Assignment 2 Solutions

Aaron Cahn
University of Wisconsin-Madison
cahn@cs.wisc.edu

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1 Solutions

1.1 Question 1

Given the matrix

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix} \tag{1}$$

1.1.1 Part A

The spectrum of A is found by finding the eigenvalues of A.

$$\begin{aligned}
A - \lambda I &= 0 && (2)\\
\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} &= 0 && (3)\\
\begin{bmatrix} 1-\lambda & 2 \\ -1 & 2-\lambda \end{bmatrix} &= 0 && (4)\\
((1-\lambda)(2-\lambda)) + 2 &= 0 && (5)\\
\lambda^2 - 3\lambda + 4 &= 0 && (6)
\end{aligned}$$

Now we simply factor to find the eigenvalues.

$$\begin{aligned}
\lambda &= \frac{3 \pm \sqrt{-7}}{2} && (7)\\
\lambda &= \frac{3 \pm \sqrt{7}i}{2} && (8)
\end{aligned}$$

Therefore the spectrum of A,

$$\boxed{\sigma(A) = \frac{3 \pm \sqrt{7}i}{2}} \tag{9}$$

The spectral radius of A is,

$$
\begin{aligned}
p(A) &= \max\{\sigma(A)\} && (10)\\
\text{Therefore, using} & && (11)\\
\lambda &= \frac{3-\sqrt{7}i}{2} && (12)\\
\text{and converting to the reals} & && (13)\\
&= \sqrt{\left(\frac{3\pm\sqrt{7}i}{2}\right)^2} && (14)\\
&= \sqrt{\frac{9}{4}+\frac{7}{4}} && (15)\\
&= \sqrt{\frac{16}{4}} && (16)\\
&= \sqrt{4} && (17)\\
\boxed{p(A)=2} & && (18)
\end{aligned}
$$

Listing 1: Matlab Commands

```
A = [1,2;-1,2];
eig(A)

ans =

    1.5000 + 1.3229i
    1.5000 - 1.3229i

abs(ans)

ans =

     2.0000
     2.0000

diary off
```

1.1.2 Part B

$$||A||_1 = \max\{2,4\} = \boxed{4} \tag{19}$$
$$||A||_{\text{inf}} = \max\{3,3\} = \boxed{3} \tag{20}$$
$$||A||_2 = \sqrt{\lambda_{\text{max}}(A'A)} \tag{21}$$

First we find the spectrum of $A'A$,

$$A'A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \tag{22}$$
$$= \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \tag{23}$$

The eigenvalues are simply the diagonal entries meaning,

$$\sigma(A'A) = \{2,8\} \tag{24}$$
$$\text{Therefore, } ||A||_2 = \sqrt{\max\{2,8\}} \tag{25}$$
$$||A||_2 = \boxed{\sqrt{8}} \tag{26}$$

Listing 2: Matlab Commands

```
norm(A,1)
ans =

     4

norm(A,2)
ans =

    2.8284

norm(A,inf)
ans =

     3
```

1.1.3 Part C

The left singular vectors of A are eigenvectors of AA'; we will denote this U. The right singular vectors of A are eigenvectors of $A'A$; we will dentote this V. The singular values of A are the square roots of the non-zero eigenvalues of $A'A$ and AA'; we will denote this S.

Note,

$$AA' = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}, A'A = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix} \tag{27}$$

First, we will find the left singular vectors of A. The eigenvalues of AA' are $\lambda = 2, 8$. We will now find eigenvectors for AA'.

Case: $\lambda = 2$,

$$\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{28}$$

$$\text{A solution: } X = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{29}$$

Case: $\lambda = 8$,

$$\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{30}$$

$$\text{A solution: } X = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{31}$$

Therefore,

$$U = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \tag{32}$$

Next, we will find the right singular vectors of A. The eigenvalues of $A'A$ are $\lambda = 2, 8$ We will now find eigenvectors for $A'A$.

Case: $\lambda = 2$,

$$\begin{bmatrix} 0 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{33}$$

$$\text{A solution: } X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{34}$$

Case: $\lambda = 8$,

$$\begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \tag{35}$$

$$\text{A solution: } X = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{36}$$

Therefore,

$$V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{37}$$

The singular values of A are the square roots of the eigenvalues of AA' and $A'A$. Therefore,

$$S = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{8} \end{bmatrix} \tag{38}$$

The singular value decomposition of A is given by $A = USV'$.

