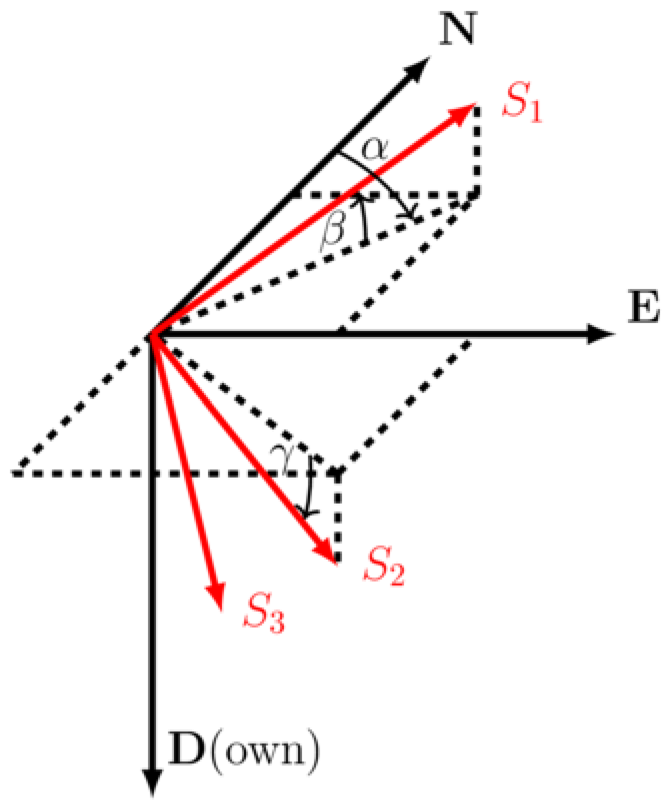
Geographical coordinate system



$$\mathbf{R}_G = \begin{bmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & -\sin\beta \\ \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \cos\beta\sin\gamma \\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

Stress in geographical coordinate system

$$\mathbf{S}_G = \mathbf{R}_G^T \mathbf{S} \mathbf{R}_G$$

Example: Strike-slip faulting

$$\mathbf{S} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$\alpha = 0^\circ \qquad \text{Azimuth of } S_{Hmax}$$

$$\beta = 0^\circ \qquad S_1 = S_{Hmax}$$

$$\gamma = 90^\circ \qquad S_2 = S_v$$

$$\mathbf{R}_G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{S}_G = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

Example: Normal faulting

$$\mathbf{S} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$\alpha = 0^\circ \qquad \text{Azimuth of } S_{hmin}$$

$$\beta = -90^\circ \qquad S_1 = S_v$$

$$\gamma = 0^\circ$$

$$\mathbf{R}_G = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{S}_G = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$

Example: Reverse faulting

$$\mathbf{S} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 20 \end{bmatrix} \qquad \begin{aligned} \alpha &= 90^\circ \\ \beta &= 0^\circ \\ \gamma &= 0^\circ \end{aligned} \qquad \begin{aligned} &\text{Azimuth of } S_{Hmax} \\ S_1 &= S_{Hmax} \\ S_2 &= S_{hmin} \end{aligned}$$

$$\mathbf{R}_G = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{S}_G = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

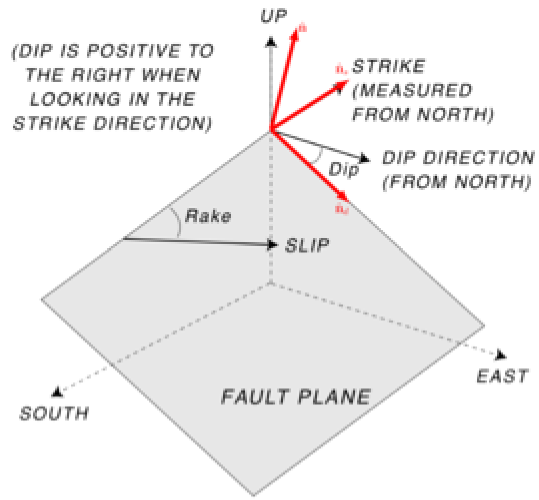
Example: Strike-slip faulting

$$\mathbf{S} = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 35 \end{bmatrix} \qquad \begin{aligned} \alpha &= 135^\circ \\ \beta &= 0^\circ \\ \gamma &= 90^\circ \end{aligned} \qquad \begin{aligned} \text{Azimuth of } S_{Hmax} \\ S_1 = S_{Hmax} \\ S_2 = S_v \end{aligned}$$

$$\mathbf{R}_G = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$\mathbf{S}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

