

Towards an Approach for Project-Library Recommendation Based on Graph Normalization

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Abstract: The performance of project-library recommendation systems depends on the choice of graph normalization techniques. This work explores two primary normalization schemes within a knowledge graph - enhanced project - library recommendation system: symmetric normalized Laplacian (SNL) and random walk normalized Laplacian (RWL). Experimental results show that RWL consistently delivers better performance in key metrics, including mean precision (MP), mean recall (MR), and mean F1 score (MF), particularly in sparse datasets. Although SNL performs well in denser datasets, its effectiveness decreases with increasing sparsity. Furthermore, loss curves for collaborative filtering (CF) and knowledge graph (KG) tasks indicate that RWL converges faster and shows greater stability. These findings establish RWL as a reliable technique for improving GNN-based recommendation systems, especially in sparse and complex project-library interaction scenarios.

1 INTRODUCTION

Deep learning has transformed numerous domains, particularly with the success of convolutional neural networks (CNNs) (LeCun et al., 1998). While CNNs excel at modeling structured grid-like data, graph neural networks (GNNs)(Wu et al., 2022) have emerged to address non-Euclidean data structures, such as graphs, enabling advances in fields such as protein structure prediction (Mastropietro et al., 2023), traffic planning (Ye et al., 2023), and recommendation systems (Isinkaye et al., 2015).

In recommendation systems, graph-based models have gained significant attention for their ability to represent and exploit complex user-item interactions. PyRec(Li et al., 2024), a framework that uses graph neural networks (GNNs)(Zhou et al., 2020) with integrated knowledge graphs, has shown promising results in third-party library recommendation (TPL) tasks. By encoding user-library relationships and using knowledge graphs for additional contextual information, PyRec(Li et al., 2024) improves the precision and relevance of recommendations. However, its reliance on GNN-based methodologies highlights a crucial challenge: the sensitivity of GNN perfor-

mance to the choice of adjacency matrix normalization schemes.

Normalization schemes are pivotal in GNN-based models as they scale node degrees, balance information propagation, and preserve graph topology during message passing. Despite their importance, the impact of normalization schemes on TPL recommendation tasks in PyRec(Li et al., 2024) has not been systematically investigated. This aspect is essential for optimizing graph-based recommendation systems and addressing challenges such as scalability, noise robustness, and sparsity.

Motivated by these considerations, this research investigates the effect of different adjacency matrix normalization techniques on PyRec(Li et al., 2024) performance. By analyzing how these schemes influence metrics such as precision, recall, and NDCG, we aim to provide information on the design of robust and efficient GNN-based recommendation systems.

The main contributions of this work are:

- We systematically explore the two primary normalization schemes, Symmetric Normalized Laplacian (SNL) and Random Walk Normalized Laplacian (RWL) - on the performance of project-library recommendation, focusing on their effects on key metrics such as precision, recall, F1 score, MAP, and MRR.
- We analyze the performance of these normaliza-

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tion schemes in varying levels of data sparsity, demonstrating the robustness and effectiveness of RWL compared to SNL in handling sparse and incomplete datasets.

- We examine loss curves for Collaborative Filtering (CF) and Knowledge Graph (KG) tasks, highlighting the faster convergence and stability of RWL over SNL.
- We provide actionable insights and practical recommendations for selecting normalization techniques to optimize graph-based recommendation systems, particularly in the context of third-party library usage and sparse datasets.

The remainder of this paper is organized as follows. Section 2 reviews the literature on graph neural networks, knowledge graphs, and normalization techniques. Section 3 describes the experimental setup and methods for evaluating normalization schemes in PyRec. Section 4 presents a detailed analysis of the results. Finally, Section 5 summarizes the findings and suggests future research directions.

