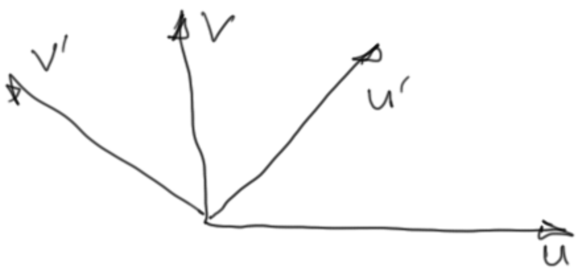
$$[t_1 \;\; t_2 \;\; t_3] = [n_1 \;\; n_2 \;\; n_3] \begin{bmatrix} \sigma_{11} & \sigma_{12} & \\ & \sigma_{22} & \\ & & \ddots \end{bmatrix}$$

$$(\vec{t}^{\,T})^T = (\hat{n}^T \sigma)^T$$

$$\boxed{\vec{t} = \sigma^T \hat{n}} \qquad \sigma^T = \sigma$$

Cauchy stress equation

$$t_i = \sigma_{ji} \hat{n}_i$$



$$u' = Ru$$
$$v' = Rv$$

$$\underline{\vec{v} = T\vec{u}} \rightarrow \vec{v}\,' = T'\vec{u}\,' \rightarrow \cancel{R^{-1}R}^{\,I}\,\vec{v} = R^{-1}T'R\vec{u}$$

$$\vec{v} = R^{-1}T'R\vec{u}$$

$$\vec{v} = \underbrace{R^T T' R}\vec{u}$$

$$T = R^T T' R$$
$$T' = R T R^T$$

$$T' = R\,T\,R^T$$

$$\sigma' = R\,\sigma\,R^T = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \qquad \sigma_I > \sigma_{II} > \sigma_{III}$$

$$Q \Lambda Q^T = D \qquad \det(\sigma - \lambda I) = 0$$

$$-\lambda^3 + \underline{I_1}\,\lambda^2 + \underline{I_2}\,\lambda + \underline{I_3} = 0$$

Invariants of stress tensor

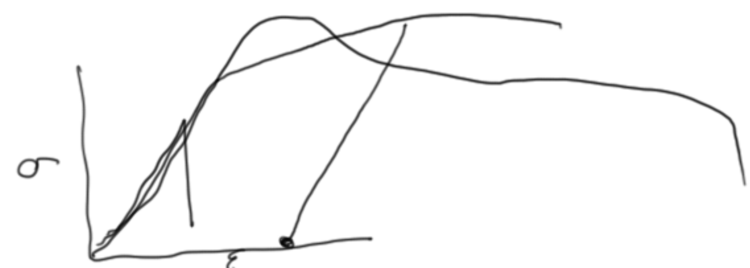
$$I_1 = \mathrm{tr}(\sigma) = \sigma_{ii} = \sigma_I + \sigma_{II} + \sigma_{III} = -3p$$

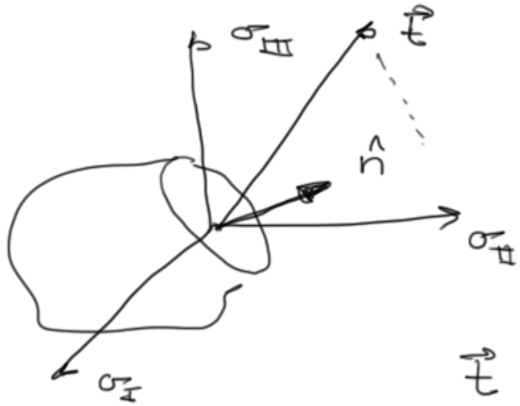
$$\boxed{I_2 = \underline{\tfrac{1}{2}\sigma_{ij}\sigma_{ij}} - \tfrac{1}{2} I_1^2} = -\left(\sigma_I \sigma_{II} + \sigma_I \sigma_{III} + \sigma_{II}\sigma_{III}\right)$$

