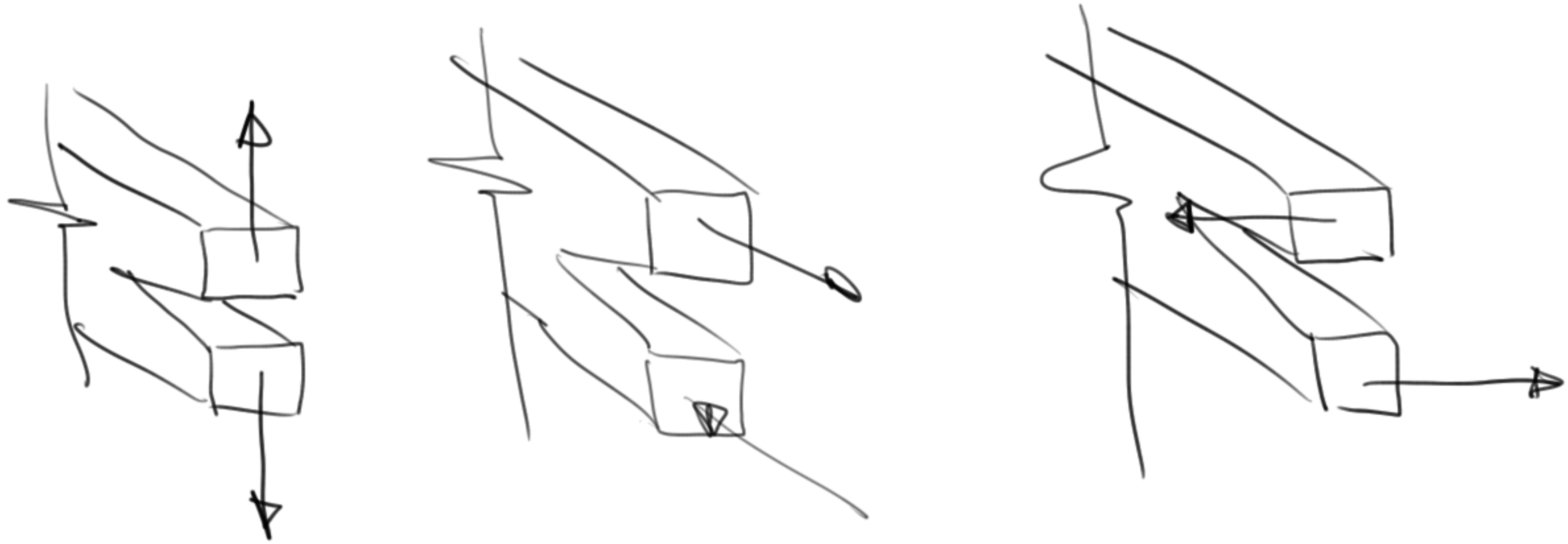
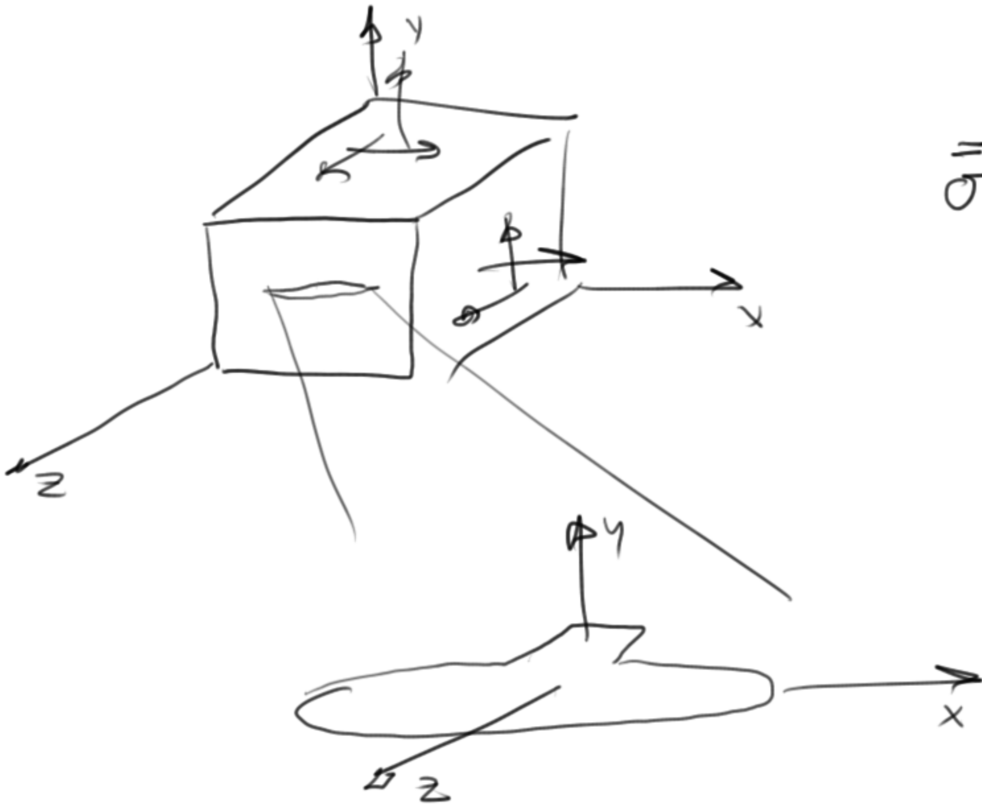
K_I

