$$\frac{d^2 u}{dx^2} - u + x^2 = 0 \qquad \text{for } 0 < x < 1$$

$$\text{subject to} \quad u(0) = 0, \quad u(1) = 0$$

$$I(u) = \frac{1}{2}\int_0^1 \left[\left(\frac{du}{dx}\right)^2 - u^2 + 2x^2 u \right] dx$$

$$u \approx u^h = \sum_{j=1}^{n} c_j \phi_j = C_1 + C_2 X + C_3 X^2$$

$$u(0) = C_1 = 0$$

$$u(1) = C_2 + C_3 = 0 \implies C_2 = -C_3$$

$$u^h(x) = -C_3 X + C_3 X^2 = C_3 (x^2 - x)$$

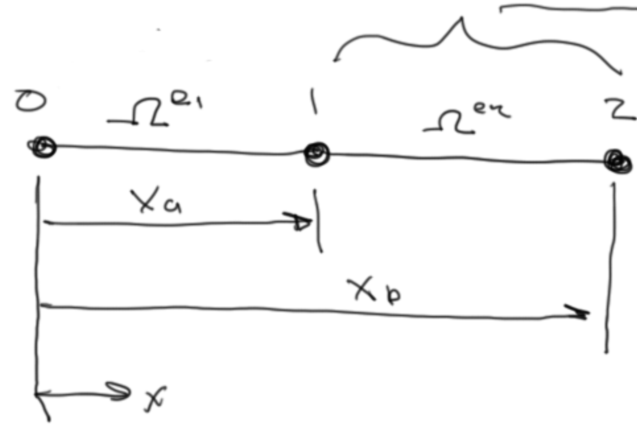
$$\frac{d}{dx}\left[EA \frac{du}{dx} \right] = 0 \qquad A(x) = A_0$$

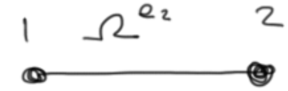
$$A(x) = A_0\left(1 - \frac{x}{2L}\right)$$

$$u(0) = 0 \qquad EA \frac{\partial u}{\partial x}\Big|_{x=L} = P$$

$$I(u) = \int_0^L \frac{EA}{2}\left(\frac{du}{dx}\right)^2 dx - Pu$$

$$u \approx u^n = c_i x^i$$





$$u_n = c_1 + c_2 x$$

$$u_n(x_a) = c_1 + c_2 x_a \equiv u_1^{e_2}$$

$$u_n(x_b) = c_1 + c_2 x_b \equiv u_2^{e_2}$$

$$\underbrace{\begin{Bmatrix} u_1^e \\ u_2^e \end{Bmatrix}}_{\vec{u}} = \underbrace{\begin{bmatrix} 1 & x_a \\ 1 & x_b \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix}}_{\vec{c}} \Rightarrow \underline{\vec{c} = A^{-1}\vec{u}}$$

$$c = \begin{bmatrix} \dfrac{u_2^e x_a - u_1^e x_b}{x_a - x_b} \\[2ex] \dfrac{u_1^e - u_2^e}{x_a - x_b} \end{bmatrix}$$

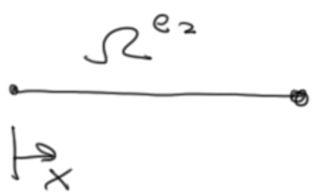
$$c_1 = \frac{u_2^e x_a - u_1^e x_b}{x_a - x_b}$$

$$c_2 = \frac{u_1^e - u_2^e}{x_a - x_b}$$

$$\boxed{u^h = \frac{x_2^e x_a - u_1^e x_b}{x_a - x_b} + \frac{u_1^e - u_2}{x_a - x_b} x}$$

$$u^h = \sum u_j \psi_j = u_1^e \psi_1 + u_2^e \psi_2$$

$$= u_1 \underbrace{\left[\frac{x - x_b}{x_a - x_b}\right]}_{\psi_1} + u_2 \underbrace{\left[\frac{x_a - x}{x_a - x_b}\right]}_{\psi_2}$$



where $x_b - x_a = L^e$

$$u^h = u_1^e \left[\frac{x - L}{L}\right] + u_2^e \left[\frac{x}{L}\right] \Rightarrow u_1^e \left[1 - \frac{x}{L}\right] + u_2^e \left[\frac{x}{L}\right]$$

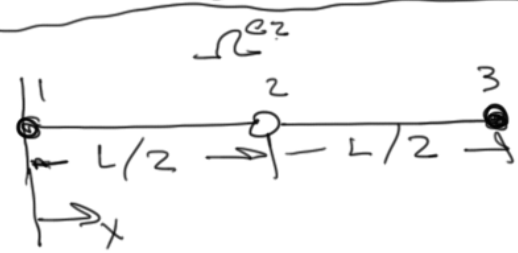
$$u^h = \psi_j u_j = \underbrace{\left[\, 1 - \frac{x}{L} \quad , \quad \frac{x}{L} \,\right]}_{\psi_j = N_j} \begin{Bmatrix} u_1^e \\ u_2^e \end{Bmatrix}$$

shape function matrix

Quadritic interpolant

$$X^T = [\, 1 \quad x \quad x^2 \,]$$

$$\boxed{u_n = N_j u_j = c_1 + c_2 x + c_3 x^2}$$



$$\underbrace{\begin{bmatrix} u_1^e \\ u_2^e \\ u_3^e \end{bmatrix}}_{\vec u} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & L/2 & (L/2)^2 \\ 1 & L & L^2 \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix}}_{\vec c}$$

$\vec u = A \vec c$

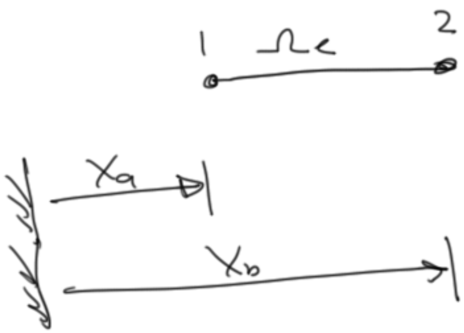
$$\boxed{u^h = \vec N^T \vec u = \vec X^T \vec c}$$

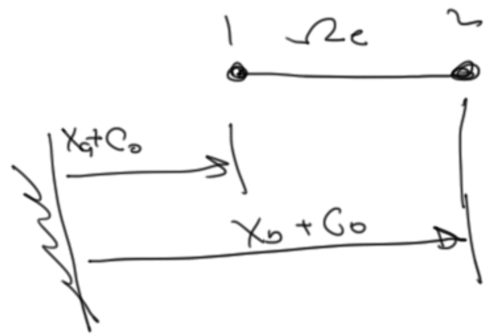
$$[\, N^T A \vec c = \vec X^T \vec c \,]\, A^{-1}$$

$$N^T A A^{-1} = \vec X^T A^{-1} \Rightarrow N^T = X^T A^{-1}$$

$$A = \begin{bmatrix} X|_{\text{NODE 1}} \\ X|_{\text{NODE 2}} \\ X|_{\text{NODE 3}} \end{bmatrix}$$

1D Element





$u_1 = c_0 \quad \& \quad u_2 = c_0 \qquad u^h = c_0$

$$u^h = \frac{c_0}{c_0} = N_1 u_1 + N_2 u_2 = \frac{N_1 c_0 + N_2 c_0}{c_0}$$

$1 = N_1 + N_2$ <u>Partition of Unity</u>

$$N_i(x_j) = \begin{Bmatrix} 0 & i \neq j \\ 1 & i = j \end{Bmatrix}$$