$$I_3 = \det(\sigma) = \sigma_I \sigma_{II} \sigma_{III}$$

$$\sigma_{ij} = \underbrace{\tfrac{1}{3}\sigma_{kk}\delta_{ij}}_{\text{spherical / hydrostatic / dilatational stress}} + \underline{S_{ij}} \rightarrow \text{deviatoric}$$

$$J_1 = 0, \quad J_2 = \tfrac{1}{2} S_{ij} S_{ij}, \quad J_3 = \det(S)$$





$$\vec{t} = \sigma^T \hat{n} = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \begin{Bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \hat{n}_3 \end{Bmatrix} = \begin{Bmatrix} \sigma_I \hat{n}_1 \\ \sigma_{II} \hat{n}_2 \\ \sigma_{III} \hat{n}_3 \end{Bmatrix}$$

$$t_n = \vec{t} \cdot \vec{n} = \sigma_I \hat{n}_1^2 + \sigma_{II} \hat{n}_2^2 + \sigma_{III} \hat{n}_3^2 \quad - 1$$

$$|\vec{t}|^2 = \cancel{\sqrt{}}\, t_s^2 + t_n^2$$

$$t^2 = t_n^2 + t_s^2 = (\sigma_I \hat{n}_1)^2 + (\sigma_{II} \hat{n}_2)^2 + (\sigma_{III} \hat{n}_3)^2 \quad - 2$$

$$|\hat{n}| = 1 = \hat{n}_1^2 + \hat{n}_2^2 + \hat{n}_3^2 \quad - 3$$

$$\begin{bmatrix} 1 & 1 & 1 \\ \sigma_I & \sigma_{II} & \sigma_{III} \\ \sigma_I^2 & \sigma_{II}^2 & \sigma_{III}^3 \end{bmatrix} \begin{Bmatrix} \hat{n}_1^2 \\ \hat{n}_2^2 \\ \hat{n}_3^2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ t_n \\ t_s^2 + t_n^2 \end{Bmatrix}$$

Solve

$$\hat{n}_1^2 = \frac{t_s^2 + (t_n - \sigma_{II})(t_n - \sigma_{III})}{\underbrace{(\sigma_I - \sigma_{II})(\sigma_I - \sigma_{III})}_{\geq 0}} \geq 0$$

$$\hat{n}_2^2 = \frac{t_s^2 + (t_n - \sigma_{III})(t_n - \sigma_I)}{\underbrace{(\sigma_{II} - \sigma_{III})(\sigma_{II} - \sigma_I)}_{\leq 0}} \leq 0$$

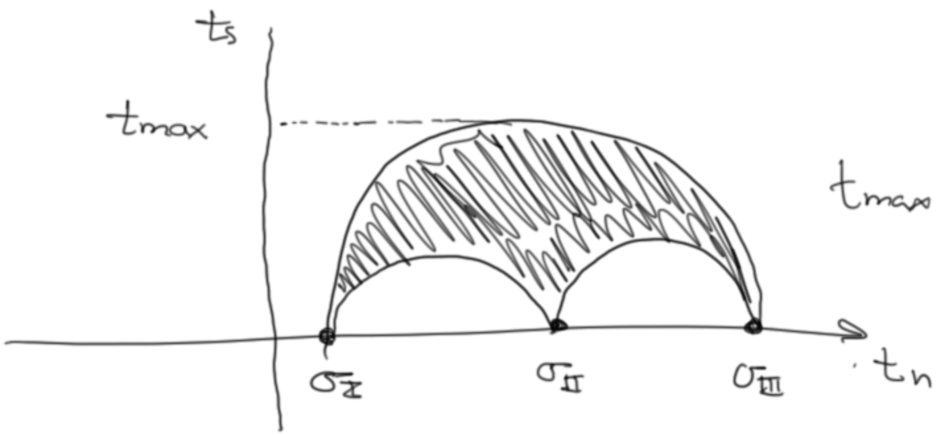
$$\hat{n}_3 = \frac{t_s^2 + (t_n - \sigma_I)(t_n - \sigma_{II})}{\underbrace{(\sigma_{III} - \sigma_I)(\sigma_{III} - \sigma_{II})}_{\geq 0}} \geq 0$$

$$\underbrace{\left[t_n - \tfrac{1}{2}(\sigma_{II} + \sigma_{III})\right]^2}_{x^2} + t_s^2 \geq \left(\tfrac{1}{2}(\sigma_{II} - \sigma_{III})\right)^2$$

