Modelling Hedge Fund Returns Using State Space Models




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STENHAM ASSET MANAGEMENT

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Firm Overview

Stenham Asset Management

25-year award winning performance

» Pioneering hedge fund investment specialists since 1980s

» US$ 2billion AUM

» 37 dedicated employees

» CIO 23 years investment experience, 17 years with Stenham

» Alignment of interest: the Stenham team are significant co-investors

» Consistently won industry recognition over many years

» Authorised and regulated by the FCA, SEC, GFSC and FSB

Industry Leading Quantitative Analysis & Risk Systems

Where MATLAB fits within our risk and quantitative analysis systems

C*NEO
Data Management & Reporting

- Robust SQL based database solution for data storage and handling
- Flexible Excel add-in analytics
- Ideal for customised reporting

RiskData
Off-the-Shelf, Returns-Based Risk System

- Robust "polymodel" approach to factor analysis and risk management
- Long-Term historical factor based Monte Carlo simulation
- Conservative "StressVAR" estimates

AlternativeSoft
Off-the-Shelf Data Analysis System

- Flexible and responsive multi-factor approach
- Highly interactive, intuitive and configurable
- Modular design mirroring stages in the investment process
- Intuitive graphical output

RiskMetrics
Off-the-Shelf, Position-Based Risk System

- Position-based portfolio transparency
- Underlying portfolios updated on a monthly basis
- Best tool for stress testing of current portfolios

MATLAB
Data Analysis and Application Development

- Technical, maths-based programming language
- Extensive libraries for financial analysis
- Ideal environment for developing proprietary models

Traditional ways of modelling hedge fund returns

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Traditional models for hedge fund returns

Basic linear factor models

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

Traditional models for hedge fund returns

Basic linear factor models

Style Analysis,
Replication and Hedging

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

Traditional models for hedge fund returns

Basic linear factor models

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Performance Measurement and Attribution

Traditional models for hedge fund returns

Basic linear factor models

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Risk Management

Traditional models for hedge fund returns

Basic linear factor models

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

Portfolio Construction

Traditional models for hedge fund returns

Basic linear factor models: what factors to use?

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Traditional models for hedge fund returns

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- CAPM (S&P)

Traditional models for hedge fund returns

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$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

- CAPM (S&P)
- Fama-French (HML, SMB)
- Carhart (Momentum)

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Traditional models for hedge fund returns

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$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

- CAPM (S&P)
- Fama-French (HML, SMB)
- Carhart (Momentum)
- Fung and Hsieh (2001)
- Agarwal and Naik (2004)

Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

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Traditional models for hedge fund returns

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- Non-linearity

Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

- Non-linearity → non-linear factors

Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

- Non-linearity
- Time dependency

Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

- Non-linearity
- Time dependency → rolling windows

Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

- Non-linearity
- Time dependency
- Serial correlation

Traditional models for hedge fund returns

(At least) three problems with basic linear factor models

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t$$

- Non-linearity
- Time dependency
- Serial correlation $\longrightarrow AR(p)$

Traditional models for hedge fund returns

Some of the questions we wanted to investigate

- Does formally allowing for time-varying parameters improve the performance of the factor models?
- Does it result in more stable parameters?
- Does inclusion of an autoregressive term provide a better fit on average?
- How does that compare with adding additional factors?
- How well do out-of-sample models perform relative to in-sample ones?
- Do the results vary across different hedge fund strategies?
- How much harder are these models to implement than a rolling window regression?

