Assignment 2 Solutions

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1 Solutions

1.1 Question 1

Given the matrix

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix} \tag{1}$$

1.1.1 Part A

The spectrum of A is found by finding the eigenvalues of A.

$$
\begin{aligned}
A - \lambda I &= 0 & (2) \\
\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} &= 0 & (3) \\
\begin{bmatrix} 1-\lambda & 2 \\ -1 & 2-\lambda \end{bmatrix} &= 0 & (4) \\
((1-\lambda)(2-\lambda)) + 2 &= 0 & (5) \\
\lambda^2 - 3\lambda + 4 &= 0 & (6)
\end{aligned}
$$

Now we simply factor to find the eigenvalues.

$$
\begin{aligned}
\lambda &= \frac{3 \pm \sqrt{-7}}{2} & (7) \\
\lambda &= \frac{3 \pm \sqrt{7}i}{2} & (8)
\end{aligned}
$$

Therefore the spectrum of A,

$$\boxed{\sigma(A) = \frac{3 \pm \sqrt{7}i}{2}} \tag{9}$$

The spectral radius of A is,

2

$$p(A) = \max\{\sigma(A)\} \tag{10}$$

Therefore, using $\qquad$ (11)

$$\lambda = \frac{3 - \sqrt{7}i}{2} \tag{12}$$

and converting to the reals $\qquad$ (13)

$$= \sqrt{\left(\frac{3 \pm \sqrt{7}i}{2}\right)^2} \tag{14}$$

$$= \sqrt{\frac{9}{4} + \frac{7}{4}} \tag{15}$$

$$= \sqrt{\frac{16}{4}} \tag{16}$$

$$= \sqrt{4} \tag{17}$$

$$\boxed{p(A) = 2} \tag{18}$$

Listing 1: Matlab Commands

```
A = [1,2;-1,2];
eig(A)

ans =

    1.5000 + 1.3229i
    1.5000 - 1.3229i

abs(ans)

ans =

    2.0000
    2.0000

diary off
```

1.1.2 Part B

$$||A||_1 = \max\{2,4\} = \boxed{4} \tag{19}$$

$$||A||_{\text{inf}} = \max\{3,3\} = \boxed{3} \tag{20}$$

$$||A||_2 = \sqrt{\lambda_{\max}(A'A)} \tag{21}$$

First we find the spectrum of $A'A$,

$$A'A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \tag{22}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \tag{23}$$

The eigenvalues are simply the diagonal entries meaning,

$$\sigma(A'A) = \{2,8\} \tag{24}$$

$$\text{Therefore, } ||A||_2 = \sqrt{\max\{2,8\}} \tag{25}$$

$$||A||_2 = \boxed{\sqrt{8}} \tag{26}$$

Listing 2: Matlab Commands

```
norm(A,1)
ans =

     4

norm(A,2)
ans =

     2.8284

norm(A,inf)
ans =

     3
```

1.1.3 Part C

The left singular vectors of A are eigenvectors of AA'; we will denote this U. The right singular vectors of A are eigenvectors of $A'A$; we will dentote this V. The singular values of A are the square roots of the non-zero eigenvalues of $A'A$ and AA'; we will denote this S.

Note,

$$AA' = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}, A'A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \tag{27}$$

First, we will find the left singular vectors of A. The eigenvalues of AA' are $\lambda = 2, 8$. We will now find eigenvectors for AA'.

Case: $\lambda = 2$,

$$\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{28}$$

$$\text{A solution: } X = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{29}$$

Case: $\lambda = 8$,

$$\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{30}$$

$$\text{A solution: } X = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{31}$$

Therefore,

$$U = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \tag{32}$$

Next, we will find the right singular vectors of A. The eigenvalues of $A'A$ are $\lambda = 2, 8$ We will now find eigenvectors for $A'A$.

Case: $\lambda = 2$,

$$\begin{bmatrix} 0 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{33}$$

$$\text{A solution: } X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{34}$$

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Case: $\lambda = 8$,

$$\begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{35}$$

$$\text{A solution: } X = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{36}$$

Therefore,

$$V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{37}$$

The singular values of A are the square roots of the eigenvalues of AA' and $A'A$. Therefore,

$$S = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{8} \end{bmatrix} \tag{38}$$

The singular value decomposition of A is given by $A = USV'$.

