Vietnam National University, Ho Chi Minh City
University of Technology
Faculty of Computer Science and Engineering

MATHEMATICAL MODELING
(CO2011)

Assignment (Semester: 231, Duration: 06 weeks)

"Stochastic Programming and Applications"

(Version 0.1, *in Preparation*)

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Abstract. In the assignment of the MM course (CO2011) this semester, students will get acquainted with a few trendy applications of stochastic optimization (SO), in particular, Stochastic Programming (SP). SP and (SO) have become extremely useful for computer scientists when they formulate optimization problems accepting uncertainty, specifically problems in Modern Urban Management with `Efficient Evacuation- Transportation, Facility Location`, and `Smart Logistics`.

Team Assignment Goals: Learn concepts, ideas and methods to

a) obtain practical experiences in **modeling** linear constraints/objectives with *uncertainty*,

b) study Two-Stage Model and its generalization,

c) use a broader class of linear stochastic programs in **various contexts**, particularly study the Stochastic Linear Program for Evacuation Planning in Disaster Responses (SLP-EPDR), a typical instance of Safe Urban Management.

1 Introduction to Stochastic Programming and Optimization

We have seen several classes of optimization problems, such as linear programming, integer programming (learned in the MM subject of our CSE curriculum), for which advanced theory for **deterministic model** exist and efficient numerical methods have been found.

This assignment will show us how mathematical concepts for modeling optimization problems involve *randomness - uncertainty*. Such problems are called `stochastic optimization problems` or shortly **Stochastic Programming (SP)**. Practical problems in S & E (science and engineering) originally are not modeled as stochastic or deterministic. The engineers and scientists determine whether to model the phenomenon as either stochastic or deterministic based on the problem to be solved. In deterministic models, the model's output is entirely determined by the parameter values and the initial conditions. On the other hand, a **stochastic model** is a tool that allows for random variation in one or more inputs over time. Randomness or uncertainty can be present in both the criterion (function) being optimized and the constraints of the problem. Briefly, SP can be viewed as mathematical programming with random parameters (e.g., **random variables**, the variables whose possible values depend on the outcomes of a chance phenomenon). We begin with `Generic SP` and `Linear SP` respectively in Sections **??** and 2. [1]

1.1 *Programming? What is Stochastic Programming? Uncertainty?*

- **Mathematical Optimization** is about decision making, mostly uses mathematical methods.
- **Stochastic Programming** (SP) is about decision making under *uncertainty*.
  View it as 'Mathematical Programming (Optimization) with *random* parameters'.
- *Stochastic linear programs* are linear programs (i.e. its objective function is linear) in which some problem data may be considered uncertain.
- *Recourse programs* are those in which some decisions or recourse (remake, modify) actions can be taken after uncertainty is disclosed.

[1] Linear SP means SLP- Stochastic linear program, whose objective function is linear.
REMINDER: Each random variable possesses a specific `probability distribution` function. The probability distributions of discrete variables [as binomial or Poisson] are specified in terms of probability **mass** functions.
The probability distributions of continuous variables [as beta, exponential or Gaussian] are specified in terms of probability **density** functions.

khai niem co ban gia dinh- dong luc

1.2 *Basic concepts, assumptions - Motivation*

Definition 1 (**Linear program (LP) with random parameters - SLP**).

A Stochastic linear program (SLP) is

$$\text{Minimize } Z = g(\boldsymbol{x}) = f(\boldsymbol{x}) = \ \boldsymbol{c}^T \cdot \boldsymbol{x}, \quad \text{s.t. } \ A\ \boldsymbol{x} = \boldsymbol{b}, \quad \text{and} \quad T\ \boldsymbol{x} \geq h$$

with $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ (decision variables),
doi voi
certain real matrix A and vector $\boldsymbol{b}$ (for deterministic constraints),
xac dinh co hoi rang buoc xs
and with random parameters T, h in $T\ \boldsymbol{x} \geq h$ define chance or probabilistic constraints. ■

don gian

♦ **EXAMPLE 1** (First simple motivation). *We consider the following optimization*

$$\text{Minimize } z = x_1 + x_2, \quad \text{subject to } \ x_1 \geq 0, x_2 \geq 0.$$

$$\begin{cases} \omega_1\ x_1 + x_2 \geq 7; \\ \omega_2\ x_1 + x_2 \geq 4. \end{cases}$$

dong nhat tuan theo
where parameters ω_1, ω_2 be uniform (random) variables following distributions
chinh xac la
$\text{Uniform}(a, b)$, precisely $\omega_1 \sim \text{Uniform}(1, 4)$, $\quad \omega_2 \sim \text{Uniform}(1/3, 1)$.

- When both $\omega_1 = \omega_2 = 1$ then the two conditons becomes

  duong cham xanh
  $x_1 + x_2 = 4$ making the red line, and $x_1 + x_2 = 7$ making the dotted blue line,
  ro rang dat duoc khu vuc kha thi chua day du mui ten hinh 1
  you obviously obtain the feasible region fully contaning the green arrow (Figure 1).

  How do we solve this problem if ω_1, ω_2 really are uniform (random) variables?

- **What do we mean by solving this problem**?

cho va xem cach tiep can gia su

1. The **wait-and-see** approach: Suppose it is possible to decide about the decision variables
   quan sat mot phan
   $\mathbf{x} = [x_1, x_2]$ after the observation of the random vector $\boldsymbol{\omega} = [\omega_1, \omega_2]$? [partially representing
   ko can
   for *data uncertainty* of the problem.] Can we solve the problem without waiting?

2. Yes, we can solve the problem but **no waiting**, i.e. we need to decide on $\mathbf{x} = [x_1, x_2]$ **before**
   phai lam gi khi
   knowing the values of $\boldsymbol{\omega} = [\omega_1, \omega_2]$?. We need to decide what to do about not knowing $\boldsymbol{\omega}$.
   ddoan
   We suggest (a) Guess at uncertainty, and (b) Probabilistic Constraints (see Definition 1).

   hop ly

   (a) Guess at uncertainty We will guess few reasonable values for $\boldsymbol{\omega}$. Three (reasonable)
   muc do rui ro
   suggestions – each of which tells us something about our level of 'risk'
   ko thien vi gia tri trug binh
   ♦ `Unbiased`: Choose mean values for each random parameters $\boldsymbol{\omega}$
   bi quan
   ♦ `Pessimistic`: Choose worst case values for $\boldsymbol{\omega}$
   lac quan
   ♦ `Optimistic`: Choose best case values for $\boldsymbol{\omega}$.

