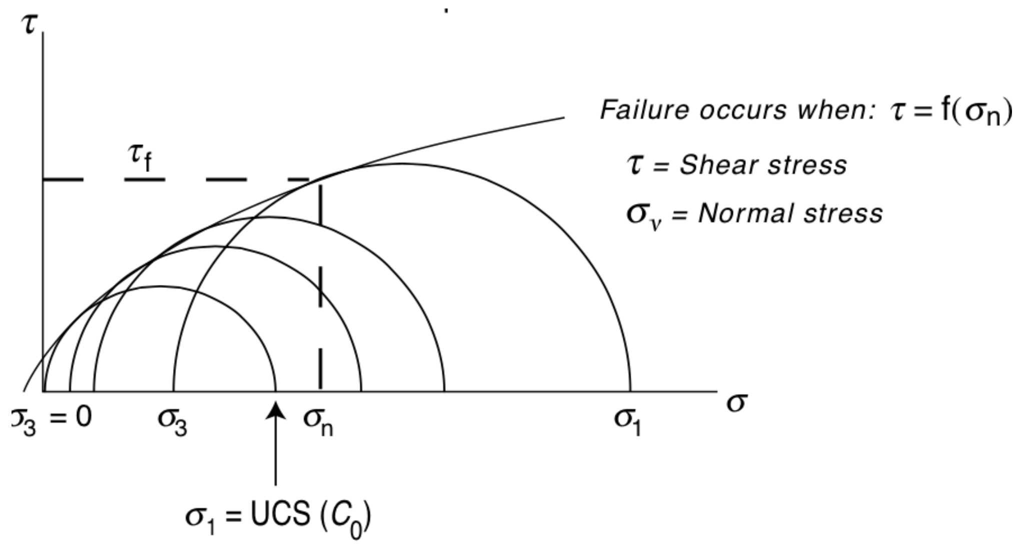
Mohr's circles

$$\tau_f = \frac{1}{2}(\sigma_1 - \sigma_3)\sin(2\beta)$$
$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos(2\beta)$$

Mohr Envelope



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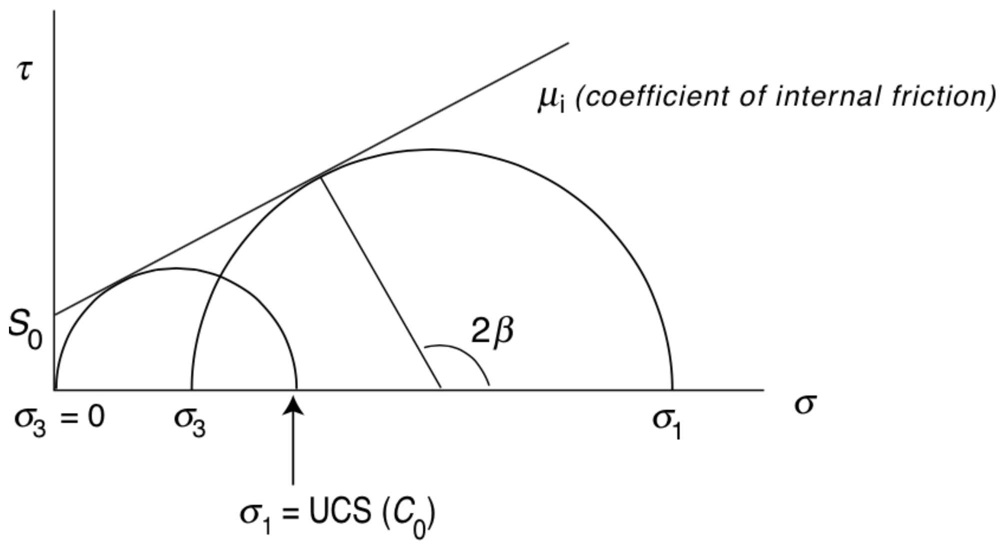
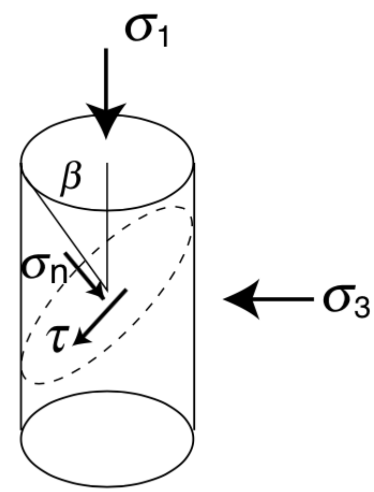
Linearized Mohr Envelope





