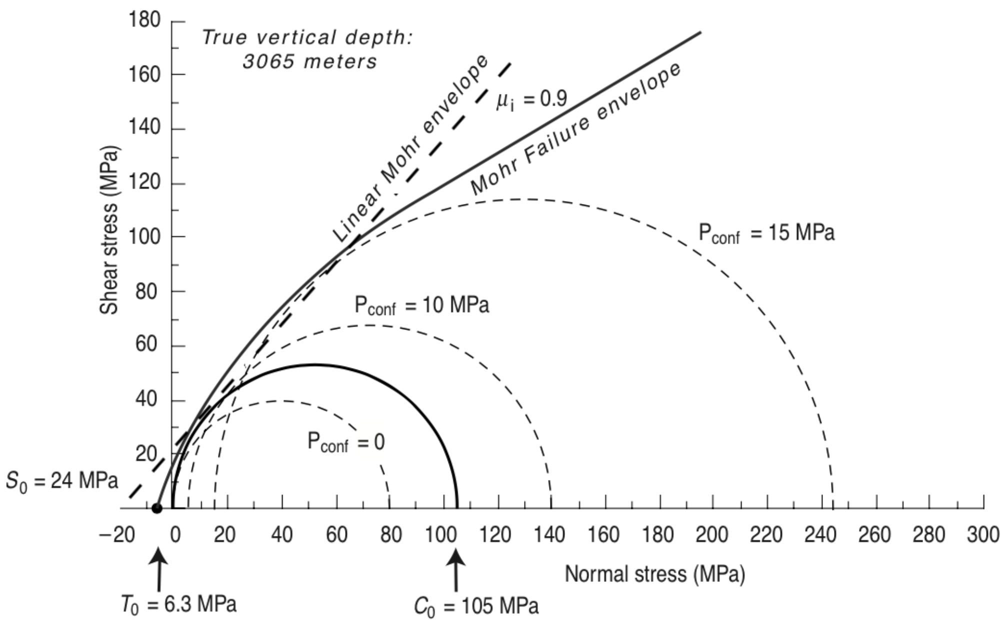
Compressive strength of rocks

Recall: Mohr Envelope for Sandstone



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Hoek-Brown criterion (parabolic fitting)

$$\sigma_1 = \sigma_3 + C_0 \sqrt{m \frac{\sigma_3}{C_0} + s}$$

m and s are fitting parameters that depend on rock properties and the degress of fracturing.

Typical values

Typical Range of m	**Types of rocks**
$5 < m < 8$	carbonate rocks (dolomite, limestone, marble)
$4 < m < 10$	lithified argillaceous rocks (sandstones, quartizite)
$15 < m < 24$	arenaceous rocks (andesite, dolerite, diabase, rhyolite)
$22 < m < 33$	course-grained polymirerallic gineous and metamorphic (amphibolite, gabbro, gneiss, norite, quartz-diorite)

Intact Rocks -- $s \rightarrow 1$

Completely Granualated -- $s \rightarrow 0$

Lade Criterion

$$\left(\frac{I_1^3}{I_3} - 27\right)\left(\frac{I_1}{p_a}\right)^{m'} = \eta_1$$

with

$I_1 = S_{ii} = S_1 + S_2 + S_3$ (first invariant of $\mathbf{S}$)

$I_3 = \det(\mathbf{S}) = S_1 S_2 S_3$ (third invariant of $\mathbf{S}$)

p_a is atmospheric pressure, m' and n_1 are material constants

Modified Lade Criterion (dependece on σ_2)

$$\left(\frac{(I_1')^3}{I_3'} \right) = 27 + \eta$$

with

$$I_1' = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S)$$

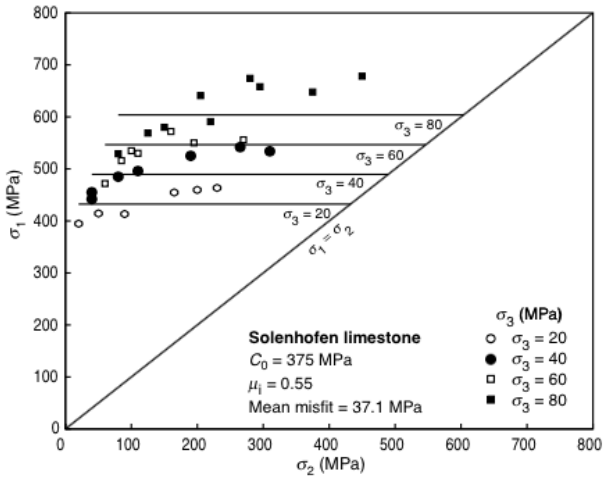
$$I_3' = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S)$$

