Problem 1

(2 points each) Circle the best answer:

(i) Which of the following is NOT a type of plate boundary?

(a) Convergent plate boundary

(b) Transform plate boundary

(c) Asthenospheric plate boundary

(ii) True or False? The eigenvectors of the stress tensor are the principle stress directions.

(a) True

(b) False

(iii) In a normal faulting regime, the hanging wall moves which way relative to the footwall?

(a) Vertically up

(b) Vertically down

(c) Horizontally

(iv) In a strike-slip faulting regime, the hanging moves which way relative to the footwall?

(a) Vertically up

(b) Vertically down

(c) Horizontally

(v) True or False? If we idealize the earth's surface as a half-space, the vertical stress due to overburden is a principle stress.

(a) True

(b) False

(vi) In the context of characterizing a fault, the *dip* angle is

(a) the angle of the fault strike with respect to north.

(b) the angle of the fault plane with respect to horizontal.

(c) the angle that defines the displacement vector of the fault slip motion measured from horizontal.

(vii) The Cauchy stress tensor is always symmetric due to what principle?

(a) Conservation of linear momentum

(b) Conservation of angular momentum

(c) Conservation of energy

(viii) According to Anderson fault classification, the principle stress magnitudes have the following ordering for a normal faulting regime.

(a) $S_v > S_{Hmax} > S_{hmin}$

(b) $S_{Hmax} > S_v > S_{hmin}$

(c) $S_v > S_{hmin} > S_{Hmax}$

Problem 1 continued on next page...

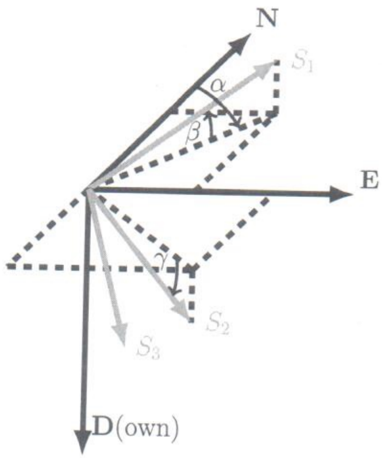
(ix) If we are given the three principle stresses S_v, S_{Hmax}, and S_{hmin}, what else is needed to fully characterize the tectonic stress field.

(a) The direction of either S_{Hmax} or S_{hmin}

(b) The dip and strike angles of the nearest fault

(c) The type of faulting regime according to Anderson classification.

(x) Consider the following figure:



If S_2 was initially in the direction of **E**, which rotation would reorient S_2 so that it was aligned with **N**orth?

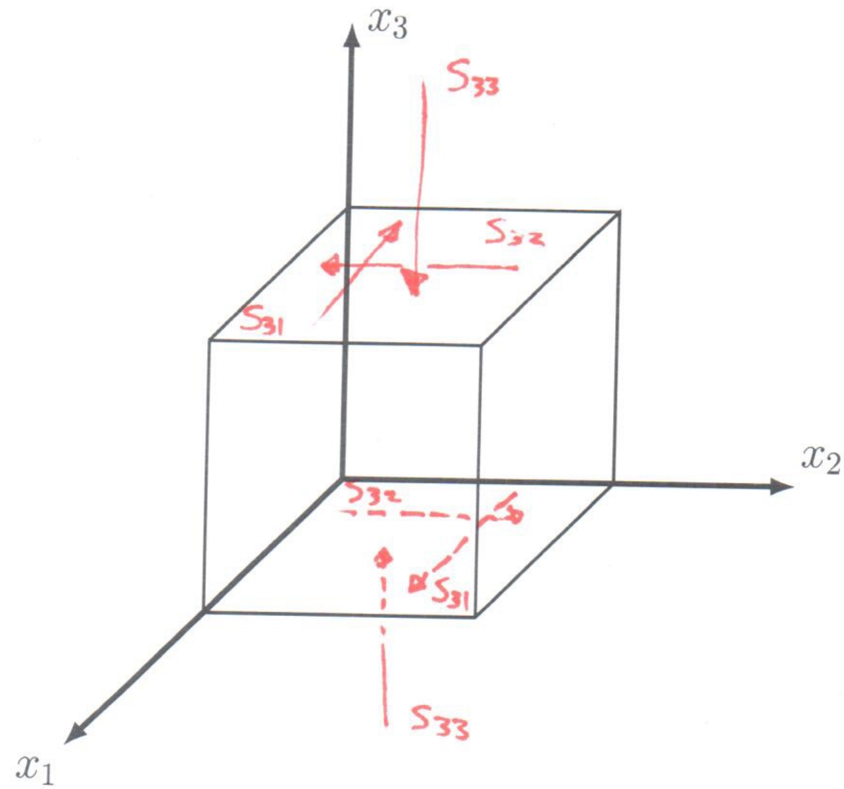
(a) $\gamma = 90°$

(b) $\beta = 90°$

(c) $\alpha = 270°$

Problem 2

(10 points) Consider the following figure:



On the figure, correctly indicate (with hand-drawn arrows) the stress components S_{33}, S_{31}, and S_{32} on both sides of the cube where they act. **Use a "compression positive" sign convention.**

Problem 3

For the following matrix $\mathbf{A}$,

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

perform the following calculations **by hand**:

(i) (**5 points**) Compute the determinate of $\mathbf{A}$.