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{8} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{39}$$

$$A = \begin{bmatrix} 1.4142 & 2.8284 \\ -1.4142 & 2.8282 \end{bmatrix} \tag{40}$$

The matrix above is in fact not A, but it is only off by a scalar. If we divide by 1.4142 we get the original matrix A. Therefore we will encorporate this scalar division into U. Now,

$$U = \begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix} \tag{41}$$

In full,

$$U = \begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}, S = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{8} \end{bmatrix}, V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{42}$$

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```
Listing 3: Matlab Commands
[U,S,V]  =  svd (A)

U  =

     0.7071     −0.7071
     0.7071      0.7071


S  =

     2.8284             0
          0      1.4142


V  =

       0      −1
       1       0
```

1.2 Question 2

1.2.1 Part A

The example symmetric matrix is,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 2 \\ 3 & 7 & 2 & 9 \\ 4 & 2 & 9 & 4 \end{bmatrix} \tag{43}$$

The eigenvalues and eigenvectors for A are thus,

$$\lambda = \begin{bmatrix} 17.4770 & 0 & 0 & 0 \\ 0 & -7.5267 & 0 & 0 \\ 0 & 0 & 3.0000 & 0 \\ 0 & 0 & 0 & -0.9502 \end{bmatrix} \tag{44}$$

$$V = \begin{bmatrix} 0.3043 & 0.0760 & 0.2673 & 0.9112 \\ 0.4769 & -0.3446 & -0.8018 & 0.1047 \\ 0.6022 & 0.7534 & -0.0000 & -0.2640 \\ 0.5633 & -0.5549 & 0.5345 & -0.2986 \end{bmatrix} \tag{45}$$

And the eigenvalues and eigenvectors for $A'A$ are thus,

$$\lambda = \begin{bmatrix} 305.4452 & 0 & 0 & 0 \\ 0 & 56.6519 & 0 & 0 \\ 0 & 0 & 9.0000 & 0 \\ 0 & 0 & 0 & 0.9030 \end{bmatrix} \tag{46}$$

$$V = \begin{bmatrix} 0.3043 & -0.0760 & 0.2673 & -0.9112 \\ 0.4769 & 0.3446 & -0.8018 & -0.1047 \\ 0.6022 & -0.7534 & 0.0000 & 0.2640 \\ 0.5633 & 0.5549 & 0.5345 & 0.2986 \end{bmatrix} \tag{47}$$

Therefore, it is clear from this example that the eigenvectors for A and $A'A$ are the same and the eigenvalues of $A'A$ are the eigenvalues of A squared. The conjecture is thus, (λ, v) is an eigenpair of $A'A$ if and only if $(\sqrt{\lambda}, v)$ is an eigenpair of A; (for symmetric A).

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```
Listing 4: Matlab Commands
A = [1,2,3,4;2,5,7,2;3,7,2,9;4,2,9,4];
[P1,D1] = eigs(A)

P1 =

    0.3043      0.0760      0.2673      0.9112
    0.4769     -0.3446     -0.8018      0.1047
    0.6022      0.7534     -0.0000     -0.2640
    0.5633     -0.5549      0.5345     -0.2986


D1 =

   17.4770           0           0           0
        0     -7.5267           0           0
        0           0      3.0000           0
        0           0           0     -0.9502

[P2,D2] = eigs(A'*A)

P2 =

    0.3043     -0.0760      0.2673     -0.9112
    0.4769      0.3446     -0.8018     -0.1047
    0.6022     -0.7534      0.0000      0.2640
    0.5633      0.5549      0.5345      0.2986


D2 =

  305.4452           0           0           0
        0     56.6519           0           0
        0           0      9.0000           0
        0           0           0      0.9030

diary off
```

9

1.2.2 Part B

Since A is a symmetric matrix we know that A can be decomposed to $A = PDP^{-1}$. Where the columns of P are the eigenvectors of A and D is a diagonal matrix where the diaganol entries are the eigenvalues of A. The proof is as follows,

$$A = PDP^{-1} \tag{48}$$
$$\text{Squaring both sides,} \tag{49}$$
$$A^2 = PDP^{-1}PDP^{-1} \tag{50}$$
$$= PD(P^{-1}P)P^{-1} \tag{51}$$
$$= PDDP^{-1} \tag{52}$$
$$= PD^2P^{-1} \tag{53}$$

We know that $A^2 = A'A$. The columns of P are the eigenvectors of $A'A$ and D^2 is a diagonal matrix where the diagonal entries are the eigenvalues of $A'A$. Therefore, the eigenvalues of $A'A$ are the eigenvalues of A squared. This concludes the proof.

1.2.3 Part C

Theorem: The 2-norm of a symmetric matrix A is as follows,

$$\|A\|_2 = p(A) \tag{54}$$
$$\text{Where, } p(A) := \max\{|\lambda| : \lambda \in \sigma(A)\} \tag{55}$$

We must check the theorm against,

$$B = \begin{bmatrix} -92 & 144 \\ 144 & -8 \end{bmatrix} \tag{56}$$

The eigenvalues of B are -200 and 100. Therefore, $\sigma(B) = \{-200, 100\}$ and $p(B) = 200$. Using Matlab we find $\|B\|_2 = 200$. Therefore, the theorem holds for B.

