Strain tensor

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad \text{for} \quad i = 1,2,3 \quad j = 1,2,3$$

Volumetric strain

$$\varepsilon_{vol} = \mathrm{tr}(\boldsymbol{\varepsilon}) = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$$

Material constants for isotropic materials

Young's modulus

$$E = \frac{S_{11}}{\varepsilon_{11}}$$

Bulk modulus

$$K = \frac{S_{11} + S_{22} + S_{33}}{3\varepsilon_{vol}}$$

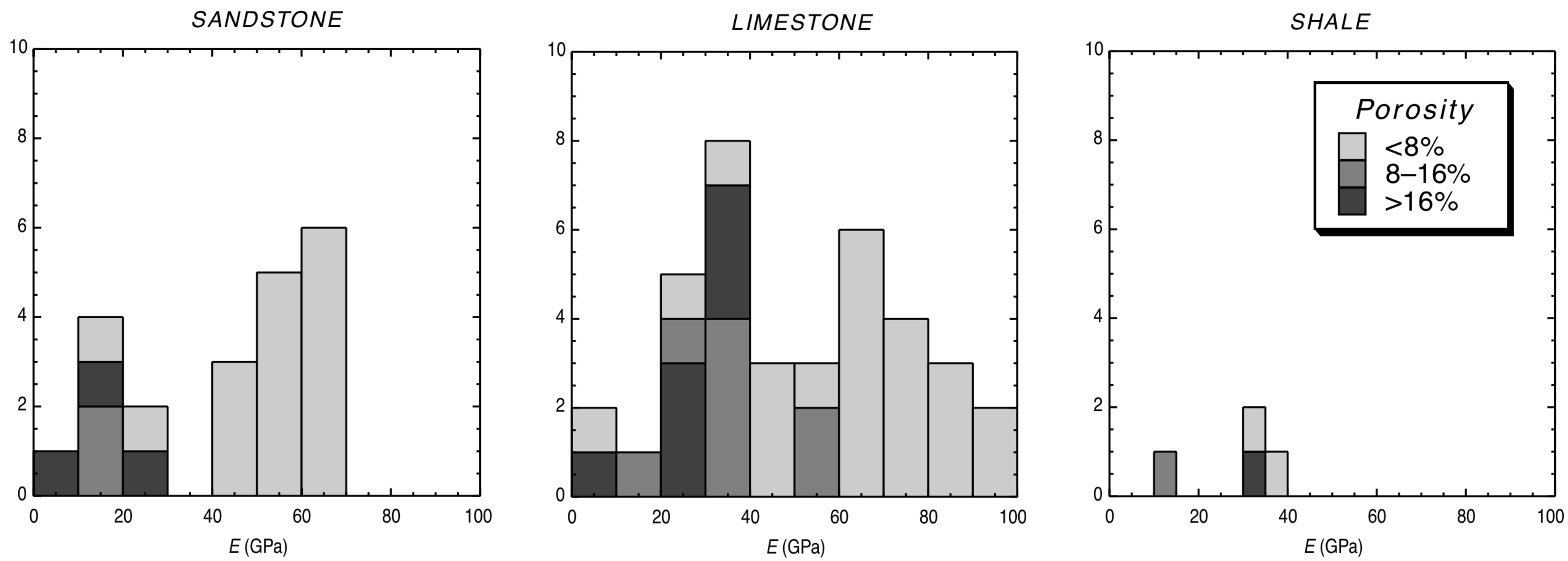
Shear Modulus

$$G = \frac{1}{2}\frac{S_{13}}{\varepsilon_{13}}$$

Poisson's ratio

$$\nu = \frac{\varepsilon_{33}}{\varepsilon_{11}}$$

Typical Young's modulus values



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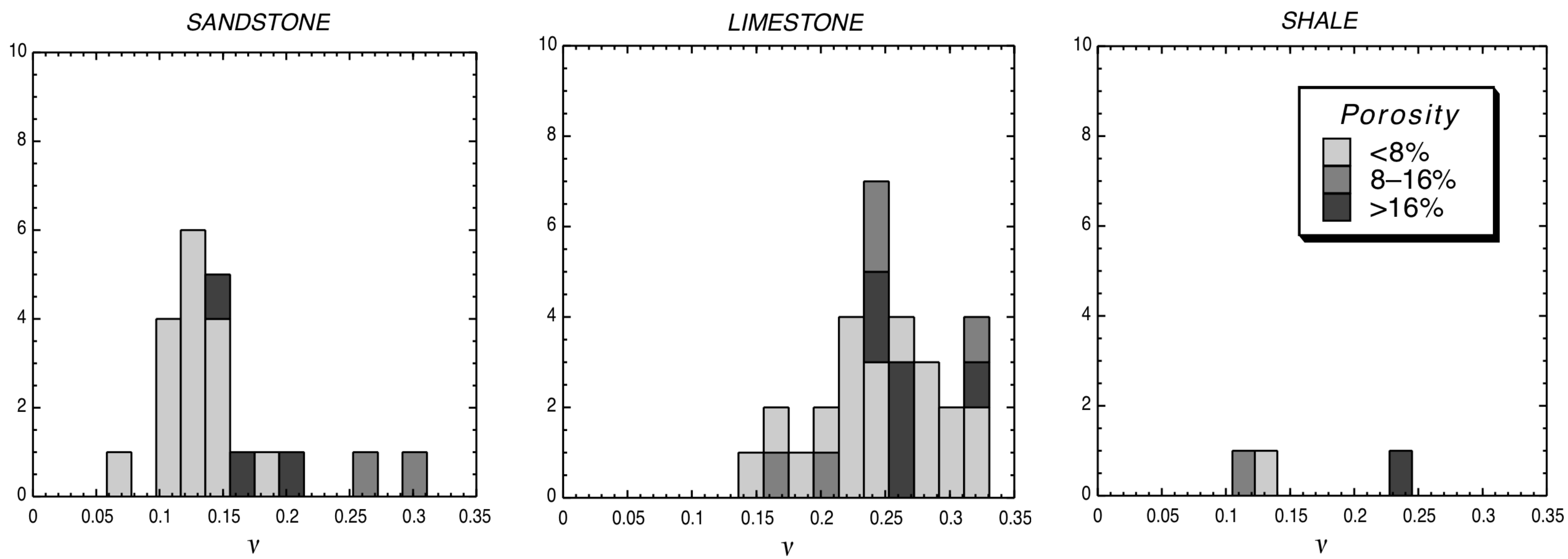
Typical Poissons' modulus values



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Generalized Hooke's law

$$\vec{\sigma} = \boldsymbol{C}\vec{\varepsilon}$$

For isotropic materials

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(1-2\nu) \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{12} \\ 2\varepsilon_{13} \\ 2\varepsilon_{23} \end{Bmatrix}$$

$$\mathbf{S} = K\varepsilon_{vol}\mathbf{I} + 2G\left(\boldsymbol{\varepsilon} - \frac{1}{3}\varepsilon_{vol}\mathbf{I}\right), \qquad G = \frac{E}{2(1+\nu)} \Rightarrow \text{shear modulus}$$

