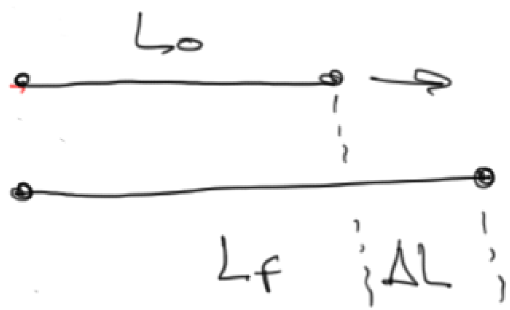


$$\varepsilon_{ENG} = \frac{\Delta L}{L_o} = \frac{L_f - L_o}{L_o} = \lambda - 1 \Rightarrow \lambda = \varepsilon_{ENG} + 1$$

Engineering strain → Lagrangian

$$\varepsilon_{LOG} = \int_{L_o}^{L_f} \frac{dL}{L} = \ln\left(\frac{L_f}{L_o}\right) = \ln(\lambda) = \ln(1+\varepsilon_{ENG})$$

Logrithrimic strain, natural strain, "True" strain

$$\varepsilon_{TR} = \frac{L_f - L_o}{L_f} = 1 - \frac{1}{\lambda}$$ ← Eulerian strain, "True strain"

Seth-Hill

$$\boxed{\varepsilon_{(m)} = \frac{1}{m}\left(\lambda^m - 1\right)}$$

$m = 1$ → Eng. strain

$m = -1$ → True stain

$m \to 0$ → Logrithrimic strain

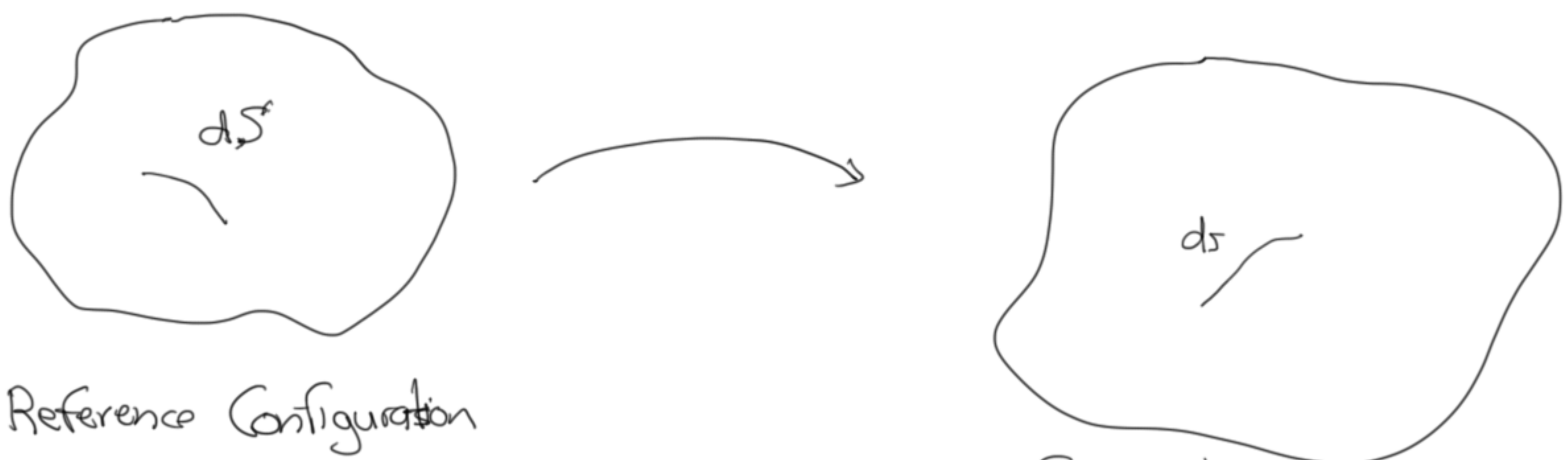
Aside

Please, please, please "label your strain!!!"