A brief introduction to State Space Models (SSM)

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Introduction to State Space Models

The general form for univariate Gaussian State Space Models

$$y_t = Z_t^{'} a_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$a_{t+1} = T_t a_t + R_t \eta_t, \qquad \eta_t \sim NID(0, Q_t)$$

$$y_t \equiv observed\ time\ series\ (t = 1, \cdots, n)$$

$$a_t \equiv (m\ x\ 1)\ unobserved\ state\ variables$$

$$Z_t \equiv (m\ x\ 1)\ observation\ vector$$

$$\varepsilon_t \equiv observation\ error\ (variance\ \sigma_\varepsilon^2)$$

$$T_t \equiv (m\ x\ m)\ transition\ matrix$$

$$R_t \equiv (m\ x\ r)\ selection\ matrix$$

$$\eta_t \equiv (r\ x\ 1)\ state\ disturbances$$

$$Q_t \equiv (r\ x\ r)\ disturbance\ covariance\ matrix$$

Introduction to State Space Models

The general form for univariate Gaussian State Space Models

Observation

$$y_t = Z_t^{'} a_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

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Transition

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Introduction to State Space Models

Example 1: the local level model

$$y_t = Z_t' a_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$a_{t+1} = T_t a_t + R_t \eta_t, \quad \eta_t \sim NID(0, Q_t)$$

$$\underbrace{a_t = \mu_t, \quad \eta_t = \xi_t, \quad Z_t = T_t = R_t = 1, \quad Q_t = \sigma_\xi^2}$$

$$y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

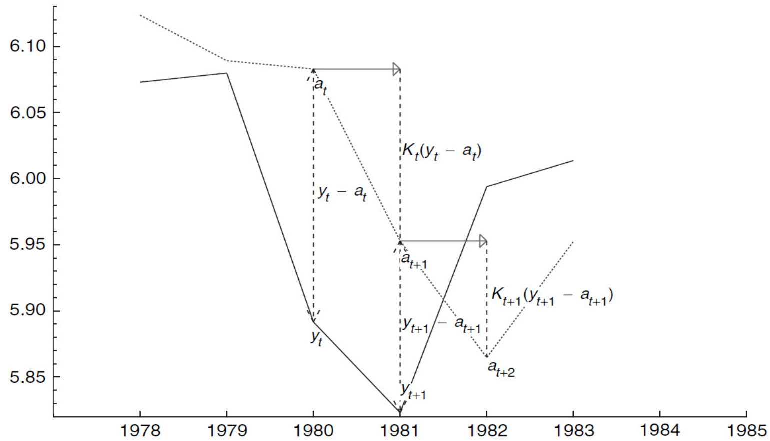
$$\mu_{t+1} = \mu_t + \xi_t, \quad \xi_t \sim NID(0, \sigma_\xi^2)$$

Introduction to State Space Models

Estimation of State Space Models using the Kalman filter and smoother

$$y_t = \mu_t + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$\mu_{t+1} = \mu_t + \xi_t, \quad \xi_t \sim NID(0, \sigma_\xi^2)$$



Source: Commandeur and Koopman (2007)

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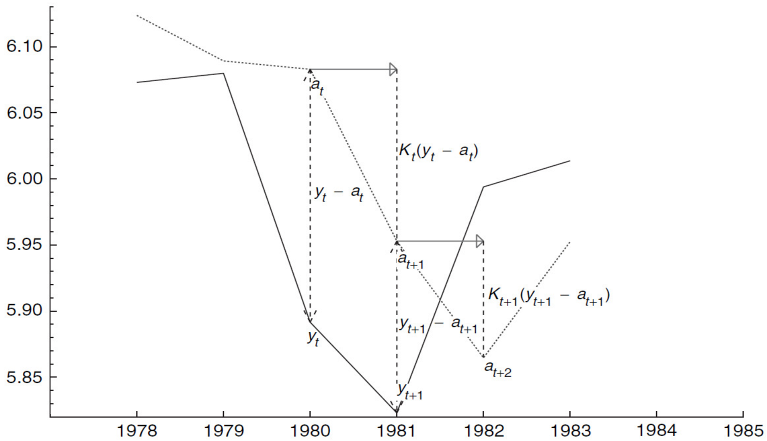
1. *Start at* $t = 1980$