$$S = \frac{S_0}{\tan \phi}$$

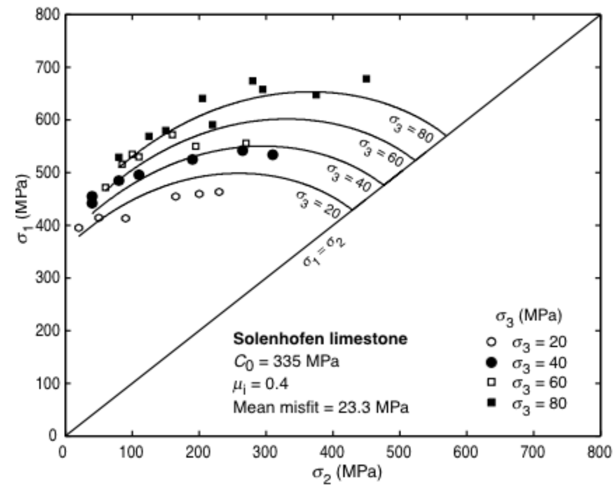
$$\eta = \frac{4(\tan \phi)^2 (9 - 7 \sin \phi)}{1 - \sin \phi)}$$

$\tan \phi = \mu_i$ and S_0 from Mohr-Coulomb criterion

Comparison



Mohr-Coulomb



modified Lade

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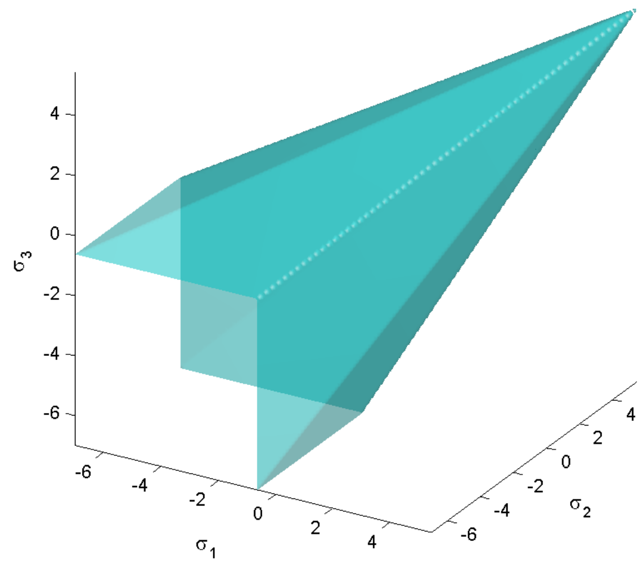
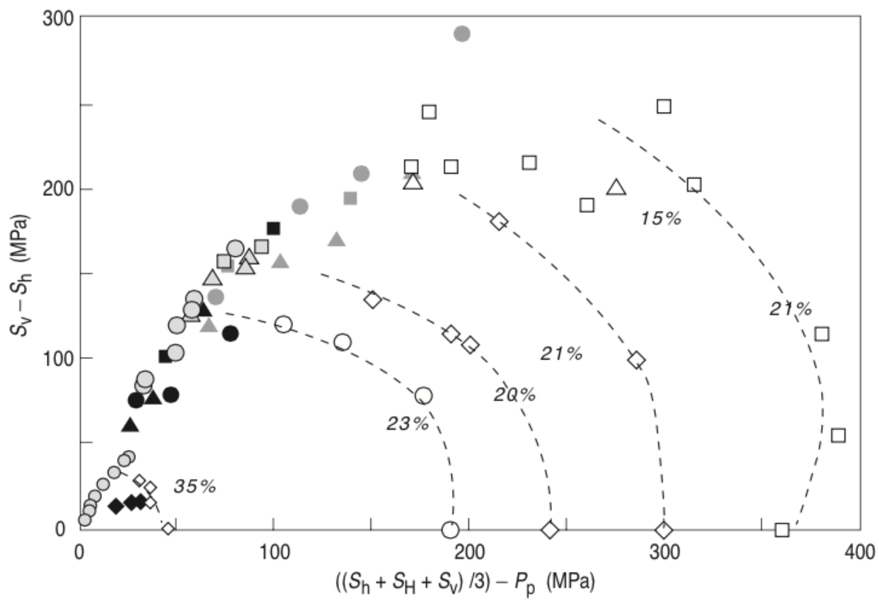
Recall: Yield surface



