1 Moffat pattern

Simulation of a gaussian pattern according to a list of parameters $\left\{x_c, y_c, \theta, \alpha_{\parallel}, \alpha_{\perp}, \beta, a, o\right\}$ where $x_c$and y_c are the coordinate of the center of the gaussian pattern, θ its orientation (in degree), $\alpha_{\parallel}$ its elongation along the parralel axis given by θ, $\alpha_{\perp}$ its elongation along the orthogonal axis given by θ, a its amplitude and o its offset:

$$
\begin{aligned}
M\left(x_c, y_c, \theta, \alpha_{\parallel}, \alpha_{\perp}, \beta, a, o\right) &= a.\left\{1+\left[\left[\frac{(x-x_c)\cos\left(\frac{\pi}{180}\theta\right)+(y-y_c)\sin\left(\frac{\pi}{180}\theta\right)}{\alpha_{\parallel}}\right]^2\right.\right.\\
&\quad \left.\left.+\left[\frac{-(x-x_c)\sin\left(\frac{\pi}{180}\theta\right)+(y-y_c)\cos\left(\frac{\pi}{180}\theta\right)}{\alpha_{\perp}}\right]^2\right]\right\}^{-\beta}+o\\
&= a.\left\{1+\left[\left(\frac{\cos^2\left(\frac{\pi}{180}\theta\right)}{\alpha_{\parallel}^2}+\frac{\sin^2\left(\frac{\pi}{180}\theta\right)}{\alpha_{\perp}^2}\right)(x-x_c)^2\right.\right.\\
&\quad +\left(\frac{\sin^2\left(\frac{\pi}{180}\theta\right)}{\alpha_{\parallel}^2}+\frac{\cos^2\left(\frac{\pi}{180}\theta\right)}{\alpha_{\perp}^2}\right)(y-y_c)^2\\
&\quad \left.\left.+2\left(x-x_c\right)\left(y-y_c\right)\cos\left(\frac{\pi}{180}\theta\right)\sin\left(\frac{\pi}{180}\theta\right)\left(\frac{1}{\alpha_{\parallel}^2}-\frac{1}{\alpha_{\perp}^2}\right)\right]\right\}^{-\beta}\\
&\quad +o\\
&= a.\left\{1+\left[\left(\frac{1}{\alpha_{\perp}^2}+\left(\frac{1}{\alpha_{\parallel}^2}-\frac{1}{\alpha_{\perp}^2}\right)\cos^2\left(\frac{\pi}{180}\theta\right)\right)(x-x_c)^2\right.\right.\\
&\quad +\left(\frac{1}{\alpha_{\perp}^2}+\left(\frac{1}{\alpha_{\parallel}^2}-\frac{1}{\alpha_{\perp}^2}\right)\sin^2\left(\frac{\pi}{180}\theta\right)\right)(y-y_c)^2\\
&\quad \left.\left.+\left(x-x_c\right)\left(y-y_c\right)\sin\left(\frac{\pi}{90}\theta\right)\left(\frac{1}{\sigma_{\parallel}^2}-\frac{1}{\sigma_{\perp}^2}\right)\right]\right\}^{-\beta}+o\\
&= a.B\left(x_c, y_c, \theta, \alpha_{\parallel}, \alpha_{\perp}\right)+o
\end{aligned}
$$

with:

$$B\left(x_c, y_c, \theta, \alpha_{\parallel}, \alpha_{\perp}\right) = \left\{1 + \left[\left[\frac{(x - x_c)\cos\left(\frac{\pi}{180}\theta\right) + (y - y_c)\sin\left(\frac{\pi}{180}\theta\right)}{\alpha_{\parallel}}\right]^2 + \left[\frac{-(x - x_c)\sin\left(\frac{\pi}{180}\theta\right) + (y - y_c)\cos\left(\frac{\pi}{180}\theta\right)}{\alpha_{\perp}}\right]^2\right]\right\}^{-\beta}$$

