第六次作业报告 PB23000141 刘彦宏

编译环境

作业使用cmake组织项目, 编译器使用msvc1942. 开发使用的IDE是Clion, VS2022当然也可以正常构建并运行.

代码运行方式与之前的作业一致, 按先前作业的批改反馈此处不再赘述.

问题描述

根据课本给出的算法(幂法和QR方法), 求解特征值和特征向量.

问题分析

因为只是根据课本算法直接求解特征值和特征向量, 略掉问题分析.

多项式的求根问题可以换成其友阵的求特征值问题.

结果展示和讨论

第一题

```
D:\Code\N_A\cmake-build-debug-visual-studio\hw6\hw6_1.exe
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(i): x^3 + 1*x^2 - 5*x^1 + 3 = 0
Largest root by power method: -3.0000000000
Verification f(lambda) = 0.0000000000

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(ii): x^3 + 0.0000000000*x^2 - 3.0000000000*x^1 - 1.0000000000 = 0
Largest root by power method: 1.8793852416
Verification f(lambda) = 0.0000000000

_____

(iii): x^8 + 101.0000000000*x^7 + 208.0100000000*x^6 + 10891.0100000000*x^5 + 9802.0800000000*x^4 + 79108.9000000000*x^3
 - 99902.0000000000*x^2 + 790.0000000000*x^1 - 1000.0000000000 = 0
Largest root by power method: -100.0000000000
Verification f(lambda) = -109.5146049929
```

正确性得到mma的验证. 最后一个求出来的根代回去值不为零应该是浮点精度损失导致的.

In[8]:= **Solve[x^3 + x^2 - 5 x + 3 == 0, x]**
解方程

Out[8]= $\{\{x \to -3\}, \{x \to 1\}, \{x \to 1\}\}$

In[9]:= **Solve[x^3 - 3 x - 1 == 0, x]**
解方程

Out[9]= $\left\{\left\{x \to \boxed{\sqrt{\ }\ -1.53...}\right\}, \left\{x \to \boxed{\sqrt{\ }\ -0.347...}\right\}, \left\{x \to \boxed{\sqrt{\ }\ 1.88...}\right\}\right\}$

In[10]:= **Solve[x^8 + 101 x^7 + 208.01 x^6 + 10 891.01 x^5 + 9802.08 x^4**
解方程
+ 79 108.9 x^3 - 99 902 x^2 + 790 x - 1000 == 0, x]

Out[10]= $\{\{x \to -100.\}, \{x \to -1. - 3. \, i\}, \{x \to -1. + 3. \, i\}, \{x \to 0. - 0.1 \, i\},$
$\{x \to 0. + 0.1 \, i\}, \{x \to 0. - 10. \, i\}, \{x \to 0. + 10. \, i\}, \{x \to 1.\}\}$

