Examen 2

1. $(1\ 1\ 1\ 1\ 1\ 1)^t \cdot (1\ 1\ 1\ 1\ 1\ 1)$

$$\underset{6\times 1}{\begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix}} \underset{1\times 6}{\begin{pmatrix}1&1&1&1&1&1\end{pmatrix}} = \underset{6\times 6}{\begin{pmatrix}1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\\1&1&1&1&1&1\end{pmatrix}}$$

2. $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \sim_{f_4-f_1} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & -1 \end{pmatrix} \sim_{f_4+f_2} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix} \sim_{f_4-2f_3} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -2 \end{pmatrix} \sim_{\frac{f_4}{-2}}$

(a) Matriz escalonada
(b) Inversa

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \underset{\substack{f_1-f_4 \\ f_2-f_4 \\ f_3-f_4}}{\sim} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim_{f_2-f_3} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim_{f_1-f_2}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim_{f_4-f_1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} \sim_{f_4+f_2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{pmatrix} \sim_{f_4-2f_3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & -2 & 1 \end{pmatrix} \sim_{\frac{f_4}{-2}}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix} \underset{\substack{f_1-f_4 \\ f_2-f_4 \\ f_3-f_4}}{\sim} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} & -1 & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix} \sim_{f_2-f_3} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & 1 & -1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix} \sim_{f_1-f_2} \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & -1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \qquad A^{-1} = \frac{1}{|A|} (\mathrm{Adj}(A))^t = \begin{pmatrix} 1/2 & -1/2 & 0 & 1/2 \\ 0 & 1 & -1 & 0 \\ -1/2 & 1/2 & 0 & 1/2 \\ 1/2 & -1/2 & 1 & -1/2 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{vmatrix} \overset{f_4 - f_1}{=} \begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & 1 & -1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{vmatrix} = -1 - 1 - (-\cancel{1} + \cancel{1}) = -2 \neq 0$$

$$\mathrm{Adj}(A) = \begin{pmatrix} -1 & 0 & 1 & -1 \\ 1 & -2 & -1 & 1 \\ 0 & 2 & 0 & -2 \\ -1 & 0 & -1 & 1 \end{pmatrix} \qquad (\mathrm{Adj}(A))^t = \begin{pmatrix} -1 & 1 & 0 & -1 \\ 0 & -2 & 2 & 0 \\ 1 & -1 & 0 & -1 \\ -1 & 1 & -2 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1 - (1) = 0 \qquad - \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -1 \qquad \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -(1) = -1 \qquad - \begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = -(1) = -1 \qquad - \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = -(1) = -1 \qquad \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1 \qquad \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = 0$$

$$- \begin{vmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -(1 - (1)) = -0 = 0 \qquad - \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -(-(1)) = 1 \qquad - \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -(-(1+1)) = 2 \qquad \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1 \qquad \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -(1+1) = -2 \qquad - \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = -(1+1) = -2 \qquad - \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -(1 + 1 - (1)) = -1$$

3. $\begin{vmatrix} \alpha^2 & 3 & 3 & \cdots & 3 \\ 3 & \alpha^2 & 3 & \cdots & 3 \\ 3 & 3 & \alpha^2 & \cdots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 3 & 3 & 3 & \cdots & \alpha^2 \end{vmatrix} = \begin{vmatrix} \alpha^2+3(n-1) & 3 & 3 & \cdots & 3 \\ \alpha^2+3(n-1) & \alpha^2 & 3 & \cdots & 3 \\ \alpha^2+3(n-1) & 3 & \alpha^2 & \cdots & 3 \\ \vdots & & & & \vdots \\ \alpha^2+3(n-1) & 3 & 3 & \cdots & \alpha^2 \end{vmatrix} = (\alpha^2+3(n-1)) \begin{vmatrix} 1 & 3 & 3 & \cdots & 3 \\ 1 & \alpha^2 & 3 & \cdots & 3 \\ 1 & 3 & \alpha^2 & \cdots & 3 \\ \vdots & \vdots & & & \\ 1 & 3 & 3 & \cdots & \alpha^2 \end{vmatrix} \begin{array}{l} \\ f_2-f_1 \\ f_3-f_1 \\ \\ f_n-f_1 \end{array}$