2 RELATED WORK

Graph Neural Networks (GNNs) have emerged as one of the most innovative approaches in machine learning and artificial intelligence, capable of addressing problems involving graph-structured data by enabling effective information exchange between graph nodes. GNNs excel at modeling complex dependencies and relationships inherent in graph representations(Zhou et al., 2020). Over time, numerous architectures have been developed to cater to diverse use cases. Prominent examples include Graph Convolutional Networks (GCNs)(Kipf and Welling, 2016), Graph Attention Networks (GATs)(Veličković et al., 2017), and Graph Isomorphism Networks (GINs)(Xu et al., 2018), all of which have been successfully applied in various domains. Zhou et al.(Zhou et al., 2020) categorized GNNs into four classes: RGNNs, GCNNs, ST-GCNs, and GraphAEs, providing a structured overview of their functionalities. Khemani et al.(Asif et al., 2021) conducted a comprehensive review of GNN architectures, offering valuable insights into their design principles, real-world applications, and benchmark datasets, laying a strong foundation for future research.

Recent breakthroughs in GNN have pointed out that low-rank approximation methods effectively enhance models’ efficiency and scalability. The low-rank approximation hypothesis assumes that complicated graph structures can be effectively represented

using lower-dimensional embeddings without significant loss of information. In addition, this approach allows for the compression of node and edge features into concise latent spaces, reducing computational overhead. For example, the LRGA mechanism was proposed to incorporate low-rank approximations into GNNs so that global attention computation can be efficiently and scalably performed using matrix-vector multiplication of low-rank matrices (Puny et al., 2020). Empirical studies (Yang et al., 2024; Zhang et al., 2018) demonstrate that this type of low-rank representation can preserve or even enhance model performance, since these methods preserve crucial structural patterns while removing redundancy. This objective has been considerably helped by using low-rank SVD and randomized low-rank matrix approximation techniques(Wu et al., 2010). It is a promising direction in developing scalable graph-based models, mainly for large-scale datasets where traditional methods can be computationally strained.

Considering these low-dimensional representations, the arising of the Laplacian matrix as a basic tool in graph theory for graph representation analysis is of great importance. In fact, regarding the encoding of graph topology, a wide range of applications can be found in spectral graph theory or machine learning. A popular variant is provided by the symmetric normalized Laplacian(SNL)(Chung, 1996), capturing the structural relationships of graphs that are caused by symmetrically normalizing the adjacency matrix. Meanwhile, the random-walk normalized Laplacian(RWL)(Schaub et al., 2020) provides a probabilistic view; hence, it is possible to investigate the random-walk dynamics on the graph nodes. These Laplacian matrices have been fundamental in applications such as trajectory analysis and personalized PageRank, complementing strengths in interpreting graph-structured data. They form the mathematical backbone of many graph learning techniques and provide a seamless bridge from traditional graph analysis to modern neural network methodologies.

In the domain of recommendation systems, the integration of GNNs with knowledge graphs opened new dimensions for improved precision in recommendations. Models like PyRec (Li et al., 2024) incorporate GNNs along with KGs to effectively model pairwise and contextual relations, improving their capture power for complex user-item interactions. However, the sensitivity of the GNN performance due to different normalization schemes applied to the adjacency matrix is relatively underexploited. This critical gap brings out the need to systematically investi-

gate the influence of various normalization techniques that might have broad ramifications in the accuracy and scalability aspects of graph-based recommendation systems.

3 METHODOLOGY

The proposed framework will investigate the impact of normalization schemes on GNNs applied to project library recommendation systems. Analyzing the behavior of GNNs in light of different normalization techniques aims to reveal their role in optimizing feature propagation and graph representation with respect to recommendation tasks. The following subsections explain these in detail, stating the theoretical basis and implementing the approach.

3.1 Preliminaries

Graph neural networks (GNNs) operate on graph-structured data, leveraging relationships between nodes to propagate and aggregate information (Li et al., 2018; Wu et al., 2020). To formalize the representation of a graph, we consider an undirected graph $G = (V, E)$, where V is the set of vertices and E is the set of edges. The adjacency matrix $A \in \mathbb{R}^{n \times n}$ and the degree matrix $D \in \mathbb{R}^{n \times n}$ are defined as follows:

$$A_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

$$D_{ii} = \sum_j A_{ij}. \quad (2)$$

Graph Laplacians are critical in the analysis of graph structures, commonly used in spectral graph theory and machine learning for tasks like clustering and node classification (Chung, 1997; Belkin and Niyogi, 2003). The Laplacian matrix L encodes the structural properties of the graph and is defined as:

$$L = D - A. \quad (3)$$

Additionally, to accommodate normalization schemes, two variants of the Laplacian are commonly employed:

Symmetric Normalized Laplacian (SNL):

$$L_{\text{sym}} = I - D^{-1/2}AD^{-1/2}, \quad (4)$$

where I is the identity matrix.

