Problem 1

(2 points each) Circle the best answer:

(i) Which of the following is NOT a type of plate boundary?

(a) Convergent plate boundary
(b) Transform plate boundary
(c) Asthenospheric plate boundary

(ii) True or False? The eigenvectors of the stress tensor are the principle stress directions.

(a) True
(b) False

(iii) In a normal faulting regime, the hanging wall moves which way relative to the footwall?

(a) Vertically up
(b) Vertically down
(c) Horizontally

(iv) In a strike-slip faulting regime, the hanging moves which way relative to the footwall?

(a) Vertically up
(b) Vertically down
(c) Horizontally

(v) True or False? If we idealize the earth's surface as a half-space, the vertical stress due to overburden is a principle stress.

(a) True
(b) False

(vi) In the context of characterizing a fault, the *dip* angle is

(a) the angle of the fault strike with respect to north.
(b) the angle of the fault plane with respect to horizontal.
(c) the angle that defines the displacement vector of the fault slip motion measured from horizontal.

(vii) The Cauchy stress tensor is always symmetric due to what principle?

(a) Conservation of linear momentum
(b) Conservation of angular momentum
(c) Conservation of energy

(viii) The range of depths for the earth's crust is

(a) 0-5 km.
(b) 0-100 km.
(c) 0-2900 km.

Problem 1 continued on next page...

(ix) According to Anderson fault classification, the principle stress magnitudes have the following ordering for a normal faulting regime.

(a) $S_v > S_{Hmax} > S_{hmin}$

(b) $S_{Hmax} > S_v > S_{hmin}$

(c) $S_v > S_{hmin} > S_{Hmax}$

(x) True or False? A good rule of thumb for estimating the overburden stress in the continental crust is that the stress increases by 0.44 psi/ft.

(a) True

(b) False

(xi) If we are given the three principle stresses S_v, S_{Hmax}, and S_{hmin}, what else is needed to fully characterize the tectonic stress field.

(a) The direction of either S_{Hmax} or S_{hmin}

(b) The dip and strike angles of the nearest fault

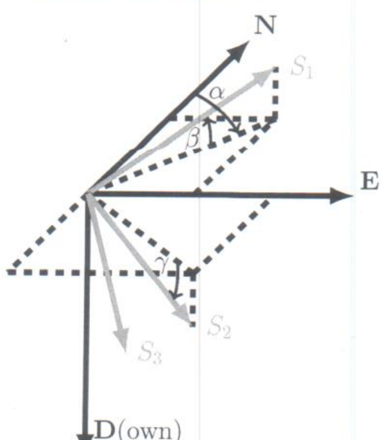
(c) The type of faulting regime according to Anderson classification.

(xii) True or False? A lithostatic pore pressure is one in which the ratio of pore pressure to S_v is exactly 0.44.

(a) True

(b) False

(xiii) Consider the following figure:



If S_1 was initially in the direction of **N**, which rotation would reorient S_1 so that it was aligned with **Down**?