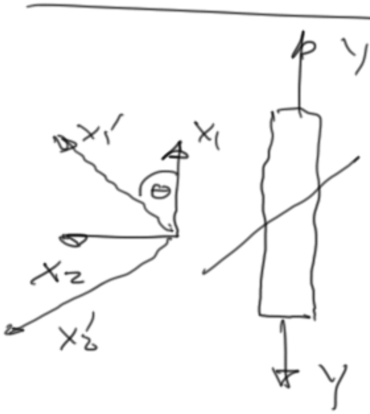
$$x^2 + y^2 \geq r^2$$

$$\left[t_n - \frac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\frac{1}{2}(\sigma_I - \sigma_{III})\right)^2$$

$$\left[t_n - \frac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\frac{1}{2}(\sigma_I - \sigma_{II})\right)^2$$



$$t_{max} = \frac{1}{2}(\sigma_I + \sigma_{III})$$

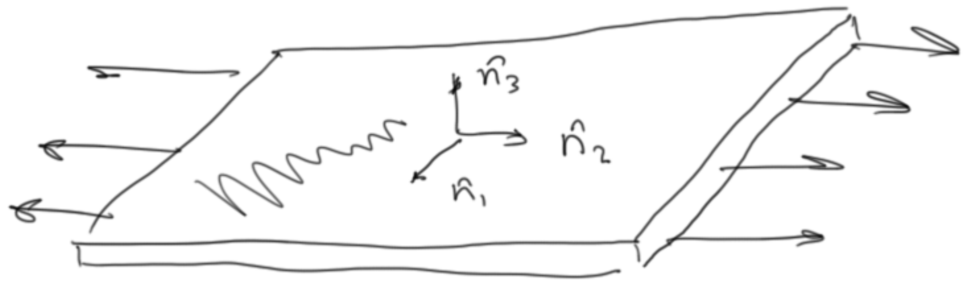


$$\sigma = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma' = R\,\sigma\,R^T$$

$$\sigma_{11}' = Y\cos\theta \qquad \sigma_{12}' = \frac{Y}{2}\sin\theta\cos\theta = \frac{Y}{2}\sin 2\theta \quad \overset{max}{} \text{ at } 45^\circ$$