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{8} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{39}$$

$$A = \begin{bmatrix} 1.4142 & 2.8284 \\ -1.4142 & 2.8282 \end{bmatrix} \tag{40}$$

The matrix above is in fact not A, but it is only off by a scalar. If we divide by 1.4142 we get the original matrix A. Therefore we will encorporate this scalar division into U. Now,

$$U = \begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix} \tag{41}$$

In full,

$$U = \begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}, S = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{8} \end{bmatrix}, V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{42}$$

Listing 3: Matlab Commands

```
[U,S,V] = svd(A)

U =

    0.7071   -0.7071
    0.7071    0.7071


S =

    2.8284         0
         0    1.4142


V =

     0    -1
     1     0
```

1.2 Question 2

1.2.1 Part A

The example symmetric matrix is,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 2 \\ 3 & 7 & 2 & 9 \\ 4 & 2 & 9 & 4 \end{bmatrix} \tag{43}$$

The eigenvalues and eigenvectors for A are thus,

$$\lambda = \begin{bmatrix} 17.4770 & 0 & 0 & 0 \\ 0 & -7.5267 & 0 & 0 \\ 0 & 0 & 3.0000 & 0 \\ 0 & 0 & 0 & -0.9502 \end{bmatrix} \tag{44}$$

$$V = \begin{bmatrix} 0.3043 & 0.0760 & 0.2673 & 0.9112 \\ 0.4769 & -0.3446 & -0.8018 & 0.1047 \\ 0.6022 & 0.7534 & -0.0000 & -0.2640 \\ 0.5633 & -0.5549 & 0.5345 & -0.2986 \end{bmatrix} \tag{45}$$

And the eigenvalues and eigenvectors for $A'A$ are thus,

$$\lambda = \begin{bmatrix} 305.4452 & 0 & 0 & 0 \\ 0 & 56.6519 & 0 & 0 \\ 0 & 0 & 9.0000 & 0 \\ 0 & 0 & 0 & 0.9030 \end{bmatrix} \tag{46}$$

$$V = \begin{bmatrix} 0.3043 & -0.0760 & 0.2673 & -0.9112 \\ 0.4769 & 0.3446 & -0.8018 & -0.1047 \\ 0.6022 & -0.7534 & 0.0000 & 0.2640 \\ 0.5633 & 0.5549 & 0.5345 & 0.2986 \end{bmatrix} \tag{47}$$

Therefore, it is clear from this example that the eigenvectors for A and $A'A$ are the same and the eigenvalues of $A'A$ are the eigenvalues of A squared. The conjecture is thus, (λ, v) is an eigenpair of $A'A$ if and only if$(\sqrt{\lambda}, v)$ is an eigenpair of A; (for symmetric A).

Listing 4: Matlab Commands

```
A = [1,2,3,4;2,5,7,2;3,7,2,9;4,2,9,4];
[P1,D1] = eigs(A)

P1 =

    0.3043    0.0760    0.2673    0.9112
    0.4769   -0.3446   -0.8018    0.1047
    0.6022    0.7534   -0.0000   -0.2640
    0.5633   -0.5549    0.5345   -0.2986


D1 =

   17.4770         0         0         0
         0   -7.5267         0         0
         0         0    3.0000         0
         0         0         0   -0.9502

[P2,D2] = eigs(A'*A)

P2 =

    0.3043   -0.0760    0.2673   -0.9112
    0.4769    0.3446   -0.8018   -0.1047
    0.6022   -0.7534    0.0000    0.2640
    0.5633    0.5549    0.5345    0.2986


D2 =

  305.4452         0         0         0
         0   56.6519         0         0
         0         0    9.0000         0
         0         0         0    0.9030

diary off
```

1.2.2 Part B

Since A is a symmetric matrix we know that A can be decomposed to $A = PDP^{-1}$. Where the columns of P are the eigenvectors of A and D is a diagonal matrix where the diaganol entries are the eigenvalues of A. The proof is as follows,

$$
\begin{aligned}
A &= PDP^{-1} && (48)\\
\text{Squaring both sides,} & && (49)\\
A^2 &= PDP^{-1}PDP^{-1} && (50)\\
&= PD(P^{-1}P)P^{-1} && (51)\\
&= PDDP^{-1} && (52)\\
&= PD^2P^{-1} && (53)
\end{aligned}
$$

We know that $A^2 = A'A$. The columns of P are the eigenvectors of $A'A$ and D^2 is a diagonal matrix where the diagonal entries are the eigenvalues of $A'A$. Therefore, the eigenvalues of $A'A$ are the eigenvalues of A squared. This concludes the proof.