假设 一个规则列表 | 或用作 *二维数组* 或 *列表*

第二题

这个输出太长, 就不贴截图了. 把输出结果我粘贴在这里.

```
D:\Code\N_A\cmake-build-debug-visual-studio\hw6\hw6_2.exe
=============================================
(2) Find all roots of x^41 + x^3 + 1 = 0
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Total 41 roots:

root[ 1] = -0.4852802393 - 0.8945383698i  (|root| = 1.0176914099)
root[ 2] = -0.4852802393 + 0.8945383698i  (|root| = 1.0176914099)
root[ 3] = 1.0143046673 + 0.0809230299i  (|root| = 1.0175276384)
root[ 4] = 1.0143046673 - 0.0809230299i  (|root| = 1.0175276384)
root[ 5] = -0.6206725055 - 0.8058893071i  (|root| = 1.0171980802)
root[ 6] = -0.6206725055 + 0.8058893071i  (|root| = 1.0171980802)
root[ 7] = -0.3369843232 - 0.9592276127i  (|root| = 1.0166986018)
root[ 8] = -0.3369843232 + 0.9592276127i  (|root| = 1.0166986018)
root[ 9] = 0.9871838580 + 0.2403543837i  (|root| = 1.0160227356)
root[10] = 0.9871838580 - 0.2403543837i  (|root| = 1.0160227356)
root[11] = -0.7391005652 - 0.6959043532i  (|root| = 1.0151613242)
root[12] = -0.7391005652 + 0.6959043532i  (|root| = 1.0151613242)
root[13] = -0.1802060432 - 0.9979622327i  (|root| = 1.0141019850)
root[14] = -0.1802060432 + 0.9979622327i  (|root| = 1.0141019850)
root[15] = 0.9336640447 + 0.3925463845i  (|root| = 1.0128283233)
root[16] = 0.9336640447 - 0.3925463845i  (|root| = 1.0128283233)
root[17] = -0.8368630050 - 0.5678256691i  (|root| = 1.0113187824)
root[18] = -0.8368630050 + 0.5678256691i  (|root| = 1.0113187824)
root[19] = -0.0197286322 - 1.0093521569i  (|root| = 1.0095449448)
root[20] = -0.0197286322 + 1.0093521569i  (|root| = 1.0095449448)
root[21] = 0.8551580276 + 0.5326335355i  (|root| = 1.0074689749)
root[22] = 0.8551580276 - 0.5326335355i  (|root| = 1.0074689749)
root[23] = -0.9105111349 + 0.4255281513i  (|root| = 1.0050396681)
root[24] = -0.9105111349 - 0.4255281513i  (|root| = 1.0050396681)
root[25] = 0.1391654350 - 0.9924767337i  (|root| = 1.0021861530)
root[26] = 0.1391654350 + 0.9924767337i  (|root| = 1.0021861530)
root[27] = 0.7537195304 - 0.6553802760i  (|root| = 0.9988075074)
root[28] = 0.7537195304 + 0.6553802760i  (|root| = 0.9988075074)
root[29] = -0.9563390113 + 0.2737762091i  (|root| = 0.9947551042)
root[30] = -0.9563390113 - 0.2737762091i  (|root| = 0.9947551042)
root[31] = 0.2898121311 - 0.9464236747i  (|root| = 0.9898024264)
root[32] = 0.2898121311 + 0.9464236747i  (|root| = 0.9898024264)
root[33] = 0.6323398271 - 0.7534011999i  (|root| = 0.9835990163)
root[34] = 0.6323398271 + 0.7534011999i  (|root| = 0.9835990163)
root[35] = -0.9681403415 - 0.1208669554i  (|root| = 0.9756559546)
root[36] = -0.9681403415 + 0.1208669554i  (|root| = 0.9756559546)
root[37] = 0.4171515783 - 0.8710673095i  (|root| = 0.9658021003)
root[38] = 0.4171515783 + 0.8710673095i  (|root| = 0.9658021003)
root[39] = 0.5075686395 - 0.8105734389i  (|root| = 0.9563760890)
root[40] = 0.5075686395 + 0.8105734389i  (|root| = 0.9563760890)
```

```
root[41] = -0.9524838752   (|root| = 0.9524838752)


============================================
(3) Find all eigenvalues of matrix A for x=0.9, 1.0, 1.1
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------------------------------------
x = 0.9000000000
Matrix A:
[
  [ 9.1000000000, 3.0000000000, 2.6000000000, 4.0000000000 ]
  [ 4.2000000000, 5.3000000000, 4.7000000000, 1.6000000000 ]
  [ 3.2000000000, 1.7000000000, 9.4000000000, 0.9000000000 ]
  [ 6.1000000000, 4.9000000000, 3.5000000000, 6.2000000000 ]
]
Eigenvalues:
  lambda[1] = 17.4396781909
  lambda[2] = 6.8195183375
  lambda[3] = 2.8704017358 + 0.6428911295i
  lambda[4] = 2.8704017358 - 0.6428911295i


------------------------------------
x = 1.0000000000
Matrix A:
[
  [ 9.1000000000, 3.0000000000, 2.6000000000, 4.0000000000 ]
  [ 4.2000000000, 5.3000000000, 4.7000000000, 1.6000000000 ]
  [ 3.2000000000, 1.7000000000, 9.4000000000, 1.0000000000 ]
  [ 6.1000000000, 4.9000000000, 3.5000000000, 6.2000000000 ]
]
Eigenvalues:
  lambda[1] = 17.4764849155
  lambda[2] = 6.7875165916
  lambda[3] = 2.8679992465 + 0.68874735553i
  lambda[4] = 2.8679992465 - 0.68874735553i


------------------------------------
x = 1.1000000000
Matrix A:
[
  [ 9.1000000000, 3.0000000000, 2.6000000000, 4.0000000000 ]
  [ 4.2000000000, 5.3000000000, 4.7000000000, 1.6000000000 ]
  [ 3.2000000000, 1.7000000000, 9.4000000000, 1.1000000000 ]
  [ 6.1000000000, 4.9000000000, 3.5000000000, 6.2000000000 ]
]
Eigenvalues:
```

```
lambda[1] = 17.5130280715
lambda[2] = 6.7560561872
lambda[3] = 2.8654578707 + 0.7321697391i
lambda[4] = 2.8654578707 - 0.7321697391i
```

因为这个41个根实在太多, 我没有仔细确定这个求解是否正确. 但是我检查了几个感觉是对的. 比如 mma的第一个根对应我求出来的第41个根, 第二个和第三个分别对应我求出来的35和36两个根.

```
In[11]:= Solve[x^41 + x^3 + 1 == 0, x]
         解方程

Out[11]= {{x → √ -0.952...}, {x → √ -0.968... - 0.121... i},
         {x → √ -0.968... + 0.121... i},
         {x → √ -0.956... - 0.274... i}, {x → √ -0.956... + 0.274... i},
         {x → √ -0.911... - 0.426... i}, {x → √ -0.911... + 0.426... i},
         {x → √ -0.837... - 0.568... i}, {x → √ -0.837... + 0.568... i},
         {x → √ -0.739... - 0.696... i}, {x → √ -0.739... + 0.696... i},
         {x → √ -0.621... - 0.806... i}, {x → √ -0.621... + 0.806... i},
         {x → √ -0.485... - 0.895... i}, {x → √ -0.485... + 0.895... i},
         {x → √ -0.337... - 0.959... i}, {x → √ -0.337... + 0.959... i},
         {x → √ -0.180... - 0.998... i}, {x → √ -0.180... + 0.998... i},
         {x → √ -0.0197... - 1.01... i}, {x → √ -0.0197... + 1.01... i},
         {x → √ 0.139... - 0.992... i}, {x → √ 0.139... + 0.992... i},
         {x → √ 0.290... - 0.946... i}, {x → √ 0.290... + 0.946... i},
         {x → √ 0.417... - 0.871... i}, {x → √ 0.417... + 0.871... i},
         {x → √ 0.508... - 0.811... i}, {x → √ 0.508... + 0.811... i},
         {x → √ 0.632... - 0.753... i}, {x → √ 0.632... + 0.753... i},
         {x → √ 0.754... - 0.655... i}, {x → √ 0.754... + 0.655... i},
         {x → √ 0.855... - 0.533... i}, {x → √ 0.855... + 0.533... i},
```

关于第三小问, 可以看得出来随着x的增加A的特征值变化不大, 这也符合课上的讨论.