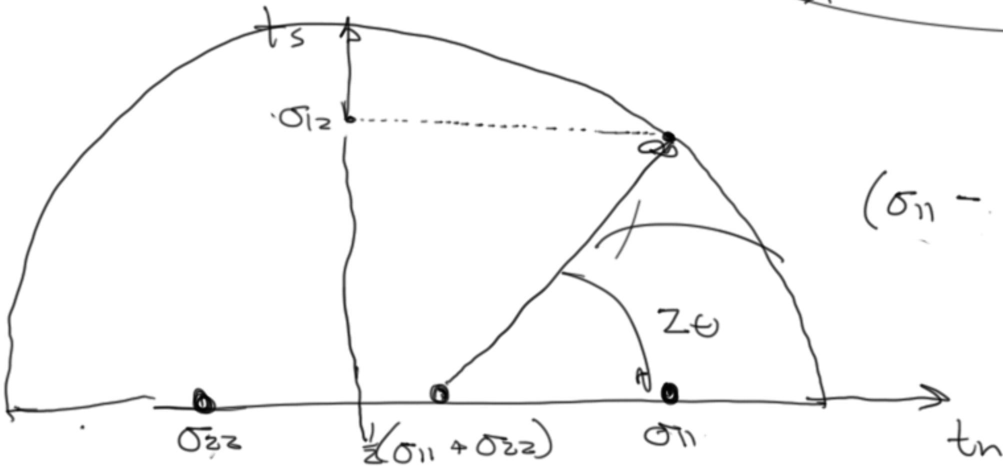
Plane stress



$$\vec{t} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$$

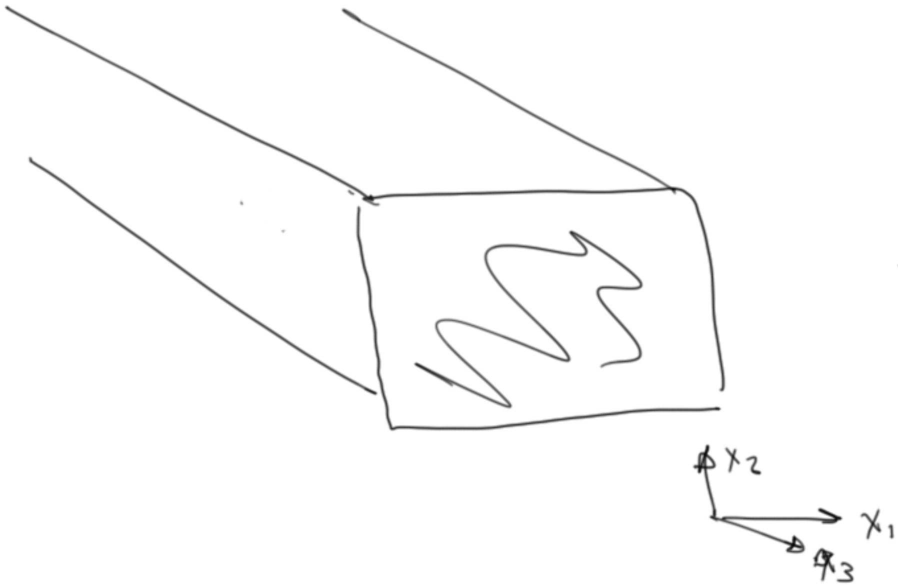
$$\sigma_{31} = \sigma_{32} = 0$$



$$\left(\sigma_{11} - \frac{\sigma_{11}+\sigma_{22}}{2}\right)^2 + \sigma_{12}^2$$

$$\tan(2\theta) = \left(\frac{2\sigma_{12}}{\sigma_{11}-\sigma_{22}}\right)$$

Plane strain



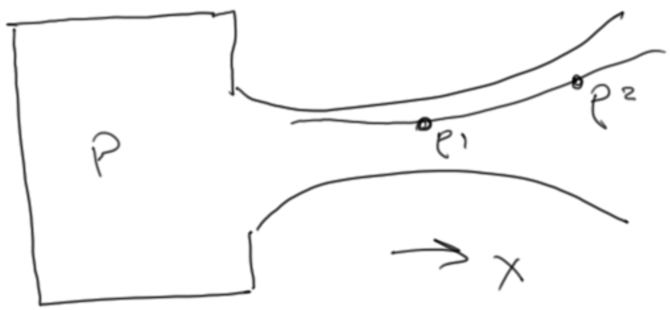
$$\varepsilon_{33} = \frac{\Delta L}{L} = 0$$

$$\varepsilon_{13} = \varepsilon_{23} = \varepsilon_{33} = 0$$

$$\varepsilon_{31} = \varepsilon_{32} = 0$$

$$\varepsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$$

$$\text{symm.}(\nabla u)$$



$$\frac{\partial \rho}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial x} \neq 0$$

$$\vec{v} = \vec{v}\left(\vec{x}(X,t),t\right)$$

$$\frac{D}{Dt}(\vec{v}) = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial x_k} v_k$$

$$= \underline{\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}}$$

$$\frac{D}{Dt}(\vec{v})$$

$$\frac{\partial v}{\partial t}$$

$$[t_1 \quad t_2 \quad t_3] = [n_1 \quad n_2 \quad n_3]\begin{bmatrix} \sigma_{11} & \sigma_{12} & \\ & \sigma_{22} & \\ & & \ddots \end{bmatrix}$$