Random-Walk Normalized Laplacian (RWL):

$$L_{\text{rw}} = I - D^{-1}A. \quad (5)$$

These matrices play a critical role in defining the propagation rule for graph neural networks (GNNs),

which iteratively update node representations by aggregating information from neighboring nodes. For a single GNN layer, the propagation rule is:

$$H^{(l+1)} = \sigma \left(L_{\text{norm}} H^{(l)} W^{(l)} \right), \quad (6)$$

where $H^{(l)} \in \mathbb{R}^{n \times d}$ denotes the node features at the l -th layer, $W^{(l)} \in \mathbb{R}^{d \times d'}$ is the trainable weight matrix, $\sigma(\cdot)$ is a non-linear activation function, and L_{norm} can be substituted with L_{sym} or L_{rw} depending on the normalization scheme.

In our investigation, these normalized Laplacians are evaluated for their impact on GNN performance in the context of project-library recommendation tasks. By analyzing the performance of models using SNL and RWL, we aim to discern their influence on the propagation of node information and overall recommendation accuracy.

3.2 Normalization Schemes on GNNs

Normalization plays a pivotal role in the design and functionality of Graph Neural Networks (GNNs). It directly affects how information is propagated across the graph, ensuring that features from neighboring nodes are appropriately aggregated without being dominated by nodes with higher degrees. Two widely adopted normalization schemes for the adjacency matrix are symmetric normalization and random-walk normalization. These schemes address the challenges posed by unbalanced node degrees and ensure effective learning from the graph structure.

The symmetric normalization scheme is represented as:

$$\hat{A}_{\text{sym}} = D^{-1/2}AD^{-1/2}$$

where D is the degree matrix and A is the adjacency matrix. This normalization method ensures that the contributions of neighboring nodes are inversely weighted by their degrees. As a result, it treats all nodes equitably, preventing over-emphasis on high-degree nodes. Symmetric normalization is particularly beneficial in tasks where the graph exhibits a high degree of heterogeneity in its node connectivity. On the other hand, the random-walk normalization scheme is defined as:

$$\hat{A}_{\text{rw}} = D^{-1}A$$

This scheme is based on the random-walk dynamics of the graph, where the transition probability from a node to its neighbors is inversely proportional to its degree. Random-walk normalization is especially useful for modeling diffusion processes and random-walk-based tasks.

In the context of project-library recommendation systems, normalization schemes influence how

Algorithm 1: Procedure for Normalization Impact Analysis (NIA)

1 . Input: Graph $G := \{A, X\}$ with adjacency matrix A and feature matrix X , normalized Laplacian matrices $\hat{A}_{\text{sym}}, \hat{A}_{\text{rw}}$, GNN model $f(H, \Theta)$, parameters λ_1, λ_2 , learning rate γ , number of epochs N , sparsity thresholds $\tau_{\text{sym}}, \tau_{\text{rw}}$.

Output: Optimized GNN and evaluation metrics.

2 Initialize: Set initial weights and adjacency matrices.

3 for $i = 0$ **to** $N - 1$ **do**

4 Forward Pass: Compute node representations:

5 $H_{\text{sym}}^{(l+1)} = \sigma(\hat{A}_{\text{sym}} H^{(l)} W^{(l)})$; *//Update project embeddings*

6 $H_{\text{rw}}^{(l+1)} = \sigma(\hat{A}_{\text{rw}} H^{(l)} W^{(l)})$; *//Update TPL embeddings*

7 Compute the loss function:

8 $L = L_{\text{GNN}}(H_{\text{sym}}, H_{\text{rw}}, Y) + \lambda_1 \|W\|_2 + \lambda_2 \|A\|_1$; *//Loss function calculation*

