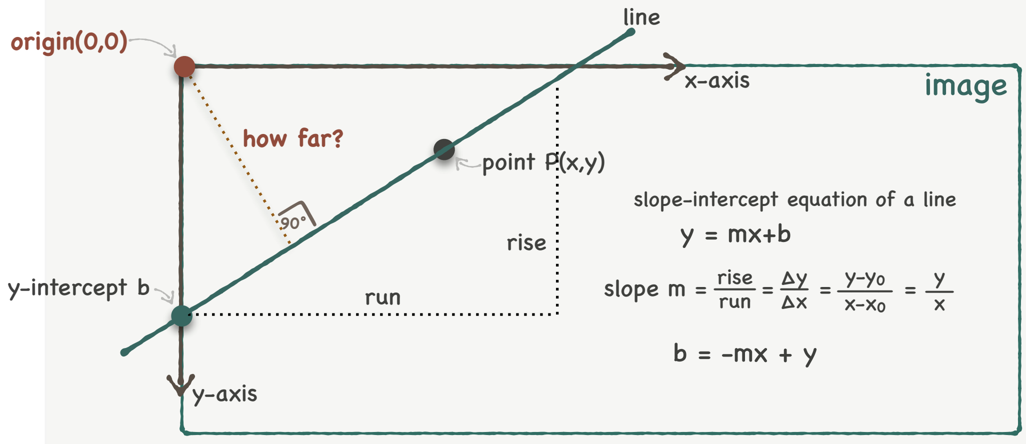
Orthogonal ⊥Distance (Polar Space)

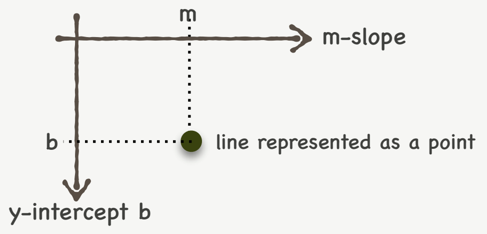
By connecting two points we can form a line.

By having one point and a slope we can also form a line.

Point P(x,y) in a Cartesian coordinate system:



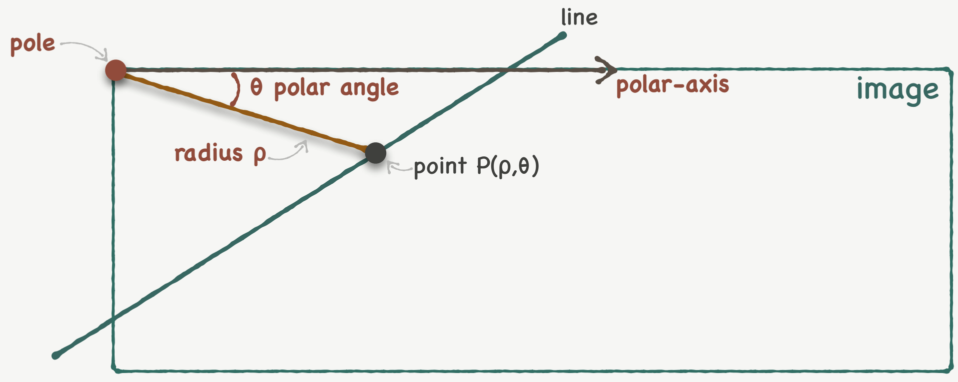
Line in a slope-intercept parameter space.



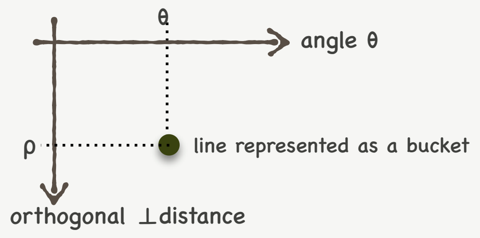
note that vertical lines can not be represented in this space because:
- slope of a vertical line is undefined (division by zero)
- vertical lines do not have y-intercept (they are parallel to y-axis)

Unfortunately, in Cartesian space, the slope of a vertical line is undefined. Thus, we need a different coordinate system, i.e., Polar Coordinate system. This system describes a point in space as an angle (polar angle, angular coordinate, azimuth, θ, ϕ, t) of rotation around the origin (pole) and a radius (radial distance, radial coordinate, r, ρ) from the origin.