$$\mathbf{S} = \lambda\varepsilon_{vol}\mathbf{I} + 2G\boldsymbol{\varepsilon}, \qquad \lambda = K - \frac{2}{3}G \Rightarrow \text{Lamé's constant}$$

Relationships between constants

	$K =$	$E =$	$\lambda =$	$G =$	$\nu =$	$M =$
(K, E)	K	E	$\frac{3K(3K-E)}{9K-E}$	$\frac{3KE}{9K-E}$	$\frac{3K-E}{6K}$	$\frac{3K(3K+E)}{9K-E}$
(K, λ)	K	$\frac{9K(K-\lambda)}{3K-\lambda}$	λ	$\frac{3(K-\lambda)}{2}$	$\frac{\lambda}{3K-\lambda}$	$3K - 2\lambda$
(K, G)	K	$\frac{9KG}{3K+G}$	$K - \frac{2G}{3}$	G	$\frac{3K-2G}{2(3K+G)}$	$K + \frac{4G}{3}$
(K, ν)	K	$3K(1 - 2\nu)$	$\frac{3K\nu}{1+\nu}$	$\frac{3K(1-2\nu)}{2(1+\nu)}$	ν	$\frac{3K(1-\nu)}{1+\nu}$
(K, M)	K	$\frac{9K(M-K)}{3K+M}$	$\frac{3K-M}{2}$	$\frac{3(M-K)}{4}$	$\frac{3K-M}{3K+M}$	M
(E, λ)	$\frac{E+3\lambda+R}{6}$	E	λ	$\frac{E-3\lambda+R}{4}$	$\frac{2\lambda}{E+\lambda+R}$	$\frac{E-\lambda+R}{2}$
(E, G)	$\frac{EG}{3(3G-E)}$	E	$\frac{G(E-2G)}{3G-E}$	G	$\frac{E}{2G} - 1$	$\frac{G(4G-E)}{3G-E}$
(E, ν)	$\frac{E}{3(1-2\nu)}$	E	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{E}{2(1+\nu)}$	ν	$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$

Relationships between constants (contd.)

