STAT-2202

Problem-01

Data $=(2,4,6,8)$

without replacement (a random sample of size 2):

$(2,4),(2,6),(2,8),(4,6),(4,8),(6,8)$

(i) Population mean, $\mu = \frac{(2+4+6+8)}{4} = 5$

sample data:

	[1]	[2]	
[1,]	2	4	→ mean = 3
[2,]	2	6	→ 4
[3,]	2	8	→ 5
[4,]	4	6	→ 5
[5,]	4	8	→ 6
[6,]	6	8	→ 7

$E = x \cdot \frac{1}{6}$

$3 \cdot \frac{1}{6} + (4 \cdot \frac{1}{6}) +$

$\therefore$ sample mean, $\bar{Y} = \frac{\cancel{3+4+5+5+6+7}}{6} \cancel{=5}$ $(3,4,5,5,6,7)$

Expected value, $E(\bar{Y}) = \sum (\bar{Y} \cdot P(\bar{Y})) = \sum 30 \times \frac{1}{6} = 5$

$\therefore \mu = E(\bar{Y})$

The sample mean is an unbiased estimate of population mean.

(ii) $V(\bar{y}) = \frac{\sigma^2}{n} \cdot \frac{N-n}{N-1}$

$V(\bar{y}) = E(\bar{Y}^2) - \{E(\bar{Y})\}^2$

$= \{(3^2+4^2+5^2+5^2+6^2+7^2) * \frac{1}{6}\} - (5)^2$

$= 1.6667$

$n = 2, N = 4$

$\sigma^2 = \frac{var(data) * (N-1)}{N} = 5$

$\frac{\sigma^2}{n} \times \frac{(N-n)}{(N-1)} = V(\bar{y}) = 1.667$

$\sigma^2 = \frac{\sum (\bar{X} - X_i)^2}{n}$

(iii) $s^2 = 2, 8, 18, 2, 8, 2$ $\qquad s = \frac{\sum(x-\bar{x})^2}{n}$

$E(s^2) = 6.67$

$\therefore \sigma^2 \neq E(s^2)$ [Biased]

$\bar{x} - \ldots$

(iv) $\alpha = 5\% = 0.05$ $\qquad Z = \frac{\bar{x} - \ldots}{s/\sqrt{n}}$

$\mu = \bar{x} + Z(s/\sqrt{n})$

For population mean,
lower bound $= 2.46$, upper bound $= 7.53$
Therefore, we are 95% confident that the population mean is between 2.46 and 7.53

For population total,
$LB = 11.23$, $UB = 28.76$
therefore, we are 95% confident that the population total is between 11.23 and 28.76

with replacement,
(2, 2), (2, 4), (2, 6), (2, 8), (4, 2), (4, 4), (4, 6), (4, 8), (6, 2), (6, 4), (6, 6), (6, 8), (8, 2), (8, 4), (8, 6), 8, 8)

$$\cancel{n^2} \times \frac{\sum(x_i - \bar{x})}{n \times n}$$

$N^2 \times$ sigma2

~~varitotal~~ $\boxed{\text{Variance total} = N^2 \times \sigma^2}$

Problem - 09:

$$H_0 : \sigma_1^2 = \sigma_2^2 \qquad n_1 = 20$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2 \qquad n_2 = 20$$

$$F = \frac{s_1^2}{s_2^2} \quad s_1^2 > s_2^2 \qquad \bar{x}_1 = \cancel{82.7}\ 4.135$$

$$\bar{x}_2 = 62.55$$

$$\frac{827}{20} = 41.35$$

$$\frac{1251}{20} = 62.55$$

$$s_1^2 = \frac{1}{n_1 - 1} \sum (x_{1i} - \bar{x}_1)^2$$

$$= \frac{1}{20-1} \left[\sum x_{1i}^2 - \frac{(\sum x_{1i})^2}{n_1} \right]$$

$$= \frac{1}{19} \left[34335 - \frac{(827)^2}{20} \right]$$

$$= 7.29$$

$$s_2^2 = \frac{1}{20-1} \left(78751 - \frac{(1251)^2}{20} \right)$$

$$= 26.365$$

$$s_2^2 > s_1^2 \therefore f_{cal} = \frac{26.365}{7.29} = 3.61$$

~~অথবা~~ $\boxed{f_{tab} = 0.33}$ (Null rejected)

p value = ?

