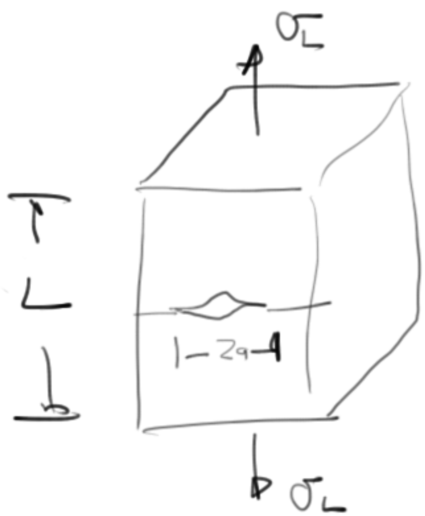
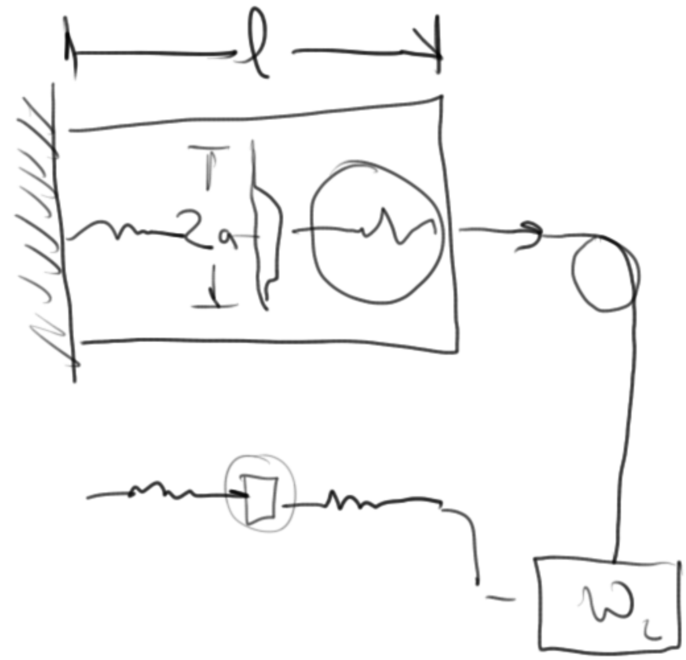
1921 - Griffith



$$C \approx \sigma_L \sqrt{a}$$



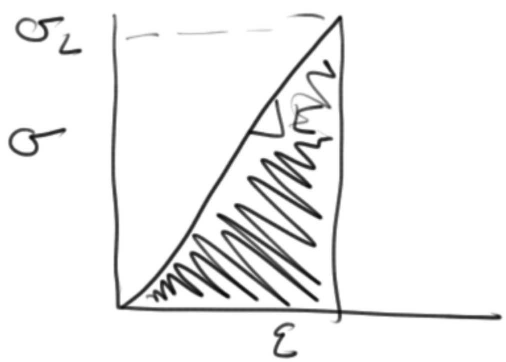
$$U_T = U_E + U_S - W_L$$

$$\frac{\partial U_T}{\partial a} = \frac{\partial U_S}{\partial a} + \frac{\partial}{\partial a}\left(U_E - W_L\right) = 0$$

$$\begin{aligned} W_L &= F \cdot \text{distance} \\ &= F \cdot \Delta L \\ &= (\sigma_L A)\cdot(\varepsilon L) \end{aligned}$$

$$\varepsilon = \frac{\Delta L}{L} \Rightarrow \Delta L = \varepsilon L$$

Strain energy



$$U_E = \frac{1}{2}\sigma_L \varepsilon A l$$

$$\frac{W_L}{U_E} = \frac{\sigma_L A \varepsilon l}{\frac{1}{2}\sigma_L \varepsilon A l} = 2 \Rightarrow W_L = 2U_E$$

$$0 = \frac{\partial U_s}{\partial a} + \frac{\partial}{\partial a}\left(-U_E\right) \Rightarrow \boxed{\frac{\partial U_s}{\partial a} = \frac{\partial U_E}{\partial a}}$$

$$U_s = 2 \cdot 2a \cdot \underbrace{\gamma}_{\text{specific surface energy}} \cdot \text{width}$$

$$\frac{U_s}{\text{width}} = 4a\gamma$$

$$\frac{U_E}{\text{width}} = \pi a^2 \sigma_L^2 \left[\frac{1-\nu^2}{E}\right] \quad (\text{Inglis},\ 1923)$$

$\cancel{\frac{\partial u}{\partial \varepsilon}}\ \varepsilon = \frac{\partial u}{\partial x}$

$\sigma = E\varepsilon$

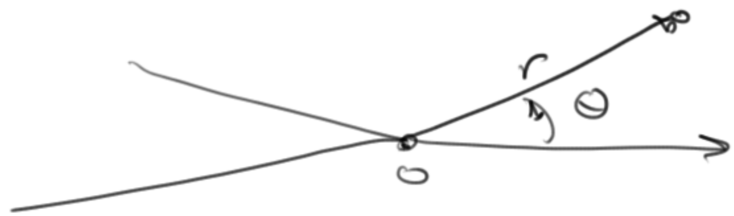
For plane strain

$$\frac{\partial}{\partial a}(4a\gamma) = \frac{\partial}{\partial a}\left(\pi a^2 \sigma_L^2 \left[\frac{1-\nu^2}{E}\right]\right)$$

$$\varepsilon_z = \frac{\Delta L}{L \rightarrow} \approx 0$$

$$G = 2\gamma = \frac{\pi a \sigma_L^2 (1-\nu^2)}{E}$$

strain energy release-rate



$$\sigma_L = \sigma_c$$

$$G_c = \underbrace{\frac{\pi a \sigma_c^2 (1-\nu^2)}{E}}$$

$$\underbrace{\sigma_L \sqrt{\pi a}} = \left[\frac{G E}{(1-\nu^2)}\right]^{1/2}$$

$$\rightarrow \underbrace{K_I} = \left[\frac{G E}{(1-\nu^2)}\right]^{1/2}$$

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta)$$