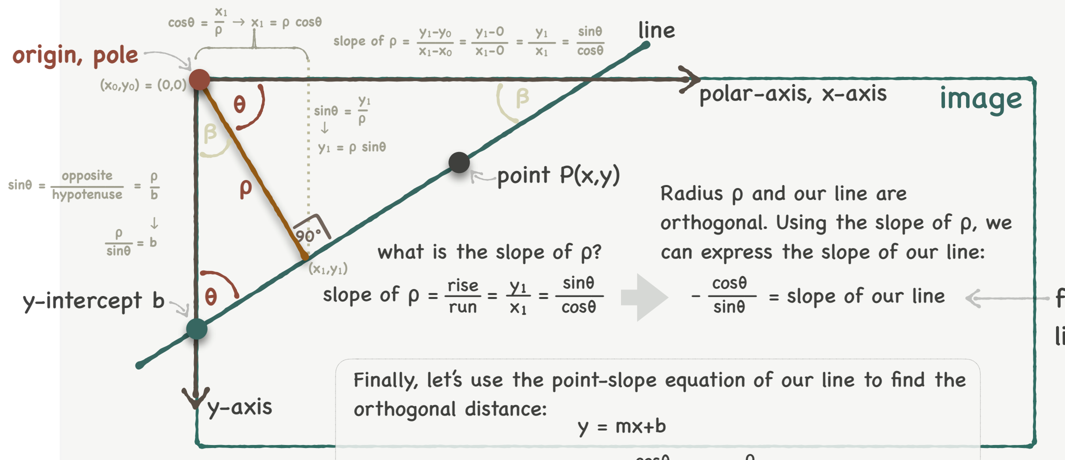
Point P(ρ,θ) in a Polar coordinate system:



Line in Hough space.



In order to represent a line in Hough Space, we need two parameters, i.e., the orthogonal ⊥distance from the origin to the line that intersects via point P, and the angle θ of that line with respect to x-axis (here our angle θ will correspond to polar angle θ). Let's use basic trigonometry and both coordinate systems to derive the ⊥distance (note that the ⊥distance will correspond to radius ρ).



fact: the slopes of two orthogonal lines are opposite reciprocals:

Finally, let's use the point-slope equation of our line to find the orthogonal distance:

$$y = mx+b$$

$$= -\frac{\cos\theta}{\sin\theta}x + \frac{\rho}{\sin\theta}$$

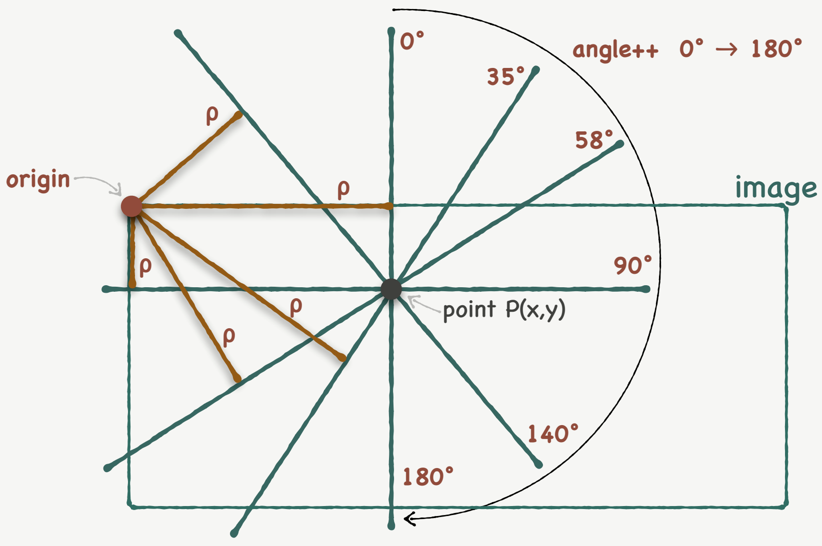
$$-\frac{\rho}{\sin\theta} = -\frac{\cos\theta}{\sin\theta}x - y \qquad | \times(-1)$$

$$\frac{\rho}{\sin\theta} = \frac{\cos\theta}{\sin\theta}x + y \qquad | \times \sin\theta$$