Consider $\lambda = 1.01000$

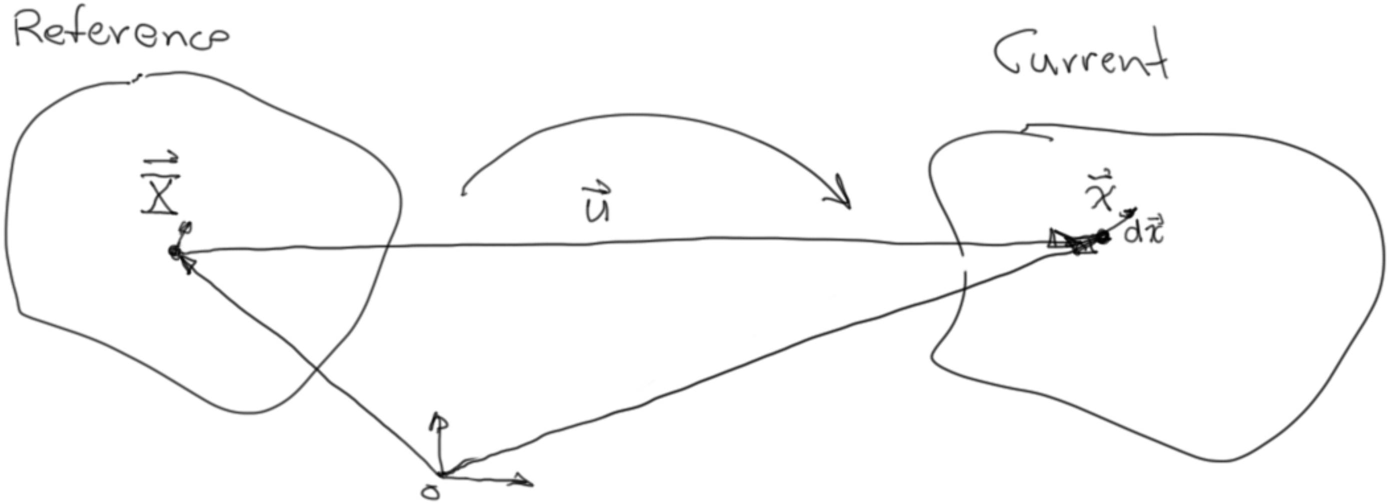
$$\varepsilon_{ENG} = \lambda - 1 = 0.01000$$

$$\varepsilon_{LOG} = \ln(\lambda) = 0.00990$$



Reference Configuration
"Undeformed" Configuration
Lagragian Configuration

Current Configuration
Deformed Configuration
Eulerian Configuration



$$\vec{x} = \vec{X} + \vec{u}$$

$$\vec{X} = X_1\,\hat{i} + X_2\,\hat{j} + X_3\,\hat{k}$$

$$\vec{X} = X_1\,\hat{e}_1 + X_2\,\hat{e}_2 + X_3\,\hat{e}_3$$

$$\vec{X} = \sum_{i=1}^{3} X_i\,\hat{e}_i$$

$$\vec{X} = X_i\,\hat{e}_i$$

$$\vec{x} = \vec{x}(\vec{X}, t)$$

$$= \begin{cases} x_1 = x_1(X_1, X_2, X_3, t) \\ x_2 = x_2(X_1, X_2, X_3, t) \\ x_3 = x_3(X_1, X_2, X_3, t) \end{cases}$$

$$dx_1 = \frac{\partial x_1}{\partial X_1} dX_1 + \frac{\partial x_1}{\partial X_2} dX_2 + \frac{\partial x_1}{\partial X_3} dX_3$$

$$dx_2 = \frac{\partial x_2}{\partial X_1} dX_1 + \frac{\partial x_2}{\partial X_2} dX_2 + \frac{\partial x_2}{\partial X_3} dX_3$$

$$dx_3 = \frac{\partial x_3}{\partial X_1} dX_1 + \frac{\partial x_3}{\partial X_3} dX_2 + \frac{\partial x_3}{\partial X_3} dX_3$$

$$\begin{Bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \cdots \\ \vdots & \ddots & \\ \frac{\partial x_3}{\partial X_1} & & \frac{\partial x_3}{\partial X_3} \end{bmatrix}}_{F_{ij} = \frac{\partial \vec{x}_i}{\partial \vec{X}_j}} \begin{Bmatrix} dX_1 \\ dX_2 \\ dX_3 \end{Bmatrix}$$