ex phan bo
For instance, use **Unbiased method**, look at EXAMPLE 1, uniform distributions $\text{Uniform}(a, b)$
ro rang co y
clearly have mean $(a + b)/2$, so $\widehat{\boldsymbol{\omega}} = (\frac{5}{2}, \frac{2}{3})$. Our program now becomes

$$\text{Minimize } z = x_1 + x_2$$

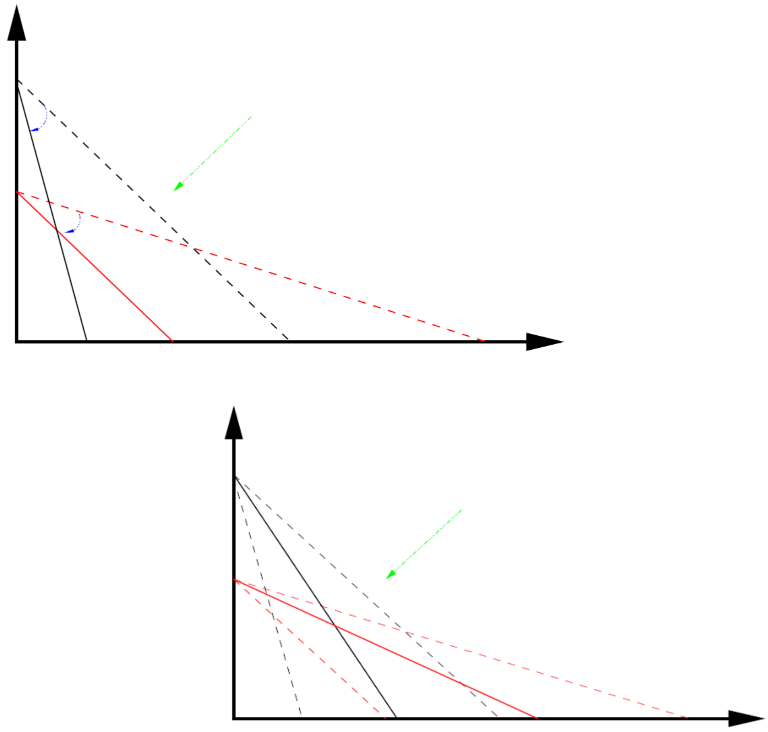


Fig. 1: *Simple SP with two decision variables*

having opt value $z_1 = \dfrac{50}{11}$ at point $\widehat{\mathbf{x}} = (\widehat{x}_1, \widehat{x}_2) = 18/11, 32/11$

$$\text{if subject to } \begin{cases} \dfrac{5}{2}\, x_1 + x_2 \geq 7; & \dfrac{2}{3}\, x_1 + x_2 \geq 4; \\ x_1 \geq 0, & x_2 \geq 0. \end{cases}$$

thi sao

nhung cach

How about `Pessimistic` and `Optimistic` ways? ■

1 giai doan

ko truy doi

2 One-Stage Stochastic linear programming - No recourse (1-SLP)

Definition 2 (**SLP with one-stage (No recourse) : 1-SLP**). *Consider the following program $LP(\boldsymbol{\alpha})$ that is parameterized by the random vector $\boldsymbol{\alpha}$:*

$$\text{Minimize } Z = g(\boldsymbol{x}) = f(\boldsymbol{x}) = \ \boldsymbol{c}^T \cdot \boldsymbol{x} = \sum_{j=1}^{n} c_j\ x_j$$

$$s.\ t.\quad A\ \boldsymbol{x} = \boldsymbol{b}, \quad \textit{(certain constraints)}$$

$$\textit{and} \quad T\ \boldsymbol{x} \geq h \quad \textit{(stochastic constraints)}$$

with assumptions that

1. matrix $T = T(\boldsymbol{\alpha})$ and (vector) $h = h(\boldsymbol{\alpha})$ express uncertainty via stochastic constraints

$$T(\boldsymbol{\alpha})\ \boldsymbol{x} \geq h(\boldsymbol{\alpha}) \iff \alpha_1\ x_1 + \cdots + \alpha_n\ x_n \geq h(\boldsymbol{\alpha})$$

2. values (T, h) not known: they are unknown before an instance of model occurs, $h(\boldsymbol{\alpha})$ depends only on random α_j;
3. **uncertainty** is expressed by probability distribution of random parameters $(\alpha_j) = \boldsymbol{\alpha}$ so deterministic LP is the degenerate case of Stochastic LP when α_j are constant,

- We deal with decision problems where the vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_n) \in \mathcal{X}$ of decision variables must be made before the realization of parameter vector $\boldsymbol{\alpha} \in \Omega$ is known.
- Often we set lower and upper bounds for $\boldsymbol{x}$ via a domain $\mathcal{X} = \{\boldsymbol{x} \in \mathbb{R}^n : \boldsymbol{l} \leq \boldsymbol{x} \leq \boldsymbol{u}\}$.

APPROACHES: In Stochastic Programming we utilize some assumption and facts.

Fundamental assumption- We know a (joint) probability distribution of data. Hence the first approach gives Probabilistic (Chance) Constraint LP.

The Scenario Analysis- not perfect, but useful, is the second approach. The scenario approach assumes that there are a finite number of decisions that nature can make (outcomes of randomness). Each of these possible decisions is called a **scenario**.

2.1 APPROACH 1 : use Chance constraint and Acceptable risk

- We can replace $T\ \boldsymbol{x} \geq h$ by probabilistic constraints $\mathbb{P}[T\ \boldsymbol{x} \geq h] \geq p$ [2]

  for some prescribed reliability level $p \in (.5, 1)$, (to be determined by problem owner.)

  The LP in Definition (2) above with random parameters $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots]$ then is called **Probabilistic Constraint LP**, or just **1-SLP**.

- Risk then is taken care of explicitly, if define an

$$\text{acceptable risk} \quad r_x := \mathbb{P}[Not\ (T\ \boldsymbol{x} \geq h)] = \mathbb{P}[T\ \boldsymbol{x} \leq h] \leq 1 - p$$

  then $(1-p)$ is `maximal acceptable risk`.

  The chance constraint $T\ \boldsymbol{x} \leq h$ implies that

  the acceptable risk r_x is less than a specified maximal $1 - p \in (0, 1)$.

Definition 3. *Stochastic LP or* ***1-SLP with Probabilistic Constraints*** *is defined by a random coefficients* $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ *in chance constraints, and a linear objective* $f(\boldsymbol{x})$*:*

$$SP: \quad \min_{\boldsymbol{x}} Z, \qquad Z = f(\boldsymbol{x}) = \boldsymbol{c}^T \cdot \boldsymbol{x} = \sum_{j=1}^{n} c_j\ x_j, \quad c_j \in \mathbb{R} \tag{1}$$

$$s.\ t. \begin{cases} A\ \boldsymbol{x} = \boldsymbol{b} & (\boldsymbol{x} = (x_1, x_2, \cdots, x_n) \text{ makes decision variables}) \\ \mathbb{P}[T\ \boldsymbol{x} = \boldsymbol{\alpha} \cdot \boldsymbol{x} \leq h] \leq (1-p) & (0 < p < 1). \end{cases}$$

NOTE: we use parameter vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots]$ in general, and denote $\boldsymbol{\omega} = [\omega_1, \omega_2, \cdots, \omega_S]$ specifically for states s called *scenarios*. We treat each scenario $\omega \in \boldsymbol{\omega}$ possibly by a combination of many random parameters α_i at once in a SP.