G_I



Mode I   Mode II   Mode III

$$\boxed{G = \frac{(1-\nu^2)}{E} K_I^2} + \frac{(1-\nu^2)}{E} K_{II} + \frac{(1+\nu)}{E} K_{III}^2$$



$$\bar{\bar{\sigma}}^{r} = \begin{bmatrix} \sigma_{xx}^{r} & \sigma_{xy}^{r} & \sigma_{xz}^{r} \\ & \sigma_{yy}^{r} & \sigma_{yz}^{r} \\ \text{sym.} & & \sigma_{zz}^{r} \end{bmatrix}$$

$$\bar{\bar{\sigma}}^{c} = \begin{bmatrix} \sigma_{xx}^{c} & \sigma_{xy}^{c} & \sigma_{xz}^{c} \\ & \sigma_{yy}^{c} & \sigma_{yz}^{c} \\ \text{sym} & & \sigma_{zz} \end{bmatrix}$$

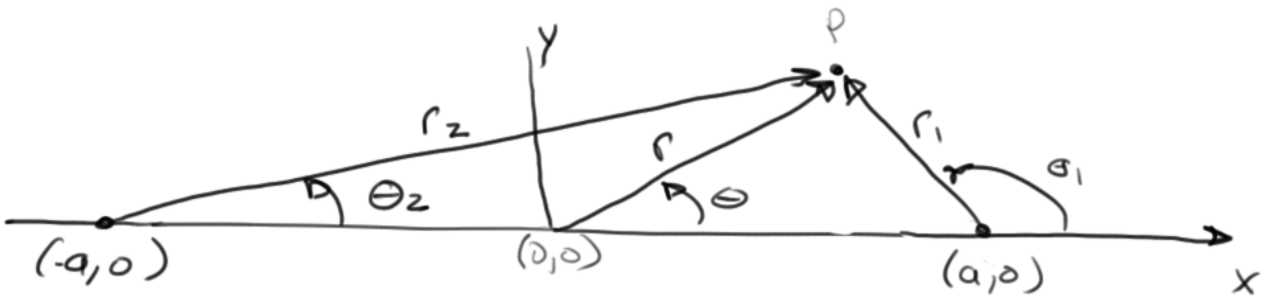
<u>Driving stresses</u>

$$\Delta\sigma_{I} = \left(\sigma_{yy}^{r} - \sigma_{yy}^{c} \right)$$

$$\Delta\sigma_{II} = \left(\sigma_{xy}^{r} - \sigma_{xy}^{c} \right)$$

$$\Delta\sigma_{III} = \left(\sigma_{xz}^{r} - \sigma_{xz}^{c} \right)$$

Follow <u>Pollard et al. 1987</u> "tri-polar coord. sys."