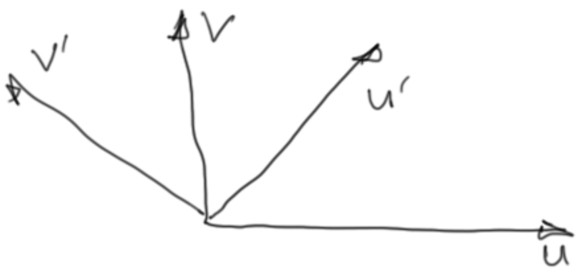
$$(\vec{t}^{\,T})^T = (\hat{n}^T \sigma)^T$$

$$\boxed{\vec{t} = \sigma^T \hat{n}}$$

$\sigma^T = \sigma$

Cauchy stress equation

$$t_i = \sigma_{ji}\,\hat{n}_i$$



$$u' = Ru$$
$$v' = Rv$$

$$\vec{v} = T\vec{u} \rightarrow \vec{v}\,' = T'\vec{u}\,' \rightarrow \underbrace{\cancel{R^{-1}R}}_{I}\vec{v} = R^{-1}T'R\vec{u}$$

$$\vec{v} = R^{-1}T'R\vec{u}$$

$$\vec{v} = \underbrace{R^{T}T'R}\,\vec{u}$$

$$T = R^T T' R$$
$$T' = R T R^T$$

$$T' = R T R^T$$

$$\sigma' = R \sigma R^T = \begin{bmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \qquad \sigma_I > \sigma_{II} > \sigma_{III}$$

$$Q \Lambda Q^T = D \qquad \det(\sigma - \lambda I) = 0$$

$$-\lambda^3 + I_1 \lambda^2 + I_2 \lambda + I_3 = 0$$

Invariants of stress tensor

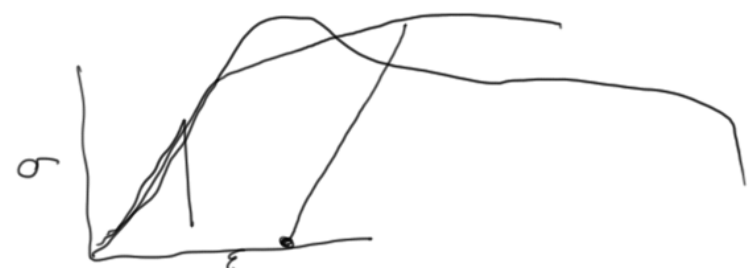
$$I_1 = tr(\sigma) = \sigma_{ii} = \sigma_I + \sigma_{II} + \sigma_{III} = -3p$$

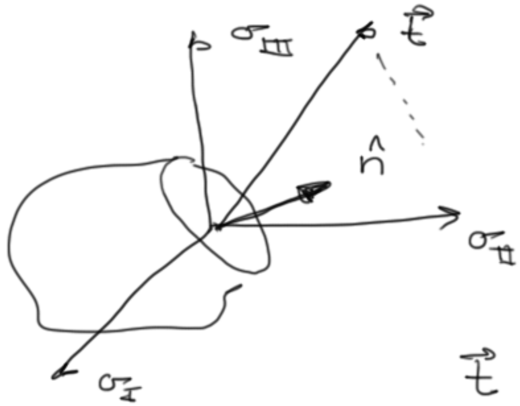
$$\boxed{I_2 = \frac{1}{2}\sigma_{ij}\sigma_{ij} - \frac{1}{2} I_1^2} = -(\sigma_I \sigma_{II} + \sigma_I \sigma_{III} + \sigma_{II} \sigma_{III})$$

$$I_3 = \det(\sigma) = \sigma_I \sigma_{II} \sigma_{III}$$

$$\sigma_{ij} = \underbrace{\frac{1}{3}\sigma_{kk}\delta_{ij}}_{\text{spherical, hydrostatic, dilatational stress}} + S_{ij} \rightarrow \text{deviatoric}$$

