

Dynamic elastic anisotropy and nonlinearity in wood and rock

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Abstract

Ultrasonic techniques are used to characterize the anisotropic and nonlinear elastic behavior of wood and rock and to interrogate the structural properties of these materials. For elastic anisotropy two types of experiments are performed, namely qP-wave velocity measurements on spherical samples in about 100 directions of propagation rather regularly sampled in space, together with S-wave birefringence measurements in a few directions on additional samples. For nonlinear elasticity, acoustoelastic experiments were conducted, consisting in P and S-wave velocity measurements under controlled confining pressure. The experimental results show that wood exhibits much larger elastic anisotropy, but much weaker elastic nonlinearity than rock. For instance, the deviations from isotropy in wood can reach 70%, whereas in rock, typically, it can hardly exceed 20%. In contrast, regarding nonlinearity the increase of P or S-wave moduli per unit confining pressure in wood is always smaller than 30, in the radial direction, or 10, in the longitudinal and transversal directions, whereas it can reach roughly one to a few hundreds in rock.

These contrasted behaviors can be simply explained by structural considerations. Thus, the exceptionally strong elastic anisotropy of wood is due to the strict structural alignment of its constituents, that is to say to the preferential orientation of the anatomical elements (tracheids, fibers, ray cells, vessels etc.) for 'textural' anisotropy, and to the cellular wall organisation for 'microstructural' anisotropy. In comparison, rock only exhibits a rough statistics of the orientation distribution function of its constituents, mainly the grain minerals, the pores and the cements. In contrast, the strikingly strong nonlinear elastic response of rock, a well-established classical observation, is due to the presence of compliant mechanical defects (cracks, microfractures, grain-joints etc.). Such features are practically nonexistent in wood which explains its weak nonlinear response. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The great division in science has always been that between the mineral materials and the vegetal materials. The majority of scientists are committed to one tradition or the other. The division arises because the properties and uses of mineral materials like rocks and vegetal materials such as timber can strongly differ. Timber is the product of a living organism, the tree, whereas most of the constituents of rocks are mineral grains or cement such as quartz, calcite or dolomite, at least in sedimentary rocks.

Rocks and wood were the building materials of almost every society for over 5000 years, from Bronze Age until the middle of the nineteenth century. Today, these are

becoming increasingly replaced by cheaper man-made materials, such as concrete and composites. Rocks and wood are porous media belonging to a broad class of disordered materials that are still important in many industrial processes, such as making rock and wood based composites, paper, etc.

However, we are inclined to think that modern material science and engineering, which are 'interdisciplinary', may shed light on the nature of the strength of materials. If we consider the mechanical properties of solids it becomes clear that, while we have some idea of 'how' materials behave, we have do not have a clear idea of 'why' the behavior is so different.

Because of environmental conditions, quite often the properties of natural composites such as rocks and wood may suffer substantial change related to their structure. This structure is the element that synergizes the properties of the constituents into a unique material. Therefore

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the task of characterization incorporates both material and structural features.

Different physical methods were developed for material characterization purposes. Between these, the ultrasonic technique provides the means to interrogate both the material and structural properties of the composite examined, in our case, rock and wood.

The aim of this paper is the characterization of rock and wood using the ultrasonic technique, with special emphasis on nonlinear elasticity and anisotropy. In Section 2 Section 3 we introduce elastic anisotropy and nonlinearity in wood and rock, firstly from a general point of view, and then from an experimental point of view in the laboratory. In Section 4 we discuss the experimental results and in Section 5 we give our conclusions and perspectives.

2. Elastic anisotropy

2.1. Anisotropy and heterogeneity

A material is elastically anisotropic if it is spatially ordered at a scale E smaller than the elastic wavelength λ (e.g. Ref. [1]). If λ is comparable or smaller than E , the elastic wave 'sees' an heterogeneous medium. Thus, anisotropy and heterogeneity are not absolute characteristics of a material, but are relative to a given physical property and to a scale length of the corresponding physical phenomenon (for instance, the wavelength λ for wave propagation). In other words, the anisotropic elastic behavior of a medium must be associated to a scale of observation. Both wood and rock have hierarchical heterogeneous structures, as shown by Figs. 1 and 2.

Wood is a natural composite and has an hierarchical structure from molecular scale to the macroscopic scale (Fig. 1). The principal constituents of wood are cellulose (50%), hemicellulose (35%), lignin (25%) and extractives [2]. The cellulosic material has a basic component, the cellulosic crystal. Wood material produced by the tree is a marvellous example of natural composite that has a basic crystal component. The hierarchical architecture of wood is mainly responsible for its high anisotropic elastic behavior, as will be discussed later.

Like wood, rock also exhibits different scales of heterogeneity. If only sedimentary rocks are considered, there are basically four scales of heterogeneity (see Fig. 2) [3]. The smallest scale is the microscopic scale, namely the scale of the grains and the pores. The typical size is <1 cm. The next scale in Fig. 2 is called the mesoscopic scale. It is the scale of petrophysical measurements on samples in the laboratory and its typical dimensions are >1 cm, but <1 m. Above the mesoscopic scale is the macroscopic scale, namely the scale of the 'elements' typical of interwell distances. It is basically the scale of the 'blocks' used in the numerical simulations of oil reservoir performance, for instance, and is typically >1 m, but <1 km. Finally the largest scale is the megascopic scale which considers elements of fieldwide character which are typically >1 km. Note, firstly, that the name of these scales are not standards; for instance, some authors use gigascopic for megascopic, and megascopic for macroscopic [4]. Secondly, note that the scales are much less 'intrinsic' in rocks than in wood since the choice of these scales is not independent of the purpose of the description. For instance, the scale description for rock is rather subjective, since it is clearly oriented towards the description of oil reservoir rocks.