(a) Determinante

(b) Si $n=9$, $\alpha=?$ para que el determinante valga 0

$$= (\alpha^2+3(n-1)) \cdot \begin{vmatrix} 1 & 3 & 3 & \cdots & 3 \\ 0 & \alpha^2-3 & 0 & \cdots & 0 \\ 0 & 0 & \alpha^2-3 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & \alpha^2-3 \end{vmatrix} = (\alpha^2+3(n-1))(\alpha^2-3)^{n-1}$$

$$n=9 \Rightarrow (\alpha^2+3(9-1))(\alpha^2-3)^{9-1} = (\alpha^2+24)(\alpha^2-3)^8 = 0 \Rightarrow (\alpha^2-3)^8 = 0 \Rightarrow \alpha^2-3=0 \Rightarrow \alpha^2=3 \Rightarrow \alpha = \pm\sqrt{3}$$

4. $\begin{cases} x+y+az = a-1 \\ x+y+z = a \\ x+(a+1)y+2z = 0 \end{cases}$

$$A=\begin{pmatrix}1&1&a\\1&1&1\\1&a+1&2\end{pmatrix} \quad |A| = \begin{vmatrix}1&1&a\\1&1&1\\1&a+1&2\end{vmatrix} = \begin{vmatrix}1&1&a\\0&0&1-a\\0&a&2-a\end{vmatrix} =$$

$$= \begin{vmatrix}0&1-a\\a&2-a\end{vmatrix} = -a(1-a) = a(a-1) = 0 \Rightarrow a=0$$
$$\Rightarrow a=1$$

(a) Compatibilidad según R-F

(b) Resolver por Crammer caso C.D

- $a \neq 0$ y $a \neq 1 \Rightarrow$ S.C.D

- $a = 0$

$$A_0 = \begin{pmatrix}1&1&0\\1&1&1\\1&1&2\end{pmatrix} \quad \begin{vmatrix}1&0\\1&1\end{vmatrix} = 1 \neq 0 \rightsquigarrow rg(A_0)=2$$

$$\begin{vmatrix}1&0&-1\\1&1&0\\1&2&0\end{vmatrix} = -2-(-1) = -1 \neq 0 \rightsquigarrow rg(A|b)=3$$

$\rightsquigarrow$ S.I

- $a = 1$

$$A_1 = \begin{pmatrix}1&1&1\\1&1&1\\1&2&2\end{pmatrix} \quad \begin{vmatrix}1&1\\1&2\end{vmatrix} = 2-1 = 1 \neq 0 \rightsquigarrow rg(A_1)=2$$

$$\begin{vmatrix}1&1&0\\1&1&1\\1&2&0\end{vmatrix} = 1-(2) = -1 \neq 0 \rightsquigarrow rg(A|b)=3$$

$\rightsquigarrow$ S.I

$$\begin{cases} x + y + az = a-1 \\ x + y + z = a \\ x + (a+1)y + 2z = 0 \end{cases}$$

$$|A| = a(a-1)$$

$$x = \frac{\begin{vmatrix} a-1 & 1 & a \\ a & 1 & 1 \\ 0 & a+1 & 2 \end{vmatrix}}{|A|} = \frac{2(a-1) + a^2(a+1) - ((a-1)(a+1) + 2a)}{a(a-1)} = \frac{\cancel{2a} - 2 + a^3 + \cancel{a^2} - (\cancel{a^2} - 1 + \cancel{2a})}{a(a-1)}$$

$$= \frac{a^3 - 1}{a(a-1)} = \frac{(a-1)(a^2+a+1)}{a(a-1)} = \boxed{\frac{a^2+a+1}{a}}$$

$$\begin{array}{rr|l} a^3 & -1 & a-1 \\ \cline{3-3} -a^3 + a^2 & & a^2+a+1 \\ \hline / \quad a^2 & -1 & \\ -a^2 + a & & \\ \hline / \quad a & -1 & \\ -a & +1 & \\ \hline / & / & \end{array}$$

$$y = \frac{\begin{vmatrix} 1 & a-1 & a \\ 1 & a & 1 \\ 1 & 0 & 2 \end{vmatrix}}{|A|} = \boxed{\frac{-a^2+a+1}{a(a-1)}}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & a-1 \\ 1 & 1 & a \\ 1 & a+1 & 0 \end{vmatrix}}{|A|} = \boxed{\frac{1}{1-a}}$$