9 Backward Pass: Update GNN weights:

10 $W^{(l+1)} \leftarrow W^{(l)} - \gamma \nabla_{W^{(l)}} L$; *//Gradient descent update for weights*

11 Update adjacency matrices:

12 $\hat{A}_{\text{sym}} \leftarrow \hat{A}_{\text{sym}} - \gamma \nabla_{\hat{A}_{\text{sym}}} L$; *//Update symmetric adjacency matrix*

13 $\hat{A}_{\text{rw}} \leftarrow \hat{A}_{\text{rw}} - \gamma \nabla_{\hat{A}_{\text{rw}}} L$; *//Update random-walk adjacency matrix*

14 Pruning: Set τ_{sym} values in \hat{A}_{sym} to 0, and others to 1; *//Apply sparsity threshold for \hat{A}_{sym}*

15 Set τ_{rw} values in \hat{A}_{rw} to 0, and others to 1; *//Apply sparsity threshold for \hat{A}_{rw}*

16 Re-training: Retrain the GNN with pruned adjacency matrices; *//Refine the model on pruned graph structure*

17 Evaluation: Measure performance metrics such as precision, recall, and F1-score; *//Evaluate final model performance*

project and library features are propagated through the graph. The symmetric normalization scheme emphasizes structural regularity by balancing node degrees, making it well suited for graphs with diverse connectivity patterns. Meanwhile, random-walk normalization better captures sequential or flow-based dynamics, which may be significant in contexts where the relationship between projects and libraries follows a temporal or directional pattern.

The propagation mechanism in GNN can be summarized as:

$$H^{l+1} = \sigma(\hat{A}H^lW^l)$$

where \hat{A} can be \hat{A}_{sym} or \hat{A}_{rw} , H^l is the node feature matrix at layer l , W^l is the trainable weight matrix, and σ is the activation function. As illustrated in Figure 1, the node features (h) and the edge features (e) are processed through successive layers of GNN, with the information iteratively propagated and updated. The features of the nodes h_i^l obtained after propagation are utilized for prediction tasks.

$$\frac{1}{|V|} \sum_{i=0}^{|V|} h_i^l$$

where $|V|$ is the total number of nodes in the graph and h_i^l is the embedding of node i at layer l . This

process ensures that the global graph representation is captured, enabling effective predictions at the graph level.

To assess the impact of these normalization schemes, the study follows the workflow described in Algorithm 1. This algorithm outlines the iterative process of updating node embeddings and adjacency matrices, ensuring consistency in the evaluation of the two normalization approaches. Figure 1 complements this by providing a graphical representation of the GNN architecture, including the input features, feature propagation, and prediction layers. Together, these elements provide a comprehensive view of how normalization schemes affect GNN performance. By systematically analyzing the performance of these normalization schemes in the project-library dataset, this work highlights their strengths and weaknesses and provides actionable insights for selecting an appropriate scheme based on the characteristics of the graph.

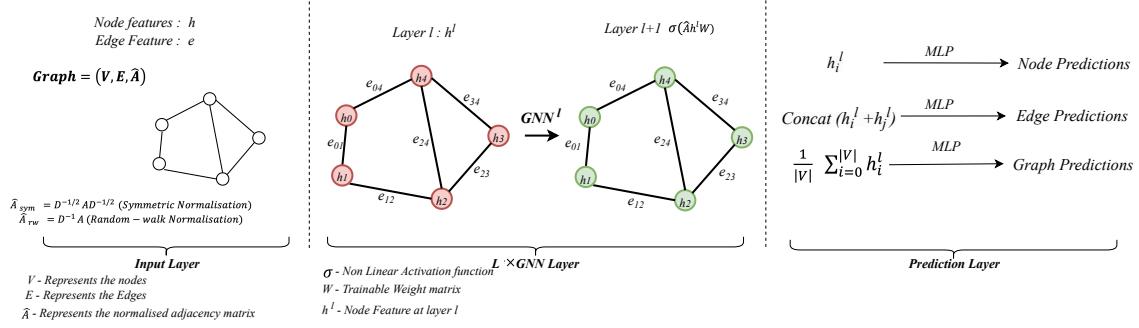


Figure 1: GNN architecture illustrating input features, feature propagation through GNN layers, and node, edge, and graph-level predictions.

