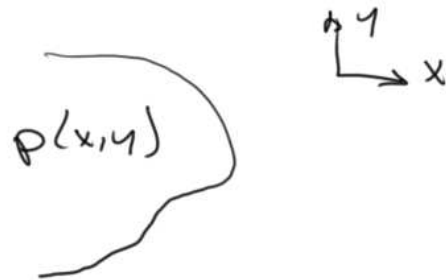


Planar 3D models

Assumption that fracture is planar and oriented perpendicular to far-field minimum in-situ stress

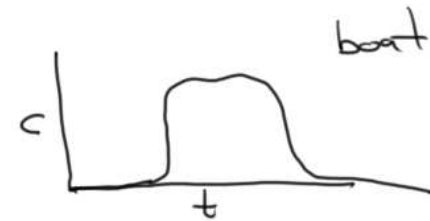
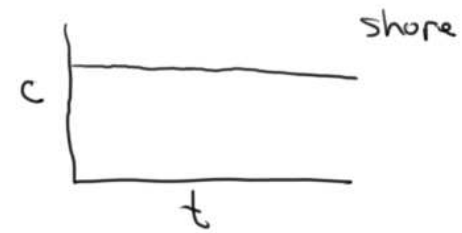
- fracture is defined by its width and shape at periphery
- width & shape vary w/ time
- - - - -
- width profile in a fracture of known shape & pressure distribution
- shape of fracture
- flow of fluid in a fracture of known shape & width



Width
Hirthe & Lotke (1968) Bui (1977)

$$w(x, y) = \iint_S f(x-x', y-y') p_{\text{net}}(x', y') dx' dy'$$

f is an elastic influence function

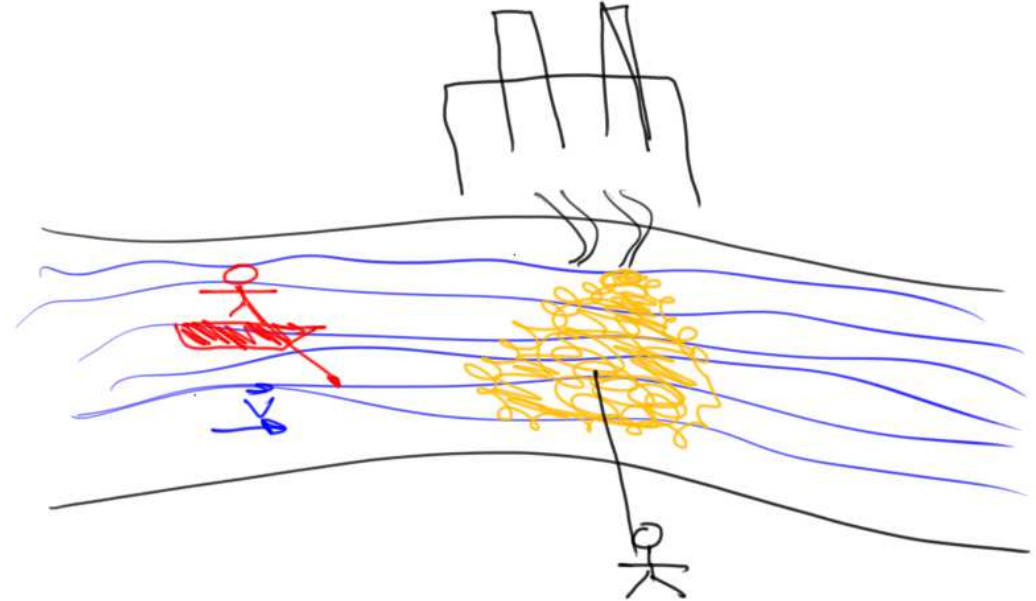


$$\frac{\partial}{\partial x}(\rho w v_x) + \frac{\partial}{\partial y}(\rho w v_y) + \frac{\partial}{\partial t}(\rho w) + 2\rho u_L = 0 \quad \leftarrow \begin{matrix} \text{Mass} \\ \{ \} \end{matrix}$$

Momentum

$$\rho \frac{D\vec{v}}{Dt} = \nabla \cdot \vec{\sigma} + \rho \vec{b}$$

$$\frac{D}{Dt}(c) = \frac{\partial}{\partial t}(c) + \vec{v} \cdot \nabla(c)$$



$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = \nabla \cdot \vec{\sigma} + \rho \vec{g}$$

$$\vec{\sigma} = \bar{\tau}_{ij} - p \bar{\mathbb{I}}$$

$$\bar{\tau} = 2\mu \left(\bar{\mathbb{D}} + \frac{1}{3} (\nabla \cdot \vec{v}) \bar{\mathbb{I}} \right)$$

$$\bar{\mathbb{D}} = \frac{1}{2} (\nabla \vec{v} + (\nabla \vec{v})^T)$$

$$D_{ij} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad i, j = 1, 2, 3$$

$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial}{\partial x} v_x + \frac{\partial}{\partial y} v_y + \frac{\partial}{\partial z} v_z = 0$$

1D flow

$$\frac{\partial^2 v_x}{\partial y^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}$$

$$v_x = \frac{1}{2\mu} \frac{\partial p}{\partial x} \left(y^2 - \left(\frac{w}{2} \right)^2 \right)$$

$$\bar{v}_x = \frac{-w^2}{12\mu} \frac{\partial p}{\partial x}$$