Listing 5: Matlab Commands
```
B = [-92,144;144,-8];
```

```
eigs (B)

ans =

  -200.0000
   100.0000

abs ( ans )

ans =

   200.0000
   100.0000

norm (B, 2 )

ans =

    200

diary  off
```

1.2.4 Part D

This theorem does not hold for the non-symmetric matrix,

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \tag{57}$$

We find $\sigma(C) = \{5.3723, -0.3723\}$ and $p(C) = 5.3723$. Using Matlab we find $||C||_2 = 5.4650$. Therefore, the theorem does not hold for C.

Listing 6: Matlab Commands
```
C = [ 1 , 2 ; 3 , 4 ];
eigs (C)

ans =
```

```
    5.3723
   −0.3723

abs(ans)

ans =

    5.3723
    0.3723

norm(C,2)

ans =

    5.4650

diary off
```

1.2.5 Part E

The singular values of a symmetric matrix *A* are the absolute values of the non-zero eigenvalues of *A*.

1.3 Question 3

1.3.1 Part A

We want to prove that (λ, v) is an eigenpair of A if and only if $(\frac{1}{\lambda}, v)$ is an eigenpair of A^{-1}. The proof is as follows,

$$
\begin{aligned}
Av &= \lambda v & (58)\\
A^{-1}Av &= A^{-1}\lambda v & (59)\\
(A^{-1}A)v &= A^{-1}\lambda v & (60)\\
Iv &= A^{-1}\lambda v & (61)\\
v &= A^{-1}\lambda v & (62)\\
\frac{1}{\lambda}v &= A^{-1}v & (63)
\end{aligned}
$$

The 'only if' portion follows by symmetry.

1.3.2 Part B

The 2-norm of A^{-1} should be 1 / (the smallest singular value of A). We will denote the singular values of A as $s(A)$. Stated formally,

$$
||A^{-1}||_2 = \max_{s_A \in s(A)} \{\frac{1}{s_a}\} \tag{64}
$$

This is true because we know from (a) that (λ, v) is an eigenpair for A if and only if $(\frac{1}{\lambda}, v)$ is an eigenpair of A^{-1}. Therefore, it must be the case that (λ, v) is an eigenpair for $A'A$ if and only if $(\frac{1}{\lambda}, v)$ is an eigenpair of $(A^{-1})'A^{-1}$ because the 2-norm is preserved. Since the λ's of $A'A$ are the singular values of A it follows that $\frac{1}{\lambda}$'s of $A'A$ are the singular values of A^{-1}. Therefore, since $||A||_2 = \max\{s(A)\}$,

$$
||A^{-1}||_2 = \max_{s_A \in s(A)} \{\frac{1}{s_a}\} \tag{65}
$$

1.4 Question 4

QR factor the matrix *Z* where matrix *Z* is,

$$
Z = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix}
\tag{66}
$$

This will require three iterations.
Iteration 1:

$$
x = \begin{bmatrix} 1 \\ 4 \\ 7 \\ 4 \\ 4 \end{bmatrix}, y = \begin{bmatrix} 9.8995 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, w = \begin{bmatrix} -0.6704 \\ 0.3013 \\ 0.5273 \\ 0.3013 \\ 0.3013 \end{bmatrix}
\tag{67}
$$

$$
H = \begin{bmatrix}
0.1010 & 0.4041 & 0.7071 & 0.4041 & 0.4041 \\
0.4041 & 0.8184 & -0.3178 & -0.1816 & -0.1816 \\
0.7071 & -0.3178 & 0.4438 & -0.3178 & -0.3178 \\
0.4041 & -0.1816 & -0.3178 & 0.8184 & -0.1816 \\
0.4041 & -0.1816 & -0.3178 & -0.1816 & 0.8184
\end{bmatrix}
\tag{68}
$$

$$
Q = \begin{bmatrix}
0.1010 & 0.4041 & 0.7071 & 0.4041 & 0.4041 \\
0.4041 & 0.8184 & -0.3178 & -0.1816 & -0.1816 \\
0.7071 & -0.3178 & 0.4438 & -0.3178 & -0.3178 \\
0.4041 & -0.1816 & -0.3178 & 0.8184 & -0.1816 \\
0.4041 & -0.1816 & -0.3178 & -0.1816 & 0.8184
\end{bmatrix}
\tag{69}
$$

$$
R = \begin{bmatrix}
9.8995 & 9.4954 & 9.6975 \\
-0.0000 & 1.6311 & 2.9897 \\
-0.0000 & 2.1044 & 1.7320 \\
-0.0000 & -1.3689 & -0.0103 \\
-0.0000 & -1.3689 & -1.0103
\end{bmatrix}
\tag{70}
$$