[2] or also replace $T\ \boldsymbol{x} \leq h$ by with $\mathbb{P}[T\ \boldsymbol{x} \leq h] \leq 1 - p$

2.2 APPROACH 2: for stochastic constraints $T(\boldsymbol{\alpha})\ \boldsymbol{x} \leq h(\boldsymbol{\alpha})$

Use Scenario analysis of $T(\boldsymbol{\alpha})\ \boldsymbol{x} \leq h(\boldsymbol{\alpha})$

For every scenario $(T^s; h^s),\ s = 1, \dots, S$, solve

$$\text{Minimize } \{f(\boldsymbol{x}) = \boldsymbol{c}^T \cdot \boldsymbol{x}; \quad \text{s.t.} \quad A\ \boldsymbol{x} = \boldsymbol{b},\ T^s\ \boldsymbol{x} \leq h^s\ \}$$

This kind of program targets a specific linear objective while accounting for a probability function associated with **various scenarios**. Hence, we find an overall solution by looking at the scenario solutions $\boldsymbol{x}^s\ (s = 1, \dots, S)$.

Advantage: Each scenario problem is an LP. / vs / **Disadvantage**: discrete distribution $\longrightarrow$ mixed-integer LP model. (In general: possibly non-convex model).

3 Generic Stochastic Programming (GSP) with RECOURSE

We focus on modeling and leave out details if not essential for understanding concepts.

Definition 4 (Stochastic program in two stages (generic **2-SP** problem))**.** *The two-stage stochastic program (2-SP) extended from Definition 2 has the form*

$$2-SP: \quad \min_{\boldsymbol{x}} g(\boldsymbol{x}) \quad \textit{with } g(\boldsymbol{x}) = f(\boldsymbol{x})\ +\ \mathbf{E}_\omega[v(\mathbf{x}, \boldsymbol{\omega})] \tag{2}$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ is the first stage decision variables,
*$f(\boldsymbol{x})$ can be linear or not, a part of the **grand objective** function $g(\boldsymbol{x})$.*

** The mean $Q(\mathbf{x}) := \mathbf{E}_\omega[v(\mathbf{x}, \boldsymbol{\omega})]$ of a function*

$$v : \mathbb{R}^n \times \mathbb{R}^S \to \mathbb{R}$$

upon influences of scenarios $\boldsymbol{\omega}$. $Q(\mathbf{x})$ is the optimal value of a certain second-stage problem

$$\min_{\boldsymbol{y} \in \mathbb{R}^p} \boldsymbol{q}\ \boldsymbol{y} \mid \textit{subject to} \quad T \cdot\ \boldsymbol{x} +\ W \cdot \boldsymbol{y} = h. \tag{3}$$

Vectors $\boldsymbol{\alpha} = \boldsymbol{\alpha}(\omega)$ and $\boldsymbol{y} = \boldsymbol{y}(\omega)$ are named `correction, tuning or recourse` decision variables, only known after the experiment **e**.

Briefly we **Minimize total expected costs** $g(\boldsymbol{x}) = f(\boldsymbol{x})\ +\ Q(\mathbf{x})$ while satisfying

$$W \cdot \boldsymbol{y}(\boldsymbol{\omega}) = h(\boldsymbol{\omega}) - T(\boldsymbol{\omega}) \cdot\ \boldsymbol{x}$$

Here W is called $m \times p$ recourse matrix, and we begin with simple case of $m = 1$,
$\boldsymbol{q}$ is the unit recourse cost vector, having the same dimension as $\boldsymbol{y}$, and $\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{\omega}) \in \mathbb{R}^p$. ■

ELUCIDATION (On **Recourse modeling issues**)

- Our grand objective $g(\boldsymbol{x})$ is built up by $f(\boldsymbol{x})$ and $Q(\mathbf{x})$. Here $\boldsymbol{y}$ is the decision vector of a second-stage LP problem, value $\boldsymbol{y}$ depends on the realization of $(T, h) := (T(\boldsymbol{\omega}), h(\boldsymbol{\omega}))$. Recourse variables $\boldsymbol{y}(\boldsymbol{\omega}) \sim$ corrective actions e.g. use of alternative production resources (overtime...)
- Quantitative risk measure: size of deviations $h(\boldsymbol{\omega}) - T(\boldsymbol{\omega}) \cdot\ \boldsymbol{x}$ is relevant.
- Here RISK is described by *expected recourse costs* $Q(\mathbf{x})$ of the decision $\mathbf{x}$.
- Model reformulation in fact is needed: Where do $\boldsymbol{q}$ and W come from?

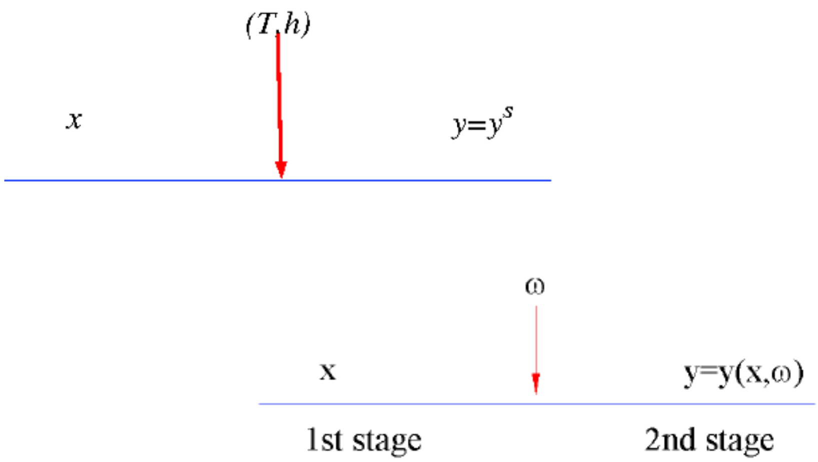


Fig. 2: *Standard view of two-stage stochastic program*
Courtesy of Maarten van der Vlerk, Univ. of Groningen, NL

4 Two-Stage Stochastic linear programming (2-SLP)

We now treat the **two-stage** stochastic LP with recourse action.

4.1 *Two-stage SLP Recourse model - (simple form)*

Definition 5 (Two-stage Stochastic LP With Recourse : 2-SLPWR). *The Two-stage Stochastic linear program With Recourse (2-SLPWR) or precisely with penalize corrective action generally described as*

$$2-SLP: \quad \min_{\boldsymbol{x}\in \mathbf{X}} \ \boldsymbol{c}^T \cdot \boldsymbol{x} + \min_{\boldsymbol{y}(\boldsymbol{\omega})\in \boldsymbol{Y}} \mathbf{E}_{\boldsymbol{\omega}}[\boldsymbol{q} \ \cdot \boldsymbol{y}]$$

or in general

$$2-SLP: \quad \min_{\boldsymbol{x}\in \mathbf{X}, \ \boldsymbol{y}(\boldsymbol{\omega})\in \boldsymbol{Y}} \mathbf{E}_{\boldsymbol{\omega}}[\boldsymbol{c}^T \cdot \boldsymbol{x} + v(\mathbf{x}, \boldsymbol{\omega})] \tag{4}$$

with $v(\mathbf{x}, \boldsymbol{\omega}) := \boldsymbol{q} \ \cdot \boldsymbol{y}$

subject to

$$A\ \boldsymbol{x} = \boldsymbol{b} \qquad \text{First Stage Constraints ,}$$

$$T(\boldsymbol{\omega}) \cdot \ \boldsymbol{x} + \ W \cdot \boldsymbol{y}(\boldsymbol{\omega}) = h(\boldsymbol{\omega}) \ \ \text{Second Stage Constraints}$$
$$\text{or shortly} \quad W \cdot \boldsymbol{y} = h(\boldsymbol{\omega}) - T(\boldsymbol{\omega}) \cdot \ \boldsymbol{x}$$

♦ This SLP program specify the above 2-SP (2) to the target - a specific *random grand objective* (function) $g(\boldsymbol{x})$ having

(1) the *deterministic* $f(\boldsymbol{x})$- being linear function, while accounting
(2) for a probability function $v(\mathbf{x}, \boldsymbol{\omega})$ associated with various scenarios $\boldsymbol{\omega}$.