From here onwards, we shall only consider the centi-

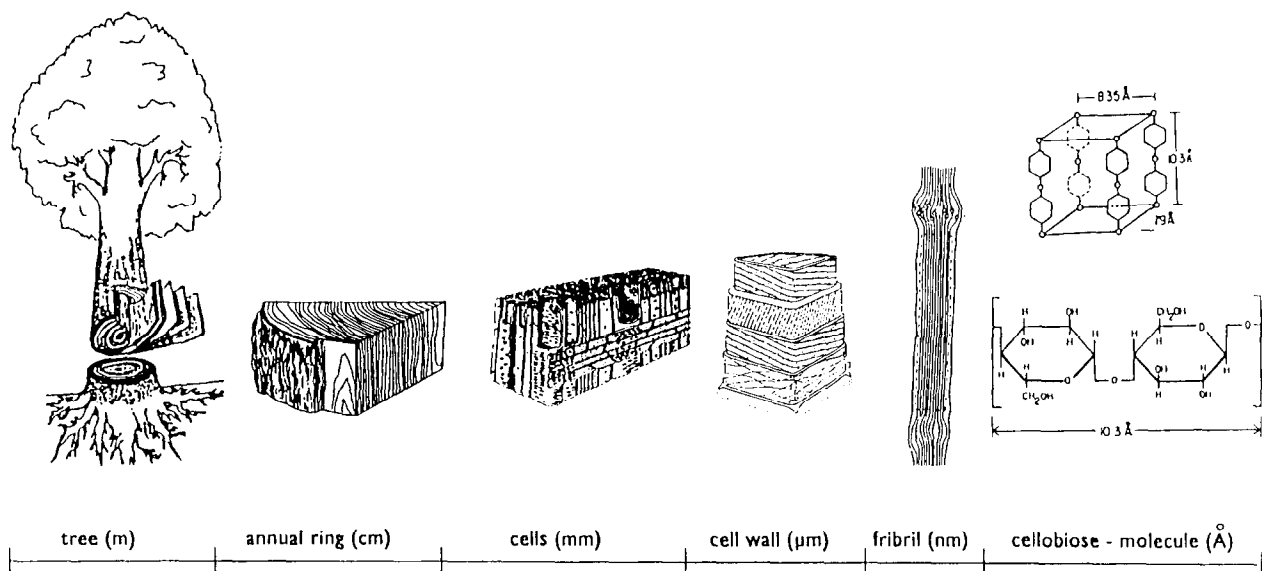


Fig. 1. Hierarchical structure of wood [23].

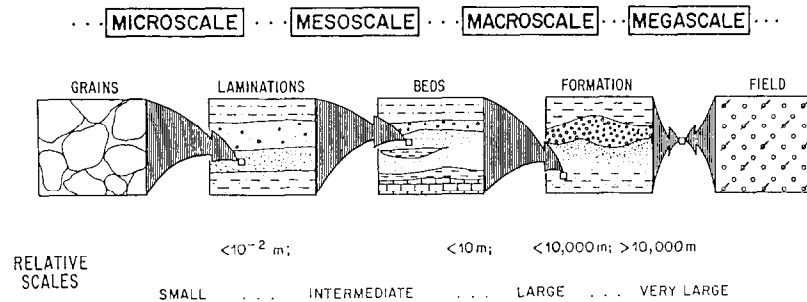


Fig. 2. Heterogeneity scales in typical sedimentary rock [3].

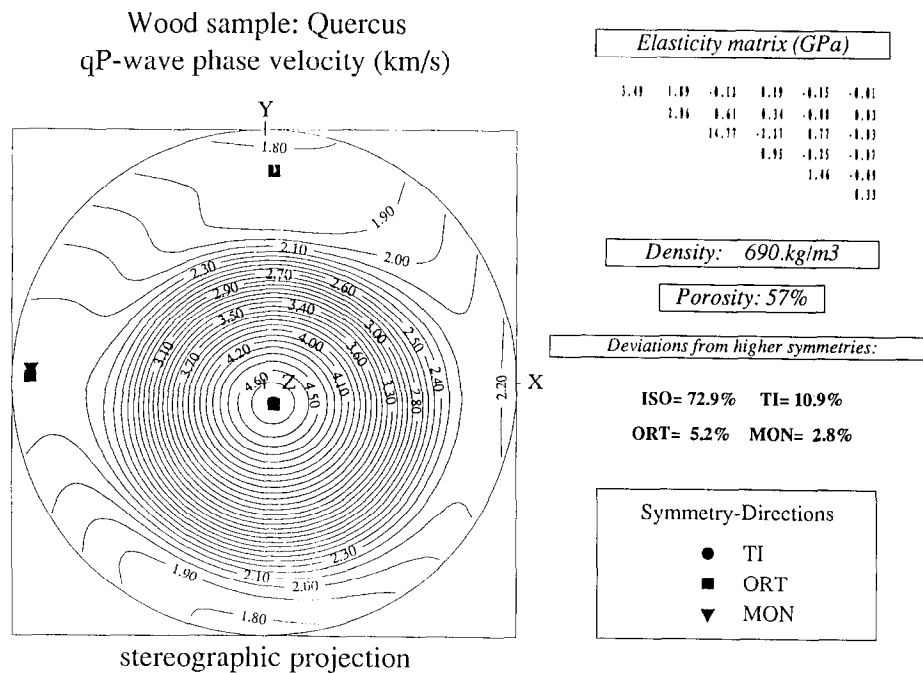


Fig. 3. Left: isolines of the qP-wave velocity surface of wood (quercus) in stereographic projection including the locations of symmetry elements in the sample. Right: complete elasticity matrix, density, porosity and deviations from high symmetries (ISO – isotropic, TI – transversely isotropic, ORT – orthotropic and MON – monoclinic).

meter scale of observation, that is to say the mesoscopic (or petrophysical) scale of Fig. 2 in rocks, or the scale of the rings on Fig. 1 in wood. At this scale, when studied in more detail, it is possible to distinguish two different ‘subscales’ for ultrasonic anisotropy, namely: For rocks

- the textural anisotropy at the scale of the grains and the pores, induced by the preferential direction of the cementation between grains during diagenesis; and
- the microstructural anisotropy at the millimetric scale, related to different components.

For wood

- the textural anisotropy at the scale of ‘fibers’ induced by the preferential orientation of the anatomical elements (tracheids, fibers, ray cells, vessels, etc.), during the life of the tree; and
- the microstructural anisotropy related to the cellular wall organisation.