4 EXPERIMENTAL SETUP AND RESULTS

This section details the experimental setup used to evaluate the applied framework, followed by the results obtained. The evaluation is based on the project library dataset.

4.1 Dataset

The study uses a large-scale heterogeneous data set specifically designed for the recommendation of the third-party library (TPL). It includes 12,421 Python projects, 963 distinct TPLs, 9,675 additional entities, 121,474 project-library interaction records, and 73,277 pieces of contextual information. The nodes represent projects, libraries, and entities, while the edges capture relationships such as library usage.

The data set supports advanced graph-based methods, and its scale (13,000+ nodes and 200,000+ edges) ensures evaluation under realistic conditions. It is publicly available for validation and reproduction of results¹. In this study, the same data splits as (Li et al., 2024) are applied.

4.2 Network Settings

Figure 1 illustrates the architecture of the GNN model used in this study. The input layer incorporates node and edge features along with the normalized adjacency matrix, supporting symmetric and random-walk normalization schemes. The GNN layer propagates features through multiple layers, using non-linear activation functions and trainable weight matrices. Finally, the prediction layer enables predictions at the node, edge, and graph levels, effectively

¹<https://github.com/Limber0117/PyRec/tree/main/datasets>

aligning with the requirements of project-library recommendation tasks.

The backbone network is trained for up to 1000 epochs using the Adam optimizer with a learning rate of 0.0001. The model uses node embeddings and relation embeddings of dimensions 128 and 64, respectively. Two aggregation layers, each with an output dimension of 64, are employed with a bi-interaction mechanism. A message dropout rate of 1% is uniformly applied across layers to enhance regularization. The adjacency matrix (Laplacian) supports two normalization schemes: symmetric and random walk, which can be specified as part of the configuration. The training process includes an early stoppage if no improvement in recall is observed in 15 consecutive epochs. Recommendations are evaluated based on metrics calculated at $K = [5, 10, 20]$, ensuring robust performance analysis. This configuration provides a flexible framework for effective project-library recommendation tasks.

4.3 Result Analysis

This study evaluates the impact of four normalization techniques: symmetric normalized Laplacian (SNL), random walk normalized Laplacian (RWN), double stochastic normalization (DSN) and unnormalized Laplacian (UNL) on the library recommendation task. The evaluation was carried out on datasets with varying levels of missing library information ($rm = 20\%$, $rm = 40\%$, and $rm = 60\%$), using metrics such as mean precision (MP), mean recall (MR), mean F1 score (MF), mean reciprocal rank (MRR) and mean average precision (MAP) at $K = 5, 10, 20$. The results are summarized in Table 1.

The main focus of the paper is on SNL and RWN, as these techniques are emphasized throughout the applied framework. However, DSN and UNL were included purely for comparative purposes to serve as

Table 1: Performance Comparison of Different Normalization Techniques.

Dataset	Norm	K = 5					K = 10					K = 20				
		MP	MR	MF	MRR	MAP	MP	MR	MF	MRR	MAP	MP	MR	MF	MRR	MAP
rm=20%	UNL	0.105	0.366	0.169	0.306	0.254	0.066	0.490	0.122	0.344	0.280	0.045	0.572	0.088	0.355	0.287
	DSN	0.112	0.373	0.169	0.313	0.258	0.079	0.488	0.128	0.340	0.377	0.049	0.584	0.092	0.349	0.273
	SNL	0.121	0.378	0.171	0.327	0.261	0.072	0.484	0.125	0.342	0.283	0.051	0.592	0.107	0.359	0.295
	RWN	0.123	0.424	0.191	0.375	0.299	0.080	0.535	0.139	0.390	0.314	0.060	0.640	0.109	0.396	0.323
rm=40%	UNL	0.201	0.303	0.251	0.507	0.257	0.130	0.429	0.214	0.489	0.274	0.104	0.489	0.138	0.514	0.284
	DSN	0.205	0.315	0.254	0.502	0.265	0.131	0.418	0.212	0.502	0.270	0.110	0.491	0.141	0.528	0.280
	SNL	0.211	0.342	0.261	0.517	0.264	0.143	0.454	0.218	0.531	0.281	0.112	0.548	0.159	0.536	0.288
	RWN	0.227	0.366	0.280	0.551	0.285	0.152	0.479	0.231	0.564	0.300	0.183	0.557	0.194	0.567	0.310
rm=60%	UNL	0.274	0.295	0.284	0.504	0.249	0.197	0.411	0.266	0.503	0.257	0.156	0.250	0.216	0.413	0.267
	DSN	0.285	0.305	0.294	0.572	0.263	0.201	0.418	0.272	0.519	0.270	0.160	0.257	0.211	0.428	0.280
	SNL	0.286	0.306	0.295	0.612	0.264	0.201	0.419	0.272	0.622	0.260	0.132	0.261	0.211	0.625	0.281
	RWN	0.288	0.309	0.298	0.621	0.271	0.203	0.421	0.274	0.611	0.275	0.160	0.291	0.242	0.634	0.292