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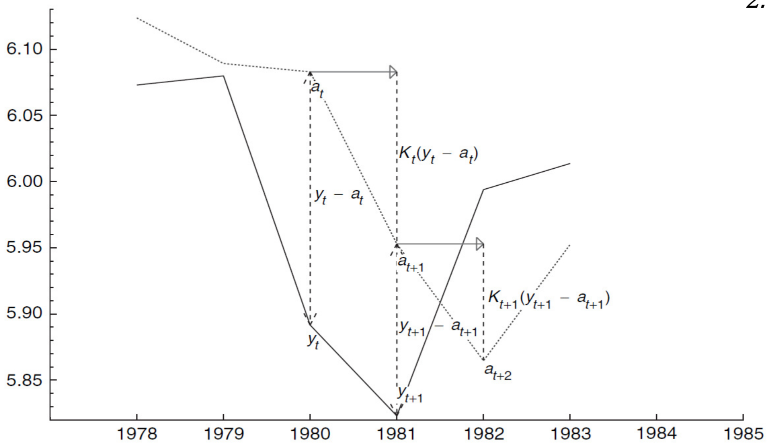

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1. *Start at* $t = 1980$
2. *You have estimated* $a_t = \mu_t$ *based on* $\{y_1, \cdots, y_{t-1}\}$

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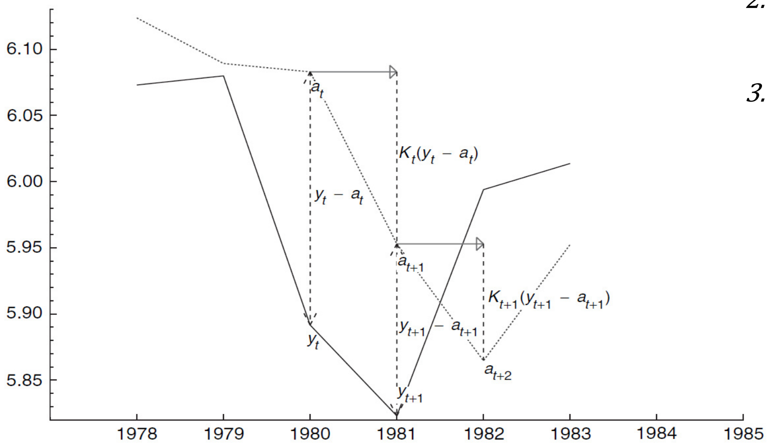

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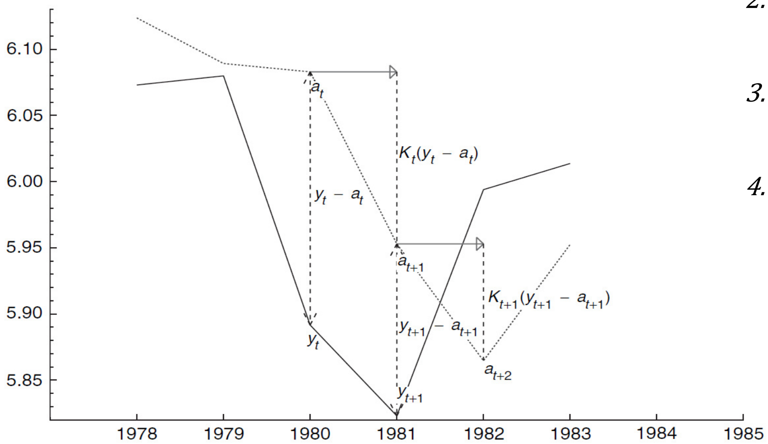

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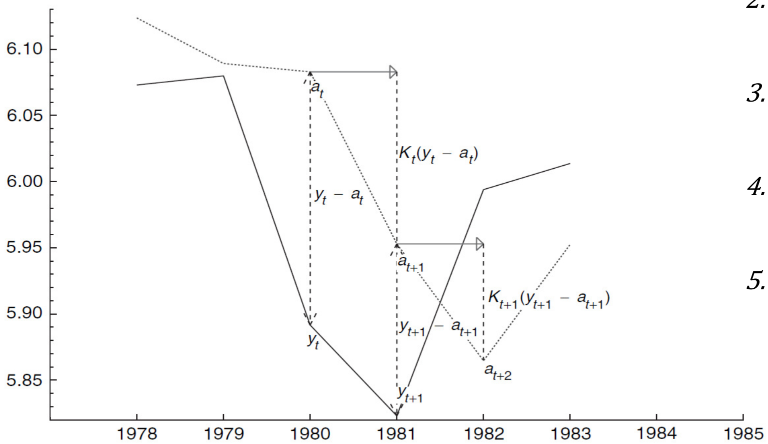