♦ $\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{x}, \omega) \in \mathbb{R}^p_+$ is named recourse action variable for decision $\boldsymbol{x}$ and realization of $\boldsymbol{\omega}$.
Recourse actions are viewed as **Penalize corrective** actions in SLP.
The Penalize correction is expressed via the mean $Q(\mathbf{x}) = \mathbf{E}_{\boldsymbol{\omega}}[v(\mathbf{x}, \boldsymbol{\omega})]$. HOW to FIND IT?
Major Approaches- APPROACH 2: Scenarios analysis again

To solve system (4-4.1) numerically, approaches are based on a random vector $\boldsymbol{\alpha}$ having a finite number of possible realizations, called *scenarios*.

Expected value $Q(\mathbf{x})$ obviously for a discrete distribution of $\boldsymbol{\omega}$!

So we take $\Omega = \{\omega_k\}$ be a finite set of size S (there are a finite number of scenarios $\omega_1, \ldots, \omega_S \in \Omega$, with respective probability masses p_k).

Since $\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{x}, \omega)$ so the expectation of $v(\boldsymbol{y}) = v(\mathbf{x}, \boldsymbol{\omega}) := q \cdot \boldsymbol{y}$ (one cost q for all y_k) is

$$Q(\mathbf{x}) = \mathbf{E}_\omega[v(\mathbf{x}, \boldsymbol{\omega})] = \sum_{k=1}^{S} p_k\ q\ y_k = \sum_{k=1}^{S} p_k\ v(\mathbf{x}, \omega_k) \tag{5}$$

where

- p_k is the density of scenario ω_k, q is single unit penalty cost,
- and $q\ y_k = v(\mathbf{x}, \omega_k)$ - the penalty cost of using y_k units in correction phase, depends on both the first-stage decision $\mathbf{x}$ and random scenarios ω_k.

4.2 *Two-stage SLP Recourse model - (canonical form)*

We now fully characterize the system (4-4.1) in the linear case.

Definition 6 (Stochastic linear program With Recourse action (2-SLPWR))**.** The canonical 2-stage **stochastic linear** program with Recourse can be formulated as

$$2 - SLP: \quad \min_{\boldsymbol{x}} g(\boldsymbol{x}) \text{ with}$$

$$g(\boldsymbol{x}) := \ \boldsymbol{c}^T \cdot \boldsymbol{x} + v(\boldsymbol{y})$$

$$\text{subject to (s. t.)} \quad A\ \boldsymbol{x} = \boldsymbol{b} \ \text{ where } \ \boldsymbol{x} \in \mathbf{X} \subset \mathbb{R}^n, \boldsymbol{x} \geq \mathbf{0}$$

$$v(\boldsymbol{z}) := \min_{\boldsymbol{y} \in \mathbb{R}^p_+} \quad \boldsymbol{q}\ \boldsymbol{y} \quad \text{subject to} \quad W \cdot \boldsymbol{y} = h(\boldsymbol{\omega}) - T(\boldsymbol{\omega}) \cdot\ \boldsymbol{x} =: \boldsymbol{z}$$

where $v(\boldsymbol{y}) := v(\mathbf{x}, \boldsymbol{\omega})$ is the second-stage value function, and
$\boldsymbol{y} = \boldsymbol{y}(\boldsymbol{x}, \omega) \in \mathbb{R}^p_+$ is a recourse action for decision $\boldsymbol{x}$ and realization of $\boldsymbol{\omega}$.

1. The expected recourse costs of the decision $\mathbf{x}$ is $Q(\mathbf{x}) := \mathbf{E}_\omega[v(\mathbf{x}, \boldsymbol{\omega})]$ by Equation (5). [precisely expected costs of the recourse $\boldsymbol{y}(\boldsymbol{\alpha})$, for any policy $\mathbf{x} \in \mathbb{R}^n$.] Hence overally we minimize total expected costs $\min_{\boldsymbol{x} \in \mathbb{R}^n,\ \boldsymbol{y} \in \mathbb{R}^p_+} \quad \boldsymbol{c}^T \cdot \boldsymbol{x} + Q(\mathbf{x})$.

2. We design the 2nd decision variables $\boldsymbol{y}(\boldsymbol{\omega})$ so that we can (tune, modify, or) react to our original constraints (4.2) in an intelligent (or optimal) way: we call it **recourse** action!

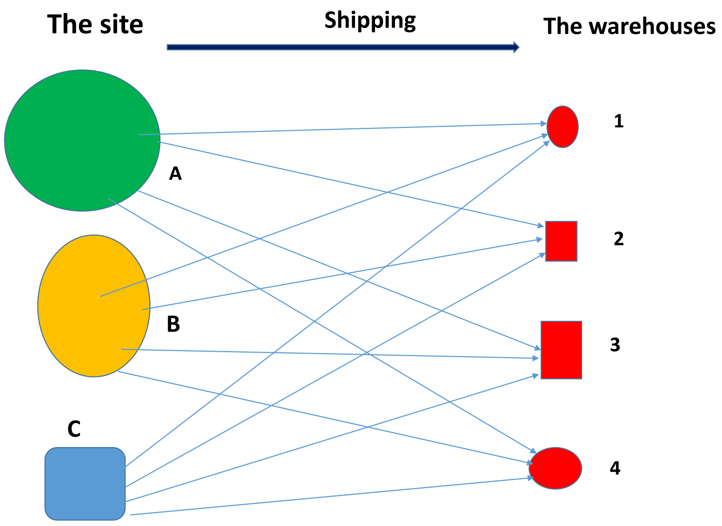
$$\mathbf{x} - - - - - - - - - - T, h, \boldsymbol{\omega} - - - - - - - -- > \boldsymbol{y}.$$

3. The optimal value of the 2nd-stage LP is $v_* = v(\boldsymbol{y}^*)$, with $\boldsymbol{y}^* = \boldsymbol{y}^*(\boldsymbol{x}, \omega)$ is its optimal solution, here $\boldsymbol{y}^* \in \mathbb{R}^p_+$. The total optimal value is $\boldsymbol{c}^T \cdot \boldsymbol{x}^* + v(\boldsymbol{y}^*)$. ■

PROBLEM 1. **[Industry- Manufacturing.]** (See [1, Chapter 1])

Consider an industrial firm **F** where a manufacturer produces n products.

There are different parts (sub-assemblies) to be ordered from in total m 3rd-party suppliers (the sites). This picture shows a transportation plan of the industrial firm **F** with $m = 3$ from suppliers and $n = 4$ production locations (products, or warehouses).



A unit of product i requires $a_{ij} \geq 0$ units of part j , where $i = 1, \ldots, n$ and $j = 1, \ldots, m$. The demand for the products is modeled as a random vector $\boldsymbol{\omega} = \mathbf{D} = (D_1, D_2, \cdots, D_n)$.

