Stress

Physics 101

$dW = \int \vec{f} \cdot d\vec{x}$ $\qquad \vec{x} = \vec{x}(\underline{X}, t)$ "current" configuration

$P = \frac{dW}{dt} = \int \vec{f} \underbrace{\left(\frac{d\vec{x}}{dt}\right)}_{\vec{v}} \left(\frac{dV}{dV}\right)$

Note $dV = A\, d\vec{x} = A_j\, dx_j$

$P = \int \underbrace{\frac{f_i}{A_j}}_{\sigma} \underbrace{\frac{dv_i}{dx_j}}_{L} dV$

$L = \text{symm}(L) + \text{antisymm}(L)$

$= \underbrace{\cancel{\tfrac{1}{2}}(L + \cancel{L^T})}_{D} + \cancel{\tfrac{1}{2}}(L - \cancel{L^T})$

$= L$ $\quad D$

$= \int \sigma : (D + W)\, dV$

$\sigma : D = \sigma_{ij} D_{ij}$

$= D + W$

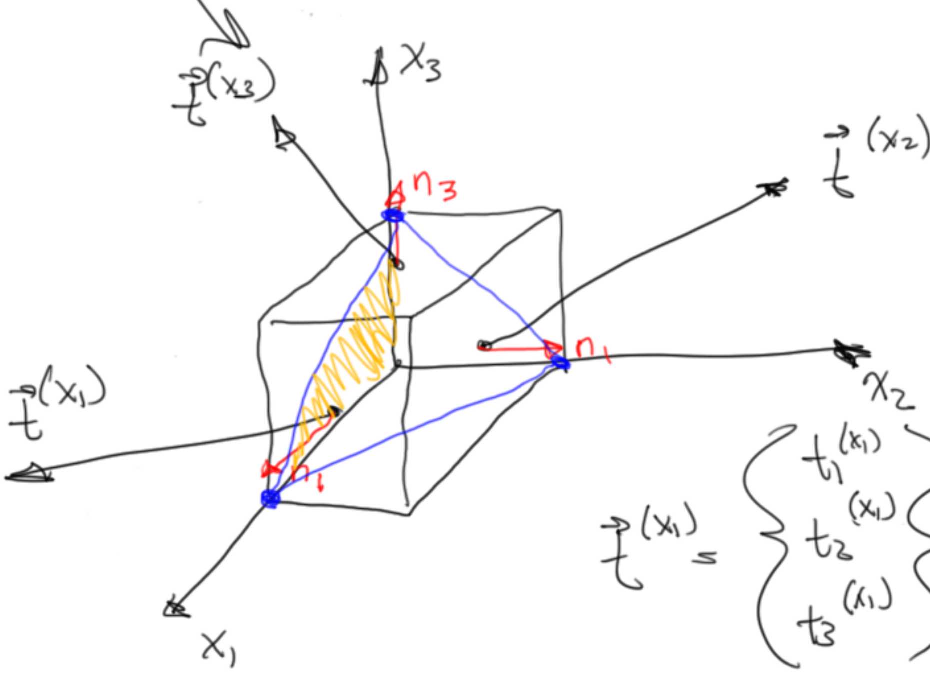
$\sigma : W = 0$

(Power)

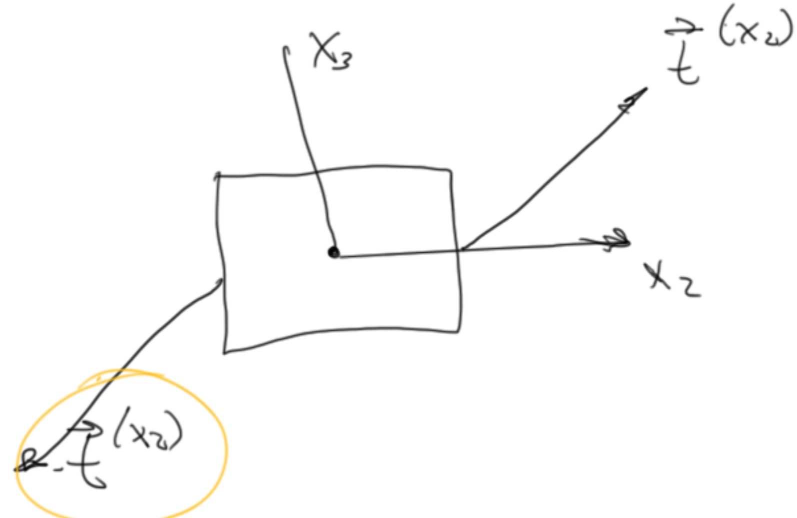
$P = \int \sigma : D\, dV \longrightarrow \int \sigma : \dot{\epsilon}\, dV$ $\quad$ Work conjugate