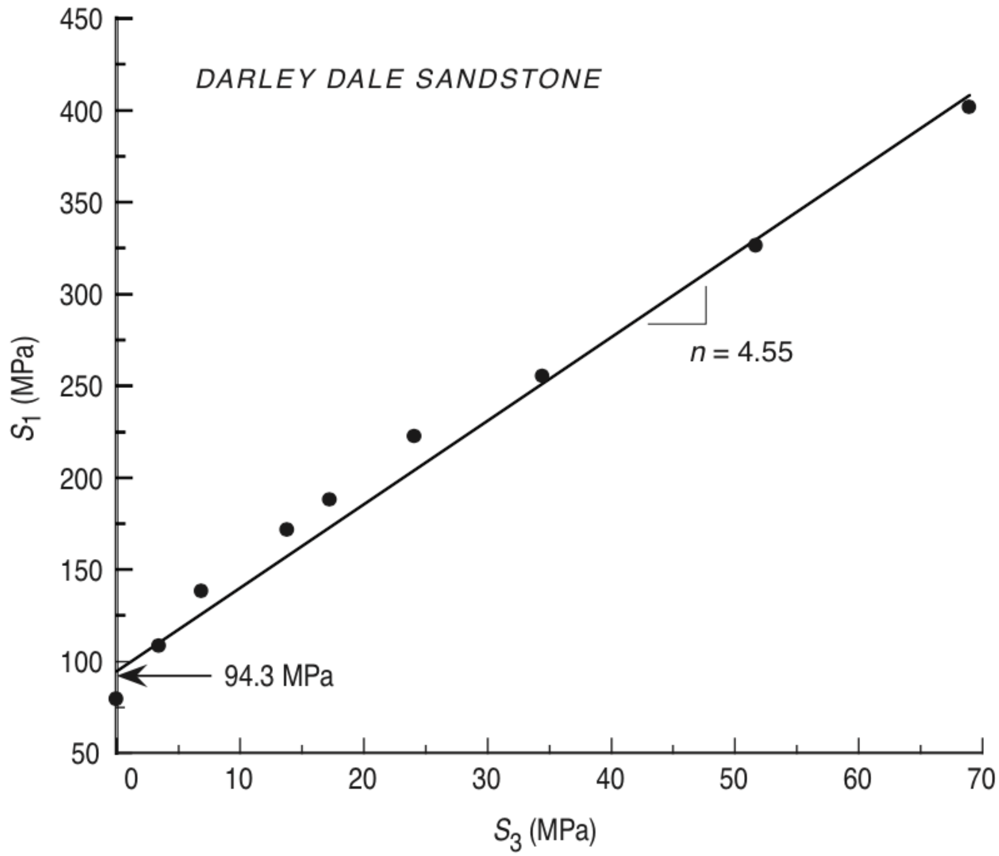
 Zoback, *Reservoir Geomechanics* (Fig. 4.2a,c pp. 88)