$$J_1 = 0, \quad J_2 = \frac{1}{2} S_{ij} S_{ij}, \quad J_3 = \det(S)$$





$$\vec{t} = \sigma^{T}\hat{n} = \begin{bmatrix} \sigma_{I} & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{bmatrix} \begin{Bmatrix} \hat{n}_1 \\ \hat{n}_2 \\ \hat{n}_3 \end{Bmatrix} = \begin{Bmatrix} \sigma_{I}\hat{n}_1 \\ \sigma_{II}\hat{n}_2 \\ \sigma_{III}\hat{n}_3 \end{Bmatrix}$$

$$t_n = \vec{t}\cdot\vec{n} = \sigma_{I}\hat{n}_1^2 + \sigma_{II}\hat{n}_2^2 + \sigma_{III}\hat{n}_3^2 \quad \backslash\ 1$$

$$|\vec{t}|^2 = t_s^2 + t_n^2$$

$$t^2 = t_n^2 + t_s^2 = (\sigma_{I}\hat{n}_1)^2 + (\sigma_{II}\hat{n}_2)^2 + (\sigma_{III}\hat{n}_3)^2 \quad \backslash\ 2$$

$$|\hat{n}| = 1 = \hat{n}_1^2 + \hat{n}_2^2 + \hat{n}_3^2 \quad \backslash\ 3$$

$$\begin{bmatrix} 1 & 1 & 1 \\ \sigma_{I} & \sigma_{II} & \sigma_{III} \\ \sigma_{I}^2 & \sigma_{II}^2 & \sigma_{III}^3 \end{bmatrix} \begin{Bmatrix} \hat{n}_1^2 \\ \hat{n}_2^2 \\ \hat{n}_3^2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ t_n \\ t_s^2 + t_n^2 \end{Bmatrix}$$

Solve

$$\hat{n}_1^2 = \frac{t_s^2 + (t_n - \sigma_{II})(t_n - \sigma_{III})}{\underbrace{(\sigma_I - \sigma_{II})(\sigma_I - \sigma_{III})}_{\geq 0}} \geq 0$$

$$\hat{n}_2^2 = \frac{t_s^2 + (t_n - \sigma_{III})(t_n - \sigma_I)}{\underbrace{(\sigma_{II} - \sigma_{III})(\sigma_{II} - \sigma_I)}_{\leq 0}} \leq 0$$

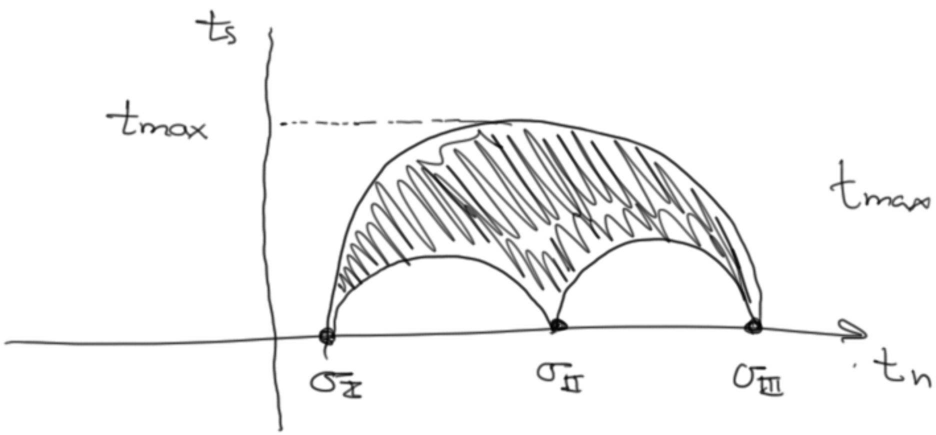
$$\hat{n}_3^2 = \frac{t_s^2 + (t_n - \sigma_I)(t_n - \sigma_{II})}{\underbrace{(\sigma_I - \sigma_{III})(\sigma_{II} - \sigma_{III})}_{\geq 0}} \geq 0$$

$$\underbrace{\left[t_n - \tfrac{1}{2}(\sigma_{II} + \sigma_{III})\right]^2}_{x^2} + \underbrace{t_s^2}_{y^2} \geq \left(\tfrac{1}{2}(\sigma_{II} - \sigma_{III})\right)^2$$

