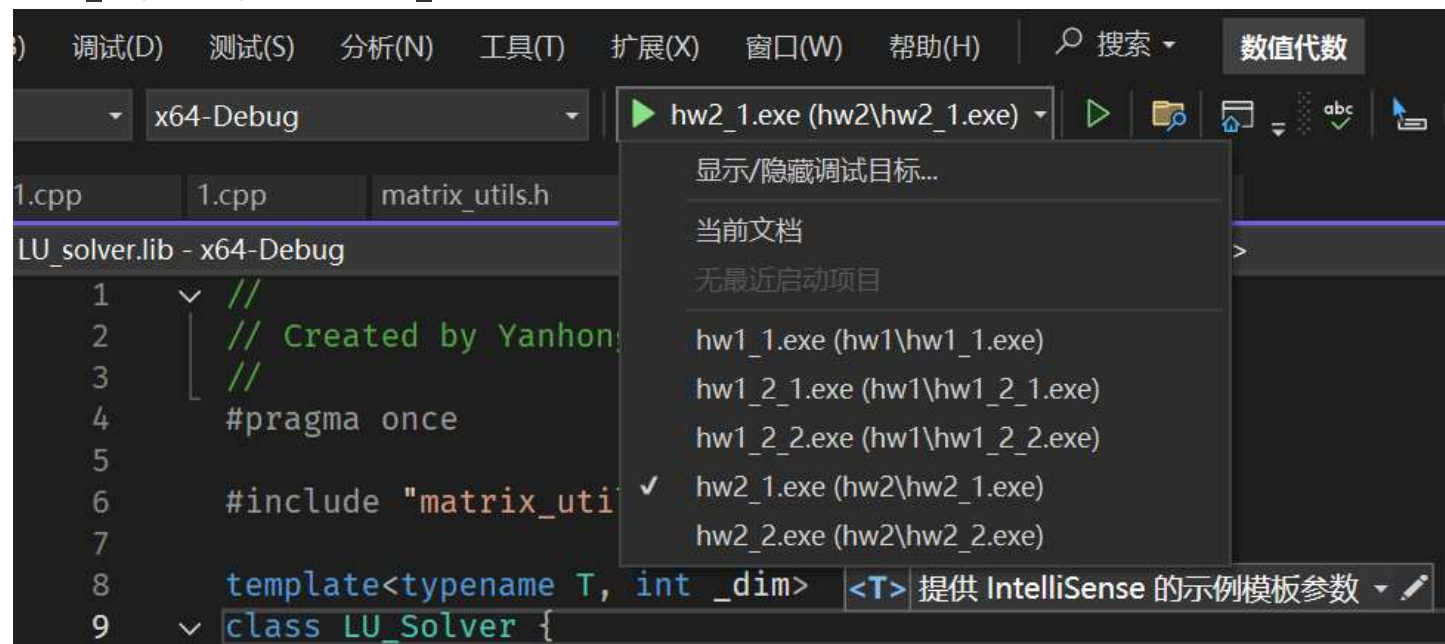


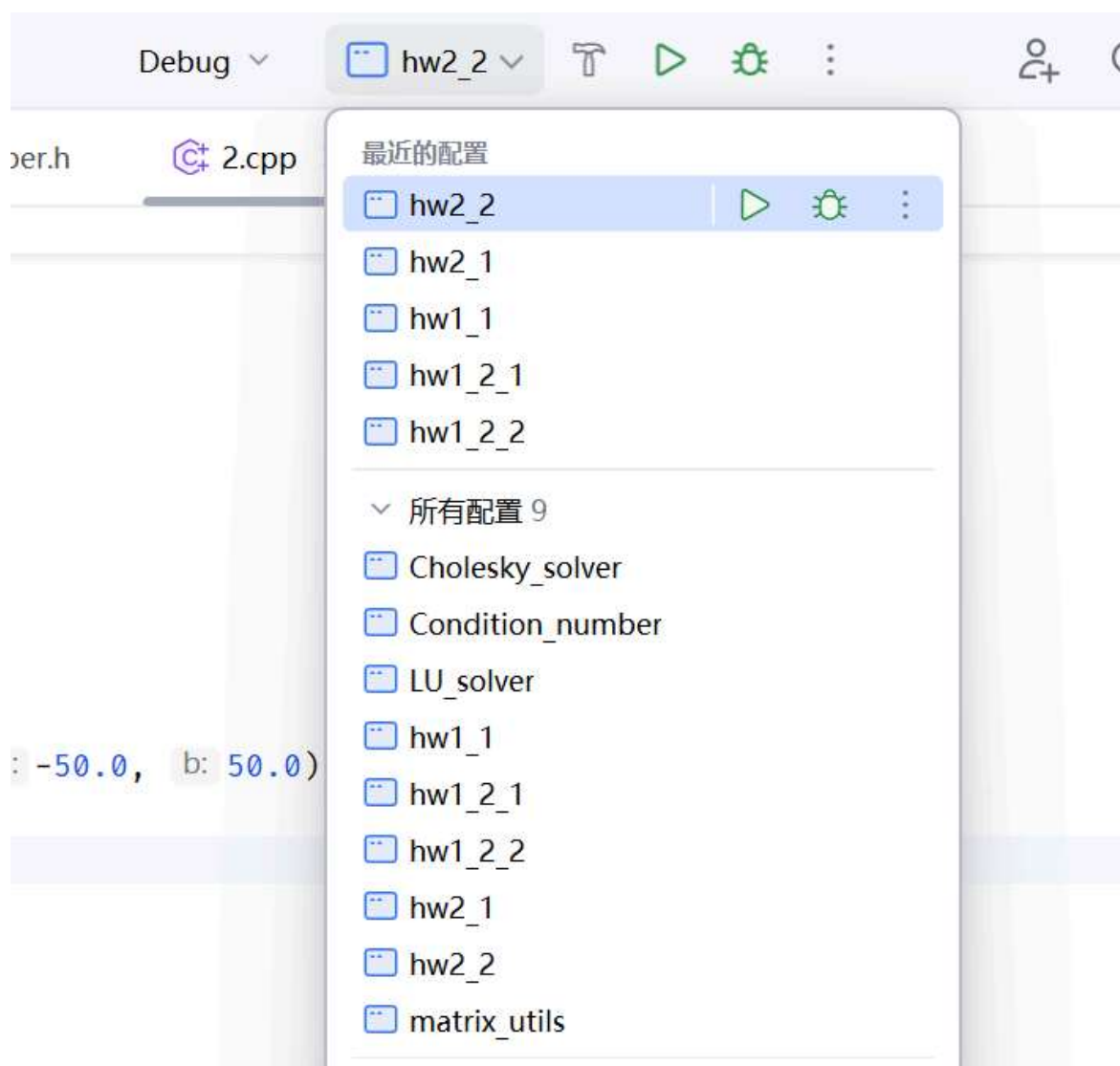
# 第二次作业报告 PB23000141 刘彦宏

## 编译环境

作业使用cmake组织项目, 编译器使用mingw64的g++. 开发使用的IDE是Clion, VS2022也可以正常构建并运行.

运行我的此次作业代码, 只需要加载根目录下的CMakeLists文件, 然后选择对应的构建目标构建后运行即可. 第一题的构建目标是hw2\_1, 第二题的构建目标是hw2\_2, 如图所示:





另外在此注明, 本次作业内容(即实现的条件数计算器)的核心代码在utils文件夹下, 程序入口在hw2文件夹下. 也就是说, 如果需要检查条件数计算器的实现, 则需要打开utils文件夹下的文件; 而如需改动方程组进行测试, 则需要hw2文件夹下进行修改.

## 问题描述

利用课本给出的算法实现对于某矩阵  $\mathbf{A}$  估计  $\mathbf{A}^{-1}$  的无穷范数, 进而得知  $\mathbf{A}$  的无穷条件数. 从而完成: 1. Hilbert矩阵的条件数计算 2. 某一给定矩阵的列主元高斯消元法求解方程组的精度估计, 并与实际的精度相比较.

## 问题分析

本次作业可以略去此部分, 因为作业的内容是实现课本上给出的算法.

## 结果展示

第一题的运行结果如下:

```
Microsoft Visual Studio 调试 × + ▾

The condition number of 5x5 Hilbert matrix is 943656
The condition number of 6x6 Hilbert matrix is 2.90703e+07
The condition number of 7x7 Hilbert matrix is 9.85195e+08
The condition number of 8x8 Hilbert matrix is 3.38728e+10
The condition number of 9x9 Hilbert matrix is 1.09965e+12
The condition number of 10x10 Hilbert matrix is 3.53538e+13
The condition number of 11x11 Hilbert matrix is 1.23062e+15
The condition number of 12x12 Hilbert matrix is 3.83175e+16
The condition number of 13x13 Hilbert matrix is 4.63576e+17
The condition number of 14x14 Hilbert matrix is 1.409e+19
The condition number of 15x15 Hilbert matrix is 1.03256e+18
The condition number of 16x16 Hilbert matrix is 1.97409e+18
The condition number of 17x17 Hilbert matrix is 1.84597e+18
The condition number of 18x18 Hilbert matrix is 9.70942e+19
The condition number of 19x19 Hilbert matrix is 3.98036e+19
The condition number of 20x20 Hilbert matrix is 2.99872e+18

D:\Code\数值代数\out\build\x64-Debug\hw2\hw2_1.exe (进程 22592)已退出，代码为 0 (0x0)。
按任意键关闭此窗口 . . .|
```