2.2. Measurement of ultrasonic anisotropy

The immersion ultrasonic technique, as described by Arts [5], was used to measure the velocity of propagation of P waves, on spherical specimens (5 cm diameter) of rock and wood in a water filled tank. The wood specimens were selected in the peripheral pan of the tree, to avoid ring curvature. In this case, the radius of curvature of the rings is very large compared with the ultrasonic wavelength. This experimental approach is also important in order to avoid description of the behavior of wood in a curvilinear coordinate system [6, 7]. For rocks, the homogeneity of the samples was carefully checked by X-ray tomodensitometry. The spherical specimens were all dry and covered with a very thin resin layer to protect against water penetration.

The ultrasonic equipment used was an ultrasonic generator Panametrics 5055 PR and standard broadband

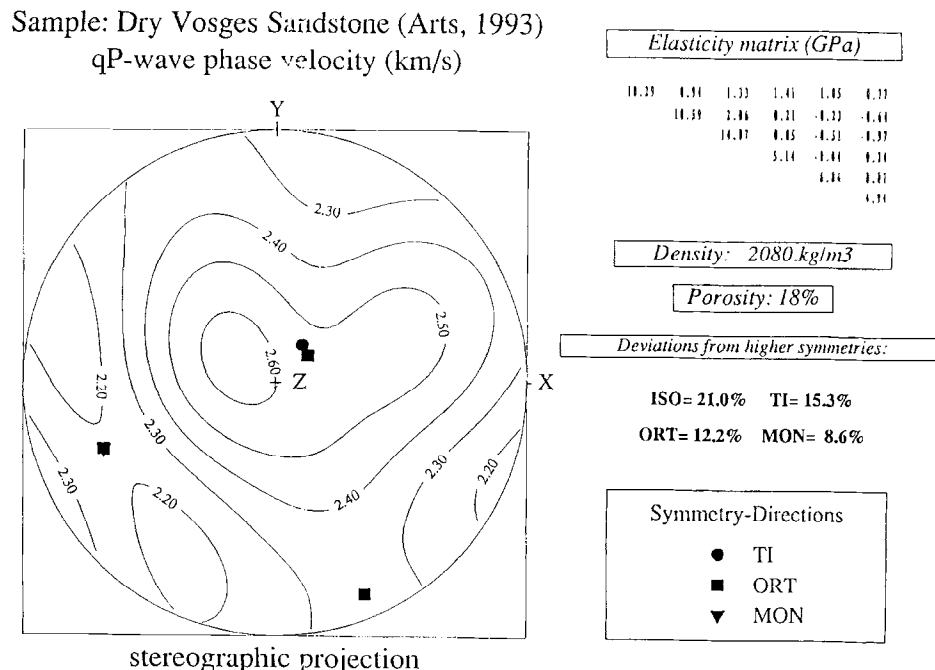


Fig. 4. Same as Fig. 3 for rock (Vosges sandstone).

piezoelectric Panametrics transducers, 25.4 mm diameter, with a central frequency of 0.5 MHz. All signals were visualized on a digital oscilloscope and recorded for further processing and inversion on a microcomputer.

Typically, about 100 directions of propagation were ausculted for velocity measurements. These were inverted in terms of the complete set of 21 components of the linear elastic stiffness tensor using the algorithm proposed by Arts et al. [8].

These stiffness tensors were also used to characterize the elastic anisotropy of the material in a more global way by identifying and orientating the best symmetry elements (axes, planes) and by quantifying the deviations from higher symmetries (isotropy, transverse isotropy, orthotropy, etc.). For this, we use the technique proposed by Arts et al. [9]. After such a procedure, a statement such as “the medium (quercus) deviates from isotropic symmetry by 73%, and from transverse isotropy by 11% with a best infinite-fold axis making an angle of less than 10° with the Z-axis” is meaningful (see Fig. 3 and the corresponding comments in the next section).

S-wave velocity measurements were also performed in the coordinate planes of additional samples of the same materials. Shear wave birefringence measurements at 500 kHz were performed using the ultrasonic pulse transmission technique on cubic wood specimens of 6 cm and on cylindric rock specimens, of 6 cm diameter and 6 cm length. This technique was largely described by Lucet and Zinszner [10].

2.3. Experimental results

The experimental results are illustrated in Figs. 3–6 and Tables 1–3.

Figs. 3 and 4 are of the same type and correspond to a wood (quercus) and rock sample (Vosges sandstone), respectively. In the left part of these figures are plotted the isolines of the qP-wave phase velocity surface in stereographic projection. On the right part of these figures are given, from the top to the bottom, the inverted elastic matrix, the independently measured density and porosity, and the quantified deviations (%) of the linear elastic tensor from isotropic (ISO), transversely isotropic (TI), orthotropic (ORT) and monoclinic (MON) symmetries using the definitions of Arts et al. [9]. Note also that we have plotted on the stereogram the direction of the best infinite-fold axis (●), the direction normal to the best symmetry plane (▲) and the directions normal to the three mutually perpendicular best planes of orthotropy (■). As the incremental value of the velocity between the successive isolines is kept constant and equal to 0.1 km on both figures, we clearly see that the wood sample exhibits much stronger elastic anisotropy than the rock sample. This is confirmed by the larger deviation from isotropy in the former (ISO=72.9%) than in the latter (ISO=21%). We also see that the symmetry level exhibited by the velocity surface of quercus is higher than that exhibited by the velocity surface of the sandstone. More precisely, although the sandstone deviates much less

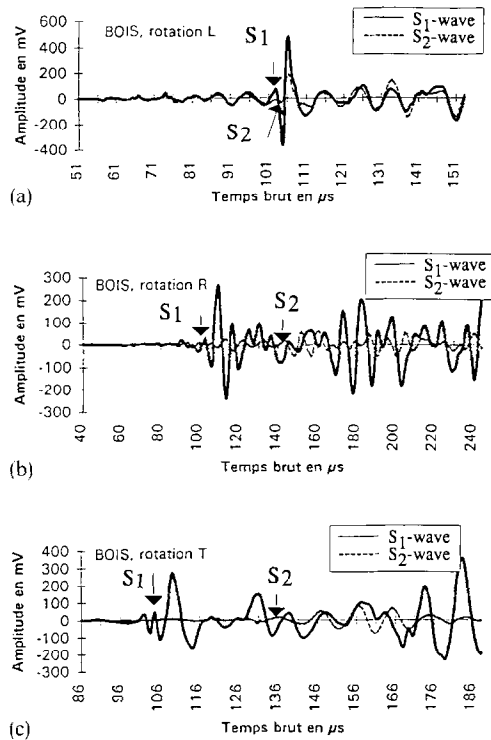


Fig. 5. Experimental evidence of shear-wave birefringence in wood (pine). Recorded ultrasonic signals corresponding to the fast shear-wave S_1 (solid line) and to the slow shear-wave S_2 (broken line). Propagation in: (a) the longitudinal L; (b) the radial R; and (c) the tangential T directions.