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3. *Before observing* y_t, *your best guess for* a_{t+1} *is* a_t
4. *Once you observe* y_t, *you can use that to update* a_{t+1}
5. *You do that by adjusting* a_t *by* $K_t(y_t - a_t)$

$$\text{where } K_t \propto \sigma_\xi^2 / \sigma_\varepsilon^2$$

Introduction to State Space Models

Estimation of State Space Models using the Kalman filter and smoother

- Predicted state variables:

$$\hat{a}_t|(y_1, \cdots, y_{t-1}; x_1, \cdots, x_{t-1})$$

- Filtered state variables:

$$\hat{a}_t|(y_1, \cdots, y_t; x_1, \cdots, x_t)$$

- Smoothed state variables:

$$\hat{a}_t|(y_1, \cdots, y_n; x_1, \cdots, x_n)$$

Testing and comparing models for hedge fund returns

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Testing and comparing models

A very basic test: how accurately can we estimate funds' market beta?

Four models, four estimation methodologies, ~6,500 funds:

$$Model\ 1: \quad y_t = \alpha_t + \beta_t x_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$Model\ 2: \quad y_t = \alpha_t + \beta_t x_t + \phi_t y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$Model\ 3: \quad y_t = \alpha_t + \beta_t x_t + \gamma_t z_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$Model\ 4: \quad y_t = \alpha_t + \beta_t x_t + \gamma_t z_t + \phi_t y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

Eq. H.F.

$Estimation\ method\ 1: 12mo\ rolling\ windows\ (out\ of\ sample)$

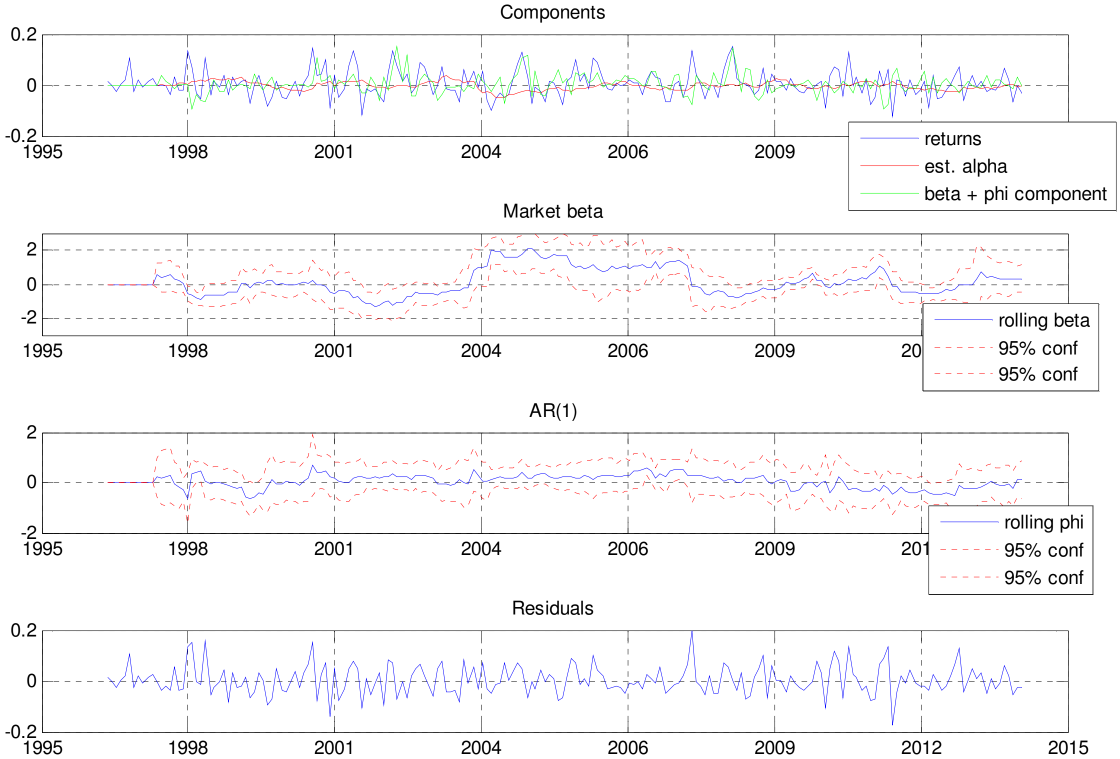
$Estimation\ method\ 2: Kalman\ filter\ (out\ of\ sample)$