$$Y = \alpha + \beta x + \epsilon$$

$H_0 : \beta = 0$, $H_1 : \beta \neq 0$

$H_0 : \rho = 0$, $H_1 : \rho \neq 0$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$t = \frac{\hat{\beta}}{\sqrt{\frac{s^2}{ss(X)}}}$$

$$r = \frac{sp(XY)}{\sqrt{ss(X) \cdot ss(Y)}}$$

$$\hat{\beta} = \frac{sp(XY)}{ss(X)}$$

$$s^2 = \frac{1}{n-2}\left[ss(Y) - \hat{\beta} \cdot sp(XY)\right]$$

$|t| > t_{0.05, n-2}$ rejected H_0

$$ss(X) = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$ss(Y) = \Sigma Y^2 - \frac{(\Sigma Y)^2}{n}$$

$$sp(XY) = \Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{n}$$

t distribut — $(n-2)$ d.f

Problem — 10:

X	Y	X^2	Y^2	XY

$\Sigma x = 1550$, $\Sigma Y = 586$, $\Sigma X^2 = 240308$

$\Sigma Y^2 = 34430$, $\Sigma XY = 90866$

extra [illegible]

$n = 10$

$$SS(X) = \sum X^2 - \frac{(\sum X)^2}{n} = 240308 - \frac{(1550)^2}{10} = 58$$

$$SS(Y) = \sum Y^2 - \frac{(\sum Y)^2}{n} = 34430 - \frac{(586)^2}{10} = 90.4$$

$$SP(XY) = \sum XY - \frac{(\sum X)(\sum Y)}{n} = 90866 - \frac{1550 \times 586}{10} = 36$$

$$\hat{\beta} = \frac{SP(XY)}{SS(X)} = \frac{36}{58} = 0.62$$

$$s^2 = \frac{1}{n-2}\left[SS(Y) - \beta^2 SP(XY)\right] = \frac{1}{10-2}\left[90.4 - 0.62 \times 36\right]$$

$$= 8.51$$

$$\therefore t = \frac{\hat{\beta}}{\sqrt{\frac{s^2}{SS(X)}}} = \frac{0.62}{\sqrt{\frac{8.51}{58}}} = 1.62$$

$$r = \frac{SP(XY)}{\sqrt{SS(X) \cdot SS(Y)}} = \frac{36}{\sqrt{58 \times 90.4}} = 0.50$$

$$\boxed{t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.50\sqrt{10-2}}{1-(0.50)^2} = 1.63}$$

$p = 0.000136$ [regression table [illegible]]

Problem-02 $\quad n_1 = 27,\ n_2 = 20$

$\bar{x} = 8.14$

$\bar{Y} = 8.57$

(i) ব্যাখ্যা

(ii) $$s_x^2 = \frac{1}{n_1 - 1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

$$s_Y^2 = \frac{1}{n_2 - 1}\left(\Sigma Y^2 - \frac{(\Sigma Y)^2}{n}\right)$$

$$\text{standard error} = \sqrt{\frac{s_x^2}{n_1} + \frac{s_Y^2}{n_2}}$$

$$\text{ratio} = \sqrt{\frac{s_x^2}{s_Y^2}} = \sqrt{\frac{\sigma_1}{\sigma_2}}$$

$$\text{var}(X-Y) = \text{var}(X) + \text{var}(Y)$$
$$= s_x^2 + s_y^2$$

Problem - 3

প্রশ্নের ব্যাপ্ত আছে উত্তরই বলেছি, তবে theory আমার ব্যাখ্যা এবং সূত্রে আসছে না।

Problem-7

$\overline{x_1} = \cancel{8415}\,33.29$

$\overline{x_2} = 34.5161$

$n_1 = 31, \quad n_2 = 31$

$\cancel{sd_1^2} = s_1^2 = 1.41 = \frac{1}{n_1 - 1}\left[x_{1i}^2 - \frac{(\Sigma x_{1i})^2}{n}\right]$

$s_2^2 = 1.09 = \frac{1}{n_2 - 1}\left[x_{2i}^2 - \frac{(\Sigma x_{2i})^2}{n}\right.$

$$z = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$= -4.34$

Pvalue = table-এ z^2 t এর মতন

Problem-8:

$a_1 = 44, \quad a_2 = 41, \quad n_1 = 80, \quad n_2 = 90$

$p_1 = \frac{a_1}{n_1} \qquad q = p - p \; ; \; p = \frac{a_1 + a_2}{n_1 + n_2}$

$p_2 = \frac{a_2}{n_2} \qquad z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$= \underline{1.22}$

p value $= \cancel{0.442}\; 0.888$

Problem-06

$\mu_0 = 37000, \bar{x} = 38100$

$H_0 : \mu \leq 37000$ $n = 48$

$H_A : \mu > 37000$ $sd = 5200$

$$z = \frac{\bar{x} - \mu}{\frac{s_d}{\sqrt{n}}} = 1.46$$

$p = 0.9279$

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