第二题的运行结果如下:

```
Microsoft Visual Studio 调试 × + ▾

n = 5 , estimated error = 1.82026e-15, real error = 6.97861e-16
n = 6 , estimated error = 2.43068e-15, real error = 2.88638e-16
n = 7 , estimated error = 6.84023e-15, real error = 1.04326e-15
n = 8 , estimated error = 5.56201e-15, real error = 9.59431e-16
n = 9 , estimated error = 1.02342e-14, real error = 1.15514e-15
n = 10 , estimated error = 2.84167e-14, real error = 6.62806e-15
n = 11 , estimated error = 2.12467e-15, real error = 3.09894e-16
n = 12 , estimated error = 2.48411e-14, real error = 2.13979e-15
n = 13 , estimated error = 2.98923e-13, real error = 4.03084e-14
n = 14 , estimated error = 2.0136e-12, real error = 2.94023e-13
n = 15 , estimated error = 4.68935e-13, real error = 4.88201e-14
n = 16 , estimated error = 6.82071e-12, real error = 9.60615e-13
n = 17 , estimated error = 6.8993e-13, real error = 7.83249e-14
n = 18 , estimated error = 2.36757e-11, real error = 1.68812e-12
n = 19 , estimated error = 4.32183e-11, real error = 4.95162e-12
n = 20 , estimated error = 3.96706e-10, real error = 2.04452e-11
n = 21 , estimated error = 4.67975e-10, real error = 4.36237e-11
n = 22 , estimated error = 4.53096e-11, real error = 4.54568e-12
n = 23 , estimated error = 1.19737e-09, real error = 4.62963e-11
n = 24 , estimated error = 5.13499e-10, real error = 2.7154e-11
n = 25 , estimated error = 4.30218e-09, real error = 3.57882e-10
n = 26 , estimated error = 4.36393e-10, real error = 5.7322e-11
n = 27 , estimated error = 5.33637e-08, real error = 2.41093e-09
n = 28 , estimated error = 1.93316e-08, real error = 1.54963e-09
n = 29 , estimated error = 1.42051e-08, real error = 1.19511e-09
n = 30 , estimated error = 4.11616e-09, real error = 3.96429e-10

D:\Code\数值代数\out\build\x64-Debug\hw2\hw2_2.exe (进程 29624)已退出，代码为 0 (0x0)。
按任意键关闭此窗口 . . .|
```

## 讨论总结

利用numpy提供的现成的函数计算Hilbert矩阵的无穷条件数结果如下:

```
1.py x
D: > Code > 数值代数报告 > hw2 > 1.py > ...
1 import numpy as np
2
3 for n in range(5, 21):
4     H = np.zeros((n, n))
5     for i in range(n):
6         for j in range(n):
7             H[i, j] = 1.0 / (i + j + 1) # 因为索引从0开始, 所以是
8                                     i+j+1
9     cond = np.linalg.cond(H, p=np.inf)
10    print(f"n={n}: κ_inf(H) = {cond}")

TERMINAL CHAT
PS C:\Users\ASUS> python -u "d:\Code\数值代数报告\hw2\1.py"
n=5: κ_inf(H) = 943656.0000017105
n=6: κ_inf(H) = 29070279.007664092
n=7: κ_inf(H) = 985194890.8606278
n=8: κ_inf(H) = 33872792587.596287
n=9: κ_inf(H) = 1099652732973.805
n=10: κ_inf(H) = 35353994546522.04
n=11: κ_inf(H) = 1231074508396630.2
n=12: κ_inf(H) = 4.003338526130967e+16
n=13: κ_inf(H) = 4.361642067529362e+18
n=14: κ_inf(H) = 7.605093403667743e+17
n=15: κ_inf(H) = 4.9373760552201094e+17
n=16: κ_inf(H) = 1.7281290513439585e+18
n=17: κ_inf(H) = 1.1948296673065009e+18
n=18: κ_inf(H) = 2.440109713523234e+19
n=19: κ_inf(H) = 4.599364003480625e+18
n=20: κ_inf(H) = 3.364984921293334e+18
PS C:\Users\ASUS>
```

与我在作业中获得的结果在n较小时基本一致, 可以认为我在作业中能够正确的完成计算无穷条件数.

在第二题中, 给出的精度估计也可以看出是相当好的, 精度也相当高, 并没有出现课本上提及的特殊情况, 由此可以看出题目给出的矩阵并不病态.