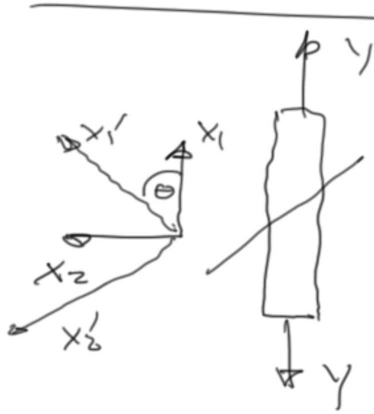
$$x^2 + y^2 \geq r^2$$

$$\left[t_n - \frac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\frac{1}{2}(\sigma_I - \sigma_{III})\right)^2$$

$$\left[t_n - \frac{1}{2}(\sigma_I + \sigma_{III})\right]^2 + t_s^2 \leq \left(\frac{1}{2}(\sigma_I - \sigma_{II})\right)^2$$



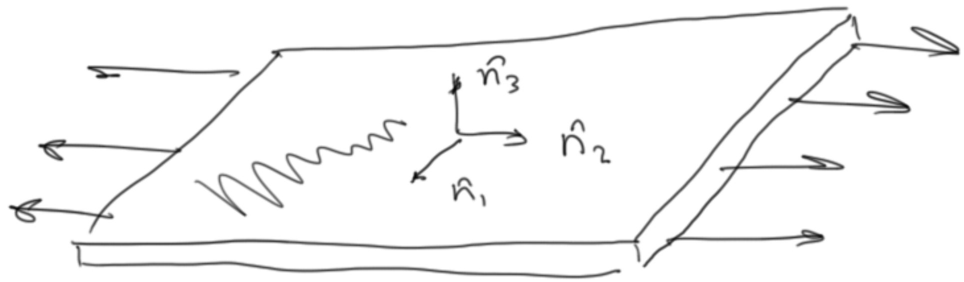
$$t_{max} = \frac{1}{2}(\sigma_I + \sigma_{III})$$



$$\sigma = \begin{bmatrix} Y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sigma' = R\,\sigma\,R^T$$

$$\sigma_{11}' = Y\cos\theta \qquad \sigma_2' = \frac{Y}{2}\sin\theta\cos\theta = \frac{Y}{2}\sin 2\theta \quad {}^{\max} \text{ at } 45^\circ$$