Mohr-Coulomb failure

$$\tau = S_0 + \sigma_n \mu_i$$

$$C_0 = 2S_0 \left(\sqrt{\mu_i^2 + 1} + \mu_i \right)$$

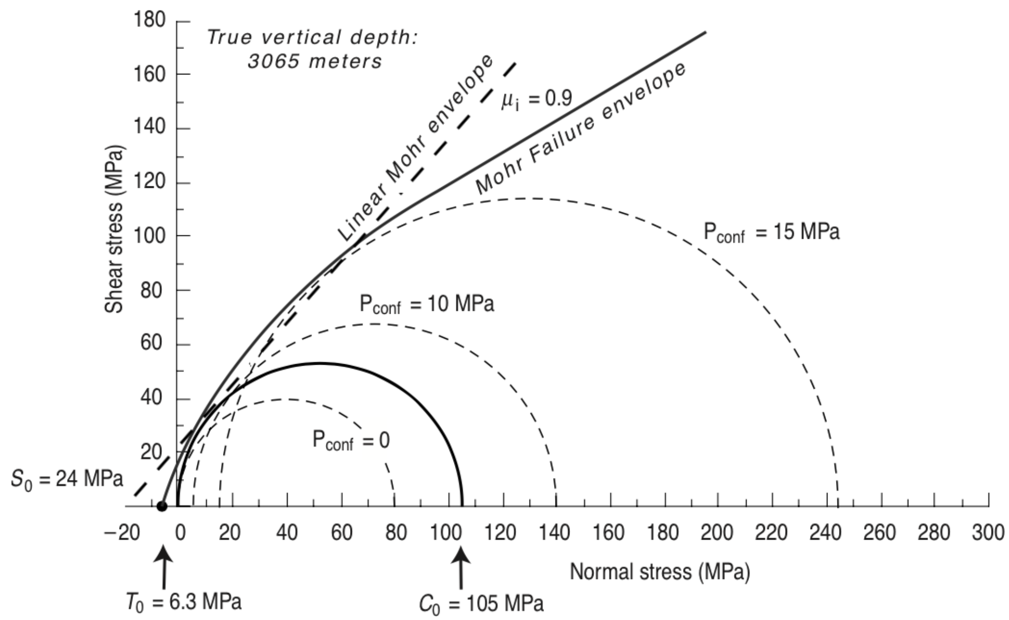
Triaxial tests on sandstone



$$\mu_i = \frac{n-1}{2\sqrt{n}}$$

 Zoback, *Reservoir Geomechanics* (Fig. 4.3a pp. 90)