1.2.3 Part C

Theorem: The 2-norm of a symmetric matrix A is as follows,

$$
\begin{aligned}
||A||_2 &= p(A) && (54)\\
\text{Where, } p(A) &:= \max\{|\lambda| : \lambda \in \sigma(A)\} && (55)
\end{aligned}
$$

We must check the theorm against,

$$B = \begin{bmatrix} -92 & 144 \\ 144 & -8 \end{bmatrix} \tag{56}$$

The eigenvalues of B are -200 and 100. Therefore, $\sigma(B) = \{-200, 100\}$ and $p(B) = 200$. Using Matlab we find $||B||_2 = 200$. Therefore, the theorem holds for B.

Listing 5: Matlab Commands

```
B = [-92,144;144,-8];
```

```
eigs(B)

ans =

  -200.0000
   100.0000

abs(ans)

ans =

   200.0000
   100.0000

norm(B,2)

ans =

    200

diary off
```

1.2.4 Part D

This theorem does not hold for the non-symmetric matrix,

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \tag{57}$$

We find $\sigma(C) = \{5.3723, -0.3723\}$ and $p(C) = 5.3723$. Using Matlab we find $||C||_2 = 5.4650$. Therefore, the theorem does not hold for C.

Listing 6: Matlab Commands

```
C = [1,2;3,4];
eigs(C)

ans =
```

```
    5.3723
   -0.3723

abs(ans)

ans =

    5.3723
    0.3723

norm(C,2)

ans =

    5.4650

diary off
```

1.2.5 Part E

The singular values of a symmetric matrix A are the absolute values of the non-zero eigenvalues of A.

1.3 Question 3

1.3.1 Part A

We want to prove that (λ, v) is an eigenpair of A if and only if $(\frac{1}{\lambda}, v)$ is an eigenpair of A^{-1}. The proof is as follows,

$$\begin{aligned} Av &= \lambda v &(58)\\ A^{-1}Av &= A^{-1}\lambda v &(59)\\ (A^{-1}A)v &= A^{-1}\lambda v &(60)\\ Iv &= A^{-1}\lambda v &(61)\\ v &= A^{-1}\lambda v &(62)\\ \frac{1}{\lambda}v &= A^{-1}v &(63) \end{aligned}$$

The 'only if' portion follows by symmetry.

1.3.2 Part B

The 2-norm of A^{-1} should be 1 / (the smallest singular value of A). We will denote the singular values of A as $s(A)$. Stated formally,

$$||A^{-1}||_2 = \max_{s_A \in s(A)} \{\frac{1}{s_a}\} \tag{64}$$

This is true because we know from (a) that (λ, v) is an eigenpair for A if and only if $(\frac{1}{\lambda}, v)$ is an eigenpair of A^{-1}. Therefore, it must be the case that (λ, v) is an eigenpair for $A'A$ if and only if $(\frac{1}{\lambda}, v)$ is an eigenpair of $(A^{-1})'A^{-1}$ because the 2-norm is preserved. Since the λ's of $A'A$ are the singular values of A it follows that $\frac{1}{\lambda}$'s of $A'A$ are the singular values of A^{-1}. Therefore, since $||A||_2 = \max\{s(A)\}$,

$$||A^{-1}||_2 = \max_{s_A \in s(A)} \{\frac{1}{s_a}\} \tag{65}$$

1.4 Question 4

QR factor the matrix Z where matrix Z is,

$$Z = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix} \tag{66}$$

This will require three iterations.
Iteration 1:

$$x = \begin{bmatrix} 1 \\ 4 \\ 7 \\ 4 \\ 4 \end{bmatrix}, y = \begin{bmatrix} 9.8995 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, w = \begin{bmatrix} -0.6704 \\ 0.3013 \\ 0.5273 \\ 0.3013 \\ 0.3013 \end{bmatrix} \tag{67}$$

$$H = \begin{bmatrix} 0.1010 & 0.4041 & 0.7071 & 0.4041 & 0.4041 \\ 0.4041 & 0.8184 & -0.3178 & -0.1816 & -0.1816 \\ 0.7071 & -0.3178 & 0.4438 & -0.3178 & -0.3178 \\ 0.4041 & -0.1816 & -0.3178 & 0.8184 & -0.1816 \\ 0.4041 & -0.1816 & -0.3178 & -0.1816 & 0.8184 \end{bmatrix} \tag{68}$$

$$Q = \begin{bmatrix} 0.1010 & 0.4041 & 0.7071 & 0.4041 & 0.4041 \\ 0.4041 & 0.8184 & -0.3178 & -0.1816 & -0.1816 \\ 0.7071 & -0.3178 & 0.4438 & -0.3178 & -0.3178 \\ 0.4041 & -0.1816 & -0.3178 & 0.8184 & -0.1816 \\ 0.4041 & -0.1816 & -0.3178 & -0.1816 & 0.8184 \end{bmatrix} \tag{69}$$

$$R = \begin{bmatrix} 9.8995 & 9.4954 & 9.6975 \\ -0.0000 & 1.6311 & 2.9897 \\ -0.0000 & 2.1044 & 1.7320 \\ -0.0000 & -1.3689 & -0.0103 \\ -0.0000 & -1.3689 & -1.0103 \end{bmatrix} \tag{70}$$

