$$[M]\ddot{u}_{n+1} + \left[\int B^T \sigma'' d\Omega\right] - [Q]p_{n+1} - f^{(1)}_{n+1} = 0$$

$$[Q]\dot{\vec{u}}_{n+1} + [H]p_{n+1} + [S]\dot{\bar{p}}_{n+1} - f^{(1)} = 0$$

Extend β method

$$\ddot{\vec{u}}_{n+1} = \ddot{u}_n + \Delta\ddot{\vec{u}}_{n+1/2}$$

$$\dot{\vec{u}}_{n+1} = \dot{\vec{u}}_n + \ddot{\vec{u}}_n \Delta t + \beta_1 \Delta\ddot{u}_n \Delta t$$

$$\bar{u}_{n+1} = \vec{u}_n + \vec{u}_n \Delta t + \frac{1}{2}\ddot{u}_n \Delta t^2 + \frac{1}{2}\beta_2 \Delta\ddot{u}_n \Delta t^2$$

$$\dot{\bar{p}}_{n+1} = \dot{\bar{p}}_n + \Delta\dot{\bar{p}}_{n+1/2}$$

$$\bar{p}_{n+1} = p_n + \dot{\bar{p}}_n \Delta t + \gamma \Delta\dot{\bar{p}}_n \Delta t$$

Unconditional stable

$$\beta_2 > \beta_1 \geqslant \gamma \geqslant \frac{1}{2}$$

$$R^{(1)} = M_{n+1}\,\Delta\ddot{\vec{u}}_{n+1/2} + P(\vec{u}_{n+1}) - Q_{n+1}\,\gamma\,\Delta t\,\Delta\dot{\bar{p}}_{n+1/2} - F_{n+1}^{(1)}$$

$$R^{(2)} = Q_{n+1}^{T}\,\beta_1\,\Delta t\,\Delta\ddot{\vec{u}}_{n+1/2} + H_{n+1}\,\gamma\,\Delta t\,\Delta\dot{\bar{p}}_{n} + S_{n+1}\,\Delta\dot{\bar{p}}_{n+1/2} - F_{n+1}^{(2)}$$

$$P(\vec{u}_{n+1}) = \int B^{T}\sigma''_{n+1}\,d\Omega = \int_{\Omega} B^{T}\Delta\sigma''_{n+1/2}\,d\Omega + P(\vec{u}_{n})$$

$$K^{T} = \begin{bmatrix} \dfrac{\partial R^{(1)}}{\partial\Delta\ddot{\vec{u}}_{n}} & \dfrac{\partial R^{(1)}}{\partial\Delta\dot{\bar{p}}} \\[2ex] \dfrac{\partial R^{(2)}}{\partial\Delta\ddot{\vec{u}}_{n}} & \dfrac{\partial R^{(2)}}{\partial\Delta\dot{\bar{p}}} \end{bmatrix}$$

use Newtons method to solve for $\Delta\ddot{\vec{u}}_{n+1/2}$, $\Delta\dot{\bar{p}}_{n+1/2}$