If axisymmetric:

$$M\left(x_c, y_c, \alpha, \beta, a, o\right) = a.\left(1 + \frac{(x - x_c)^2 + (y - y_c)^2}{\alpha^2}\right)^{-\beta} + o$$

and

$$B\left(x_c, y_c, \alpha, \beta\right) = \left(1 + \frac{(x - x_c)^2 + (y - y_c)^2}{\alpha^2}\right)^{-\beta} = A\left(x_c, y_c, \alpha\right)^{-\beta}$$

$$A\left(x_c, y_c, \alpha\right) = 1 + \frac{(x - x_c)^2 + (y - y_c)^2}{\alpha^2}$$

2 Partial derivatives (axisymmetric)

$$\frac{\partial A^{-\beta}}{\partial A} = -a\beta A^{-\beta-1}$$

$$\frac{\partial M}{\partial x_c} = \frac{2a\beta}{\alpha^2}\left(x - x_c\right) A\left(x_c, y_c, \alpha\right)^{-\beta-1}$$

$$\frac{\partial M}{\partial y_c} = \frac{2a\beta}{\alpha^2}\left(y - y_c\right) A\left(x_c, y_c, \alpha\right)^{-\beta-1}$$

$$\frac{\partial M}{\partial \alpha} = 2a\beta\frac{(x - x_c)^2 + (y - y_c)^2}{\alpha^3} A\left(x_c, y_c, \alpha\right)^{-\beta-1}$$

$$\frac{\partial M}{\partial \beta} = \frac{\partial e^{-\beta.\ln A(x_c, y_c, \alpha)}}{\partial \beta} = -a\ln A\left(x_c, y_c, \alpha\right) A\left(x_c, y_c, \alpha\right)^{-\beta}$$

$$\frac{\partial M}{\partial a} = B\left(x_c, y_c, \alpha, \beta\right)$$

$$\frac{\partial M}{\partial o} = \mathbf{1}$$

3 Normalization

$$\begin{aligned}
M\left(x_c, y_c, \alpha, \beta, a, o\right) &= a.\frac{\beta-1}{\pi\alpha^2}\left(1+\frac{\left(x-x_c\right)^2+\left(y-y_c\right)^2}{\alpha^2}\right)^{-\beta}+o \\
&= a.\frac{\beta-1}{\pi\alpha^2}A\left(x_c, y_c, \alpha\right)^{-\beta}+o \\
&= a.\frac{\beta-1}{\pi\alpha^2}B\left(x_c, y_c, \alpha, \beta\right)+o
\end{aligned}$$

and

$$B\left(x_c, y_c, \alpha, \beta\right) = \left(1+\frac{\left(x-x_c\right)^2+\left(y-y_c\right)^2}{\alpha^2}\right)^{-\beta} = A\left(x_c, y_c, \alpha\right)^{-\beta}$$

$$A\left(x_c, y_c, \alpha\right) = 1+\frac{\left(x-x_c\right)^2+\left(y-y_c\right)^2}{\alpha^2}$$

and then

$$\frac{\partial M}{\partial x_c} = \frac{2a\beta}{\alpha^2}\frac{\beta-1}{\pi\alpha^2}\left(x-x_c\right)A\left(x_c, y_c, \alpha\right)^{-\beta-1}$$

$$\frac{\partial M}{\partial y_c} = \frac{2a\beta}{\alpha^2}\frac{\beta-1}{\pi\alpha^2}\left(y-y_c\right)A\left(x_c, y_c, \alpha\right)^{-\beta-1}$$

$$\frac{\partial M}{\partial \alpha} = \frac{2a\left(\beta-1\right)}{\pi\alpha^3}\left(\frac{\beta}{\alpha^2}\left(\left(x-x_c\right)^2+\left(y-y_c\right)^2\right)-A\left(x_c, y_c, \alpha\right)\right)A\left(x_c, y_c, \alpha\right)^{-\beta-1}$$

$$\frac{\partial M}{\partial \beta} = \frac{\partial e^{-\beta.\ln A\left(x_c, y_c, \alpha\right)}}{\partial \beta} = \frac{a}{\pi\alpha^2}.\left(1-\left(\beta-1\right)\ln A\left(x_c, y_c, \alpha\right)\right)A\left(x_c, y_c, \alpha\right)^{-\beta}$$

$$\frac{\partial M}{\partial a} = \frac{\beta-1}{\pi\alpha^2}B\left(x_c, y_c, \alpha, \beta\right)$$

$$\frac{\partial M}{\partial o} = \mathbf{1}$$