Iteration 2:

$$x = \begin{bmatrix} 9.4954 \\ 1.6311 \\ 2.1044 \\ -1.3689 \\ -1.3689 \end{bmatrix}, y = \begin{bmatrix} 9.4954 \\ 3.2919 \\ 0 \\ 0 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ -0.5023 \\ 0.6364 \\ -0.4140 \\ -0.4140 \end{bmatrix} \tag{71}$$

$$H = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0.4955 & 0.6393 & -0.4158 & -0.4158 \\ 0 & 0.6393 & 0.1900 & 0.5269 & 0.5269 \\ 0 & -0.4158 & 0.5269 & 0.6572 & -0.3428 \\ 0 & -0.4158 & 0.5269 & -0.3428 & 0.6572 \end{bmatrix} \tag{72}$$

$$Q = \begin{bmatrix} 0.1010 & 0.3162 & 0.8185 & 0.3316 & 0.3316 \\ 0.4041 & 0.3534 & 0.2714 & -0.5649 & -0.5649 \\ 0.7071 & 0.3906 & -0.4537 & 0.2661 & 0.2661 \\ 0.4041 & -0.5580 & 0.1590 & 0.5082 & -0.4918 \\ 0.4041 & -0.5580 & 0.1590 & -0.4918 & 0.5082 \end{bmatrix} \tag{73}$$

$$R = \begin{bmatrix} 9.8995 & 9.4954 & 9.6975 \\ -0.0000 & 3.2919 & 3.0129 \\ -0.0000 & -0.0000 & 1.7026 \\ -0.0000 & 0.0000 & 0.0089 \\ -0.0000 & 0.0000 & -0.9911 \end{bmatrix} \tag{74}$$

Iteration 3:

$$x = \begin{bmatrix} 9.6975 \\ 3.0129 \\ 1.7026 \\ 0.0089 \\ -0.9911 \end{bmatrix}, y = \begin{bmatrix} 9.6975 \\ 3.0129 \\ 1.9701 \\ 0 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 0 \\ 0 \\ -0.2606 \\ 0.0086 \\ -0.9654 \end{bmatrix} \tag{75}$$

$$H = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 0.8642 & 0.0045 & -0.5031 \\ 0 & 0 & 0.0045 & 0.9999 & 0.0167 \\ 0 & 0 & -0.5031 & 0.0167 & -0.8641 \end{bmatrix} \tag{76}$$

$$Q = \begin{bmatrix} 0.1010 & 0.3162 & 0.5420 & 0.3408 & -0.6928 \\ 0.4041 & 0.3534 & 0.5162 & -0.5730 & 0.3422 \\ 0.7071 & 0.3906 & -0.5248 & 0.2684 & 0.0028 \\ 0.4041 & -0.5580 & 0.3871 & 0.5006 & 0.3534 \\ 0.4041 & -0.5580 & -0.1204 & -0.4825 & -0.5273 \end{bmatrix} \tag{77}$$

$$R = \begin{bmatrix} 9.8995 & 9.4954 & 9.6975 \\ -0.0000 & 3.2919 & 3.0129 \\ -0.0000 & -0.0000 & 1.9701 \\ -0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 \end{bmatrix} \tag{78}$$

Therefore the QR factorization of Z is

$$Q = \begin{bmatrix} 0.1010 & 0.3162 & 0.5420 & 0.3408 & -0.6928 \\ 0.4041 & 0.3534 & 0.5162 & -0.5730 & 0.3422 \\ 0.7071 & 0.3906 & -0.5248 & 0.2684 & 0.0028 \\ 0.4041 & -0.5580 & 0.3871 & 0.5006 & 0.3534 \\ 0.4041 & -0.5580 & -0.1204 & -0.4825 & -0.5273 \end{bmatrix} \tag{79}$$

$$R = \begin{bmatrix} 9.8995 & 9.4954 & 9.6975 \\ -0.0000 & 3.2919 & 3.0129 \\ -0.0000 & -0.0000 & 1.9701 \\ -0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 \end{bmatrix} \tag{80}$$

