For a given element, the objective is to fit a law depending on the different integration times t_i

$$\left(\tilde{a}, \tilde{b}\right) = \underset{a,b}{\operatorname{argmin}} \Delta\left(a, b\right) = \sum_{t_i} W_i \left[d_i - \left(a t_i + b\right)\right]^2$$

1 Linear law

Thus, $\tilde{b} = 0$:

$$\begin{aligned}\tilde{a} =& \underset{a}{\operatorname{argmin}} \left[\Delta\left(a, 0\right) = \sum_{t_i} W_i \left(d_i - a t_i\right)^2 = a^2 \sum_{t_i} W_i t_i^2 - 2a \sum_{t_i} W_i t_i d_i + \sum_{t_i} W_i d_i^2\right] \\ =& \frac{\sum_{t_i} W_i t_i d_i}{\sum_{t_i} W_i t_i^2}\end{aligned}$$

2 Affine law

$$\Delta\left(a, b\right) = \sum_{t_i} W_i \left[d_i - a t_i - b\right]^2$$

Its gradient is:

$$\nabla\Delta\left(a, b\right) = -2 \begin{pmatrix} \sum_{t_i} W_i t_i \left(d_i - a t_i - b\right) \\ \sum_{t_i} W_i \left[d_i - a t_i - b\right] \end{pmatrix}$$

And:

$$\nabla\Delta\left(a, b\right) = 0 \Leftrightarrow 0 = \begin{pmatrix} a \sum_{t_i} W_i t_i^2 + b \sum_{t_i} W_i t_i - \sum_{t_i} W_i t_i d_i \\ -a \sum_{t_i} W_i t_i - W_s b + \sum_{t_i} W_i d_i \end{pmatrix}$$

with:

$$W_s = \sum_{t_i} W_i$$

Thus:

$$b = \frac{1}{W_s} \left(\sum_{t_i} W_i d_i - a \sum_{t_i} W_i t_i\right)$$

And :

$$0 = a\sum_{t_i} W_i t_i^2 + \frac{\sum_{t_i} W_i t_i}{W_s}\left(\sum_{t_i} W_i d_i - a\sum_{t_i} W_i t_i\right) - \sum_{t_i} W_i t_i d_i$$

Which gives:

$$a = \frac{1}{\sum_{t_i} W_i t_i^2 - \frac{\left(\sum_{t_i} W_i t_i\right)^2}{W_s}}\left(\sum_{t_i} W_i t_i d_i - \frac{1}{n_t}\left(\sum_{t_i} W_i t_i\right)\left(\sum_{t_i} W_i d_i\right)\right)$$

And:

$$\begin{aligned} b =& \frac{1}{W_s}\left(\sum_{t_i} W_i d_i - \frac{\sum_{t_i} W_i t_i}{\sum_{t_i} W_i t_i^2 - \frac{\left(\sum_{t_i} W_i t_i\right)^2}{W_s}}\left(\sum_{t_i} W_i t_i d_i - \frac{1}{n_t}\left(\sum_{t_i} W_i t_i\right)\left(\sum_{t_i} W_i d_i\right)\right)\right) \\ =& \frac{\sum_{t_i} W_i t_i^2 \sum_{t_i} W_i d_i - \sum_{t_i} W_i t_i \sum_{t_i} W_i t_i d_i}{W_s \sum_{t_i} W_i t_i^2 - \left(\sum_{t_i} W_i t_i\right)^2} \end{aligned}$$

In total:

$$a = \frac{W_s \sum_{t_i} W_i t_i d_i - \sum_{t_i} W_i t_i \sum_{t_i} W_i d_i}{W_s \sum_{t_i} W_i t_i^2 - \left(\sum_{t_i} W_i t_i\right)^2}$$

$$b = \frac{\sum_{t_i} W_i t_i^2 \sum_{t_i} W_i d_i - \sum_{t_i} W_i t_i \sum_{t_i} W_i t_i d_i}{W_s \sum_{t_i} W_i t_i^2 - \left(\sum_{t_i} W_i t_i\right)^2}$$