$$R = \sqrt{r_1 r_2}$$

$$\Gamma = \left(\frac{\theta_1 + \theta_2}{2}\right)$$

$$u_x = \frac{1}{2\mu}\Big\{ \Delta\sigma_{II} \big\{ 2(1-\nu)(R\Gamma - r\sin\theta) + r\sin\theta\left[r R^{-1}\cos(\theta-\Gamma) - 1\right]\big\}$$

$$+ \Delta\sigma_{I} \big\{ (1-2\nu)(R\cos\Gamma - r\cos\theta) - r\sin\theta \cdot \left[r R^{-1}\sin(\theta-\Gamma)\right]\big\}\Big\}$$

$$u_y = \frac{1}{2\mu}\Big\{ \sigma_{I} \big\{ 2(1-\nu)(R\sin\Gamma - r\sin\theta) - r\sin\theta\left[r R^{-1}\cos(\theta-\Gamma) - 1\right]\big\}$$

$$- \Delta\sigma_{II} \big\{ (1-2\nu)(R\cos\Gamma - r\cos\theta) + r\sin\theta \cdot \left[r R^{-1}\sin(\theta-\Gamma)\right]\big\}\Big\}$$

$$u_z = \frac{1}{2\mu}\Big\{ 2\,\Delta\sigma_{III}(R\sin\Gamma - \sin\theta)\Big\}$$

On crack surface

$r = |x|$ $\qquad \theta = 0, \pi$ $\qquad R = (a^2 - x^2)^{1/2}$

$r_1 = a - x$ $\qquad \theta_1 = -\pi, \pi$ $\qquad \Gamma = \frac{\pi}{2}, \frac{3\pi}{2}$

$r_2 = a + x$ $\qquad \theta_2 = 0, 2\pi$

$$\begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \pm \begin{Bmatrix} \Delta\sigma_{II} \\ \Delta\sigma_{I} \\ \Delta\sigma_{III} \end{Bmatrix} \frac{1-\nu}{\mu} (a^2 - x^2)^{1/2} + \begin{Bmatrix} -\Delta\sigma_{I} \\ \Delta\sigma_{II} \\ 0 \end{Bmatrix} \frac{(1-2\nu)}{2\mu} x$$

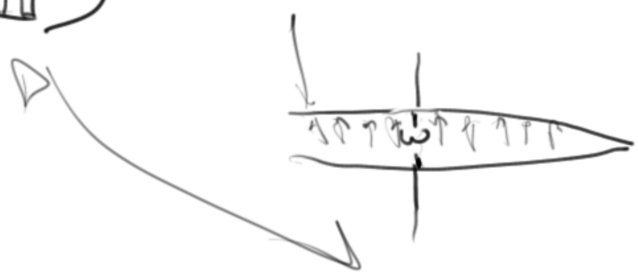
$\pm$ refers $y = 0^+$, $y = 0^-$

$$[\![u_m]\!] = (u_i^+ - u_i^-)$$

$i = x, y, z$

$m = I, II, III$

$$\begin{Bmatrix} [[u_I]] \\ [[u_{II}]] \\ [[u_{III}]] \end{Bmatrix} = \begin{Bmatrix} \Delta\sigma_I \\ \Delta\sigma_{II} \\ \Delta\sigma_{III} \end{Bmatrix} \frac{2(1-\nu)}{\mu}\left(a^2 - x^2\right)^{1/2}$$



$$\frac{\partial}{\partial t}(\rho w) = \frac{\partial}{\partial x}\left(\frac{\rho w^2}{12\mu}\frac{\partial p}{\partial x}\right)$$

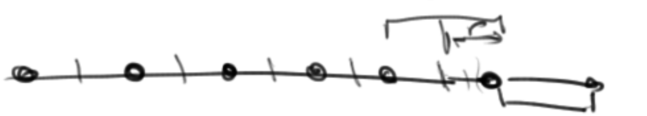
Rice (1969)

$$K_I = \frac{\mu}{4(1-\nu)}\sqrt{\frac{2\pi}{r}}\,[[u_I]]$$

$$K_{II} = \frac{\mu}{4(1-\nu)}\sqrt{\frac{2\pi}{r}}\,[[u_{II}]]$$

$$\boxed{\begin{matrix} K_I > K_{Ic} \\ K_{II} > K_{IIc} \end{matrix}}$$

$r \to 0$



Max circumferential stress theory

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos^2 \frac{\theta_0}{2} \left(K_I \cos \frac{\theta_0}{2} - 3 K_{II} \sin \frac{\theta_0}{2} \right)$$

