Characterizing cemented sandstones with physics-based and machine learning approaches

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# Abstract

## Introduction

Sandstone is one of the most common types of reservoir rocks, contributing about 30% to the stratigraphic total of sedimentary rocks (Pettijohn et al., 1972, p. 4). Therefore, it is of great interest to predict the reservoir properties of sandstones. In this paper, we will focus on explaining the factors that influence sandstone permeability. Two approaches are available for permeability prediction of sandstones: 1) physics-based models such as the Carman-Kozeny equation and 2) empirical models developed using statistical or machine learning tools that assume no particular physical laws linking predictors and permeability. There are a number of physics-based and empirical models; Dullien (2012) is a nice review of both model types. We will apply a hybrid approach that considers both the physical intuition encapsulated in the Carman-Kozeny equation and data-centric models.  
Kozeny (1927) and Carman (1937) developed an equation linking permeability to three factors: porosity, hydraulic tortuosity, and specific surface area.  
Porosity and permeability are routinely measured during core analysis, but hydraulic tortuosity (as opposed to electrical tortuosity) and specific surface area are not commonly evaluated. However, tortuosity and specific surface area arise from geologic processes that can be modeled and distributed throughout the reservoir. Therefore, understanding the magnitude and effect of proxies for tortuosity and surface area can aid in building accurate porosity-permeability transformations and applying these transformations in geomodels.  
Panda and Lake (1995) developed a mathematical framework for estimating tortuosity and specific surface area for real rocks that had undergone diagenesis. With this framework, permeability can be predicted from the intergranular porosity, average grain diameter, grain size distribution, and the amounts of various types of cements.  
Machine learning is a tool that can be used to understand how useful proxies for tortuosity and specific surface area are for predicting permeability. With advanced non-parametric machine learning (such as the gradient boosting machine developed by Friedman, 2001), there is no requirement to assume *a priori* a functional form between these proxies and the predicted quantity. With the recent derivation of a consistent feature attribution system for explaining tree-based models (Lundberg et al., 2018), the functional form can be visualized after modeling, helping petrophysicists understand the mechanisms controlling permeability.  
Given the physics-based model and advanced machine learning approaches, we propose a hybrid approach, combining the best qualities of each approach.  
In this study, we develop estimates for the permeability of the Garn sandstone reservoir (Ehrenberg, 1990), using the data from that study. We compare different methods for calculating the tortuosity and specific surface area from core description, and we find the most important determinants of porosity-permeability transforms in this case.

**XXX More results**

## Methods

### Physical models

Perhaps the most well-known physics-based approach to estimating permeability was developed by Kozeny (1927) and later modified by Carman (1937). In its modern form, the equation is written as

which, for simplicity, we will write as

where permeability is , porosity is , tortuosity is , the specific surface area is , and the Carman-Kozeny void fraction is . For an uncemented sandstone, tortuosity can be calculated following the derivation in Appendix A, which comes from Panda and Lake (1994). For a cemented sandstone (Appendix B), the tortuosity changes because of cements blocking and forcing modification of the flow paths.  
Specific surface area for an uncemented sandstone can be estimated from the particle size distribution, after assuming spherical particles. After cementation, the cement distribution is important to how the surface area changes. Some cements will coat the pores walls, slightly decreasing the specific surface area. Other cements will line or bridge the pores, moderately to greatly increasing the specific surface area.  
A competing hypothesis is that pore throat sizes are the most important determinant of permeability-porosity transforms. This hypothesis is implicitly included in the Winland relations that follow the form

where is the pore throat radius (see Kolodzie, 1980 and Di and Jensen, 2015). Winland’s