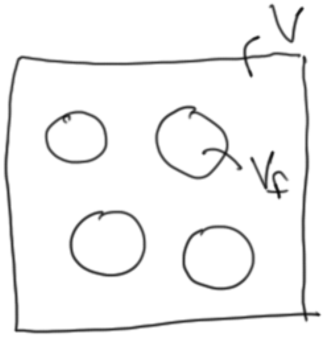


$$n = \lim_{V \to 0} \frac{V_f}{V} \Rightarrow \text{"porosity"}$$

In the case 2 phase solid + fluid

$$\rho = n \rho_f + (1-n) \rho_s$$

Momentum for solid fluid mixture "full saturated"

$$1 \longrightarrow \frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = \rho \dot{v}_i + \underbrace{\rho_f \left(\dot{w}_i + w_j \frac{\partial w_i}{\partial x_j} \right)}_{\text{fluid acceleration w.r.t. to solid in general}}$$

$$-\frac{1}{\rho}\nabla p + \vec{b} = \frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}$$ → nonviscous fluid

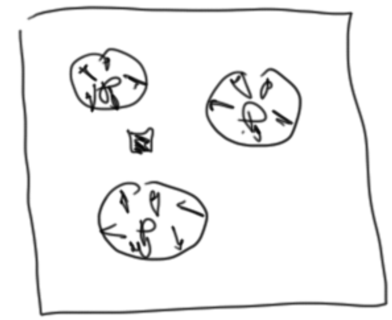
$$2 \qquad -\frac{\partial p}{\partial x_i} + \rho_f b_i - \rho_f\left[\frac{\partial w_i}{\partial t} + w_j \frac{\partial w_i}{\partial x_j}\right] - \rho_f \dot{v}_i - R_i = 0$$

$$K_{ij} R_j = w_i$$

$$\underline{\underline{K}} \rightarrow \frac{L^3 t}{m} \qquad K = \frac{K'}{\rho g} \qquad \frac{L^2}{t}$$

$$\frac{m}{L^3 t} : \frac{L}{t} = \frac{m}{L^2 t^2}$$

Balance of mass for fluid accounting for:



$$\frac{D\rho}{Dt} + \rho \nabla \vec{v} = 0$$

1.) increase in volume due to volumic strain

$$\frac{d\varepsilon_{ii}}{dt}$$

2.) additional volume stored by compression of fluid due to fluid pressure increase

$$n \frac{dp}{dt} \frac{1}{K_f}$$

3.) additional volume stored by compression of solid

$$(1-n) \frac{dp}{dt} \frac{1}{K_s}$$

4.) Change in volume of solid due to fluid pressure

$$= -\frac{1}{3} \delta_{ij} \frac{d\sigma}{dt} \frac{1}{K_s} = -\frac{K_T}{K_s} \left(\frac{d\varepsilon}{dt} + \frac{dp}{dt} \frac{1}{K_s} \right)$$

$$\frac{\partial w_i}{\partial x_j} + \frac{\partial \varepsilon_{ii}}{\partial t} + \frac{n}{K_f}\frac{\partial p}{\partial t} + \frac{(1-n)}{K_s}\frac{\partial p}{\partial t} - \frac{K_T}{K_s}\left(\frac{\partial \varepsilon_{ii}}{\partial t} + \frac{\partial p}{\partial t}\frac{1}{K_s}\right)$$

$$+ n \frac{\partial \rho_f}{\partial t}\frac{1}{\rho_f} = 0$$

using $\alpha = 1 - \frac{K_T}{K_s}$

$$\boxed{\frac{\partial w_i}{\partial x_j} + \alpha \dot{\varepsilon}_{ii} + \frac{\dot{p}}{Q} + n \frac{\dot{\rho}_f}{\rho_f} = 0} \qquad 3$$

$v = \dot{u}$

$\dot{v} = \ddot{u}$

where $\frac{1}{Q} = \frac{n}{K_f} + \frac{\alpha - n}{K_s}$

p – pressure fluid

w – velocity fluid

u – displacement solid

neglect covective term

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i = \rho \cancelto{0}{\ddot{u}_i}$$

$$K_{ij}\left[-\frac{\partial p}{\partial x_j} - R_j - \rho_f \ddot{u}_j + \rho_f b_j\right] = 0 \qquad K_{ij} R_j = w_i$$

$$\underbrace{K_{ij}\left[-\frac{\partial p}{\partial x_j} - \rho_f \ddot{u}_j + \rho_f b_j\right]}_{= w_i} - w_i = 0$$

$$\frac{\partial}{\partial x_i}\left[K_{ij}\left(-\frac{\partial p}{\partial x_j} - \rho_f \ddot{u}_j + \rho_f b_j\right)\right] + \alpha \dot{\varepsilon}_{ii} + \frac{\dot{p}}{Q} + \frac{n \dot{\rho}}{K_s \rho_f} = 0$$

$u - p$

$$\rightarrow \quad \frac{\partial}{\partial x_i}\left(-K_{ij}\frac{\partial p}{\partial x_j}\right) + \frac{\dot{p}}{Q} + \alpha \dot{\varepsilon}_{ii} = 0$$

$$\alpha \approx 1$$

$$\alpha \simeq \frac{2}{3}$$

$$\approx 0.5$$