(ii) (**5 points**) Compute the eigenvalues of $\mathbf{A}$.

(iii) (**5 points**) Compute the eigenvectors of $\mathbf{A}$, put them in unit-vector form.

(i) $\det\left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\right) = (1)(3) - (2)(4) = \underline{\underline{-5}}$

(ii) $\det\left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}\right) = 0$

$(1-\lambda)(3-\lambda) - 8 = 0$

$\lambda^2 - 4\lambda - 5 = 0$

$(\lambda - 5)(\lambda + 1) = 0 \Rightarrow \lambda_{1,2} = \{5, -1\}$ ///

(iii)

$\lambda = 5 \rightarrow \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix} \xrightarrow{R1+R2} \begin{bmatrix} -4 & 2 \\ 0 & 0 \end{bmatrix} \xrightarrow{R1 \times -\frac{1}{4}} \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix}$

$x_1 = +\frac{1}{2} x_2$

$x_2 \Rightarrow$ free, choose $= 2$

$V_1 = \begin{Bmatrix} +1 \\ 2 \end{Bmatrix} = \frac{1}{\sqrt{5}} \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$

This page provided for additional calculations

$$\lambda = -1 \rightarrow \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \xrightarrow{-2\times R1 + R2} \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \xrightarrow{\frac{1}{2}\times R1} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$x_1 = -x_2$

$x_2 \Rightarrow$ free, choose $= 1$

$$V_2 = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \frac{1}{\sqrt{2}} \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

Problem 4

(**20 points**) Given the following principle stresses under *normal* faulting (using Anderson classification). What is the geographical stress tensor, $\mathbf{S}_G$, if S_{Hmax} is oriented exactly southwest?

$$S_1 = 60 \text{ MPa}, \quad S_2 = 45 \text{ MPa}, \quad S_3 = 40 \text{ MPa}$$

Normal faulting $\Rightarrow$ $S_1 = S_v$, $S_2 = S_{Hmax}$, $S_3 = S_{hmin}$

Using Fig. from Problem 1 (x).

To move S_1 from N to D : $\beta = 270°$

To move S_2 from E to SW : $\alpha = 135°$

$\gamma = 0°$

Using Matlab code:

$$S_G = \begin{bmatrix} 42.5 & 2.5 & 0 \\ 2.5 & 42.5 & 0 \\ 0 & 0 & 60 \end{bmatrix} \text{ MPa}$$

This page provided for additional calculations

Problem 5

Given the geographical stress,

$$\mathbf{S}_G = \begin{bmatrix} 47.5 & -12.5 & 0 \\ -12.5 & 47.5 & 0 \\ 0 & 0 & 40 \end{bmatrix} \text{ MPa}$$

(i) (**10 points**) For a fault strike oriented east-west and a dip 65° from south, determine the normal and shear stresses acting on the fault plane.

(ii) (**5 points**) If the fault where to slip, what type of fault slip would it be?

(iii) (**10 points**) Faults typically slip when the ratio of shear to normal effective stress exceeds 0.6, i.e. $\tau/\sigma_n^{eff} > 0.6$. Using this criterion, calculate the critical injection pressure in petroleum engineering operations that one should not exceed to prevent fault slippage. You can assume that the fault is very near the injector and steady state conditions, such that the pore pressure is equal to the injection pressure.

(i) strike = 90°

dip = 65°

From Matbab:

$$\vec{t} = S_g \cdot \hat{n} = \begin{Bmatrix} -43.0496 \\ 11.3288 \\ -16.9047 \end{Bmatrix}$$

$$\sigma_n = \vec{t} \cdot \hat{n} = 46.2 \text{ MPa}$$

$$\tau_s = \vec{t} \cdot \hat{n}_s = 11.32 \text{ MPa}$$

$$\tau_d = \vec{t} \cdot \hat{n}_d = 2.87 \text{ MPa}$$

$$\tau = \sqrt{\tau_s^2 + \tau_d^2} = 11.7 \text{ MPa}$$

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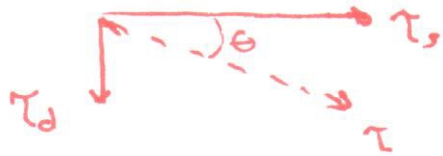
(ii)



$$\theta = \arctan\left(\frac{\tau_d}{\tau_s}\right) = 14.2^\circ$$

Mostly strike-slip, but dipping at angle of 14.2° ///

(iii) $$\frac{\tau}{(\sigma_n - P_p)} > 0.6 \Rightarrow \frac{\tau}{(\sigma_n - P_c)} = \mu$$

$$P_c = \frac{\mu \sigma_n - \tau}{\mu} = \frac{(0.6)(46.2) - 11.7}{0.6} = 26.7 \text{ MPa} \;///$$