$Estimation\ method\ 3: static\ linear\ regression\ (in\ sample)$

$Estimation\ method\ 4: Kalman\ smoother\ (in\ sample)$

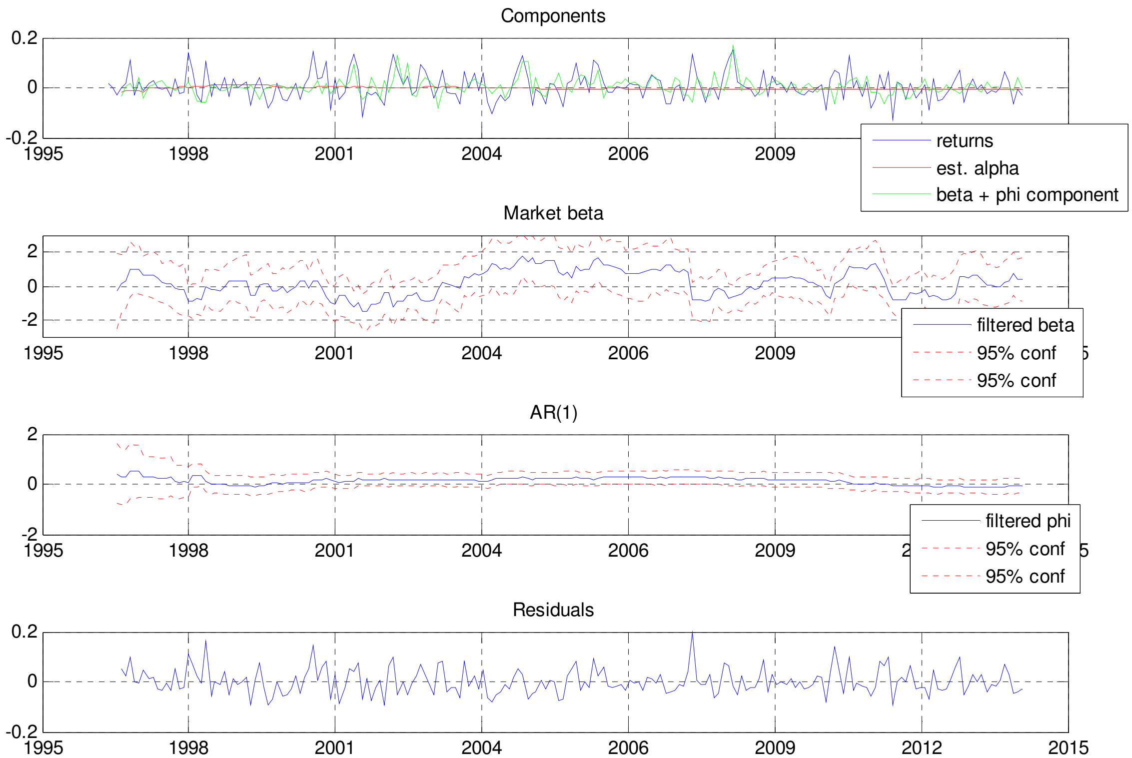
Testing and comparing models

Results for a sample fund – Model 2 – 12mo rolling window



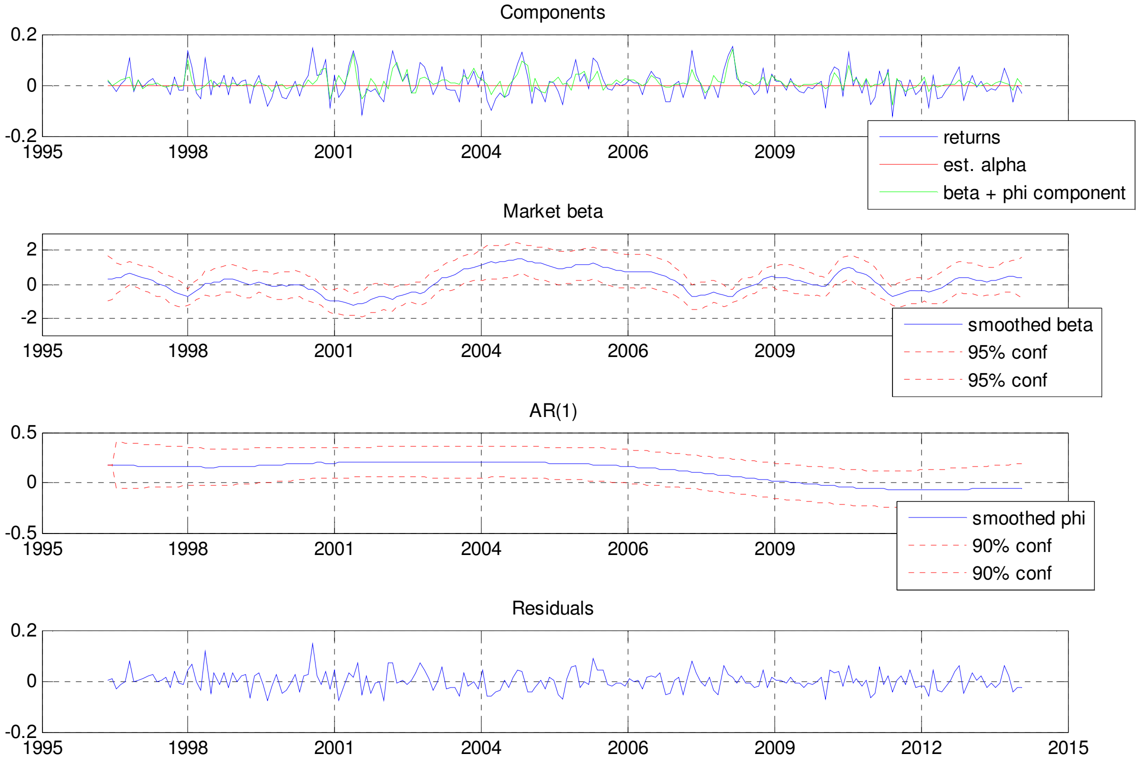
Testing and comparing models

Results for a sample fund – Model 2 – Kalman filter



