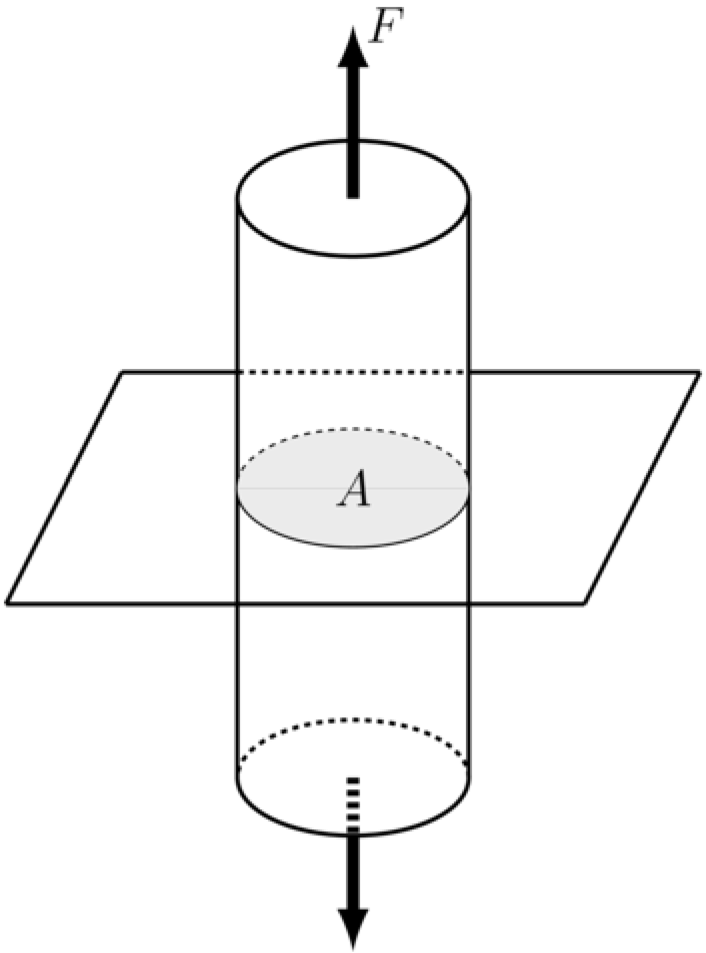
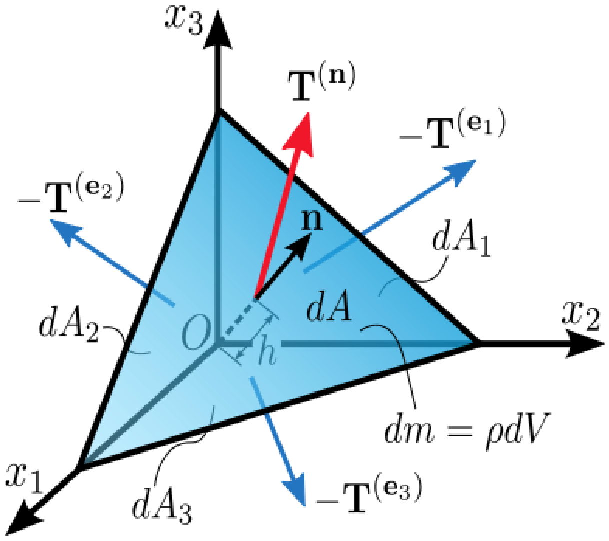
Stress



$$\sigma = \frac{F}{A}$$

Cauchy tetrahedron

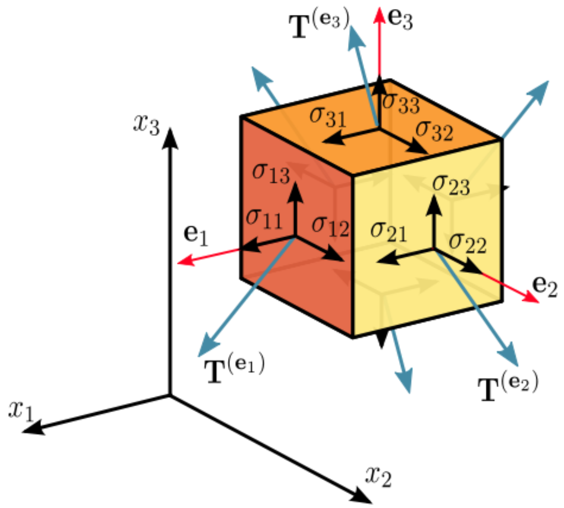


TEXAS
The University of Texas at Austin

Stress tensor

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Visual definition



- First subscripted index refers to the index of the unit vector that is normal to the face.
- Second subscripted index refers to the component of traction vector.

Stress tensor (Zoback book notation)

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Due to **conservation of angular momentum**: $S_{12} = S_{21}, S_{13} = S_{31}$ and $S_{32} = S_{23}$.

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

Principle stresses and directions

$$\mathbf{S}' = \mathbf{Q}^{-1}\mathbf{S}\mathbf{Q}$$

$$\mathbf{S}' = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix}$$

with $S_1 > S_2 > S_3$ where the S_i's are the eigenvalues of $\mathbf{S}$

$$\mathbf{Q} = [\vec{v}_1 \mid \vec{v}_2 \mid \vec{v}_3]$$

where $\vec{v}_1$ is the eigenvector corresponding to S_1, $\vec{v}_2$ is the eigenvector corresponding to S_2, and $\vec{v}_3$ is the eigenvector corresponding to S_3.

Example

Determine the principle stresses and directions given:

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

$$S_1 = 4, \quad S_2 = 2, \quad S_3 = 1$$

$$\mathbf{Q} = [\vec{v}_1 \mid \vec{v}_2 \mid \vec{v}_3] = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$