$$\underbrace{\sigma_{\theta\theta}^{c} \sqrt{2\pi r}}_{}$$

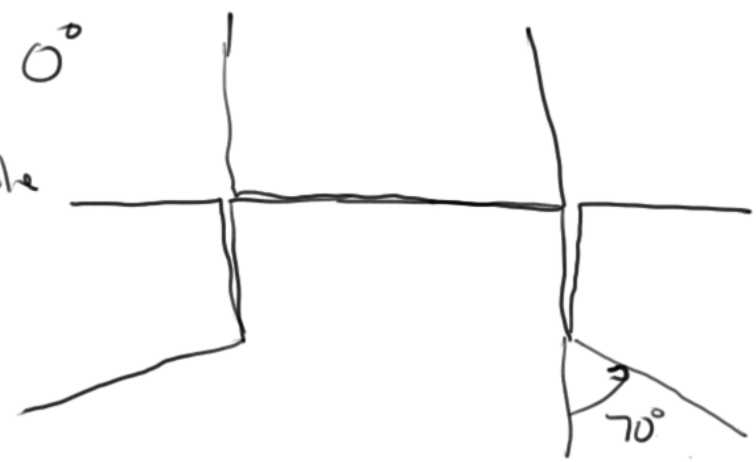
$$K_{Ic}$$

$$\theta_0 = \sin^{-1} \left[\frac{K_I}{K_{Ic}} \cos \left[\tan^{-1} \left(3 \frac{K_{II}}{K_{Ic}} \right) \right] \right]$$

For pure mode I

$$K_I \sin \theta = 0 \quad \therefore \quad \theta = 0^\circ$$

Kalthoff – Winkler



For pure mode II

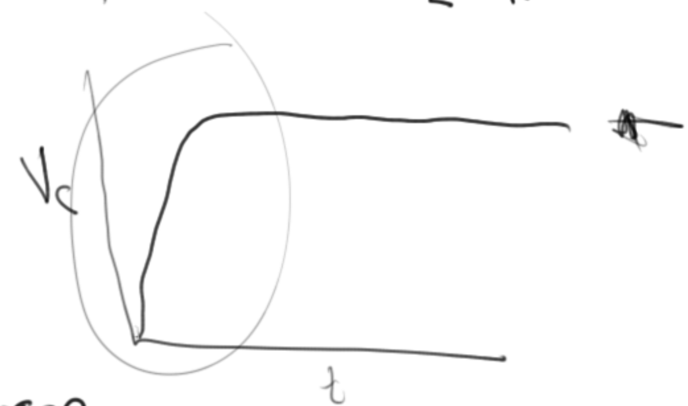
$$K_{II} (3 \cos \theta - 1) \rightarrow \theta = 70.5^\circ$$

Compute

w ~~to~~ $[[u_y]]$, p from Lubrication theory $\rightarrow$ K_I + K_{II} $\rightarrow$ θ_0

extend crack

extend crack at known vel, $v \sim \frac{1}{2} c_R$



Limitations

- Assume crack kinetic rel.
  - how fast crack prop.
- Hetero. material prop.
- Leakoff ? $\rightarrow$ Carter leakoff

  P3D $\rightarrow$ correction for fracture height

Review exam

Study Exam I & II

- Structural geology
  - ~~B~~ Plate boundaries
  - Anderson fault classification
- Linear algebra
  - Eigenvalues
- Stress
  - Principle stresses
  - Overburden calc.
  - Eff. stress
    - → Pore pressure
- Elasticity theory

Rotations for stress resolution

In situ → Geo → Wellbore

→ fault

Rock failure

- Failure models
  - Mohr - Coulomb
  - Hoek-Brown
  - Cap models

Slip on faults

- calculate if slip will occur

Wellbore Geomechanics

- Compute breakouts will occur
- Kirsh solution
- Where → breakouts → tensile fractures
- Deviated wellbores

Diagnostics

- DFIT, mini-frac
- Well logging - viz. breakouts

Sand productions

- Mechanism for limiting
- Causes

Reservoir depletion

- Production induced faulting
  - why?
- Reservoir space plot
  (know how interpret)
- Stress reorientation
- Porosity reduction

HF

- Overview

Fracture Mechanics

- Modes
- Energy release rate
- Fracture toughness
- Compute SIF (K) from [[U]]