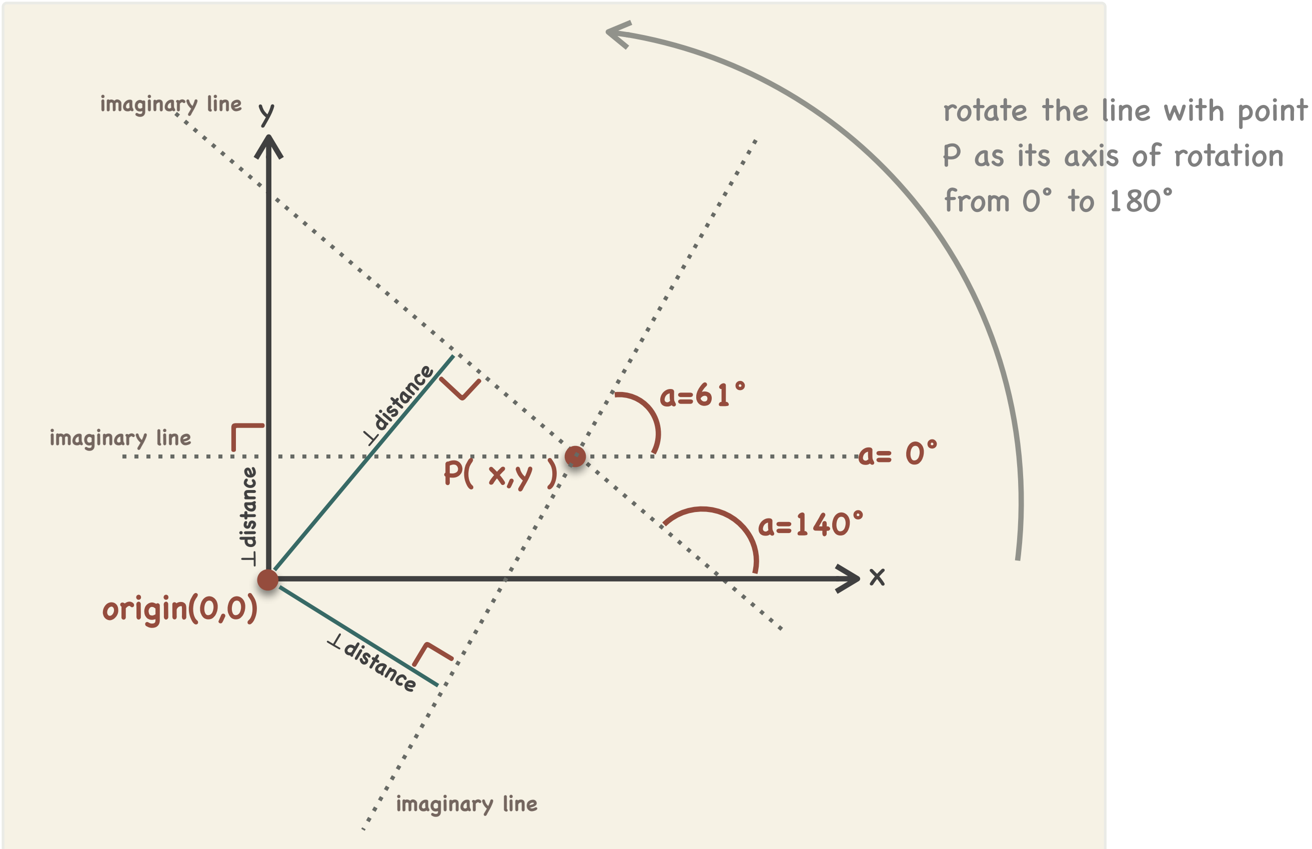
$$\rho = x\cos\theta + y\sin\theta$$

At last, we can express the orthogonal ⊥distance from the origin to any line (at any angle) transecting via point P:

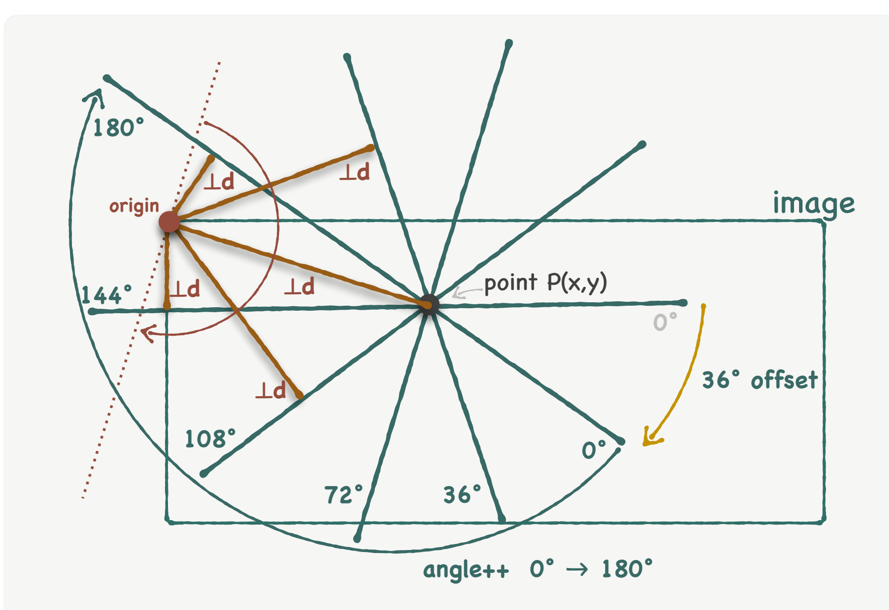
$$\rho = x\cos\theta + y\sin\theta$$



Notes are based on the lectures given by Dr. Tsaiyun Phillips in Spring 2019



translation



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