from isotropic symmetry than quercus, the former deviates more from transverse isotropy ($TI=15.3\%$), from orthotropy ($ORT=12.2\%$) and from monoclinic symmetry ($MON=8.6\%$) than the latter ($TI=10\%$, $ORT=5.2\%$ and $MON=2.8\%$). In other words, quercus exhibits much stronger anisotropy, or more precisely deviates much more from isotropy, than the sandstone, but the symmetry level of the wood sample is higher than that of the rock sample. This is a general result, as illustrated by Table 1, where we have reported the deviations from higher symmetries in different samples of wood (pine, oak and sapelli) and rock (marble, Vosges sandstone and andesite). We see that all the wood samples exhibit much larger deviations from isotropy than the rock samples, but have comparable, and sometimes smaller (e.g. oak sample), deviations from transverse isotropy, orthotropy or monoclinic symmetry. It is worth noting from Figs. 3 and 4 the great differences in density (a ratio of about 3) and porosity (a ratio of about 3) between wood and rock.

Figs. 5 and 6 illustrate S-wave birefringence measurements in wood (pine) and rocks (marble, Vosges sandstone and andesite). Fig. 5(a)–(c) in pine represent the recorded ultrasonic signals corresponding to the fast and slow shear waves (superimposed on the same figure) propagating in the longitudinal (L), radial (R) and

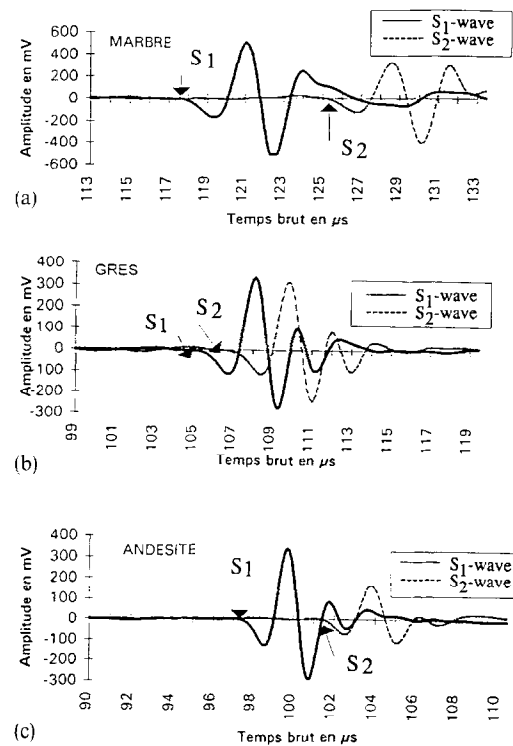


Fig. 6. Same as Fig. 5 for different rock samples, namely: (a) marble; (b) sandstone; and (c) andesite. The directions of propagation are roughly the directions of maximum shear-wave birefringence.

tangential (T) directions, respectively. We clearly see that the longitudinal direction (L) exhibits very weak S-birefringence and the strongest birefringence is exhibited by the transverse direction (T) for which the S_1 -wave and the S_2 -wave are separated by roughly two periods. Note the great contrast between the strong amplitude of the quickest S-wave and the small amplitude of the slowest S-wave, which makes this latter very difficult to extract from experimental noise.

Fig. 6(a)–(c) are the same types of signals in rock samples, namely marble, Vosges sandstone and andesite, respectively. Here, the direction of propagation is roughly the direction of maximum shear-wave birefringence. Note that the time lag between the two S-waves is much smaller in rocks than in wood (values are comparable, since the sample lengths are identical) and never exceeds 1.5 period (in marble, which is rather exceptional). Note also that the amplitudes of the two S-waves are less contrasted.

Tables 2 and 3 summarize P-wave anisotropy and S-wave birefringence measurements, respectively, in different samples of wood (pine, oak and sapelli) and rock (marble, Vosges sandstone and andesite). In these tables, and from now on, V_{ij} designates the velocity of a wave propagating in the i direction and polarized along the j direction. In Table 2, the first three rows give the P-velocities along the coordinate axes, and the three remaining rows give the P-wave anisotropy coefficients

Table 1

Deviations from high symmetries (ISO—*isotropy*, TI—*transverse isotropy*, ORT—*orthotropy*, MON—*monoclinic*) of the stiffness tensor in rock and wood

Sample		Deviations (%) from high symmetries of the stiffness tensor			
		ISO	TI	ORT	MON
Rock	Marble	12.7	4.1	2.6	1.2
	Sandstone	21.0	15.3	12.2	8.6
	Andesite	10.6	6.0	4.1	2.7
Wood	Pine	74.2	19.5	14.8	8.0
	Oak	72.9	10.9	5.2	2.8
	Sapelli	69.5	23.3	13.4	6.4

Table 2

P-wave anisotropy coefficients of wood and rock at atmospheric pressure

	Wood			Rock		
	Pine	Oak	Sapelli	Marble	Sandstone	Andesite
P-wave velocity (m/s)						
V_{xx}	2434	2033	2305	4834	2536	3633
V_{yy}	2009	1942	1862	4664	2389	3607
V_{zz}	5214	4699	4406	5242	2479	3212
P-wave anisotropy coefficients ^a						
XY-plane	0.17	0.04	0.19	0.035	0.062	0.007
XZ-plane	0.53	0.57	0.48	0.077	−0.023	−0.131
YZ-plane	0.62	0.59	0.58	0.110	0.036	−0.123

^aThe P-wave anisotropy coefficient in the (*ij*) plane is defined by: $A_p = (V_{ii} - V_{jj})/V_{ii}$.