Fault orientation



 Zoback, *Reservoir Geomechanics* (Fig. 5.5, pp. 150)

Fault traction and stress

Traction on fault plane

$$\vec{t} = \mathbf{S}_G \cdot \hat{\mathbf{n}}$$

Normal stress to plane

$$S_n = \vec{t}^{\intercal} \cdot \hat{\mathbf{n}}$$

Shear stress in dip direction

$$\tau_d = \vec{t}^{\intercal} \cdot \hat{\mathbf{n}}_d$$

Shear stress in strike direction

$$\tau_s = \vec{t}^{\intercal} \cdot \hat{\mathbf{n}}_s$$

Example: Strike-slip faulting

$$\mathbf{S}_G = \begin{bmatrix} 30 & -8.66 & 0 \\ -8.66 & 40 & 0 \\ 0 & 0 & 30 \end{bmatrix}$$

$$strike = 60^\circ$$
$$dip = 90^\circ$$

$$\hat{\mathbf{n}} = \begin{bmatrix} -0.866 \\ 0.5 \\ 0 \end{bmatrix} \qquad \hat{\mathbf{n}}_s = \begin{bmatrix} 0.5 \\ 0.866 \\ 0 \end{bmatrix} \qquad \hat{\mathbf{n}}_d = \begin{bmatrix} 0 \\ 0 \\ 1.0 \end{bmatrix}$$

$$S_n = 40 \quad \tau_d = 0 \quad \tau_s = 8.66$$

Example: Normal faulting

$$\mathbf{S}_G = \begin{bmatrix} 4000 & 0 & 0 \\ 0 & 3000 & 0 \\ 0 & 0 & 5000 \end{bmatrix}$$

$$\begin{aligned} strike &= 45^\circ \\ dip &= 60^\circ \end{aligned}$$

$$\hat{\mathbf{n}} = \begin{bmatrix} -0.612 \\ 0.612 \\ -0.5 \end{bmatrix} \qquad \hat{\mathbf{n}}_s = \begin{bmatrix} 0.707 \\ 0.707 \\ 0 \end{bmatrix} \qquad \hat{\mathbf{n}}_d = \begin{bmatrix} -0.3535 \\ 0.3535 \\ 0.866 \end{bmatrix}$$

$$S_n = 3870 \quad \tau_d = -649 \quad \tau_s = -433$$

Example: Normal faulting

$$\mathbf{S}_G = \begin{bmatrix} 5000 & 0 & 0 \\ 0 & 4000 & 0 \\ 0 & 0 & 3000 \end{bmatrix}$$

$$strike = 225^\circ$$
$$dip = 60^\circ$$

$$\hat{\mathbf{n}} = \begin{bmatrix} 0.612 \\ -0.612 \\ -0.5 \end{bmatrix} \quad \hat{\mathbf{n}}_s = \begin{bmatrix} -0.707 \\ -0.707 \\ 0 \end{bmatrix} \quad \hat{\mathbf{n}}_d = \begin{bmatrix} 0.3535 \\ -0.3535 \\ 0.866 \end{bmatrix}$$

$$S_n = 3870 \quad \tau_d = -649 \quad \tau_s = -433$$

Example: Revese faulting

$$\mathbf{S}_G = \begin{bmatrix} 2100 & -520 & 0 \\ -520 & 1500 & 0 \\ 0 & 0 & 1000 \end{bmatrix}$$

$$strike = 120^\circ$$
$$dip = 70^\circ$$

$$\hat{\mathbf{n}} = \begin{bmatrix} -0.814 \\ -0.470 \\ -0.342 \end{bmatrix} \qquad \hat{\mathbf{n}}_s = \begin{bmatrix} -0.5 \\ 0.866 \\ 0 \end{bmatrix} \qquad \hat{\mathbf{n}}_d = \begin{bmatrix} 0.2961 \\ -0.1710 \\ 0.9396 \end{bmatrix}$$

$$S_n = 1441 \quad \tau_d = 160 \quad \tau_s = 488$$