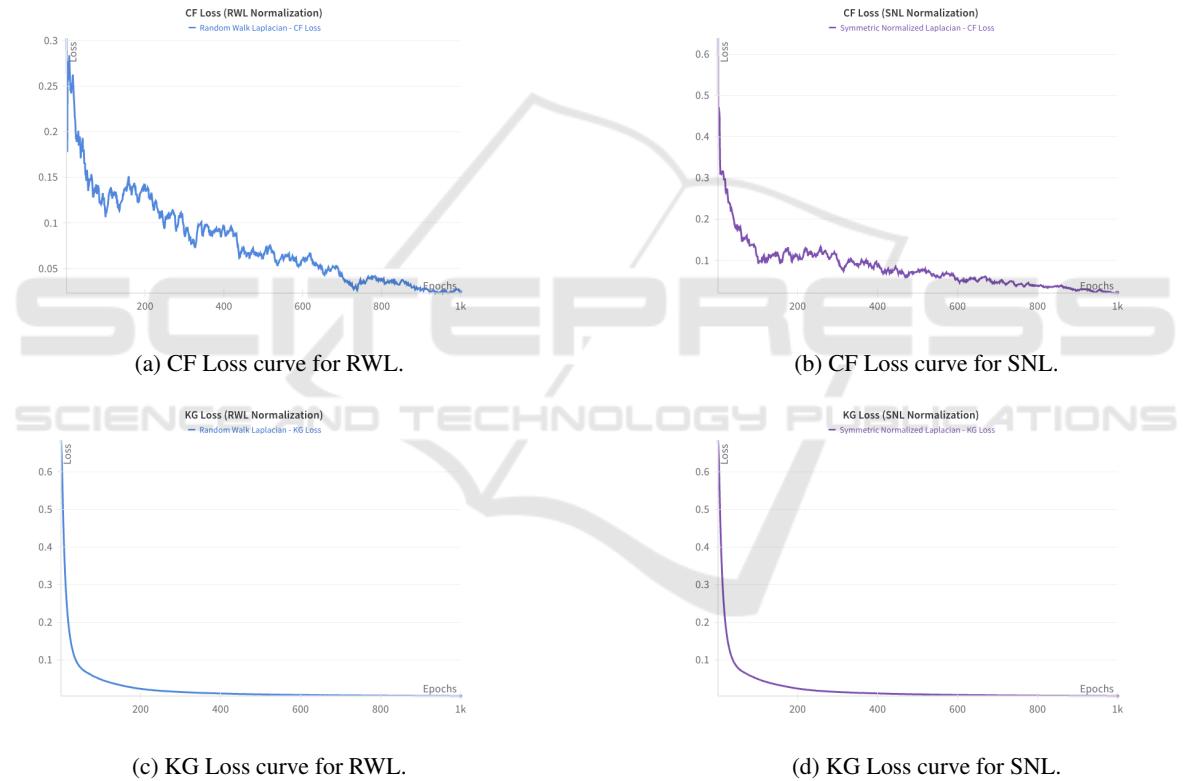


Figure 2: Loss curves for Collaborative Filtering(CF) and Knowledge Graph (KG) under Symmetric Normalized Laplacian (SNL) and Random Walk Laplacian (RWL) normalization schemes.

benchmarks, highlighting the strengths of SNL and RWN. The inclusion of DSN and UNL is justified by their widespread use in graph-based learning tasks, offering a broader perspective on performance.