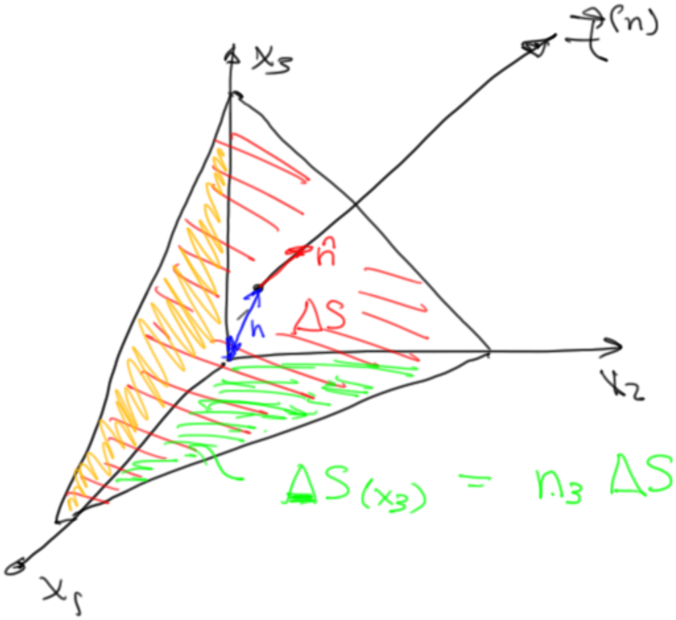
stress

Surface forces → tractions → $\frac{\text{force}}{\text{area}}$



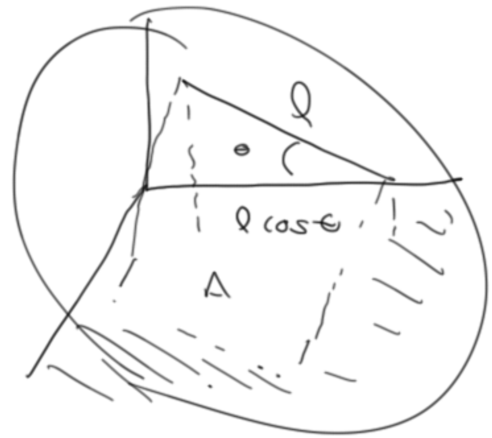
$$\vec{t}^{(x_1)} = \begin{Bmatrix} t_1^{(x_1)} \\ t_2^{(x_1)} \\ t_3^{(x_1)} \end{Bmatrix}$$





$\Delta S_{(x_3)} = n_3 \Delta S$

$$\hat{n} = \begin{Bmatrix} \cos(\hat{n}, x_1) \\ \cos(\hat{n}, x_2) \\ \cos(\hat{n}, x_3) \end{Bmatrix}$$



$\vec{F} = m\vec{a}$

$a_p = A \cos\theta$

$$\vec{F} = \vec{t}^{(n)} \cancel{\Delta S} - \vec{t}^{(x_2)} n_2 \cancel{\Delta S} - \vec{t}^{(x_1)} n_1 \cancel{\Delta S} - t^{(x_3)} n_3 \cancel{\Delta S} = \rho \cancel{\tfrac{1}{3} h} \cancel{\Delta S} \vec{a}$$

0

For small tet. $h \to 0$

$$\vec{t}^{(n)T} = \underline{t}^{(x_1)T} n_1 + t^{(x_2)T} n_2 + t^{(x_3)T} n_3$$

$$t^{(n)T} = (n_1 \quad n_2 \quad n_3) \underbrace{\begin{bmatrix} \vec{t}^{(x_1)T} \\ \vec{t}^{(x_2)T} \\ \vec{t}^{(x_3)T} \end{bmatrix}}_{\sigma \text{ stress tensor}}$$

tractions in the coord. directions

$$t_i^{(n)} = \sigma_{ji}\,\hat{n}_j$$

$$\vec{t}^{(n)} = \hat{n}\,\sigma$$

$$\vec{t}^{(n)} = \sigma^T\,\hat{n}$$

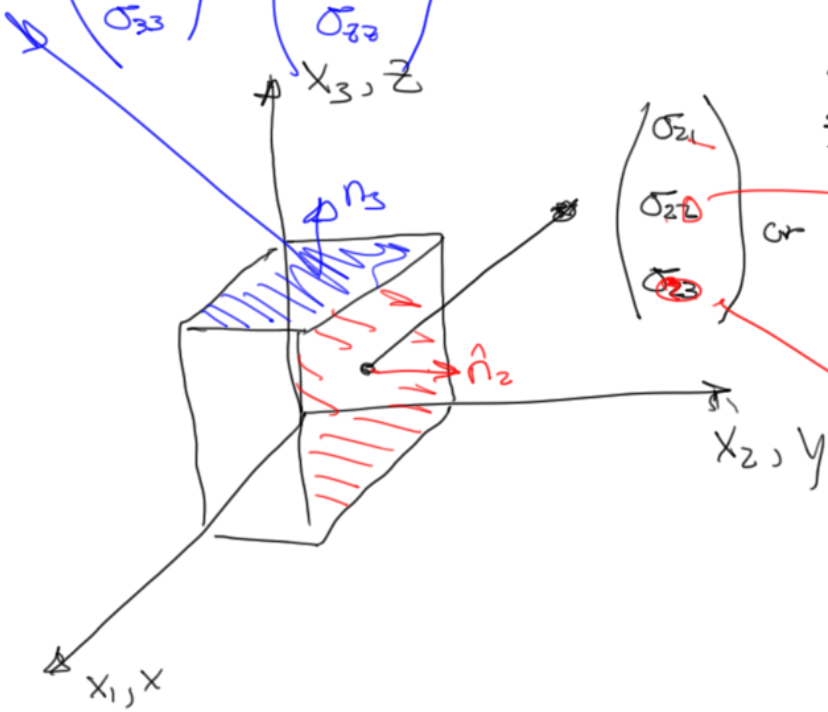
$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\begin{pmatrix} \sigma_{31} \\ \sigma_{32} \\ \sigma_{33} \end{pmatrix} \text{ or } \begin{pmatrix} \sigma_{zx} \\ \sigma_{zy} \\ \sigma_{zz} \end{pmatrix}$$

$1 \rightarrow x$

$2 \rightarrow y$

$3 \rightarrow z$



$$\begin{pmatrix} \sigma_{21} \\ \sigma_{22} \\ \sigma_{23} \end{pmatrix} \text{ or } \rightarrow \begin{pmatrix} \sigma_{yx} \\ \sigma_{yy} \\ \sigma_{yz} \end{pmatrix}$$

vector component

face defined by normal n_y

n_z