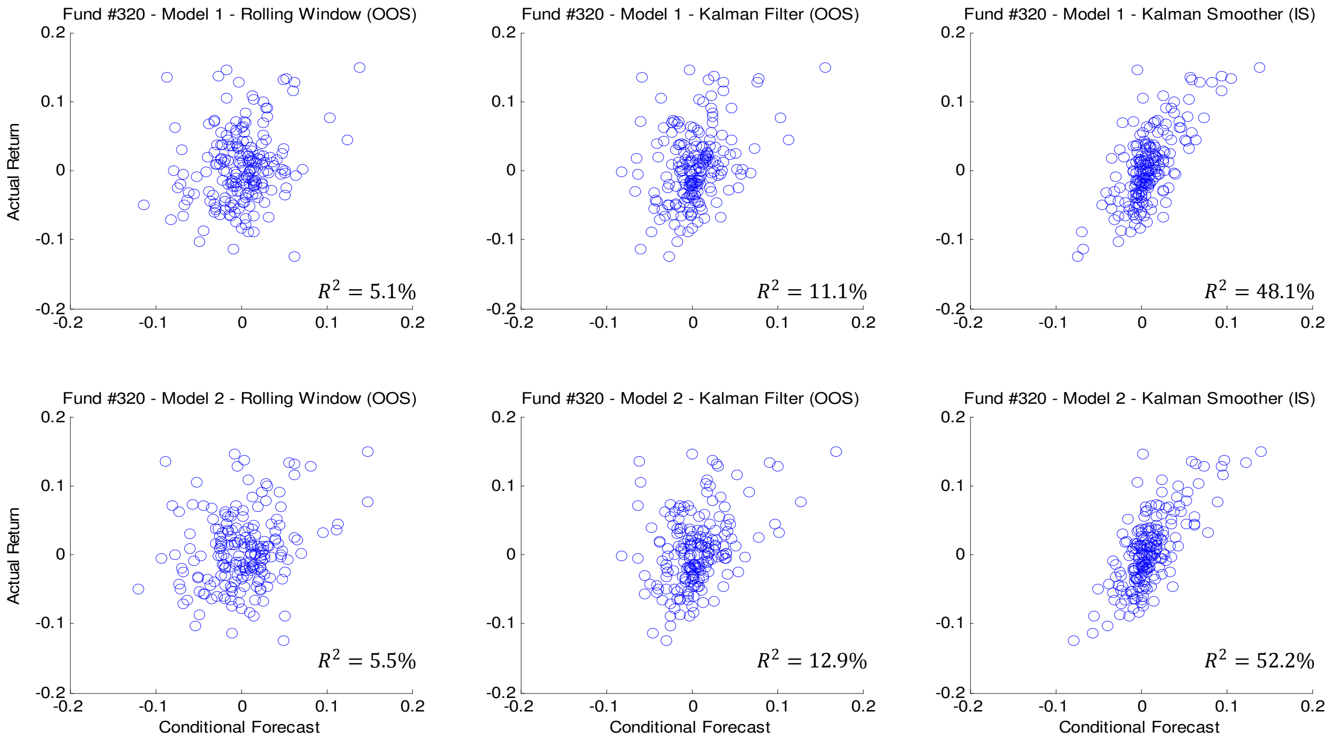
Testing and comparing models

Results for a sample fund – Model 2 – Kalman smoother



Testing and comparing models

Results for a sample fund: comparing across models and estimation techniques



$$Cond.Forecast \equiv \{\hat{y}_t | \hat{\alpha}_t; \hat{\beta}_t; \hat{\gamma}_t; \hat{\phi}_t; \hat{x}_t; \hat{z}_t; y_{t-1}\} = y_t - \varepsilon_t$$

Testing and comparing models

Results across ~6,500 funds: median R^2 of conditional forecast

	Equities	Eq. + AR(1)	Eq. + HFF	All
Rolling Window (OOS)	11.6%	10.3%	13.6%	12.3%
Kalman Filter (OOS)	14.5%	19.1%	21.6%	25.5%
Static Linear Regression (IS)	14.5%	18.8%	20.9%	25.2%
Kalman Smoother (IS)	34.6%	39.9%	45.6%	49.0%

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Testing and comparing models