Mohr Coulomb Yield Surface 3Da. Licensed under CC BY-SA 3.0 via Wikipedia

Shear enhanced compaction



Porosity loss in sandstone

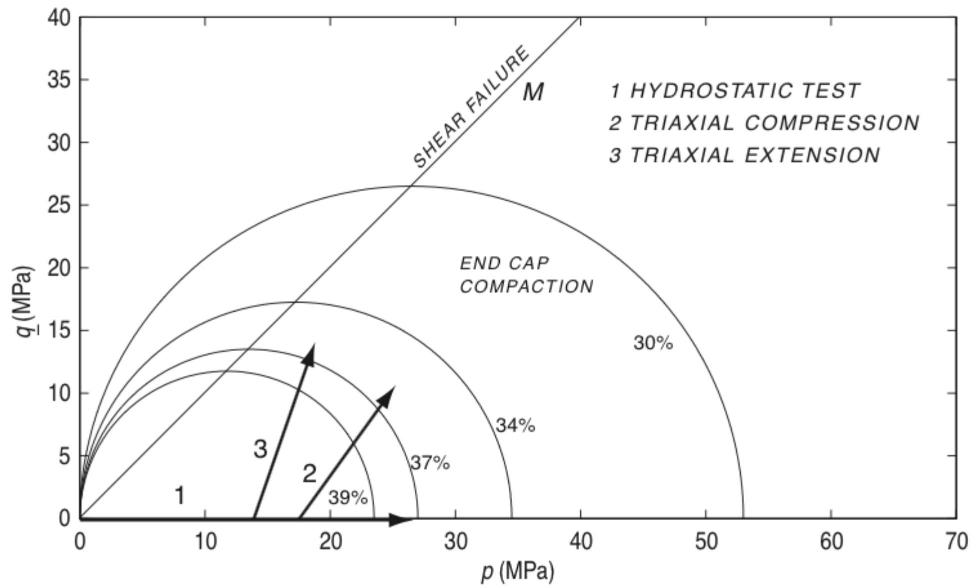
Cam-Clay model

$$M^2p^2 - M^2p_0p + q^2 = 0$$

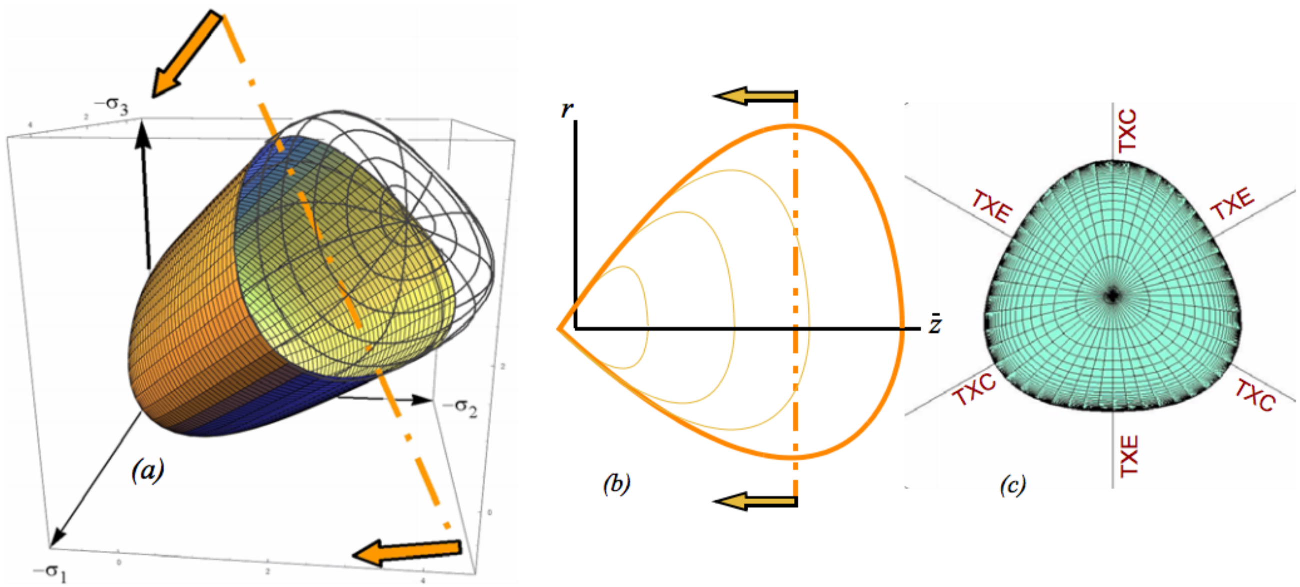
with

$$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$