Mohr Envelope for Sandstone



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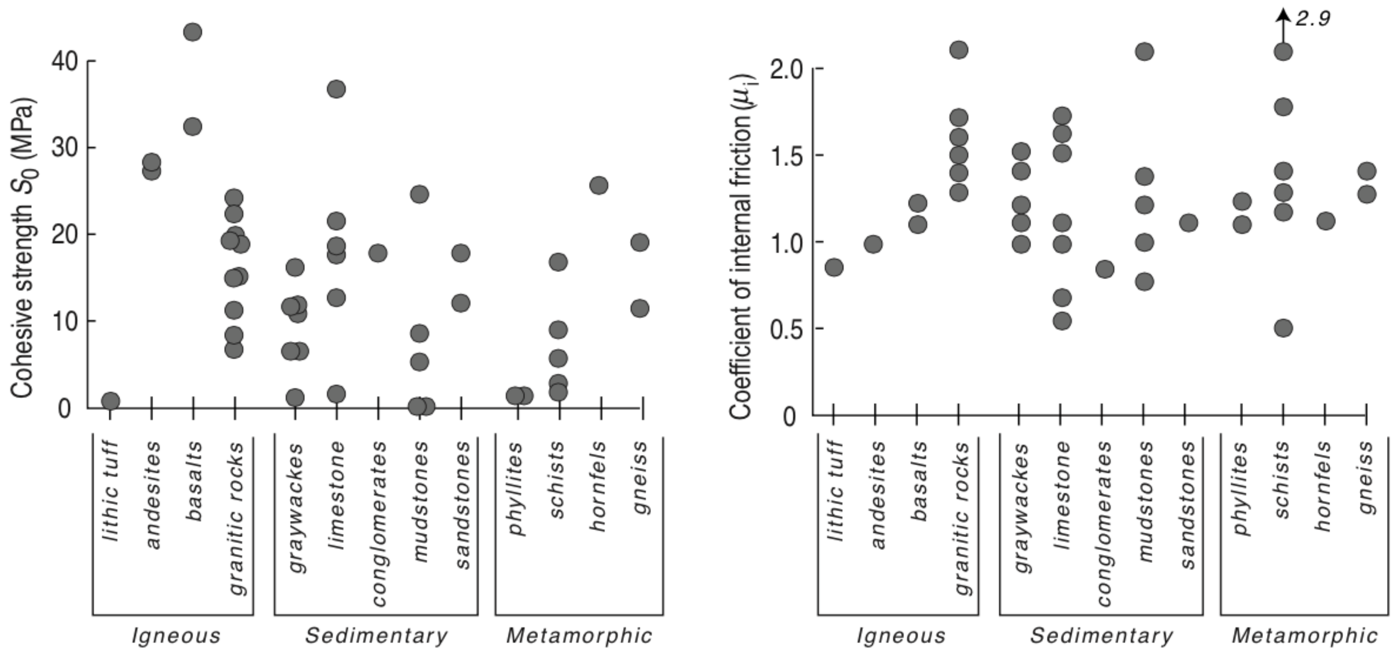
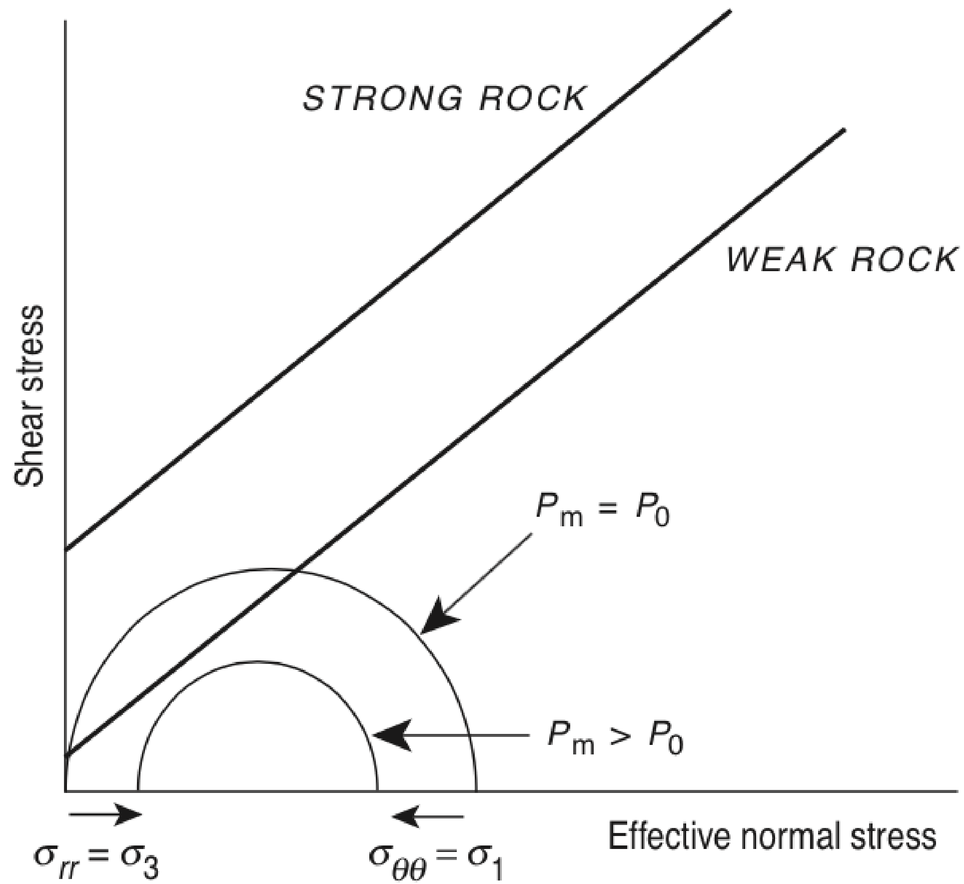
Cohesion and internal friction data



 Zoback, *Reservoir Geomechanics* (Fig. 4.4, pp. 91)