Listing 7: Matlab Commands

```
Z=[1,2,3;4,5,6;7,8,7;4,2,3;4,2,2];
R=Z;

% Iteration 1
x=R(:,1);
a=norm(x(1:5),2);
y=[x(1:0)' a zeros(1,4)]';
w=(x-y)/norm(x-y,2);
H=eye(5)-2*w*w';
Q=eye(5)*H;
R=H*R;

% Iteration 2
x=R(:,2);
a=norm(x(2:5),2);
y=[x(1:1)' a zeros(1,3)]';
w=(x-y)/norm(x-y,2);
H=eye(5)-2*w*w';
Q=Q*H;
R=H*R;

% Iteration 3
x=R(:,3);
a=norm(x(3:5),2);
y=[x(1:2)' a zeros(1,2)]';
w=(x-y)/norm(x-y,2);
H=eye(5)-2*w*w';
Q=Q*H;
R=H*R;
```

1.5 Question 5

1.5.1 Part A

The formula to compute a Householder matrix H_i is $H_i = I - 2ww'$. H_i is then multiplied by a matrix A_i to generate A_{i+1}. The problem is the computation of A_{i+1} is $O(m^3)$. Therefore, we want to compute $A_{i+1} = H_iA_i$ without explicitly calculating H_i first. This can be done by utilizing the formula for computing H_i.

$$\begin{aligned} A_{i+1} &= H_iA_i && (81)\\ &= (I - 2ww')A_i && (82)\\ &= IA_i - 2ww'A_i && (83)\\ &= A_i - 2ww'A_i && (84)\\ &= \boxed{A_i - (2w)(w'A_i)} && (85) \end{aligned}$$

This requires $O(m^2$ operations because it only multiplies a matrix by a vector and a vector by a vector. Note, $IA_i = A_i$ so this matrix multiplication is done implicitly and not explicitly. The intuition is made explicit below,

$$A_{i+1} = \underbrace{A_i}_{\text{mxm}} - \underbrace{(2w)}_{\text{mx1}}\underbrace{(w'A_i)}_{\text{1xn}} \qquad (86)$$

The complexity of each operation in isolation is

$$\begin{aligned} IA_i &\Rightarrow O(1) && (87)\\ 2w &\Rightarrow O(m) && (88)\\ w'A_i &\Rightarrow O(m^2) && (89)\\ (2w)(w'A_i) &\Rightarrow O(m^2) && (90)\\ A_i - ((2w)(w'A_i)) &\Rightarrow O(m) && (91) \end{aligned}$$

Therefore the complexity is dominated by the $O(m^2)$ terms and the complexity of the operation is $O(m^2)$.

1.5.2 Part B

Listing 8: Matlab Commands

```
ops = 0;
n=size(A);
R=A;
Q=eye(n(1));

% QR Factorization
for i=1:n(2),
    % ‘x’ is the current i’th column of R
    x=R(:,i);
    ops = ops+n(1);
    % each execution of this line is n
    a=norm(x(i:n),2);
    ops = ops+n(1);
    % each execution of this line is n
    y=[x(1:i-1)' a zeros(1,n(1)-i)]';
    ops = ops+n(1);
    % each execution of this line is 2n
    w=(x-y)/norm(x-y,2);
    ops = ops+2*n(1);
    % each execution of this is 2n^2+2n
    Q=Q-(2*(Q*w))*w';
    ops = ops + 2*n(1)^2 + 2*n(1);
    % each execution of this is 2n^2+2n
    R=R-(2*w)*(w'*R);
    ops = ops + 2*n(1)^2 + 2*n(1);
end

% solve a system
% n operations
b=rand(n(1),1);
% n^2 operations
y=Q'*b;
% use back substitution n^2 operations
R\y
```

1.5.3 Part C

Listing 9: Matlab Commands

```
A=rand(5,4);
linear_qr

ans =

    0.8981    0.3008    0.5534    0.4524

A\b

ans =

    0.8981    0.3008    0.5534    0.4524

A=rand(9,7);
linear_qr

ans =

  0.7455  0.6301  0.5719  0.7018  0.6683  0.6107  0.6380

A\b

ans =

  0.7455  0.6301  0.5719  0.7018  0.6683  0.6107  0.6380
```

1.5.4 Part D

For a matrix A_{mxn}, if we add the complexity of each line of the algorithm from (c) we have,

$$m+m+2m+2m^2+2m+2m^2+2m+m+m^2+m^2 \quad (92)$$

$$\boxed{6m^2+9m} \quad (93)$$

Therefore, the complexity of the algorithm is $O(m^2)$.