Iteration 2:

14

$$x = \begin{bmatrix} 9.4954 \\ 1.6311 \\ 2.1044 \\ -1.3689 \\ -1.3689 \end{bmatrix}, y = \begin{bmatrix} 9.4954 \\ 3.2919 \\ 0 \\ 0 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ -0.5023 \\ 0.6364 \\ -0.4140 \\ -0.4140 \end{bmatrix} \tag{71}$$

$$H = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0.4955 & 0.6393 & -0.4158 & -0.4158 \\ 0 & 0.6393 & 0.1900 & 0.5269 & 0.5269 \\ 0 & -0.4158 & 0.5269 & 0.6572 & -0.3428 \\ 0 & -0.4158 & 0.5269 & -0.3428 & 0.6572 \end{bmatrix} \tag{72}$$

$$Q = \begin{bmatrix} 0.1010 & 0.3162 & 0.8185 & 0.3316 & 0.3316 \\ 0.4041 & 0.3534 & 0.2714 & -0.5649 & -0.5649 \\ 0.7071 & 0.3906 & -0.4537 & 0.2661 & 0.2661 \\ 0.4041 & -0.5580 & 0.1590 & 0.5082 & -0.4918 \\ 0.4041 & -0.5580 & 0.1590 & -0.4918 & 0.5082 \end{bmatrix} \tag{73}$$

$$R = \begin{bmatrix} 9.8995 & 9.4954 & 9.6975 \\ -0.0000 & 3.2919 & 3.0129 \\ -0.0000 & -0.0000 & 1.7026 \\ -0.0000 & 0.0000 & 0.0089 \\ -0.0000 & 0.0000 & -0.9911 \end{bmatrix} \tag{74}$$

Iteration 3:

$$
x = \begin{bmatrix} 9.6975 \\ 3.0129 \\ 1.7026 \\ 0.0089 \\ -0.9911 \end{bmatrix}, \; y = \begin{bmatrix} 9.6975 \\ 3.0129 \\ 1.9701 \\ 0 \\ 0 \end{bmatrix}, \; w = \begin{bmatrix} 0 \\ 0 \\ -0.2606 \\ 0.0086 \\ -0.9654 \end{bmatrix} \tag{75}
$$

$$
H = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0.8642 & 0.0045 & -0.5031 \\ 0 & 0 & 0.0045 & 0.9999 & 0.0167 \\ 0 & 0 & -0.5031 & 0.0167 & -0.8641 \end{bmatrix} \tag{76}
$$

$$
Q = \begin{bmatrix} 0.1010 & 0.3162 & 0.5420 & 0.3408 & -0.6928 \\ 0.4041 & 0.3534 & 0.5162 & -0.5730 & 0.3422 \\ 0.7071 & 0.3906 & -0.5248 & 0.2684 & 0.0028 \\ 0.4041 & -0.5580 & 0.3871 & 0.5006 & 0.3534 \\ 0.4041 & -0.5580 & -0.1204 & -0.4825 & -0.5273 \end{bmatrix} \tag{77}
$$

$$
R = \begin{bmatrix} 9.8995 & 9.4954 & 9.6975 \\ -0.0000 & 3.2919 & 3.0129 \\ -0.0000 & -0.0000 & 1.9701 \\ -0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 \end{bmatrix} \tag{78}
$$

Therefore the *QR* factorization of *Z* is

$$
Q = \begin{bmatrix} 0.1010 & 0.3162 & 0.5420 & 0.3408 & -0.6928 \\ 0.4041 & 0.3534 & 0.5162 & -0.5730 & 0.3422 \\ 0.7071 & 0.3906 & -0.5248 & 0.2684 & 0.0028 \\ 0.4041 & -0.5580 & 0.3871 & 0.5006 & 0.3534 \\ 0.4041 & -0.5580 & -0.1204 & -0.4825 & -0.5273 \end{bmatrix} \tag{79}
$$

