Conservation of mass

$$\frac{\partial(\cancel{\rho}\, w\, V_x)}{\partial x} + \cancel{\frac{\partial(\rho w V_y)}{\partial y}} + \frac{\partial}{\partial t}(\cancel{\rho}\, w) + 2\,\cancel{\rho}\, u_L = 0$$

$$V_x = -\frac{w^2}{12\mu}\frac{\partial p}{\partial x}$$

$$w(x,y) = \iint f(x,y,x',y')\, p(x',y')\, dx'dy'$$

$$-\frac{\partial}{\partial x}\left(\frac{w^3}{12\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial w}{\partial t} + 2u_L = 0$$

NO LEAKOFF

$$-\frac{1}{12\mu}\left[\frac{\partial w^3}{\partial x}\frac{\partial p}{\partial x} + w^3\frac{\partial^2 p}{\partial x^2}\right] + \frac{\partial w}{\partial t} + \cancel{2u_L} = 0$$

$$f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + \frac{1}{2!}\frac{\partial^2 f}{\partial x^2}\Delta x^2 + \dots$$

$$\frac{\partial f}{\partial x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} - \frac{1}{2!}\frac{\partial^2 f}{\partial x^2}\Delta x + \dots$$

$$\frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \text{error} \sim O(\Delta x)$$

Forward finite-difference

$$f(x-\Delta x) = f(x) - \frac{\partial f}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Delta x^2 + \dots$$

$$\frac{\partial f}{\partial x} \approx \frac{f(x) - f(x-\Delta x)}{\Delta x} \qquad \text{Error} \sim \mathcal{O}(\Delta x)$$

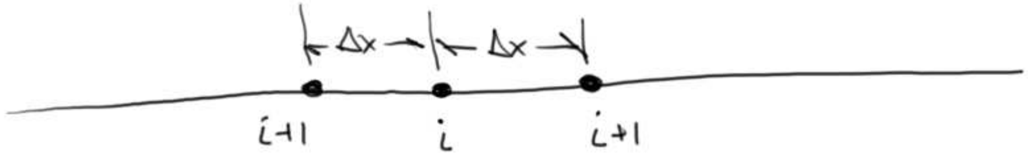
FD − BD (Taylor expansions)

$$\frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\,\Delta x} \qquad \text{error } \mathcal{O}(\Delta x^2)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\dfrac{f(x+\Delta x) - f(x)}{\Delta x} - \dfrac{f(x) - f(x-\Delta x)}{\Delta x}}{\Delta x}$$

$$= \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}$$

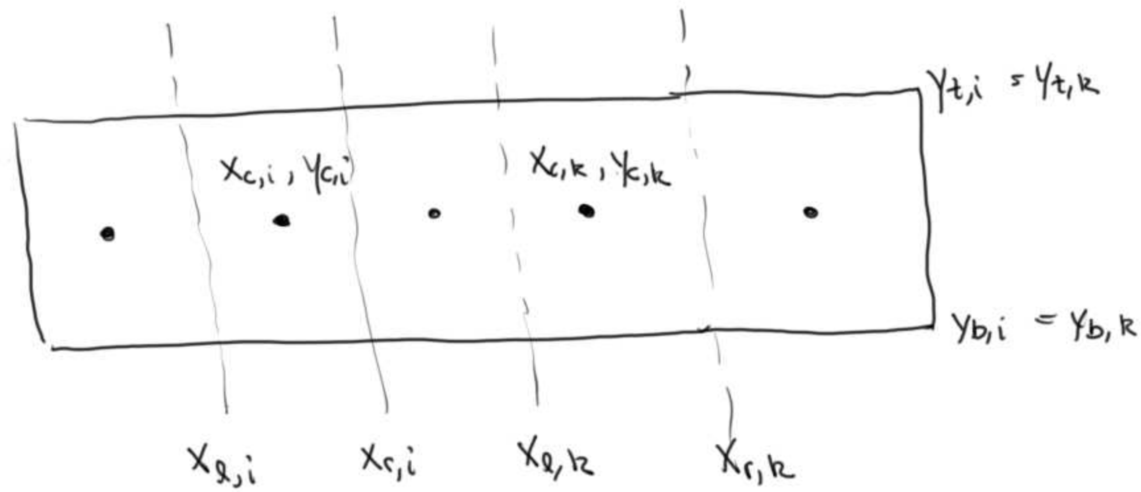
error $\mathcal{O}(\Delta x^2)$



$t = 0\Delta t, 1\Delta t, 2\Delta t, \ldots, N\Delta t$

$$-\frac{1}{12\mu}\left[\frac{w_{i+1}^3 - w_{i-1}^3}{2\Delta x} \cdot \frac{p_{i+1} - p_{i-1}}{2\Delta x} + w_i^3\left[\frac{p_{i+1} - 2p_i + p_{i-1}}{\Delta x^2}\right]\right] + \frac{w_i^{N+1} - w_i^N}{\Delta t} = 0$$

Crouch & Starfield (1983)



$$w(x,y) = \iint f(x,y,x',y')\, p(x',y')\, dx'dy'$$

$$= \sum_k f(x_i, y_i, x_k, y_k)\, p(x_k, y_k)$$

$$p(x_i) = \sum_k A_{ik} w_k$$

shear modulus

$$A_{ik} = \frac{G}{4\pi(1-\nu)}\, I(x_{c,i}, y_{c,i}, x_{l,k}, y_{t,k}, x_{r,k}, y_{b,k})$$

$$I = \frac{\left[(x_{c,i} - x_{r,k})^2 + (y_{c,i} - y_{t,k})^2 \right]^{1/2}}{(x_{c,i} - x_{r,k})(y_{c,i} - y_{t,k})}$$

$$- \frac{\left[(x_{c,i} + x_{l,k})^2 + (y_{c,i} - y_{t,k})^2 \right]^{1/2}}{(x_{c,i} + x_{l,k})(y_{c,i} - y_{t,k})}$$

$$p(x_i) = A_{ik} \omega_k$$

$$+ \frac{\left[(x_{c,i} + x_{r,k})^2 + (y_{c,i} + y_{b,k})^2 \right]^{1/2}}{(x_{c,i} + x_{r,k})(y_{c,i} + y_{b,k})}$$

$$- \frac{\left[(x_{c,i} - x_{r,k})^2 + (y_{c,i} + y_{b,k})^2 \right]^{1/2}}{(x_{c,i} - x_{r,k})(y_{c,i} + y_{b,k})}$$

$$-\frac{1}{12\mu}\left[\frac{w_{i+1}^3-w_{i-1}^3}{2\Delta x}\cdot\frac{A_{(i+1)k}w_k-A_{(i-1)k}w_k}{2\Delta x}+w_i^3\left[\frac{A_{(i+1)k}w_k-2A_{ik}w_k+A_{(i-1)k}w_k}{\Delta x^2}\right]\right]$$

$$+\frac{w_i^{N+1}-w_i^N}{\Delta t}=0$$