The second-stage problem:

For an observed value (a realization) $\boldsymbol{d} = (d_1, d_2, \cdots, d_n)$ of the above random demand vector $\mathbf{D}$, we can find the best production plan by solving the following stochastic linear program (SLP) with decision variables $\boldsymbol{z} = (z_1, z_2, \cdots, z_n)$ - the number of units produced,

and **other** decision variables $\boldsymbol{y} = (y_1, y_2, \cdots, y_m)$ - the number of parts left in inventory

$$LSP: \quad \min_{\boldsymbol{z},\boldsymbol{y}} \; Z = \sum_{i=1}^{n} (l_i - q_i)\, z_i - \sum_{j=1}^{m} s_j y_j, \tag{6}$$

where $s_j < b_j$ (defined as pre-order cost per unit of part j), and

$x_j, \; j = 1, \ldots, m$ are the numbers of parts to be ordered before production.

$$\text{subject to} \begin{cases} y_j = x_j - \displaystyle\sum_{i=1}^{n} a_{ij}\, z_i, \quad j = 1, \ldots, m \\ 0 \leq z_i \leq d_i, \quad i = 1, \ldots, n; \quad y_j \geq 0, \; j = 1, \ldots, m. \end{cases}$$

The whole model (of the second-stage) can be equivalently expressed as

$$MODEL = \begin{cases} \min_{\boldsymbol{z},\boldsymbol{y}} \; Z = \; \boldsymbol{c}^T \cdot \boldsymbol{z} - \boldsymbol{s}^T \cdot \boldsymbol{y} \\ \text{with} \quad \boldsymbol{c} = (c_i := l_i - q_i) \text{ are cost coefficients} \\ \boldsymbol{y} = \boldsymbol{x} - A^T \; \boldsymbol{z}, \quad \text{where } A = [a_{ij}] \; \text{ is matrix of dimension } n \times m, \\ 0 \leq \boldsymbol{z} \leq \boldsymbol{d}, \quad \boldsymbol{y} \geq 0. \end{cases} \tag{7}$$

Observe that the solution of this problem, that is, the vectors $\boldsymbol{z}, \boldsymbol{y}$ depend on realization $\boldsymbol{d}$ of the random demand $\boldsymbol{\omega} = \boldsymbol{D}$ as well as on the 1st-stage decision $\boldsymbol{x} = (x_1, x_2, \cdots, x_m)$.

The first-stage problem:

The whole 2-SLPWR model is based on a popular rule that **production $\geq$ demand**.

Now follow distribution-based approach, we let $Q(\boldsymbol{x}) := \mathbf{E}[Z(\boldsymbol{z}, \boldsymbol{y})] = \mathbf{E}_{\omega}[\boldsymbol{x}, \boldsymbol{\omega}]$ denote the optimal value of problem (6). Denote

$\boldsymbol{b} = (b_1, b_2, \cdots, b_m)$ built by preorder cost b_j per unit of part j (before the demand is known). The quantities x_j are determined from the following optimization problem

$$\min g(\mathbf{x}, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{b}^T \cdot \boldsymbol{x} + Q(\boldsymbol{x}) = \boldsymbol{b}^T \cdot \boldsymbol{x} + \mathbf{E}[Z(\boldsymbol{z})] \tag{8}$$

where $Q(\boldsymbol{x}) = \mathbf{E}_{\omega}[Z] = \sum_{i=1}^{n} p_i\ c_i\ z_i$ is taken w. r. t. the probability distribution of $\boldsymbol{\omega} = \boldsymbol{D}$.

The first part of the objective function represents the **pre-ordering cost** and $\boldsymbol{x}$. In contrast, the second part represents the **expected cost** of the optimal production plan (7), given by the updated ordered quantities $\boldsymbol{z}$, already employing random demand $\boldsymbol{D} = \boldsymbol{d}$ with their densities.

ELUCIDATION

- Decision variables include vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^m$, and also $\boldsymbol{z} \in \mathbb{R}^n$.
- After the demand D is observed, the manufacturer may decide which portion of the demand is to be satisfied so that the available numbers of parts are **not** exceeded. It costs additionally l_i to satisfy a unit of demand for product i, and the unit selling price of this product is q_i.
- After the demand D becomes known, we determine how much of each product to make. The parts not used are assessed salvage values s_j, giving vector $\boldsymbol{s} = (s_1, s_2, \cdots, s_m)$. $\triangleright$

SUMMARY

1. Problem (6)–(8) is an example of a **two-stage stochastic programming** problem, where (6) is called the *second-stage* problem and (8) is called the *first-stage* problem. As (6) contains random demand $\boldsymbol{D}$, its optimal value $Q(\boldsymbol{x}, \boldsymbol{d})$ is a random variable.
2. The 1st-stage decisions $\boldsymbol{x}$ should be made before a realization of the random data $\boldsymbol{D}$ becomes available and hence should **be independent** of the random data. The $\boldsymbol{x}$ variables are often referred to as `here-and-now` decisions.
3. The second-stage decision variables $\boldsymbol{z}$ and $\boldsymbol{y}$ in (6)

   are made after observing the random data and are functions of the data $\boldsymbol{d}$. They are referred to as `wait-and-see` decisions (solution).
4. The problem (6) is feasible for every possible realization of the random data $\boldsymbol{d}$; for example, take $\boldsymbol{z} = 0$ and $\boldsymbol{y} = \boldsymbol{x}$.

5 APPLICATION I: Stochastic Linear Program for Evacuation Planning In Disaster Responses (SLP-EPDR)

AIM - MOTIVATION

We want to utilize two-stage SLP models to evacuate the affected people to safe areas during disaster response. The case study is based on researches of Li Wang [3] and Esra Koca [4].

The main goal of **emergency response** is to provide shelter and assistance to affected people as soon as possible. The optimal evacuation plan for affected people is one of the dominant components in emergency response after a disaster, and lots of scholars have denote their efforts into this interesting problem.

APPROACHES: The stochastic programming with recourse (**Dantzig**, [5]) is very popular method for dealing with randomness of factors, and this method is to find non-anticipative decisions that have to be made before knowing the realizations of random variables. According to the number of stages, the stochastic programming with recourse problem is generally referred to

as two-stage/multi-stage stochastic programming.