$$q^2 = \frac{1}{2}\left((S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_1 - S_3)^2\right)$$
$$M = \frac{q}{p}$$

Cam-Clay model



Sandia geomodel (Kayenta)



R.M. Brannon, A.F. Fossum, and O.E. Strack: Kayenta: Theory and User's Guide. Tech. rep. Sandia National Laboratories, 2009.

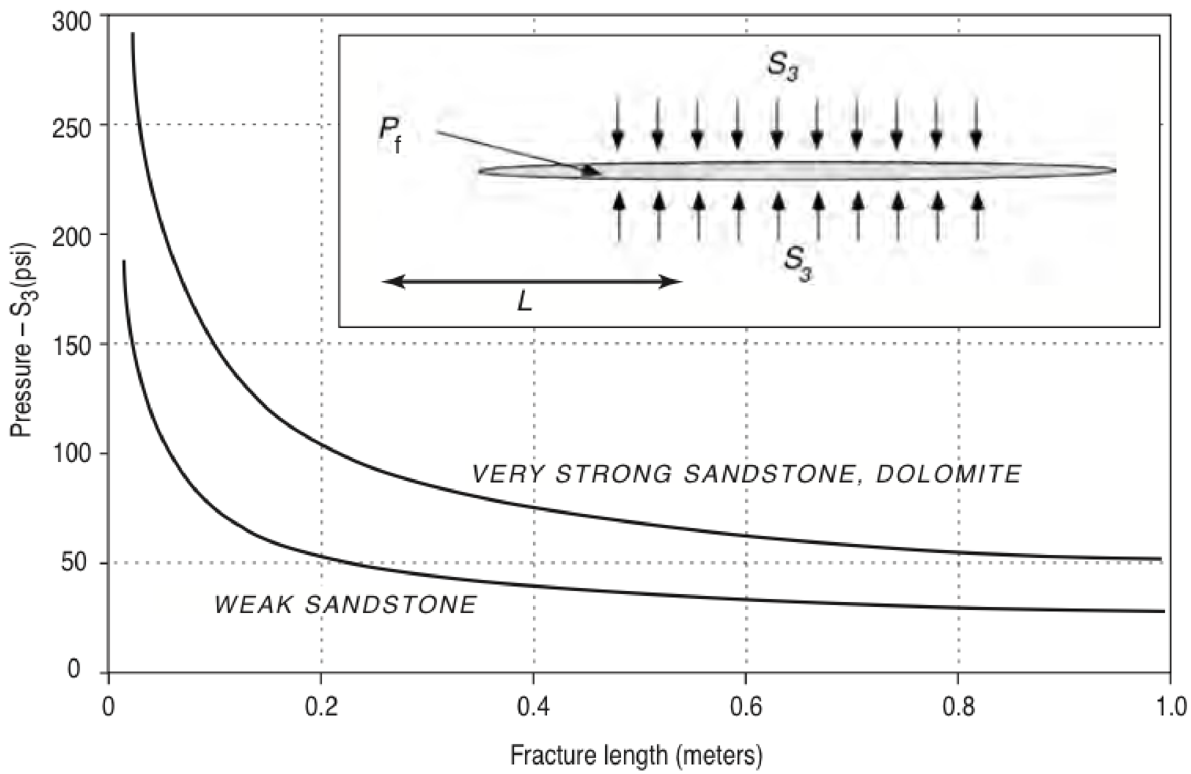
Tensile strength of rocks

- Relatively unimportant!