	$K =$	$E =$	$\lambda =$	$G =$	$\nu =$	$M =$
(E, M)	$\frac{3M-E+S}{6}$	E	$\frac{M-E+S}{4}$	$\frac{3M+E-S}{8}$	$\frac{E-M+S}{4M}$	M
(λ, G)	$\lambda + \frac{2G}{3}$	$\frac{G(3\lambda+2G)}{\lambda+G}$	λ	G	$\frac{\lambda}{2(\lambda+G)}$	$\lambda + 2G$
(λ, ν)	$\frac{\lambda(1+\nu)}{3\nu}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$	λ	$\frac{\lambda(1-2\nu)}{2\nu}$	ν	$\frac{\lambda(1-\nu)}{\nu}$
(λ, M)	$\frac{M+2\lambda}{3}$	$\frac{(M-\lambda)(M+2\lambda)}{M+\lambda}$	λ	$\frac{M-\lambda}{2}$	$\frac{\lambda}{M+\lambda}$	M
(G, ν)	$\frac{2G(1+\nu)}{3(1-2\nu)}$	$2G(1+\nu)$	$\frac{2G\nu}{1-2\nu}$	G	ν	$\frac{2G(1-\nu)}{1-2\nu}$
(G, M)	$M - \frac{4G}{3}$	$\frac{G(3M-4G)}{M-G}$	$M - 2G$	G	$\frac{M-2G}{2M-2G}$	M
(ν, M)	$\frac{M(1+\nu)}{3(1-\nu)}$	$\frac{M(1+\nu)(1-2\nu)}{1-\nu}$	$\frac{M\nu}{1-\nu}$	$\frac{M(1-2\nu)}{2(1-\nu)}$	ν	M

Siesmic wave velocity

$$V_p = \sqrt{\frac{M}{\rho}}, \qquad V_s = \sqrt{\frac{G}{\rho}}$$

Poroelasticity

Poroelasticity Assumptions

1. There is an interconnected pore system uniformly saturated with fluid.
2. The total volume of the pore system is small compared to the volume of the rock.
3. The pore pressure, the total stress acting on the rock externally, and the stresses acting on the grains are statistically defined.

Effective stress

Terzaghi definition

$$\boldsymbol{\sigma}' = \mathbf{S} - P_p \mathbf{I}$$

"Exact" effective stress

$$\boldsymbol{\sigma}'' = \mathbf{S} - \alpha P_p \mathbf{I}$$

α is called Biot's coefficient

Biot's coefficeint

$$\alpha = 1 - \frac{K_T}{K_S}$$

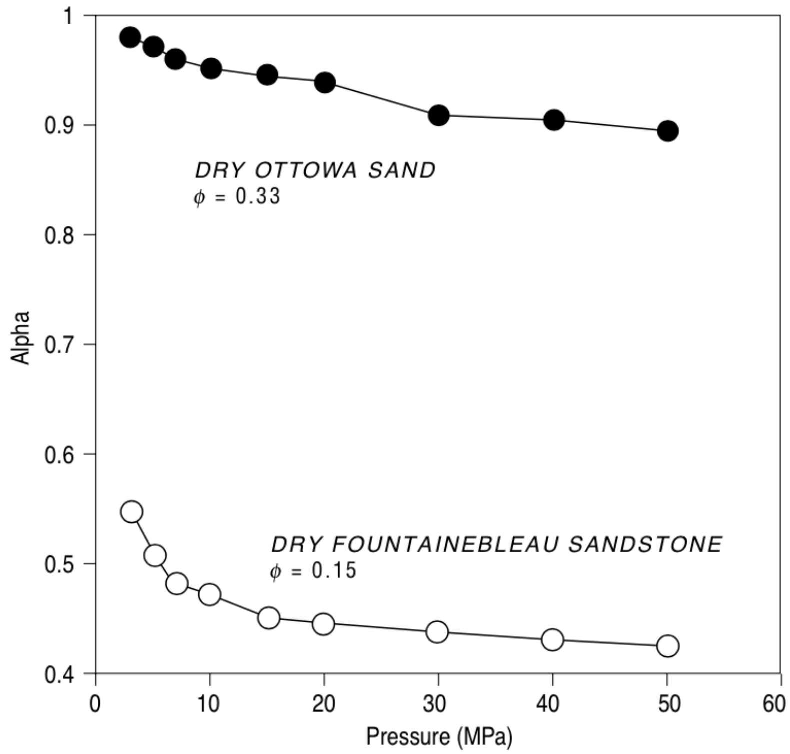
For sand

$$K_S >> K_T \qquad \alpha \approx 1$$

For rocks

$$\alpha \approx \frac{2}{3}$$

Biot's coefficient (cont.)



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