Cohesion and internal friction data



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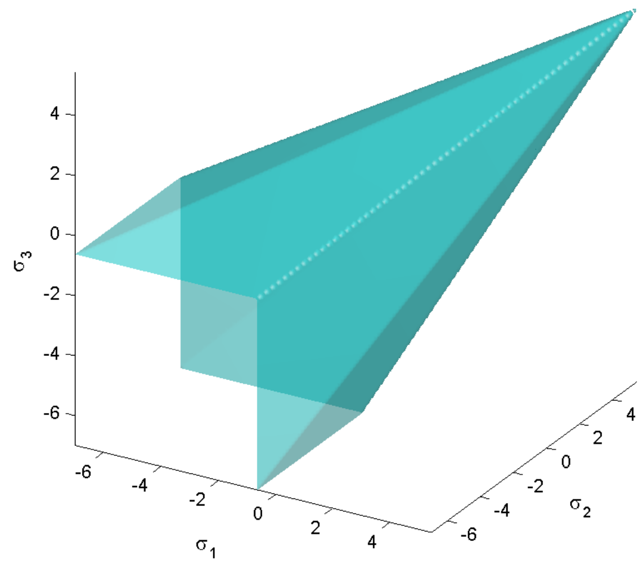
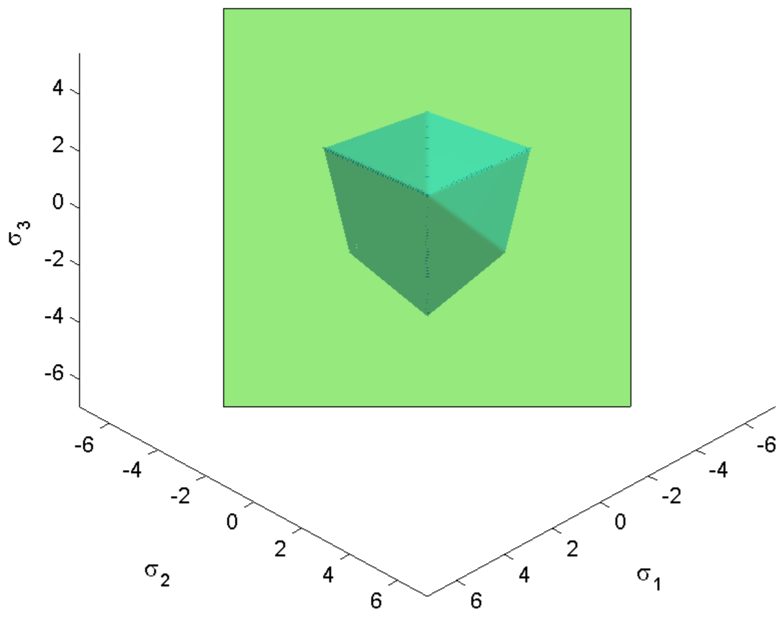
Yield surface



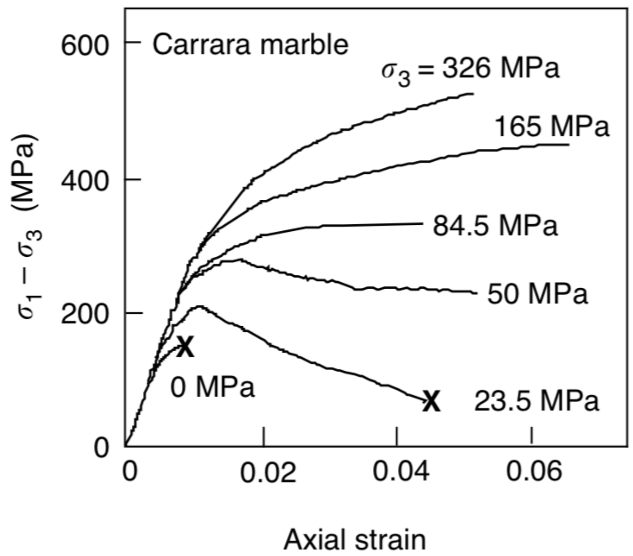
Mohr Coulomb Yield Surface 3Da. Licensed under CC BY-SA 3.0 via Wikipedia

π-plane



Mohr Coulomb Yield Surface 3Db. Licensed under CC BY-SA 3.0 via Wikipedia

Pressure dependence



© Blackwell Publishing Jaeger, et al., *Fundamentals of Rock Mechanics* (Fig. 4.5, pp. 86)