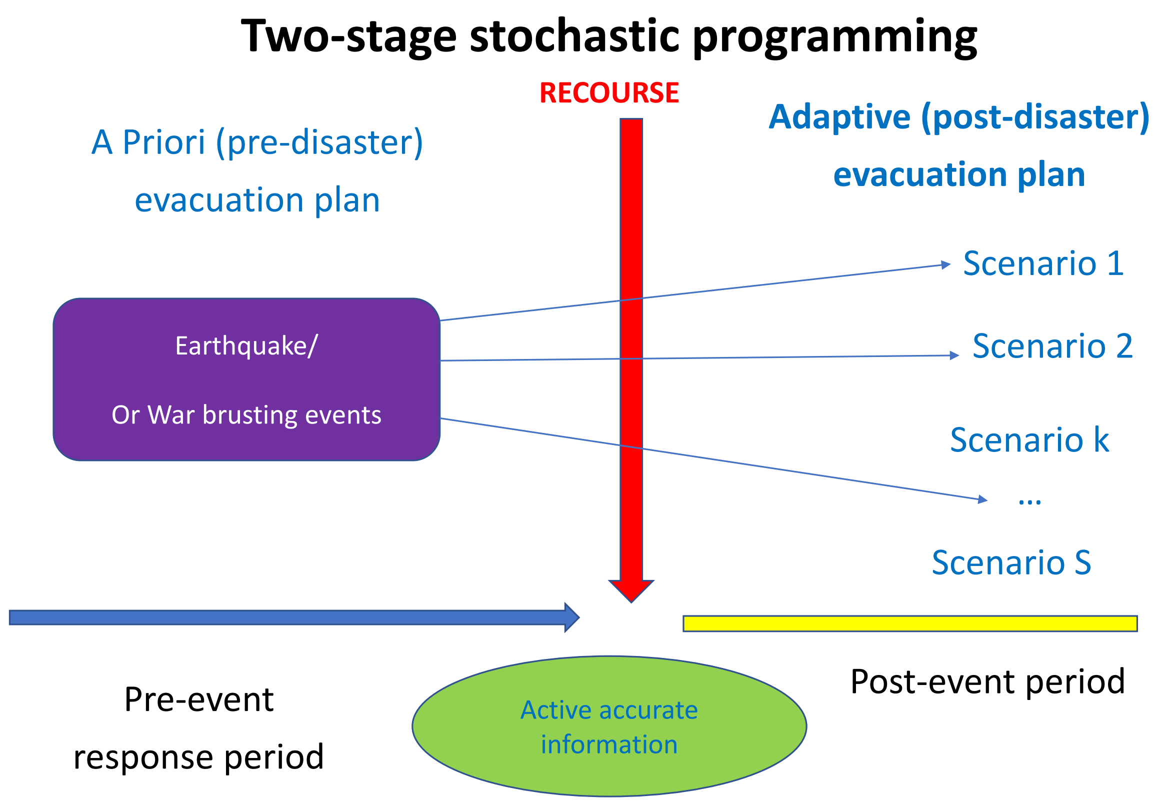


Fig. 3: *An Illustration for Occurrence of Earthquake or War brusting events*

[3] A two-stage stochastic programming framework for evacuation planning in disaster responses, **Journal of Computers & Industrial Engineering**, vol 145, 2020 Elsevier;

[4] Two-stage stochastic facility location problem with disruptions and restricted shortages **Journal of Computers & Industrial Engineering**, vol 183, 2023 Elsevier.

[5] Dantzig, G. (1955), Linear Programming under Uncertainty. Management Science, 1, 197–206.

5.1 *BACKGROUND - OPEN ISSUSES*

A two-stage stochastic scenario-based programming model should be proposed to evacuate affected people in disaster areas. The first-stage decisions are the robust and reliable evacuation plan for all levels of disaster. The second-stage decisions involve the evacuation plan for affected people in response to specific scenario-based road conditions

We would use a set of discrete scenarios to represents potential magnitudes of the disaster, which tries to formulate a model that combines **pre-event emergency evacuation plan** with scenario-based evacuation plan for affected people after an event. Specifically, a part of transportation roads may be destroyed during the event, causing stochastic travel times and capacities when traveling on the road. In other words, non-anticipative first-stage decisions are made in the advance of realization of uncertainty.

The 2nd-stage decisions (recourse), which are conditional on the 1st-stage decisions, are made after the realization of stochastic travel times and capacities. Therefore, the objective is to make the optimal pre-event evacuation plan in the first stage, which is under uncertainty conditions to be faced in the 2nd stage.

5.2 *ANALYSIS- Problem statements*

Representation of the evacuation problem

As a 2-SLP (two-stage stochastic programming)

We need to characterize the **evacuation process** under certain assumptions, and to make sure that **the evacuation phase** should be divided into two stages according to the acquisition time of accurate information. Briefly, the objective is to make

the optimal evacuation planning in the 1st stage under uncertainty to be faced in the 2nd stage.

5.3 *MODEL FORMULATION To ALGORITHMIC SOLUTION*

We have to properly define and discuss decision variables, system constraints and the objective function with relevant notations used in the mathematical formulation.

HINT: use Table 1, 2 of

Ref. 1 = Li Wang, *A two-stage stochastic programming framework for evacuation planning in disaster responses*, Journal of Computers & Industrial Engineering, vol 145, 2020 **Elsevier**

Two-stage stochastic evacuation planning model

The evacuation's objective is to obtain

(1) a **robust** evacuation plan in the first stage by

(2) the evaluation of **adaptive** evacuation plans in the second stage.

We should evaluate the evacuation plan of the first stage with
the **expected overall evacuation time** of each scenario's adaptive evacuation path, and the probability of occurrence of each scenario s is assumed as $p_s = \mu_s, \; s = 1, 2, \ldots, S$.

- The teams might employ model (9) and its equivalent models [in **Ref. 1**]-

  they are called time-dependent and stochastic two-stage evacuation planning models.

- Make sure that you fully explain system constraints and the objective function of models.

MODELING APPROACHES for MM-HCMUT-2023 Assignment

In this assignment we focus on the strategy of combining
a priori (pre-disaster) and adaptive (post-disaster) path selection, which can be achieved by the `two-stage stochastic programming`, to determine the evacuation plan for affected people, either upon earthquake event or war brusting events.

The teams of at most 5 students of HCMUT should

1. Understand PROBLEM 1 [produce n products satisfying **production** $\geq$ **demand**] via a numerical instance.

2. Employ the two-stage stochastic programming model that considers both a **priori** (pre-disaster) and **adaptive** (post-disaster) path selection to provide a priori evacuation plan for the affected people from dangerous areas to safe areas.

3. Formulate the explicit movement process of affected people when a disaster occurs, this paper proposed a min-cost flow model based on two-stage stochastic programming

6 QUESTIONS for ASSIGNMENT 2023

Students prepare a report consisting of the theoretic answer and computational solution following requests.

STANDARD WORK- two tasks for HCMUT MM intake 2023:

1. **To PROBLEM 1** [produce n products satisfying **production** $\geq$ **demand**]. (4 points)

   Use the 2-SLPWR model given in Equations 7 and 8 when $n = 8$ products, the number of scenarios $S = 2$ with density $p_s = 1/2$, the number of parts to be ordered before production $m = 5$, we randomly simulate data vector $\boldsymbol{b}$, $\boldsymbol{l}$, $\boldsymbol{q}, \boldsymbol{s}$ and matrix A of size $n \times m$.

   We also assume that the random demand vector $\boldsymbol{\omega} = \mathbf{D} = (D_1, D_2, \cdots, D_n)$ where each ω_i with density p_i follows the binomial distribution $\text{Bin}(10, 1/2)$.

   REQUEST: build up the nummerical models of Equations 7 and 8 with simulated data. Find the optimal solution $\mathbf{x}, \boldsymbol{y} \in \mathbb{R}^m$, and $\boldsymbol{z} \in \mathbb{R}^n$ by suitable soft (as GAMSPy)

2. **To the SLP-EPDR: Algorithmic Solutions** (6 points)

   Few Solution algorithms are given in Section 4 of **Ref. 1**, and this year 20023 CSE - HCMUT students may try only the first approach (Algorithm 1) based on Min-cost Flow Problem.