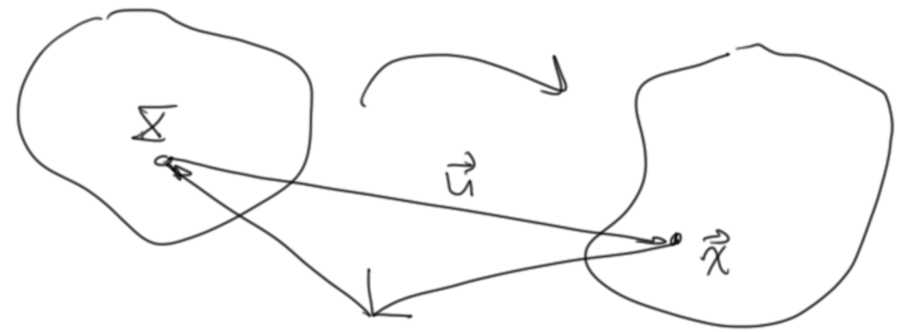
$\leftarrow \vec{x} = \vec{x}(\vec{X})$

$$d\vec{x} = F\, d\vec{X}$$

$$dx_i = \sum_{j=1}^{3} F_{ij}\, dX_j \quad \text{for } i = 1, 2, 3$$

$$dx_i = F_{ij}\, dX_j$$

$$\frac{\partial X_1}{\partial X_1} = 1 \qquad \frac{\partial X_1}{\partial X_2} = 0$$



$$\vec{x} = \vec{X} + \vec{u}$$

$$\frac{\partial x_i}{\partial X_j} = \frac{\partial X_i}{\partial X_j} + \frac{\partial u_i}{\partial X_j}$$

$$F_{ij} = \delta_{ij} + \underbrace{\frac{\partial u_i}{\partial X_j}}_{\nabla_X \vec{u}}$$

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}$$

$$F = I + \nabla_X \vec{u}$$

$$(ds)^2 = |d\vec{x}| = \left(\sqrt{dx_1^2 + dx_2^2 + dx_3^2}\right)^2$$

$$(ds)^2 = dx_1^2 + dx_2^2 + dx_3^2 = \begin{bmatrix} dx_1 & dx_2 & dx_3 \end{bmatrix} \begin{Bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{Bmatrix}$$

$$= d\vec{x}^T d\vec{x}$$

$$d\vec{x} = \underline{F\, d\vec{X}}$$

$$= (F\, d\vec{X})^T (F\, d\vec{X})$$

$$= \underline{d\vec{X}^T F^T F\, d\vec{X}}$$

$$(dS)^2 = d\vec{X}^T d\vec{X}$$

$$\underbrace{(ds)^2 - (dS)^2} = d\vec{X}^T F^T F\, d\vec{X} - d\vec{X}^T I\, d\vec{X}$$

$$= d\vec{X}^T \underbrace{\left(F^T F - I\right)}_{2E} d\vec{X}$$

$E = \frac{1}{2}\left(F^T F - I\right) \leftarrow$ Lagragian or Green strain

$$E = \frac{1}{2}(F^T F - I) \qquad \text{recall} \quad F = I + \nabla u$$

$$= \frac{1}{2}\left(\nabla_X \vec{u} + (\nabla_X \vec{u})^T + (\nabla_X \vec{u})^T (\nabla_X u)\right)$$

$$\rightarrow \boxed{E = \boxed{\frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right.} + \left.\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right)}$$

$$\|\nabla u\| << I$$

$$\Downarrow$$

$$\boxed{\varepsilon = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

LINEAR STRAIN → CAUCHY

$$(ds)^2 - (dS)^2 = d\vec{x}^T d\vec{x} - d\vec{X}^T d\vec{X} = d\vec{x}^T d\vec{x} - (F^{-1} d\vec{x})^T (F^{-1} d\vec{x})$$

$$F^{-1} d\vec{x} = \cancel{F^{-1} F}\, d\vec{X}$$

$$F^{-1} d\vec{x} = d\vec{X}$$

$$= d\vec{x}^T d\vec{x} - d\vec{x}^T F^{-T} F^{-1} d\vec{x}$$

$$= d\vec{x}^T \underbrace{\left(I - F^{-T} F^{-1}\right)}_{2e} d\vec{x}$$

$$F = I + \nabla u$$

$$\rightarrow \boxed{e = \boxed{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right.} - \left.\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right)}$$

$$e = \frac{1}{2}\left(I - F^{-T} F^{-1}\right)$$

Eulerian strain → Almansi strain