- Reasons:
  - Tensile strength is low compared to compressive strength.
  - When a large enough volume of rock is considered, flaws are bound to exist making the tensile strength near zero.
  - *In situ* stress at depth is never tensile.

Opening mode fracture (Mode I)

$$K_{Ic} \geq K_I = (P_f - S_3)\pi\sqrt{L}$$



 Zoback, *Reservoir Geomechanics* (Fig. 4.21, pp. 122)

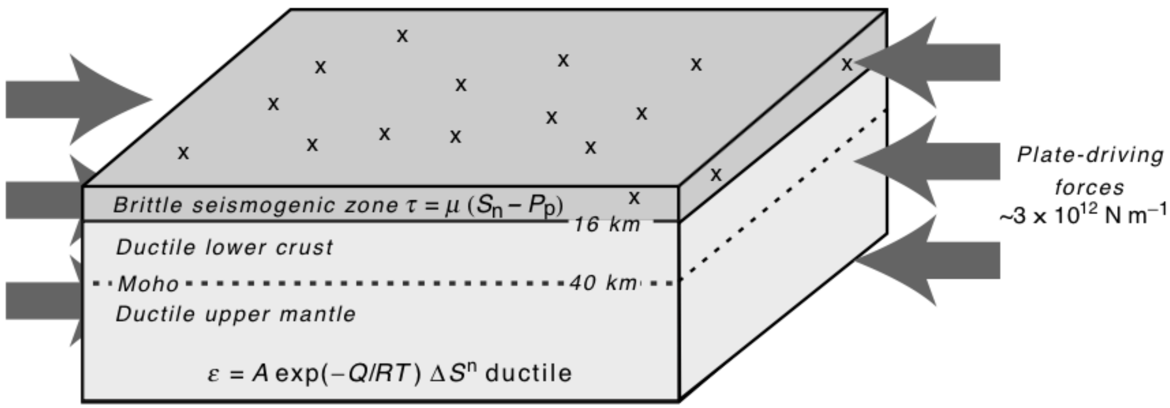
Recall: Slip on faults

$$\frac{\tau}{\sigma_n} = \mu$$

Coulomb failure function

$$f = \tau - \mu\sigma_n \leq 0$$

Critically stressed crust



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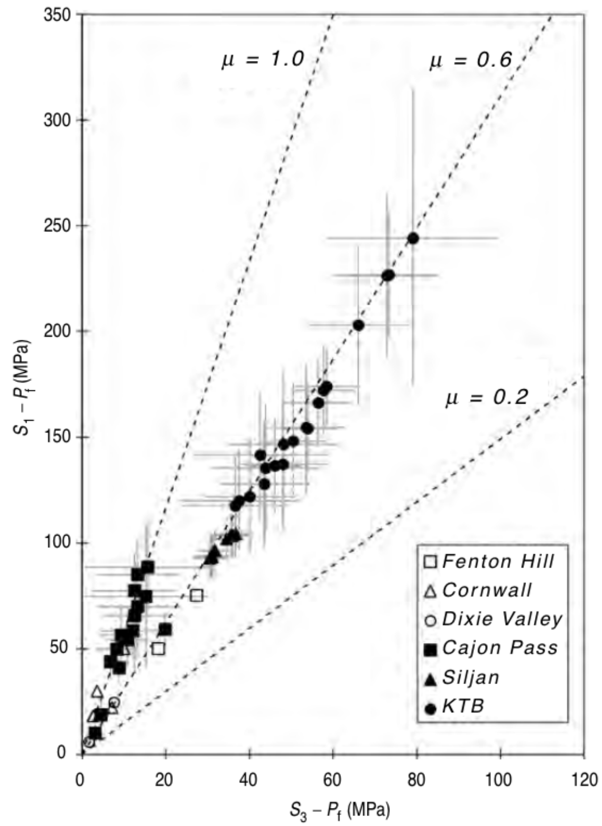
Stress magnitudes controlled by frictional strength