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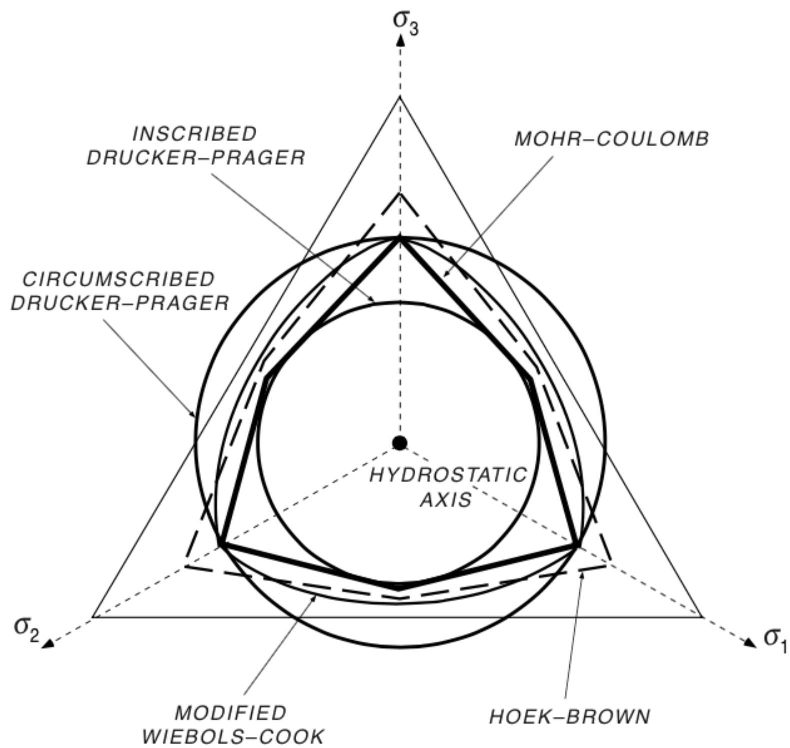
Other failure criteria



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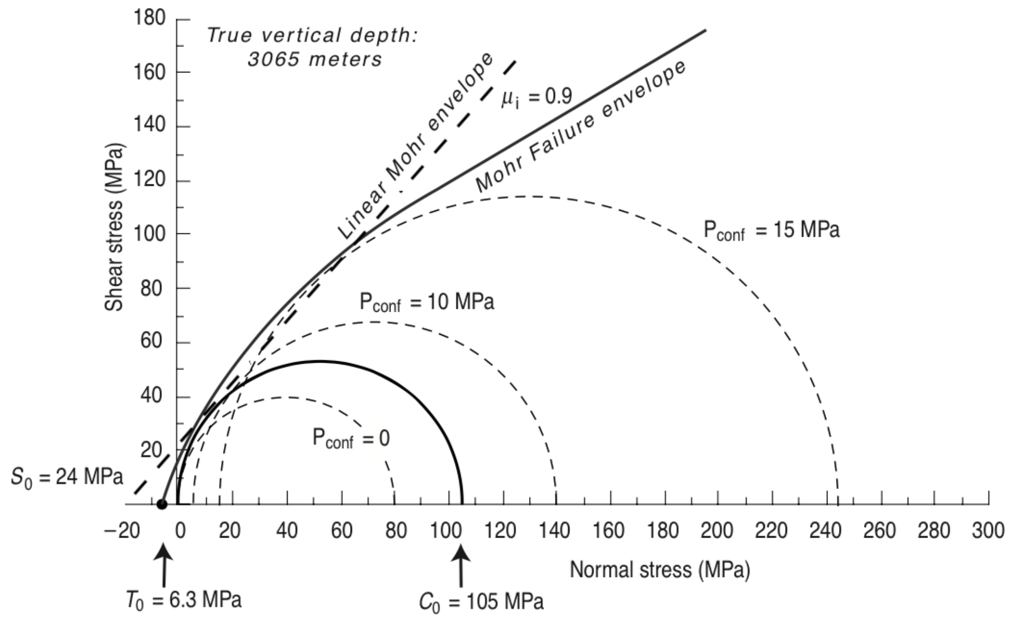
Recall: Mohr Envelope for Sandstone



 Zoback, *Reservoir Geomechanics* (Fig. 4.3a, pp. 88)

Hoek-Brown criterion (parabolic fitting)

$$\sigma_1 = \sigma_3 + C_0 \sqrt{m \frac{\sigma_3}{C_0} + s}$$

m and s are fitting parameters that depend on rock properties and the degress of fracturing.