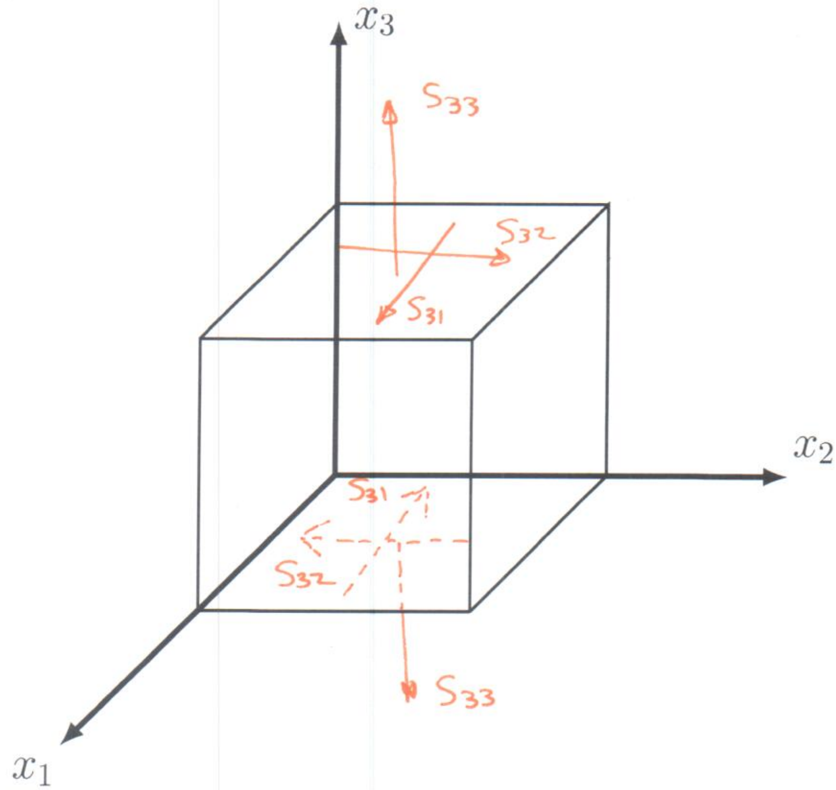
(a) $\gamma = 90°$

(b) $\beta = 90°$

(c) $\beta = 270°$

Problem 2

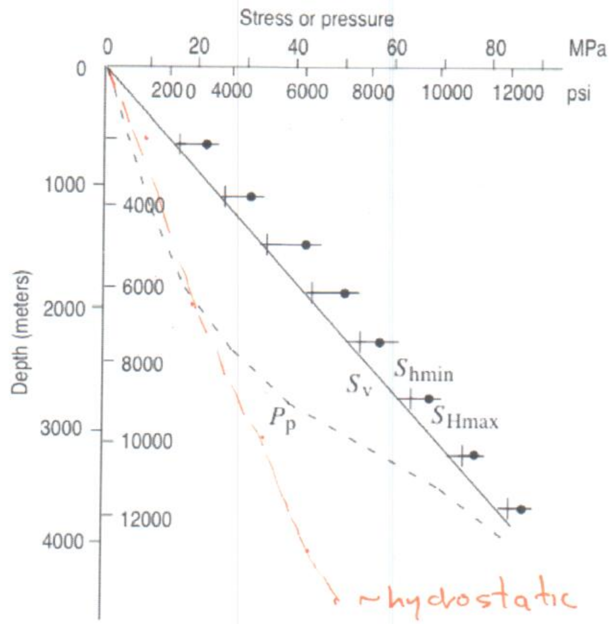
(4 points) Consider the following figure:



On the figure, correctly indicate (with hand-drawn arrows) the stress components S_{33}, S_{31}, and S_{32} on both sides of the cube where they act.

Problem 3

(2 points each) Consider the figure, and circle the best answers to the questions:



(i) The pore pressure profile between 3000 m and 4000 m is

- (a) hydrostatic.
- (b) lithostatic.
- (c) overpressured.
- (d) underpressured.

(ii) The minimum principle stress is

- (a) S_v.
- (b) S_{hmin}.
- (c) S_{Hmax}.

Problem 4

For the following matrix **A**,

$$\mathbf{A} = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}$$

perform the following calculations **by hand**:

(i) (**2 points**) Compute the determinate of **A**.

(ii) (**2 points**) Compute the eigenvalues of **A**.

(iii) (**2 points**) Compute the eigenvectors of **A**, put them in unit-vector form.

(i) $4(-3) - (-5)(2) = -12 + 10 = \boxed{-2 = \det(A)}$ m

(ii) $\det(A - \lambda I) = \det\left(\begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{bmatrix}\right) = 0$

$(4-\lambda)(-3-\lambda) + 10 = 0$

$\lambda^2 - \lambda - 2 = 0$

$\lambda_{1,2} = \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2} = \dfrac{1}{2} \pm \dfrac{3}{2}$

$\boxed{\lambda_{1,2} = \{2, -1\}}$ m

(iii) For $\lambda = 2$

$\begin{bmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 2 & -5 & 0 \\ 2 & -5 & 0 \end{array}\right] \xrightarrow{-R1+R2} \left[\begin{array}{cc|c} 2 & -5 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$2x_1 = 5x_2$

$x_2 = \text{free} \Rightarrow 1$

$x_1 = \dfrac{5}{2}$

$x_2 = 1$

$\sqrt{\left(\frac{5}{2}\right)^2 + (1)^2} = \dfrac{\sqrt{29}}{2}$

$\hat{x}_1 = \dfrac{5}{2} \cdot \dfrac{2}{\sqrt{29}} = \dfrac{5}{\sqrt{29}}$

$\hat{x}_2 = 1 \cdot \dfrac{2}{\sqrt{29}} = \dfrac{2}{\sqrt{29}}$

$\hat{v}_1 = \begin{bmatrix} 5/\sqrt{29} \\ 2/\sqrt{29} \end{bmatrix}$ m

This page provided for additional calculations

For $\lambda = -1$

$$\begin{bmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -5 \\ 2 & -2 \end{bmatrix} \xrightarrow{\frac{1}{5}R_1 \; + \; \frac{1}{2}R_2} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$x_1 = x_2$

$x_2 = \text{free} \rightarrow 1$

$x_1 = 1$

$x_2 = 1$

$\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

$\hat{x}_1 = \frac{1}{\sqrt{2}}$

$\hat{x}_2 = \frac{1}{\sqrt{2}}$

$$\hat{v}_2 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

Problem 5

(**20 points**) Given the following principle stresses under strike-slip faulting (using Anderson classification). What is the geographical stress tensor, $\mathbf{S}_G$, if S_{Hmax} is oriented exactly northwest?