Table 3

S-wave birefringence in wood and rock at atmospheric pressure

	Pine	Oak	Sapelli	Marble	Sandstone	Andesite
Velocities (m/s)						
V_{xy}	650	647	812	2135	1534	1829
V_{yx}	685	782	827	1835	1459	1672
V_{zx}	1326	1559	1377	—	1703	—
V_{xz}	1355	1482	1345	2843	1710	2083
V_{zy}	1199	1252	1200	—	1570	—
V_{yz}	1118	1250	1225	—	1585	—
Birefringence coefficients ^a						
Direction						
X radial	0.52	0.56	0.40	0.25	0.10	0.12
Y tangential	0.39	0.37	0.32	—	0.08	—
Z longitudinal	0.09	0.20	0.13	—	0.08	—

^aThe shear-wave birefringence coefficient is defined by $B_s = (V_{s1} - V_{s2})/V_{s1}$, where V_{s1} and V_{s2} are the velocities of the fast and slow shear wave, respectively, in the considered direction of propagation.

A_{ij} , defined by:

$$A_{ij} = \frac{V_{ii} - V_{jj}}{V_{ii}}. \quad (1)$$

In Table 2, the first six rows give the velocities of the S-waves propagating along the coordinate axes, and the three remaining rows give the S-wave birefringence B_s

along the coordinate axes, B_s being defined by:

$$B_s = \frac{V_{s1} - V_{s2}}{V_{s1}}, \quad (2)$$

where V_{s1} and V_{s2} denote the velocities of the fast and the slow shear waves, respectively, in the considered direction.

3. Nonlinear elasticity

3.1. Nonlinear acoustics and acoustoelasticity

The fundamentals of nonlinear elastodynamics are developed in many reference textbooks (e.g. Refs. [11,12]). Linear elasticity is characterized by 21 coefficients in the most general symmetry type and by only two constants (e.g. the bulk modulus K and the shear modulus μ) in isotropic media. Nonlinear elasticity (at least up to the third order in energy) is characterized by 56 elastic coefficients in triclinic media and by only three (for instance, Murnaghan's coefficients l , m and n , or Landau's coefficients A , B and C ; see Ref. [12]) in isotropic media [13]. Regarding wave propagation in nonlinear elastic media, two cases are generally considered in the literature.

The first case is when the waves are not perturbative, but have finite amplitude. This is the field of nonlinear acoustics for which the elastic velocities depend on the strain level (e.g. Refs. [14,15]). For a P-wave propagating in a one-dimensional isotropic medium,

$$V_p(\epsilon) \approx V_p(\epsilon=0)[1 + \beta\epsilon], \quad (3)$$

where $V_p(\epsilon=0)$ and $V_p(\epsilon)$ designate the linear and the nonlinear velocities, respectively. The nonlinear acoustic coefficient β is given by:

$$\beta = \frac{3}{2} + \frac{l+2m}{K+\frac{4}{3}\mu}, \quad (4)$$

where l and m are two of the previously introduced Murnaghan nonlinear coefficients. Note that the magnitude of the nonlinear response is quantified not by the absolute value of the nonlinear coefficients (i.e. l and m), but by the ratio between these coefficients and the linear elastic coefficients (K and μ).

The second case usually considered in the literature is when a small wave perturbation is superimposed on a static pre-deformation due to the presence of a static prestress (e.g. Ref. [16]). It is the field of acoustoelasticity which is the acoustical analog of photoelasticity in optics. The analytical expressions of the variations of the wave moduli of the three bulk waves in anisotropic media of arbitrary symmetry under uniaxial or hydrostatic states of stress were derived by Thurston and Brugger [17]. In the particular case of isotropic media, Hughes and Kelly [18] give the following expressions for the derivatives of the P and the S-waves moduli with respect to the hydrostatic pressure p :

$$\frac{\partial M_p}{\partial p} = -\frac{18l+12m+21K+16\mu}{9K} \quad (5)$$

and

$$\frac{\partial M_s}{\partial p} = -\frac{6m-n+6K+8\mu}{6K}, \quad (6)$$

where M_p and M_s denote the 'natural' P-wave and S-wave moduli. More precisely, these moduli are equal to the zero-pressure density of the material multiplied by the square of the natural velocity. The natural velocity is defined by the length of the 'acoustical path' in the unstressed state divided by the wave traveltime in the stressed state. Note that here, as for nonlinear acoustics, the magnitude of the nonlinear response is quantified not by the absolute value of the nonlinear coefficients, but by the ratio between these coefficients and the linear elastic coefficients which, like β , is a dimensionless quantity. Such equations were used by Johnson and Rasolofosaon [19] to invert the nonlinear elastic coefficients of various types of rocks and to relate them with the stress-induced P-wave anisotropy and S-wave birefringence. We are not aware of similar work in wood. In a certain way, this work will try to partially fill this gap.

3.2. Measurement of the nonlinear response

We used acoustoelastic experiments to quantify the nonlinear response of the samples. Cylindrical specimens of 6 cm diameter were used for ultrasonic experiments. In order to perform measurements under confining pressure the samples were jacketed using impermeable heat shrink tubing. Confining pressure was applied by helium gas. Ultrasonic velocity of P and S wave (central frequency of the transducers was 500 kHz) under confining pressure was measured from 1 to 19 MPa by the pulse transmission technique. The experimental procedure is described in detail in Ref. [20].

The density of all the samples were measured independently, and, as described in the previous section, the slope of the curve natural wave moduli versus hydrostatic pressure were computed to quantify the magnitude of the nonlinear response.