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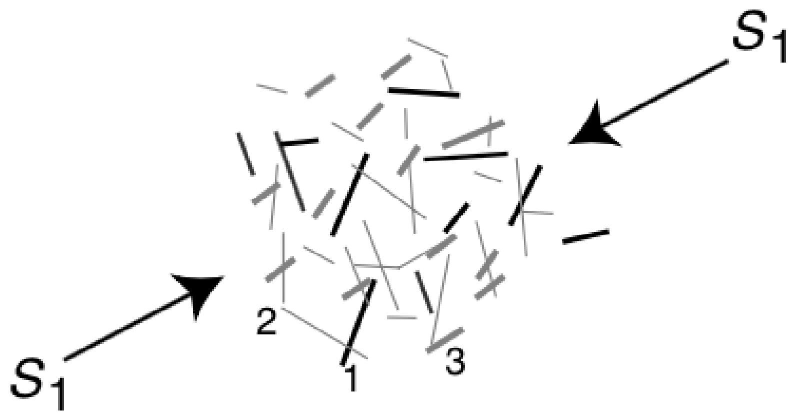
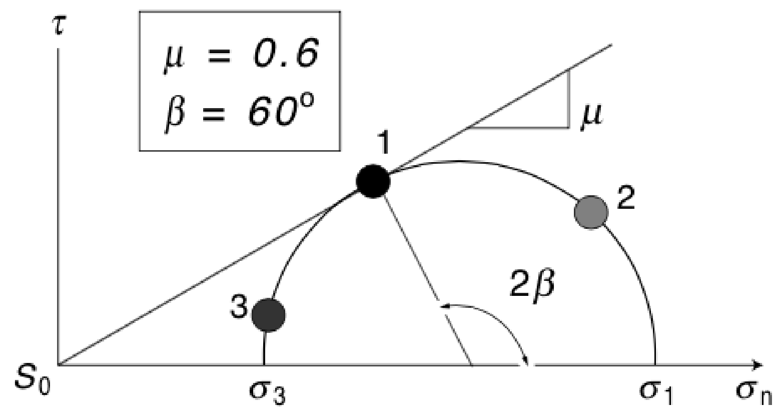
Limits on *in situ* stress





Optimal angle for frictional sliding:

$$\beta = \frac{\pi}{4} + \frac{1}{2}\tan^{-1}\mu$$

 Zoback, *Reservoir Geomechanics* (Fig. 4.27b,c, pp. 131)

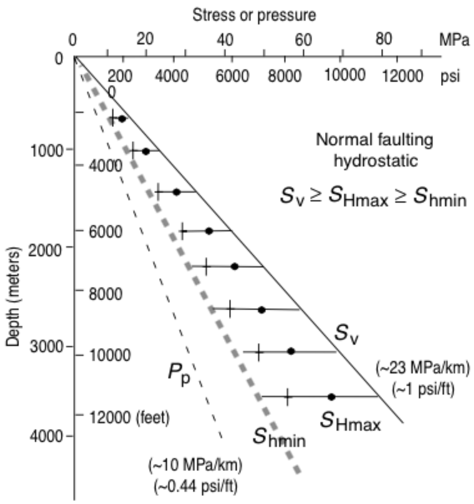
Principle stress ratio

$$\frac{\sigma_1}{\sigma_3} = \frac{S_1 - P_p}{S_3 - P_p} = \left(\sqrt{\mu^2 + 1} + \mu^2\right)^2$$