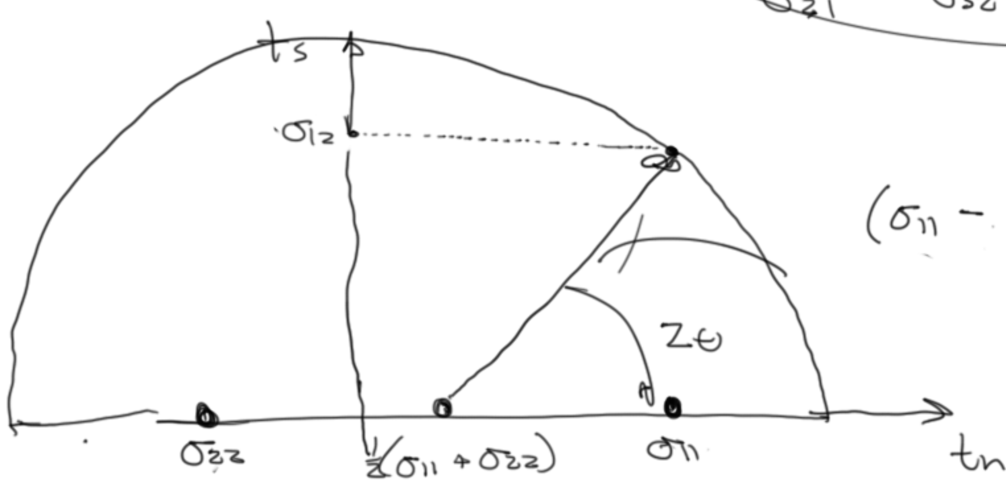


Plane stress

$$\vec{t} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$$

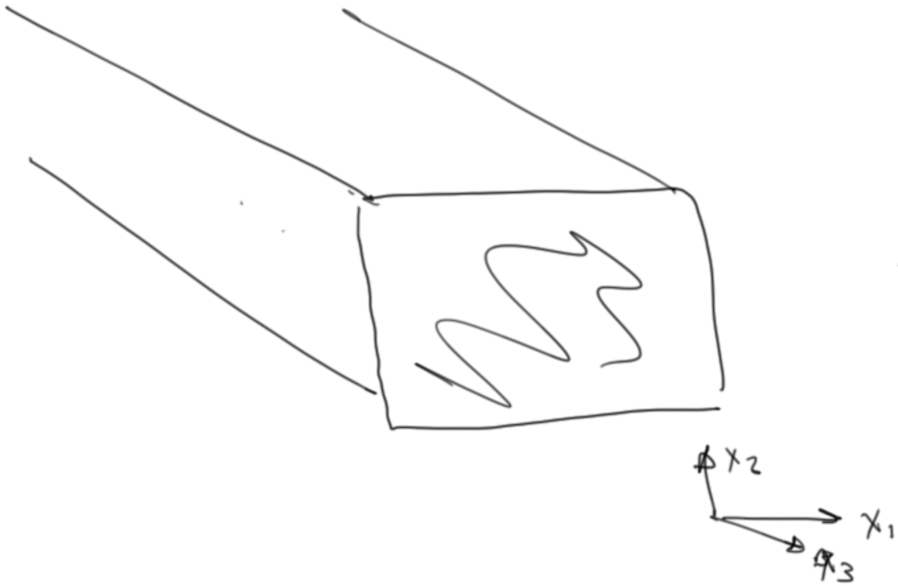
$$\sigma_{21} = \sigma_{32} = 0$$



$$\left(\sigma_{11} - \frac{\sigma_{11} + \sigma_{22}}{2}\right)^2 + \sigma_{12}^2$$

$$\tan(2\theta) = \left(\frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}}\right)$$

Plane strain



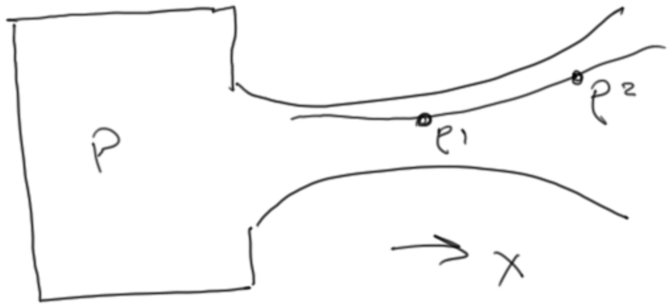
$$\varepsilon_{33} = \frac{\Delta L}{L} = 0$$

$$\varepsilon_{13} = \varepsilon_{23} = \varepsilon_{33} = 0$$

$$\varepsilon_{31} = \varepsilon_{32} = 0$$

$$\varepsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right)$$

$$\text{symm.}(\nabla u)$$



$$\frac{\partial \rho}{\partial t} = 0 \qquad \frac{\partial \rho}{\partial x} \neq 0$$

$$\vec{v} = \vec{v}\left(\vec{x}(X,t),t\right)$$

$$\frac{D}{Dt}(\vec{v}) = \frac{\partial \vec{v}}{\partial t} + \frac{\partial \vec{v}}{\partial x_k} v_k$$

$$= \underline{\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}}$$

$$\frac{D}{Dt}(\vec{v})$$

$$\frac{\partial v}{\partial t}$$