3.3. Experimental results

The experimental results on nonlinear properties are illustrated by Figs. 7 and 8 and Table 4. We have plotted in Figs. 7 and 8 the confining pressure dependence of the P and S-waves, respectively, in wood (cherry) and in two rock samples (Vosges sandstone and andesite). Fig. 7(a) corresponds to the velocities of the P-wave propagating along the three principal directions, namely R for the radial or X-axis direction, T for the tangential or Y-axis and L for the longitudinal or Z-axis direction. Fig. 7(b) corresponds to some velocities (namely V_{LR} , V_{RL} and V_{TL}) of S-wave propagating along the three

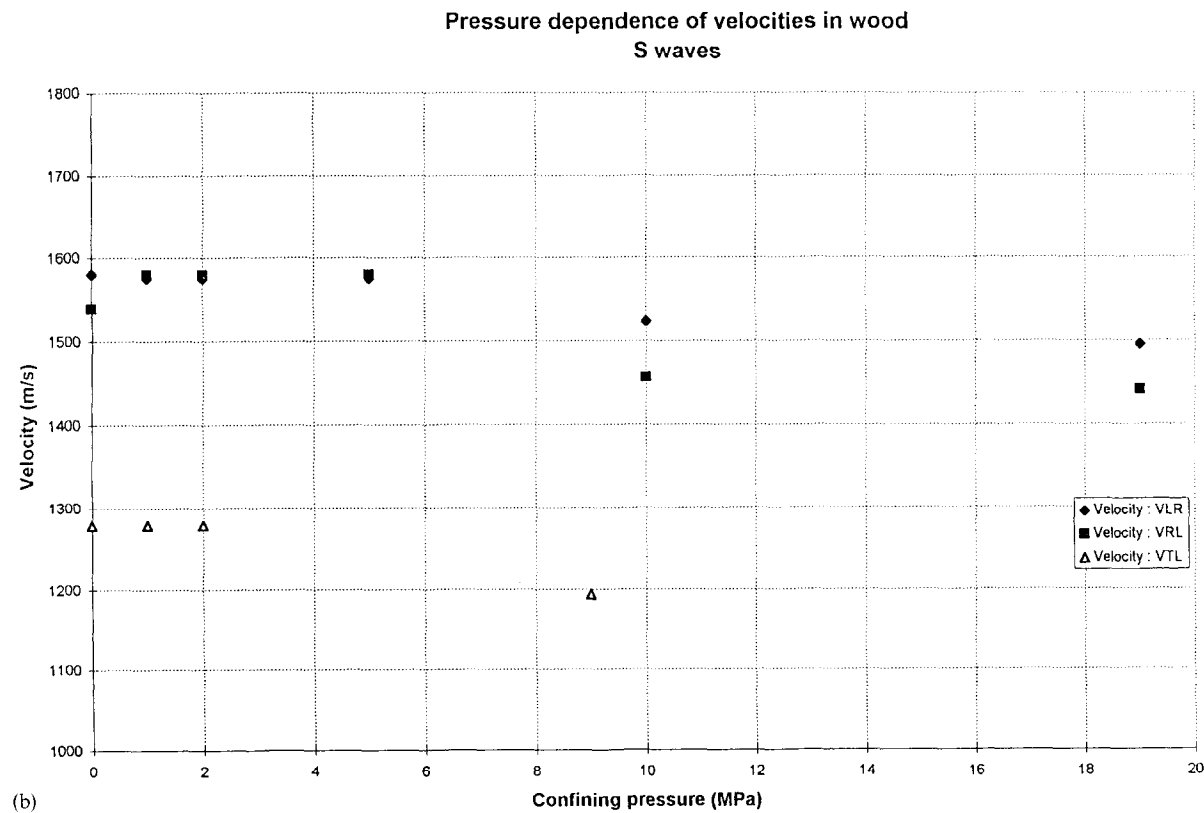
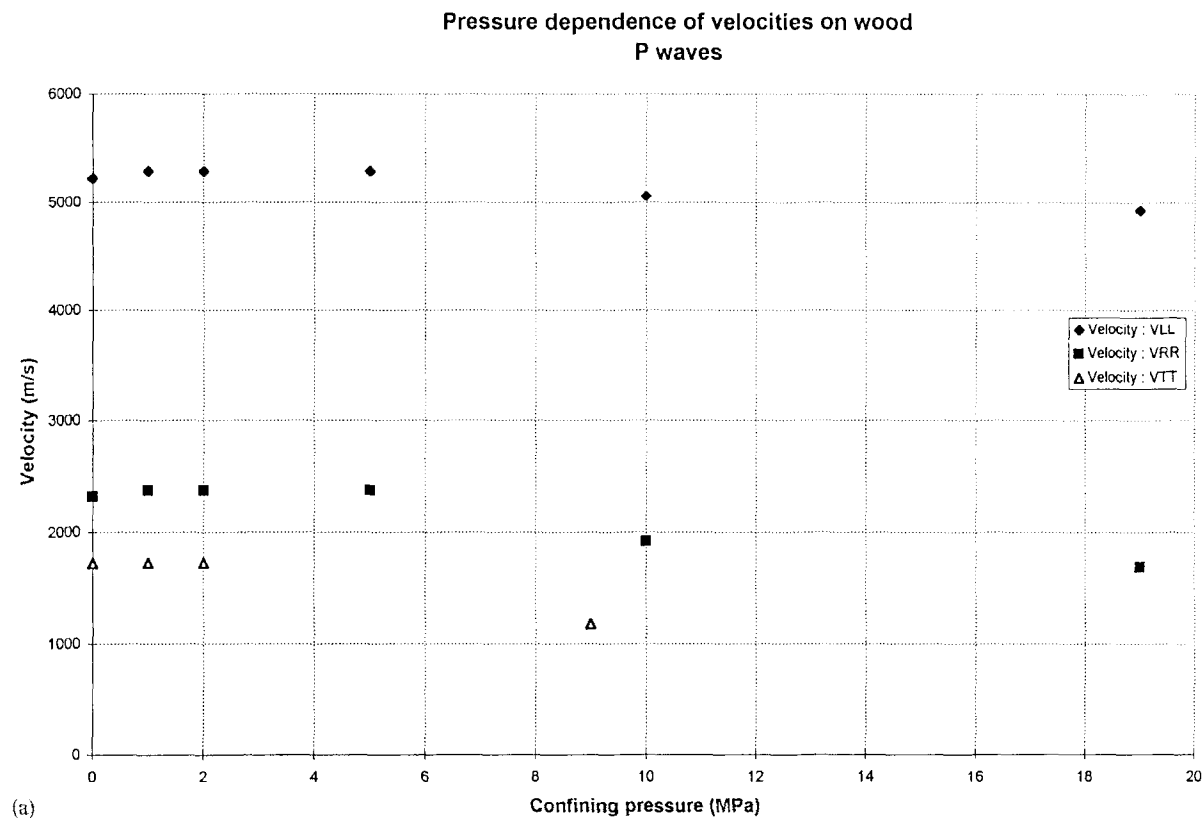


Fig. 7. Confining pressure-dependence of: (a) P-wave; and (b) S-wave ultrasonic velocities in wood (cherry tree).