For $rm = 20\%$, SNL achieved an MF of 0.191 at $K = 5$, demonstrating its stability for smaller values K . However, it was outperformed by RWN in all metrics. For example, RWN achieved an MF of 0.280 at $K = 5$ and an MAP of 0.299, compared to 0.261 for

SNL. DSN and UNL also performed reasonably well, and UNL achieved an MF of 0.169 at $K = 5$, indicating its limitations in quality classification compared to the other techniques.

As the level of missing data increased to $rm = 40\%$, RWN demonstrated its robustness with an MF of 0.280 at $K = 5$, significantly outperforming SNL, which achieved an MF of 0.264. Both DSN and UNL lagged behind, and DSN showed better MAP

and MRR values compared to UNL, particularly at higher values K . RWN's ability to maintain superior MAP and MRR scores highlights its capacity to handle sparse and incomplete datasets effectively.

For $rm = 60\%$, RWN continued to outperform other normalization schemes, achieving an MF of 0.298 at $K = 5$ and an MAP of 0.292 at $K = 20$. SNL achieved an MF of 0.264 and an MAP of 0.281 at $K = 20$, showing its limitations in sparse scenarios. DSN and UNL exhibited declining performance as sparsity increased, strengthening the robustness of RWN in high-sparsity environments.

The loss curves presented in Figure 2 further validate these findings. The Collaborative Filtering (CF) and Knowledge Graph (KG) loss curves indicate faster convergence and stability for RWN compared to SNL. Specifically, RWN achieved lower overall CF and KG losses, reflecting its ability to learn more effectively. Figure 3 compares the runtime efficiency of SNL and RWN, showing that while RWN required slightly more computation time, its superior performance justifies the trade-off.

In general, the results emphasize the superiority of Random Walk Normalized Laplacian (RWN) over Symmetric Normalized Laplacian (SNL) for the library recommendation task. Although DSN and UNL provided additional information, their inclusion was purely for comparative purposes. The ability of RWN to capture higher-order interactions and deliver superior ranking performance makes it a preferred choice, particularly in sparse settings. These findings validate the design choices of the applied framework and highlight the critical role of normalization schemes in enhancing the quality of recommendations and retrieval performance.

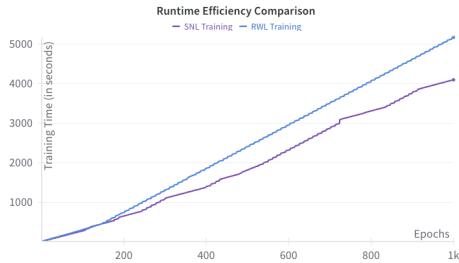


Figure 3: Runtime Efficiency Comparison between SNL and RWN.

5 CONCLUSIONS

This study evaluated the impact of the Symmetric Normalized Laplacian (SNL) and the Random Walk Normalized Laplacian (RWN) on the performance of

the PyRec recommendation model in data sets with varying levels of missing library information ($rm = 20\%, 40\%, 60\%$). The results show that RWN consistently outperforms SNL in all sparsity levels and evaluation metrics, particularly in sparse scenarios ($rm = 60\%$), highlighting its robustness and ability to improve ranking quality. Although SNL performs reasonably well on dense datasets ($rm = 20\%$), its effectiveness diminishes with increasing sparsity.

Future work will explore additional normalization techniques, such as Doubly Stochastic Normalization and Unnormalized Laplacian, to further improve the adaptability of PyRec. Investigating the interaction between normalization schemes, hyperparameter configurations, and temporal dynamics will also be prioritized to improve the scalability and generalizability of the model.

These findings emphasize the critical role of normalization techniques in improving the performance of graph-based recommendation systems and open avenues for further advancements in handling sparse and complex data sets.

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