   Learn, implement and verify the effectiveness of the studied Algorithm or solving the two-stage stochastic evacuation planning model on a small grid network only, with Experiment Design approach. Precisely the simulated would have max of 50 nodes and 100 links ([in Section 5.1 of **Ref. 1**]).

7 COMPLEMENTS: Software For Stochastic Optimization

7.1 *Soft and programming languages For Stochastic Optimization*

Few well-known programming languages for stochastic modeling and analysis are

1. **GAMS/DECIS**: GAMS stands for General Algebraic Modeling Language, and is one of the most widely used modeling languages.

   DECIS is a system for solving *large-scale stochastic programs*, i.e. programs that include parameters (coefficients and right-hand sides) that are **not known** with certainty, but are assumed to be known by their probability distribution. It employs Benders decomposition and advanced *Monte Carlo sampling* techniques.

   DECIS includes a variety of solution strategies, such as solving the universe problem (all scenarios), the expected value problem, Monte Carlo sampling within the Benders decomposition algorithm. see `https://www.gams.com/latest/docs/S-DECIS.html`

   and `https://infanger.com/software/decis.html`

   * **GAMSPy** = GAMS + Python, `https://gamspy.readthedocs.io/en/latest/user/index.html`

2. **Stochastic Modeling Interface** (SMI),

   get from link `https://github.com/coin-or/Smi`, works on Linux OS

3. `R` : We can use `ARIMA` command in `R` to develop stochastic models.

   The background of `ARIMA` is complex, but you can try **? arima** in command line section to learn practical usages via manual.

   Also we must combine 3 soft WEKA, Rapid Miner and `R` in once to deal with SLP practically.

   Rapid Miner for academic use is available at `https://rapidminer.com/platform/educational/`

   A good research text in SHM is *Using R, WEKA and RapidMiner in Time Series Analysis of Sensor Data for Structural Health Monitoring*

   H. Kosorus, Jürgen Hönigl, J. Küng 22nd International Workshop... 29 August 2011 Computer Science International Workshop on Database and Expert Systems Applications, link

   `https://ieeexplore.ieee.org/document/6059835`

7.2 *Software Requirements For Stochastic Optimization*

Here are some key requirements for Stochastic Optimization, and SLP particularly.

1. Deterministic and stochastic together in the same model

2. End user selects distribution function:

   Stochastic definitions, including the specification of the probability distribution function and parameters, should be entirely data driven. The end user should be free to select the required distribution amongst a list of choices.

3. Coefficients and constraints can be made stochastic

   End users eventually ask for stochastic definition support for something not currently in their model, called **out-of-the-box support**. Stochastic variability to all types of data should be sought out that typically include:

- Costs, including materials, labor
- Input distributions and Output yields
- Process rates and downtime factors
- Minimum and maximum limits ...

4. Distribution functions and parameters defined at variable level.

   Stochastic definitions should be flexible enough to allow definition for each variable explicitly. A data-driven approach lets the end user assign a different distribution function to each and every coefficient, if necessary.

5. Individual stochastic definitions can be temporarily enabled/disabled. [Compare with Item 1.]

6. Visualizations to identify stochastic definitions should be taken into account.

7. Data checks identify conflicting data:

   Look for an optimization platform with built-in data checking. Generating stochastic values, like those used for variable constraints, can easily cause data errors. The best optimization modeling platforms include a large library of built-in data checks, which execute between the time the random values are generated and when the model is solved.

References

[1] A. Shapiro, D. Dentcheva, and A. Ruszczynski, *Lectures on stochastic programming: modeling and theory*. SIAM, 2021.

[2] S. W. Wallace and W. T. Ziemba, *Applications of stochastic programming*. SIAM, 2005.

[3] L. Wang, "A two-stage stochastic programming framework for evacuation planning in disaster responses," *Computers & Industrial Engineering*, vol. 145, p. 106458, 2020.

8 Instructions and requirements

Students have to follow the instructions and comply with the requirements below. *Lecturers do not solve the cases arising because students do not follow the instructions or do not comply with the requirements.*

8.1 Instructions

Students must work closely with the other members of their team of 3-5 members. The deadline for teaming up is October 22, 2023, via the BKeL class site. Team members must be in the same Group/Class, and you are not allowed to team up with other students from other Group/Class.

All aspects of this assignment will be quizzed (about 10 - 12 of 25 multiple-choice questions) in the subject's final exam. Therefore, team members must work together so that all of you understand all aspects of the assignment. The team leader should organize the work team to meet this requirement.

If you have any questions about the assignment during work, **please post that question on the class forum on BKeL.**

Regarding the background knowledge related to the topic, students should refer to all the references. However, it would be best to put all of them in the reference section of your report.

8.2 Requirements

- Deadline for submission: **December 04, 2023**. Students have to answer each question clearly and coherently.
- Write a report by using LaTeX in accordance with **the layout as in the template file** (you can find it on https://tinyurl.com/mt29ftrd).
- Each team when submitting its report **need to submit also a log file (diary)** in which clearly state: **weekly work progress for all 06 week(s)**, tasks, content of opinions exchanged of the members, ...
- Programming languages: C++/Python/Java/...(at your team preference)

8.3 Submission

- Students must submit their team report via the BK-eLearning system (to be opened by October 23, 2023): compress all necessary files (.pdf file, .tex file, coding file, ...) into a single file named *"Assignment-CO2011-CSE231-Team_name.zip"* and submit it to the assignment submission site.
- All of the meeting minutes of your group must be combined into one .txt file and consist of the % of each member's effort on the whole (total effort is 100%) on the first line.
- Noting that for each team, **only the leader will submit the report of the team.**

9 Evaluation and cheating treatment

9.1 Evaluation

Each assignment will be evaluated as in Table 1.

Table 1: Evaluation

Criteria	**Score (%)**
- Analyze, answer coherently, systematically, focus on the goals of the questions and requests	30%
- The programs are neatly written and executable	30%
- Correct, clear, and intuitive graphs & diagrams	20%
- Background section is well written, correct, and appropriate	15%
- Well written report and correct	5%

9.2 Cheating treatment

The assignment has been done by each group separately. Students in a group will be considered as cheating if:

- There is an unusual similarity among the reports (especially in the background section). In this case, ALL similar submissions are considered cheating. Therefore, the students of a group must protect their group's work.
- They do not understand the works written by themselves. You can consult from any source, but make sure that you know the meaning of everything you have written.

Students will be judged according to the university's regulations if the article is cheating.

10 APPLICATION II: SP in Telecommunications

NOTE: This proposal is aimed for graduate work, to master students in their second year only who want to study and research in Mathematical Optimization integrated with ML and Data Analytics.

OVERVIEW: `Telecommunications` problems have a long tradition of application of **advanced mathematical modeling** methods. Besides being a consumer of mathematical modeling, telecommunications motivated the development of applied mathematics, statistics, and computing areas. Essential chapters of the **theory of random-stochastic processes** have their roots in the work of telecommunication engineers.

10.1 Design Problems in Telecommunication

This section briefly presents different optimization problems under uncertainty that arise in telecommunications. Three levels of decisions are distinguished:

1. design of **structural elements** of telecommunication networks,
2. **top level design** of telecommunication networks, and
3. **design of optimal policies** of a telecommunication enterprise.

