Bui (1977)



$$w(x,y) = \iint_S f(x-x', y-y')\, p_{net}(x',y')\,dx'dy'$$

f is an elastic influence function

$$\frac{\partial}{\partial x}(\rho w v_x) + \frac{\partial}{\partial y}(\rho w v_y) + \frac{\partial}{\partial t}(\rho w) + 2\rho u_L = 0 \leftarrow \text{Mass}$$



Momentum

$$\rho \frac{D\vec{v}}{Dt} = \nabla \cdot \bar{\bar{\sigma}} + \rho \vec{b}$$

$$\frac{D}{Dt}(c) = \frac{\partial}{\partial t}(c) + \vec{v} \cdot \nabla(c)$$

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\,\nabla\cdot\vec{v}\right) = \nabla\cdot\bar{\bar{\sigma}} + \rho\vec{g}$$

$$\bar{\bar{\sigma}} = \bar{\bar{\tau}}_{ij} - p\,\bar{\bar{I}}$$

$$\bar{\bar{\tau}} = 2\mu\left(\bar{\bar{D}} + \frac{1}{3}\left(\cancelto{0}{\nabla\cdot\vec{v}}\right)\right)$$

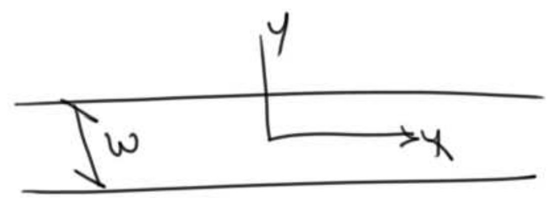
$$\nabla\cdot\vec{v} = 0$$

$$\frac{\partial}{\partial x}v_x + \frac{\partial}{\partial y}v_y + \frac{\partial}{\partial z}v_z = 0$$

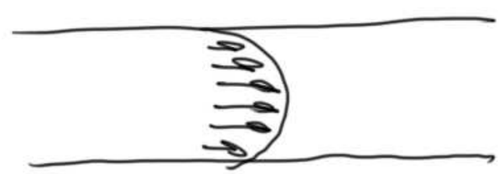
$$\bar{\bar{D}} = \frac{1}{2}\left(\nabla\vec{v} + (\nabla\vec{v})^{T}\right)$$

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \qquad i,j = 1,2,3$$

1D flow



$$\frac{\partial^2 v_x}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$



$$v_x = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2 - (w/2)^2\right)$$

$$\bar{v}_x = \frac{-w^2}{12\mu}\frac{\partial p}{\partial x}$$