Typical values

Typical Range of m	**Types of rocks**
$5 < m < 8$	carbonate rocks (dolomite, limestone, marble)
$4 < m < 10$	lithified argillaceous rocks (sandstones, quartizite)
$15 < m < 24$	arenaceous rocks (andesite, dolerite, diabase, rhyolite)
$22 < m < 33$	course-grained polymineurallic gineous and metamorphic (amphibolite, gabbro, gneiss, norite, quartz-diorite)

Intact Rocks -- $s \rightarrow 1$

Completely Granualated -- $s \rightarrow 0$

Lade Criterion

$$\left(\frac{I_1^3}{I_3} - 27\right)\left(\frac{I_1}{p_a}\right)^{m'} = \eta_1$$

with

$I_1 = S_{ii} = S_1 + S_2 + S_3$ (first invariant of $\mathbf{S}$)

$I_3 = \det(\mathbf{S}) = S_1 S_2 S_3$ (third invariant of $\mathbf{S}$)

p_a is atmospheric pressure, m' and n_1 are material constants

Modified Lade Criterion (dependece on σ_2)

$$\left(\frac{(I_1')^3}{I_3'}\right) = 27 + \eta$$

with

$$I_1' = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S)$$

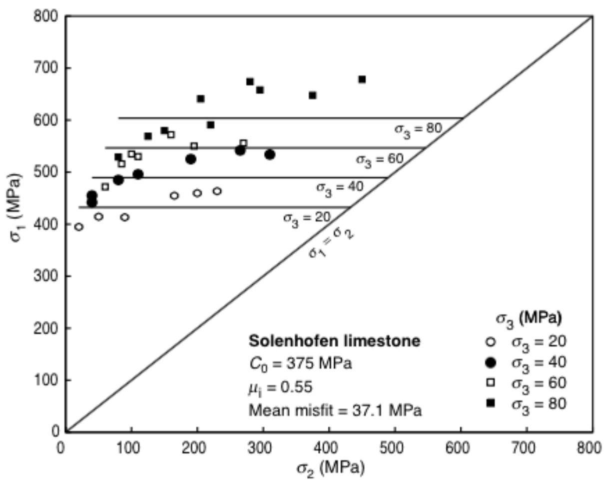
$$I_3' = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S)$$

$$S = \frac{S_0}{\tan\phi}$$

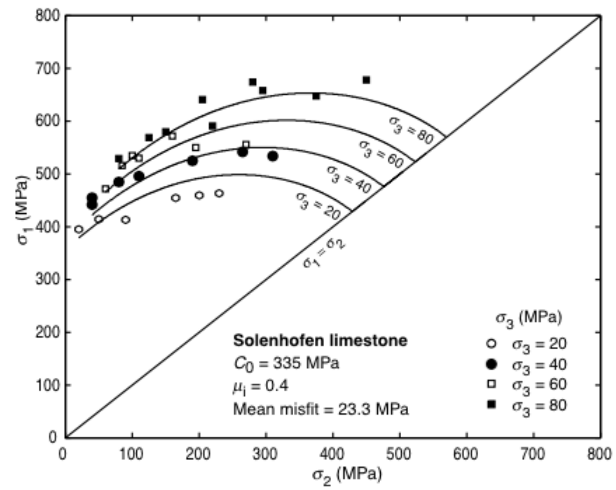
$$\eta = \frac{4(\tan\phi)^2(9 - 7\sin\phi)}{1 - \sin\phi)}$$

$\tan\phi = \mu_i$ and S_0 from Mohr-Coulomb criterion

Comparison



Mohr-Coulomb



modified Lade

 Zoback, *Reservoir Geomechanics* (Fig. 4.8b and 4.9b, pp. 96-97)

Others

- modified Wiebols-Cook
- Druker-Prager
- many more!