$$
R = \begin{bmatrix} 9.8995 & 9.4954 & 9.6975 \\ -0.0000 & 3.2919 & 3.0129 \\ -0.0000 & -0.0000 & 1.9701 \\ -0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 \end{bmatrix} \tag{80}
$$

```
Listing 7: Matlab Commands
Z=[1 ,2 ,3;4 ,5 ,6;7 ,8 ,7;4 ,2 ,3;4 ,2 ,2];
R=Z;

% Iteration 1
x=R(: ,1);
a=norm(x(1:5) ,2);
y=[x(1:0)' a zeros(1 ,4)]';
w=(x−y)/norm(x−y ,2);
H=eye(5)−2*w*w';
Q=eye(5)*H;
R=H*R;

% Iteration 2
x=R(: ,2);
a=norm(x(2:5) ,2);
y=[x(1:1)' a zeros(1 ,3)]';
w=(x−y)/norm(x−y ,2);
H=eye(5)−2*w*w';
Q=Q*H;
R=H*R;

% Iteration 3
x=R(: ,3);
a=norm(x(3:5) ,2);
y=[x(1:2)' a zeros(1 ,2)]';
w=(x−y)/norm(x−y ,2);
H=eye(5)−2*w*w';
Q=Q*H;
R=H*R;
```

1.5 Question 5

1.5.1 Part A

The formula to compute a Householder matrix H_i is $H_i = I - 2ww'$. H_i is then multiplied by a matrix A_i to generate A_{i+1}. The problem is the computation of A_{i+1} is $O(m^3)$. Therefore, we want to compute $A_{i+1} = H_i A_i$ without explicitly calculating H_i first. This can be done by utilizing the formula for computing H_i.

$$
\begin{align}
A_{i+1} &= H_i A_i \tag{81} \\
&= (I - 2ww')A_i \tag{82} \\
&= IA_i - 2ww'A_i \tag{83} \\
&= A_i - 2ww'A_i \tag{84} \\
&= \boxed{A_i - (2w)(w'A_i)} \tag{85}
\end{align}
$$

This requires $O(m^2$ operations because it only multiplies a matrix by a vector and a vector by a vector. Note, $IA_i = A_i$ so this matrix multiplication is done implicitly and not explicitly. The intuition is made explicit below,

$$
A_{i+1} = \underbrace{A_i}_{\text{mxm}} - \underbrace{(2w)}_{\text{mx1}}\underbrace{(w'A_i)}_{\text{1xn}} \tag{86}
$$

The complexity of each operation in isolation is

$$
\begin{align}
IA_i &\Rightarrow O(1) \tag{87} \\
2w &\Rightarrow O(m) \tag{88} \\
w'A_i &\Rightarrow O(m^2) \tag{89} \\
(2w)(w'A_i) &\Rightarrow O(m^2) \tag{90} \\
A_i - ((2w)(w'A_i)) &\Rightarrow O(m) \tag{91}
\end{align}
$$

Therefore the complexity is dominated by the $O(m^2)$ terms and the complexity of the operation is $O(m^2)$.

1.5.2 Part B

```matlab
ops = 0;
n=size(A);
R=A;
Q=eye(n(1));

% QR Factorization
for i=1:n(2),
    % 'x' is the current i'th column of R
    x=R(:,i);
    ops = ops+n(1);
    % each execution of this line is n
    a=norm(x(i:n),2);
    ops = ops+n(1);
    % each execution of this line is n
    y=[x(1:i-1)' a zeros(1,n(1)-i)]';
    ops = ops+n(1);
    % each execution of this line is 2n
    w=(x-y)/norm(x-y,2);
    ops = ops+2*n(1);
    % each execution of this is 2n^2+2n
    Q=Q-(2*(Q*w))*w';
    ops = ops + 2*n(1)^2 + 2*n(1);
    % each execution of this is 2n^2+2n
    R=R-(2*w)*(w'*R);
    ops = ops + 2*n(1)^2 + 2*n(1);
end

% solve a system
% n operations
b=rand(n(1),1);
% n^2 operations
y=Q'*b;
% use back substitution n^2 operations
y\R
```

1.5.3 Part C

```
Listing 9: Matlab Commands
A=rand(5,4);
linear_qr

ans =

    0.8981      0.3008      0.5534      0.4524

b\A

ans =

    0.8981      0.3008      0.5534      0.4524

A=rand(9,7);
linear_qr

ans =

  0.7455  0.6301  0.5719  0.7018  0.6683  0.6107  0.6380

b\A

ans =

  0.7455  0.6301  0.5719  0.7018  0.6683  0.6107  0.6380
```

1.5.4 Part D

For a matrix A_{mxn}, if we add the complexity of each line of the algorithm from (c) we have,

$$m + m + 2m + 2m^2 + 2m + 2m^2 + 2m + m + m^2 + m^2 \tag{92}$$

$$\boxed{6m^2 + 9m} \tag{93}$$

Therefore, the complexity of the algorithm is $O(m^2)$.