Examples of typical problems from each level show that the **stochastic programming paradigm** is a powerful approach for solving telecommunication design problems. Different types of uncertainty come into play at different levels. We distinguish three distinct scale levels: (a) technological, (b) network, and (c) enterprise.

The technological level corresponds to the smallest scale, and
the enterprise level matches with the largest and the most aggregated scale.

The technological level deals with the design of different elements of telecommunication networks, including switches, routers, and multiplexers. The critical decisions are the engineering decisions that define the structure for blueprints of these elements. Such blueprints depend on several parameters which should be chosen from the views of performance and quality of service.

Traditionally, performance evaluation of the elements of telecommunication networks was the domain of **Queueing Theory** [a crucial part of system performance evaluation in general].

10.2 The Technological level from the Statistical Viewpoint

The team carries out their project, with the theory shortly given in this section.

The central problem is to find the design parameters of a piece of telecommunication equipment which will ensure a given level of performance for a specified class of traffic patterns. In interesting cases, the performance (of a device or system) is practically measured by the function.

$$F(\boldsymbol{x},\ H) = \mathbf{E}_H[f(\boldsymbol{x},\ \xi)] = \int f(\boldsymbol{x},\ \xi)\ dH(\xi) \tag{9}$$

where

- $\boldsymbol{x}$ is the vector of *design parameters*,
- ξ denotes the values of a stochastic process $\{U(t) = \xi\}$ defined on an appropriate probability space which describes the interaction of traffic with the device, and
- H is the stationary distribution of the process $U(t)$.

- The function $f(\boldsymbol{x},\ \xi)$ with two arguments describes the performance of the device (equipment) rooted from the interaction of a given traffic value ξ and a given value of design parameters $\boldsymbol{x}$.

- $\mathbf{E}_H[.]$ is the expected value concerning H.

CONCEPTS: What is a stationary distribution?

Definition 7 (Strong stationarity).

Mathematically, strongly or *strictly stationary process* means for every $k \in \mathbb{N}$, and time points $t_i, h \in \mathbb{R}^+, i = 1..k$, for any time realization $\boldsymbol{z} = (t_1, \dots, t_k)$, the distributions of sequences

$Y_{\boldsymbol{z}} = (Y_{t_1}, \cdots, Y_{t_k})$ and $Y_{\boldsymbol{z}+h} = (Y_{t_1+h}, \cdots, Y_{t_k+h})$ are the same. That is, the joint cdf $F(.)$ of the process $\boldsymbol{Y}$ satisfies $F_{\boldsymbol{z}} = F_{\boldsymbol{z}+h}$, for any time lag $h \in \mathbb{R}^+$, that is

$$F_{\boldsymbol{z}}(\boldsymbol{b}) = F_{t_1,\dots,t_k}(b_1, \dots, b_k) = F_{\boldsymbol{z}+h} = F_{t_1+h,\dots,t_k+h}(b_1, \dots, b_k), \text{ for all point } (b_1, \dots, b_k) \in \mathbb{R}^k$$

where $F_{t_1,\dots,t_k}(b_1, \dots, b_k) = \mathbb{P}[Y_{t_1} \leq b_1, \dots, Y_{t_k} \leq b_k]$.

Strict stationarity is a very strong assumption; we often use the following.

Definition 8 (Weak stationarity).

A process $\{Y_i\} = Y_1, Y_2, \dots$ is *weakly stationary*, *second order* stationary, *covariance stationary*, or just *stationary* if the first two moments (mean and covariance) are unchanged by time shifts, or invariant as a function of time, meaning that $\mathbf{E}[Y_i]$ and $\mathrm{Cov}[Y_i, Y_{i+k}]$ do not depend on index i. Mathematically, $\{Y_i\}$ is a weakly stationary process if

- $\mathbf{E}[Y_i] = \mu$ (a constant) for all i; [named *mean stationarity*], and

- the *covariance* satisfies [*covariance stationarity*]

$$\mathrm{Cov}[Y_i, Y_{i+k}] = \mathbf{E}\big[(Y_i - \mu)(Y_{i+k} - \mu)\big] = \gamma(|k|) \tag{10}$$

  for all i and k and some function $\gamma(.)$.

NOTE: The values of the performance measure F in (9) should belong to the set of *admissible values* AV, which describes requirements for the grade of service, symbolized as $AV = [\phi_{min}, \phi_{max}]$, so the design parameters $\boldsymbol{x}$ should satisfy

$$\phi_{min} \leq F(\boldsymbol{x},\ H) \leq \phi_{max}.$$

DESIGN PROBLEMS- In conclusion, stochastic optimization shortly can be applied to other design problems. It is often important to carefully exploit each case's special structure to obtain approximations of performance measures. The design process follows the path

$$\text{Technological design} \implies \text{Network design}$$

10.3 The Network level

Briefly, the objective of the network level is to develop a design for the telecommunication network with a **given capability** to provide **a set of services** to a population of end users.

Results of technological design are used as the inputs to the network design.

This design should serve different and often conflicting purposes, e.g.,
satisfaction of demand, cost-effectiveness, or maintenance of a given service quality.

Important decisions to make at this level include placement and dimensioning of processing nodes and transmission links.

10.4 Two Learning Objectives

To learn and solve complex problems in urban areas where telecommunications is the essential backbone nowadays, by mathematical modeling (MM) methods, we propose a study of one of two planning problems using SP for Telecommunications, namely planning an Internet-based information service and evacuation during disasters while maintaining telecommunications.

- **P1: Planning of an Internet-based information service**
  - We consider the problem of deployment of an Internet-based information service on some territory, which can be a country, a region, a metropolis, or a city.
  - We assume that the network itself exists already, and the decision consists of deploying servers at the nodes of this network and assigning demand generated in different geographical locations to these servers. Among various aspects of the problem, we should consider *geographical dimension; uncertainty of demand and costs*; cost structure, which includes fixed and variable costs; competition and substitution between services; and relations between different market actors, e.g., network providers and service providers.
- **P2: Planning for evacuation upon disasters while maintaining telecommunications**

  Extreme events (man-made or natural disasters) might seriously influence telecommunications networks and services in metropolitan areas. Evacuation planning to properly n such disasters is a critical action and should be carefully thought of (early) and performed (on time), particularly when uncertainty exists.

HOW to carry out the project on SP for Telecommunications?

The team of at least four students will choose only one problem and present the model building with decisions being made. They should perform experimental work (with programs written in `Python` or `R`) for a proposed problem with small simulations or realistic data.

Objective 1 for Problem 1: SP for planning an Internet-based information service.

The team only learned how to plan an Internet-based information service (see Gaivoronski, [2, Subsection 32.3.1]) and write a report on major steps of such model development, using the theory of `Stochastic programming and Markov decision process`, namely

Step 1: Single-period deterministic cost minimization model

Step 2: Two period stochastic cost minimization model

Step 3: Two-period stochastic profit maximization model with pricing.

Objective 2 for Problem 2: SP for evacuation planning in disaster responses.

War, climate change impacts, man-made or natural disasters ... may strike a community with little warning and leave much damage and many casualties. Telecommunications service could be broken under such bad scenarios. How do you deal with such extremes?

The team learns how to build a two-stage stochastic programming to deal with that extreme cases, see recent research [3].