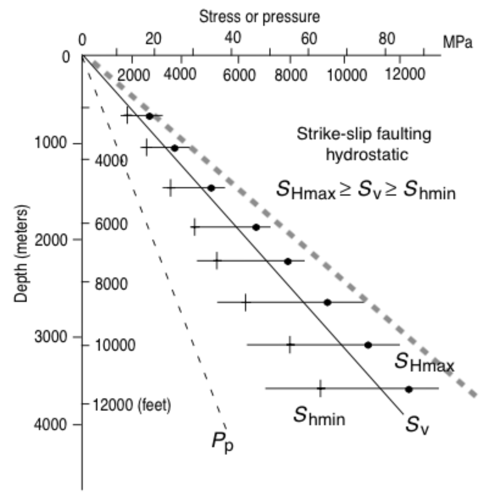
Asuming $\mu = 0.6$

$$\frac{\sigma_1}{\sigma_3} = 3.1$$

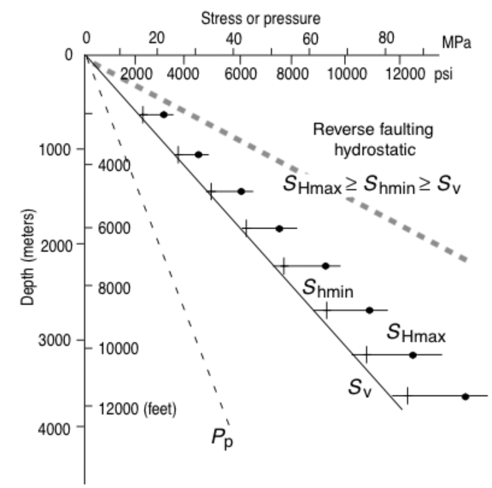
Stress bounds



$$\frac{S_v - P_p}{S_{hmin} - P_p} \leq \left(\sqrt{\mu^2 + 1} + \mu^2\right)$$

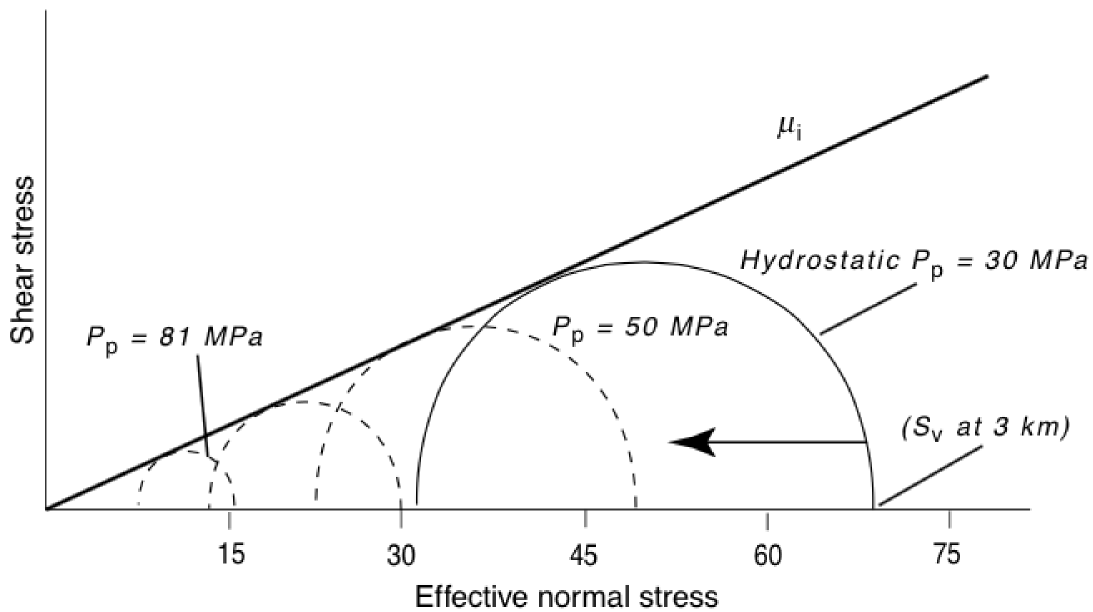


$$\frac{S_{Hmax} - P_p}{S_{hmin} - P_p} \leq \left(\sqrt{\mu^2 + 1} + \mu^2\right)$$



$$\frac{S_{Hmax} - P_p}{S_v - P_p} \leq \left(\sqrt{\mu^2 + 1} + \mu^2\right)$$

Pore pressure, stress difference, and fault slip



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