$$S_1 = 60 \text{ MPa}, \quad S_2 = 45 \text{ MPa}, \quad S_3 = 40 \text{ MPa}$$

Anderson strike-slip $S_{Hmax} > S_v > S_{hmin}$

$S_1 \rightarrow S_{Hmax}$

$S_2 \rightarrow S_v$

$S_3 \rightarrow S_{hmin}$

Using figure from Problem 1 (xiii) need to rotate S_2 to D so $\underline{\gamma = 90^\circ}$ and rotate S_1 to NW so $\underline{\alpha = -45^\circ}$ $(\beta = 0^\circ)$

$$R_G = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & -\sin\beta \\ \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \cos\beta\sin\gamma \\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \cos\beta\cos\gamma \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \end{bmatrix}$$

$$S_G = R_G^T S R_G \qquad S R_G = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 45 & 0 \\ 0 & 0 & 40 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \end{bmatrix} = \begin{bmatrix} 30\sqrt{2} & -30\sqrt{2} & 0 \\ 0 & 0 & 45 \\ -20\sqrt{2} & -20\sqrt{2} & 0 \end{bmatrix}$$

$$= R_G^T (S R_G) = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 30\sqrt{2} & -30\sqrt{2} & 0 \\ 0 & 0 & 45 \\ -20\sqrt{2} & -20\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 50 & -10 & 0 \\ -10 & 50 & 0 \\ 0 & 0 & 45 \end{bmatrix}$$

Problem 6

Given the geographical stress,

$$\mathbf{S}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix} \text{ MPa}$$

(i) **(10 points)** For a fault strike oriented to the northeast and a dip 70° from southeast, determine the normal and shear stresses acting on the fault plane.

(ii) **(5 points)** If the fault where to slip, what type of fault slip would it be?

(iii) **(10 points)** Faults typically slip when the ratio of shear to normal effective stress exceeds 0.6, i.e. $\tau/\sigma_n^{eff} > 0.6$. Using this criterion, calculate the critical injection pressure in petroleum engineering operations that one should not exceed to prevent fault slippage. You can assume that the fault is very near the injector and steady state conditions, such that the pore pressure is equal to the injection pressure.

(i) strike = 45°

dip = 70°

$$\hat{n} = \begin{bmatrix} -\sin(st)\sin(dip) \\ \cos(st)\sin(dip) \\ -\cos(dip) \end{bmatrix} = \begin{bmatrix} -0.664463 \\ 0.664463 \\ -0.34202 \end{bmatrix}$$

$$\vec{t} = S_G \cdot \hat{n} = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix} \begin{bmatrix} -0.6644 \\ 0.6644 \\ -0.34202 \end{bmatrix} = \begin{bmatrix} -39.8677 \\ 39.8677 \\ -13.6808 \end{bmatrix}$$

$$\hat{n}_s = \begin{bmatrix} \cos(st) \\ \sin(st) \\ 0 \end{bmatrix} = \begin{bmatrix} 0.707106 \\ 0.707106 \\ 0 \end{bmatrix}$$

$$\sigma_n = \vec{t}^T \cdot \hat{n} = 57.66 \text{ MPa}$$

$$\tau_s = \vec{t}^T \cdot \hat{n}_s = 0.0 \text{ MPa}$$

$$\hat{n}_d = \begin{bmatrix} -\sin(st)\cos(dip) \\ \cos(st)\cos(dip) \\ \sin(dip) \end{bmatrix} = \begin{bmatrix} -0.241844 \\ 0.241844 \\ 0.93969 \end{bmatrix}$$

$$\tau_d = \vec{t}^T \cdot \hat{n}_d = 6.42787 \text{ MPa}$$

This page provided for additional calculations

(ii) If the fault slips it would slip in the positive dip direction, so in a normal faulting.

(iii) $\sigma_n^{eff} = \sigma_n - P_p$

$$\frac{\tau_d}{\sigma_n^{eff}} \geq 0.6 \Rightarrow \frac{6.42787}{(57.66 - P_p)} \geq 0.6 \Rightarrow 6.42787 \geq 0.6(57.66 - P_p)$$

$$\frac{-6.42787 + (\cancel{0.6})(57.66)}{\cancel{0.6}} \geq P_p \Rightarrow \boxed{46.947 \text{ MPa} = P_{p(crit)}}$$

Problem 7

(**15 points**) Given the geographical stress,

$$\mathbf{S}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix} \text{ MPa}$$

For a wellbore deviated 30° from vertical along an azimuth oriented directly to the north, find the wellbore stress tensor, $\mathbf{S}_B$.

$\phi = 30^\circ$

$\delta = 0^\circ$

$$R_B = \begin{bmatrix} \cos\delta\cos\phi & \sin\delta\cos\phi & -\sin\phi \\ -\sin\delta & \cos\delta & 0 \\ \cos\delta\sin\phi & \sin\delta\sin\phi & \cos\phi \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$S_B = R_B S R_B^T \qquad (S R_B^T) = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 & +\frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 41.1362 & -12.5 & 23.75 \\ -10.82 & 47.5 & -6.25 \\ -20 & 0 & 34.64 \end{bmatrix}$$

$$S_B = R_B (S R_B)^T = \begin{bmatrix} 45.625 & -10.825 & 3.247 \\ -10.825 & 47.5 & -6.25 \\ 3.247 & -6.25 & 4.875 \end{bmatrix} \text{ MPa}$$