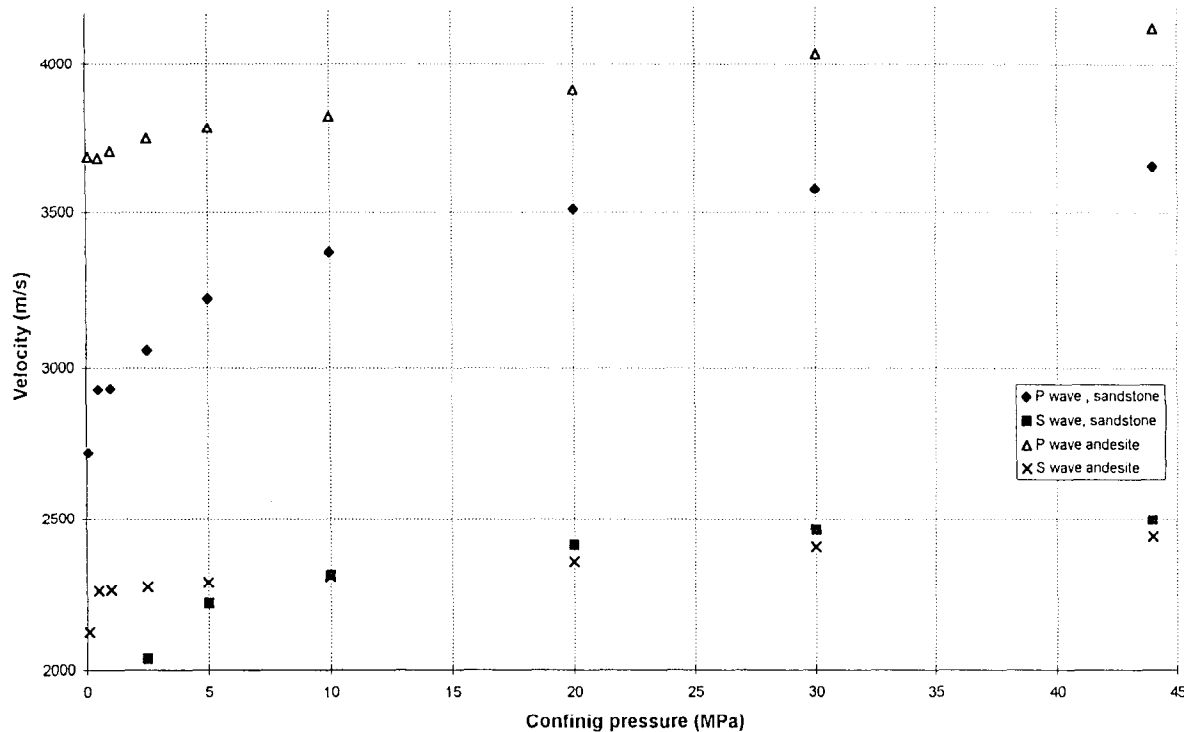


Fig. 8. Confining pressure-dependence of P and S-waves ultrasonic velocities in wood samples of: (a) Vosges sandstone; and (b) andesite.

Table 4
Nonlinear elastic coefficients of wood and rock from acoustoelastic experiments under confining pressure

Specimen	Density (kg/m ³)	Nonlinear coefficients	
		P wave	S wave
Wood cylinder ^a L	600	8.70	1.9
Wood cylinder ^a R	600	29.9	14.9
Wood cylinder ^a T	600	2.0	0.3
Marble	2850	436.7	263.6
Sandstone	2067	280.9	85.7
Andesite	2190	169.7	71.4

The nonlinearity coefficient is: $\rho(V_{\text{final}}^2 - V_{\text{atmospheric}}^2)/\Delta P$ where V_{final} is the velocity at the confining pressure $\Delta P = 5$ MPa in wood and $\Delta P = 40$ MPa in rock.

^aCylindrical specimens of cherry (*Prunus* spp.).

principal directions L, R and T. Fig. 8(a) and (b) show plots of the P-wave and S-wave velocities as functions of the confining pressure in Vosges sandstone and in andesite, respectively.

We can clearly see from Fig. 7 that the pressure dependence of the velocities in the wood sample is very weak, at least if one does not exceed a critical pressure of about 5 MPa beyond which all the velocities tend to decrease. In rocks (Fig. 8) all the velocities increase with pressure with a larger rate at lower pressure than at high pressure, towards an asymptotic value at high pressure. The zero-pressure slope is highly variable in the different rock types, namely steeper in Vosges sandstone than in andesite. In other words, this means that the nonlinear response of wood is very weak, whereas

rock can exhibit small to very strong nonlinearity. These are very general results found in rocks (e.g. Refs. [21,22]).

Table 4 summarizes values of the average slope of the curves of the natural P- and S-wave moduli, previously designated by M_p and M_s , respectively, versus the confining pressure in the three principal directions of wood (cherry) and in rock samples (marble, sandstone and andesite). These quantities are roughly equal to the quantities $\partial M_p / \partial p$ and $\partial M_s / \partial p$ defined by Eqs. (5) and (6), respectively. The averages are taken over the interval 0–5 MPa for the wood sample and over the interval 0–40 MPa in rocks. As for Figs. 7 and 8 we see that wood always exhibits negligible nonlinearity (slopes smaller than 30) when compared with rocks. Rocks can

exhibit rather weak nonlinearity (slope of roughly 70 in andesite) to a very large nonlinear response (slope of more than 400 in marble).

