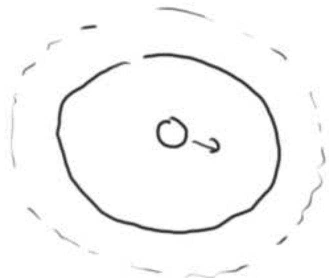
$$p_{net} = (p_f - \sigma_3) = \frac{K_{Ic}}{\sqrt{\pi L}}$$ $\Rightarrow$ center-cracked infinite plate

Sack (1946)

$$p_{net} = \sqrt{\frac{\pi G_{Ic} E}{4(1-\nu^2)R}} = \sqrt{\frac{\pi}{4R}}\, K_{Ic}$$



$$G_{Ic} = \frac{K_{Ic}^2}{E}(1-\nu^2)$$

combine - eliminate R

Sneddon (1946)

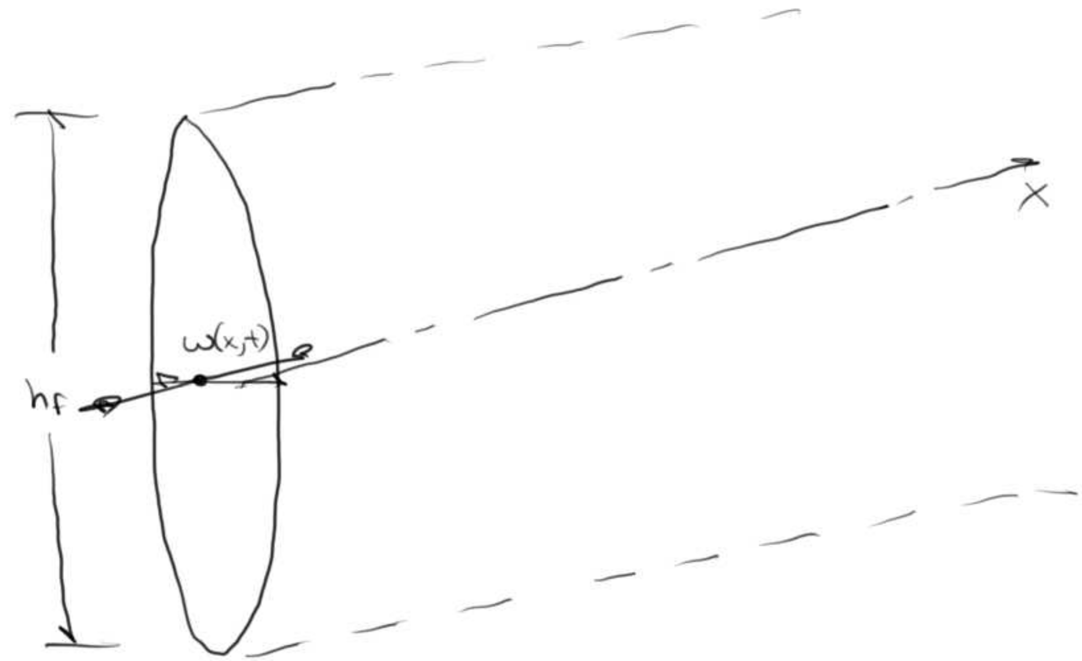
$$V = \frac{16(1-\nu^2)R^2(p_{net})}{3E}$$

$$p_{net} = \left[\frac{\pi^3 G^3 E^2}{12(1-\nu^2)^2 V}\right]$$

1961 Perkins - Kern

Lamb 1932

$$\frac{\partial p}{\partial x} = -\frac{64\, q\, \mu}{\pi\, h_f\, \omega^3}$$



$q \rightarrow$ flow rate

$\mu \rightarrow$ fluid viscosity

Newtonian fluid in elliptical cross section

Sneddon + Elliot (1946)

$$\omega = \frac{2\, p_{net}\, h_f\, (1-\nu^2)}{E}$$

Plane strain in direction of h_f

where $E' = \frac{E}{(1-\nu^2)}$

substitute q by $q_i/2$

$$\int_0^{p_{net}} p^3\, dp = \int_0^{L} -\frac{4}{\pi}\, \frac{\mu\, q_i\, E'^3}{h_f}\, dx$$

$$p_{net} = \left[\frac{16\mu\, q_i\, E'^3}{\pi\, h_f} L \right]^{1/4}$$

$$w = 3 \left[\frac{\mu\, q_i\, (L-x)}{E'} \right]^{1/4}$$

$\Bigg\rangle$ Perkins - Kern 1961

- Ignores Fracture Mechanics
- Plane Strain vertical direction
- Leakoff neglected
- Fixed height

Carter

$$u_L = \frac{C_L}{\sqrt{t - t_{exp}}}$$

$C_L \rightarrow$ leakoff coef.

$t_{exp} \rightarrow$ exposure time

mass balance

$$q_i = q_L + q_f$$

$q_L \rightarrow$ leakoff rate over whole fracture

$q_f \rightarrow$ volume rate of storage in the fracture

$$q_i = \underbrace{2\int_0^{A_f(t)} u_L \, dA_f}_{q_L} + \bar{\omega}\frac{\partial A_f}{\partial t}$$

$\bar{\omega} \rightarrow$ assumed constant

$$q_i = 2\int_0^t u_L(t-\lambda)\frac{\partial A_f}{\partial \lambda}\, d\lambda + \bar{\omega}\frac{\partial A_f}{\partial t}$$

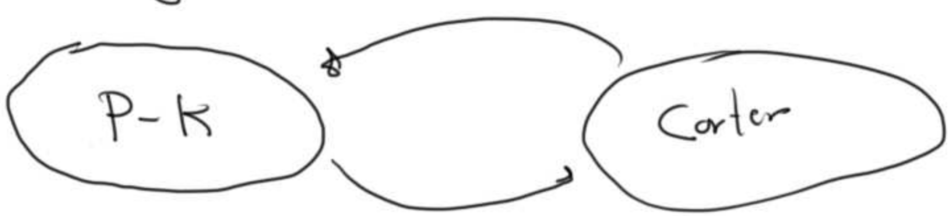
Harrington & Hannah (1975)

$$A_f = \frac{q_i \bar{\omega}}{4\pi C_L^2}\left(e^{S^2}\operatorname{erfc}(S) + \frac{2}{\sqrt{\pi}}S - 1 \right) \leftarrow \qquad A_f = \frac{q_i t}{\bar{\omega} + 2C_L\sqrt{2t}}$$

with

$$S = \frac{2C_L\sqrt{\pi t}}{\bar{\omega}} \qquad \operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\, dt$$

Fracture length $L(t)$ can be obtained by dividing $A(t)$ by 2 & by h_f



Norgren

$$\frac{\partial q}{\partial x} + q_L + \frac{\partial A}{\partial t} = 0$$

q is volume flow rate through cross-section

$q_L = 2 h_f u_L$ leakoff per unit length

$$\frac{E'}{128 \mu h^f} \frac{\partial^2 w^4}{\partial x^2} = \frac{8 C_L}{\pi \sqrt{t - t_{exp}}} + \frac{\partial w}{\partial t} \qquad (\text{PKN})$$