4. Discussion

Firstly, the experimental results show that wood exhibits exceptionally large anisotropy, but weak nonlinear response. In contrast, rocks presents relatively weak anisotropy but, can exhibit a strikingly strong nonlinear response. We think that the main explanation of these results can be found in the characteristic structural organisation of these materials. Elastic anisotropy of wood is due to its texture, which is highly ordered in longitudinal direction by cellular disposition and in radial direction by the presence of annual rings. In spite of very high porosity, the elastic constants of wood are practically independent of pressure (at least below the threshold pressure of porosity collapse, typically around 5–7 MPa) due to the practical absence of compliant features (e.g. cracks, microfractures, grain joints etc.). In contrast, these mechanical defects are often ubiquitous in rocks which explains the strong nonlinear response of these materials. Weak elastic anisotropy in rock can be explained by the coarse statistical spatial distribution of the orientation of its constituents (i.e. essentially minerals, pores and cement).

Regarding anisotropy, and to input more detail, we note from Table I that the deviations from isotropy (ISO) in wood are much larger than in rock. However, in contrast, we also see that the deviations from transverse isotropy (TI), from orthotropy (OT) or from monoclinic (MON) symmetry are comparable in wood and in rock, and even sometimes smaller in wood. It is worth noting that the direction of the gravity at the time of the genesis of both materials practically corresponds to the ‘best axis of transverse isotropy’. The fact that transversely isotropic symmetry is more marked in wood than in rock is probably due to the fact that the vertical direction is always the direction of maximum stress in wood and, as a consequence, corresponds to the direction of maximum stiffening of wood during cells growth. In a greater detail, the presence of fibers running along the growth axis of the tree, the array of medullary rays running from the heart to the bark and the randomly distributed structure in tangential direction to the annual rings explain the systematic ordering [23]. In contrast, in sedimentary rock, mechanical processes due to gravity forces superimposed with tectonic forces, together with complicate diagenetic physico-chemical processes [24], tend to smooth the orientation distribution functions of the causes of elastic anisotropy (for instance mineral or microfracture alignment), thus resulting in a relatively weak elastic anisotropy in a coarsely ordered material [25, 1]. Note that these consid-

erations are direct consequences of a very general principle, outlined by Curie, regarding the influence of symmetry on physical phenomena [26]. In summary, Curie’s principle states that effects are at least as symmetric as their causes. Or, in other words, and in modern jargon, the symmetry group of the causes (i.e. mainly the gravity field for wood, and the gravity field, the tectonic forces and other diagenetic processes for rock) is a subgroup of the symmetry group of the effects (i.e. the elastic anisotropy of the considered material). Thus, the marked elastic anisotropy of wood is essentially due to the quasi unicity of the cause (namely the gravity field which exhibits a vertical infinite-fold symmetry axis), whereas the relative weak elastic anisotropy in rock is primarily due to the diversity of the causes, which do not necessarily share the same symmetry elements.

Note also that the XYZ directions were chosen in both materials following visual criterions. More precisely, for wood the X, Y and Z-axes are parallel to the radial, the tangential and the longitudinal directions, respectively, which are unambiguously visible. Regarding rocks, the Z-axis is chosen perpendicular to the layering of the sedimentary rocks and most of the time the orientations of the remaining axes are arbitrary. Note that the eigen directions recovered by the ultrasonic techniques are very close to the directions chosen on the basis of visual criterions which, firstly, is a consistent result, and secondly, demonstrates that acoustic methods are valuable for symmetry characterization of materials. For instance, the best axis of transverse isotropy recovered acoustically (see ● in Fig. 4) is practically perpendicular to the layering of the rock, or in other words, nearly parallel to the ‘visual’ Z-axis. Moreover, the X, Y, Z-axes of the wood samples corresponding to the visual radial, tangential and longitudinal directions, respectively, are all well-recovered acoustically and essentially corresponds to the best axes of orthotropy (■ in Fig. 3).

In regard to porosity and stiffness, we note that, in spite of their very large porosities (as a consequence of low densities), the elastic moduli of wood, at least in longitudinal direction, is comparable to that of rock. With wood, nature has succeeded in building a remarkable corpus with a stiff frame, but a minimum of material. Some other natural materials, such as bone, share such mechanical quality. Wood and bone both have large stiffness lines along the lines of maximum stress solicitation, that is to say in the vertical direction in wood and femoral bones, for instance.

Regarding nonlinearity, the elastic domain in wood is limited to confining pressure smaller than roughly 5 MPa. Note that in rocks the limit of elasticity can reach a few 100 MPa in confining pressure. Hitherto, experimental results on nonlinear properties of wood (in the elastic domain) has not been published before.

5. Conclusions

In this paper, we have used ultrasonic techniques to characterize the anisotropic and nonlinear elastic behavior of wood and rock and to interrogate the structural properties of these materials.

We have demonstrated that wood exhibits exceptionally large anisotropy, but weak nonlinear response. In contrast, rock presents relatively weak anisotropy, but can exhibit a strikingly strong nonlinear response. The main explanation of this behavior is related to the characteristic structural organisation of these materials. The elastic anisotropy of wood is due to its texture which is highly ordered in longitudinal direction by cellular disposition and in radial direction by the presence of annual rings. In spite of very high porosity, the elastic constants of wood are virtually independent of pressure (at least below the threshold pressure of collapsing of the porous network typically around 5–7 MPa) due to the practical absence of compliant features (e.g. cracks, microfractures, grain joints etc.). In contrast, these mechanical defects are often ubiquitous in rocks, which explains the strong nonlinear response of these materials. Weak elastic anisotropy in rock can be explained by the coarse statistical spatial distribution of the orientation of its constituents (i.e. essentially minerals, pores and cement).

Elastic anisotropy has already been well studied in both materials (e.g. Bucur [23] for wood, and Bourbié et al. [27] and Arts [5] for rock). In contrast, nonlinear elastic properties are only starting to be extensively studied in rocks (e.g. Refs. [19,21]) and is still ignored in wood, at least if one refers to the literature. Thus, we encourage future experiments, above all, on nonlinear elasticity, and on anisotropy, in order to check the generality of our results.

The nonlinear and/or anisotropic response of wood and rock, either as a man-made material or in their natural configuration, may be used for characterization purposes. In addition, characterization of material property change by monitoring nonlinear and/or anisotropic response may be of value. For instance, these changes include variations in saturation (drying or fluid invasion), change in response to variations in stress, change induced by fatigue damage, etc.

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