Results across ~6,500 funds: percentage of funds with higher R^2

		Rolling Window (OOS)				Kalman Filter (OOS)				Static Linear Regression (IS)				Kalman Smoother (IS)			
		Eq.	Eq.+AR	Eq.+HFF	All	Eq.	Eq.+AR	Eq.+HFF	All	Eq.	Eq.+AR	Eq.+HFF	All	Eq.	Eq.+AR	Eq.+HFF	All
Rolling Window	Eq.	0.0%	66.4%	45.5%	51.1%	31.9%	20.1%	17.0%	11.3%	36.6%	29.9%	24.6%	18.7%	11.2%	5.9%	5.5%	2.6%
	Eq.+AR	33.6%	0.0%	34.6%	45.2%	30.0%	16.8%	16.9%	9.4%	34.4%	25.6%	22.9%	15.9%	11.7%	4.8%	6.0%	2.2%
	Eq.+HFF	54.5%	65.4%	0.0%	65.1%	44.5%	33.5%	18.0%	12.5%	46.0%	39.3%	26.2%	20.5%	18.1%	12.6%	5.0%	2.7%
	All	48.9%	54.8%	34.9%	0.0%	41.3%	29.1%	18.2%	10.5%	42.8%	34.5%	25.6%	17.3%	17.3%	10.6%	5.7%	2.1%
Kalman Filter	Eq.	68.1%	70.0%	55.5%	58.7%	0.0%	24.6%	18.0%	12.0%	47.3%	39.5%	29.4%	23.5%	7.6%	6.0%	3.1%	2.4%
	Eq.+AR	79.9%	83.2%	66.5%	70.9%	75.4%	0.0%	38.3%	22.7%	63.6%	50.9%	41.1%	31.6%	23.4%	11.5%	11.3%	5.6%
	Eq.+HFF	83.0%	83.1%	82.0%	81.8%	82.0%	61.7%	0.0%	29.3%	71.3%	61.3%	50.0%	39.8%	32.2%	24.8%	6.7%	5.4%
	All	88.7%	90.6%	87.5%	89.5%	88.0%	77.3%	70.7%	0.0%	80.4%	71.8%	61.8%	49.2%	42.7%	32.3%	17.6%	8.3%
Static Linear Regression	Eq.	63.4%	65.6%	54.0%	57.2%	52.7%	36.4%	28.7%	19.6%	0.0%	43.1%	26.7%	18.6%	17.8%	10.4%	6.5%	4.0%
	Eq.+AR	70.1%	74.4%	60.7%	65.5%	60.5%	49.1%	38.7%	28.2%	56.9%	0.0%	36.0%	26.4%	25.6%	12.3%	13.1%	5.3%
	Eq.+HFF	75.4%	77.1%	73.8%	74.4%	70.6%	58.9%	50.0%	38.2%	73.3%	64.0%	0.0%	41.7%	33.1%	27.3%	9.8%	6.8%
	All	81.3%	84.1%	79.5%	82.7%	76.5%	68.4%	60.2%	50.8%	81.4%	73.6%	58.3%	0.0%	40.7%	32.1%	19.2%	8.6%
Kalman Smoother	Eq.	88.8%	88.3%	81.9%	82.7%	92.4%	76.6%	67.8%	57.3%	82.2%	74.4%	66.9%	59.3%	0.0%	25.9%	15.3%	18.7%
	Eq.+AR	94.1%	95.2%	87.4%	89.4%	94.0%	88.5%	75.2%	67.7%	89.6%	87.7%	72.7%	67.9%	74.1%	0.0%	37.7%	24.6%
	Eq.+HFF	94.5%	94.0%	95.0%	94.3%	96.9%	88.7%	93.3%	82.4%	93.5%	86.9%	90.2%	80.8%	84.7%	62.3%	0.0%	33.9%
	All	97.4%	97.8%	97.3%	97.9%	97.6%	94.4%	94.6%	91.7%	96.0%	94.7%	93.2%	91.4%	81.3%	75.4%	66.1%	0.0%

Testing and comparing models

Results across ~6,500 funds: median standard deviation of beta

	Equities	Eq. + AR(1)	Eq. + HFF	All
Rolling Window (OOS)	23.7%	24.7%	24.6%	25.8%
Kalman Filter (OOS)	15.6%	13.6%	15.0%	10.9%
Static Linear Regression (IS)	0.0%	0.0%	0.0%	0.0%
Kalman Smoother (IS)	7.7%	1.0%	7.0%	0.0%

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Results across ~6,500 funds: median standard deviation of beta

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Kalman Filter (OOS)	15.6%	13.6%	15.0%	10.9%
Static Linear Regression (IS)	0.0%	0.0%	0.0%	0.0%
Kalman Smoother (IS)	7.7%	1.0%	7.0%	0.0%

Testing and comparing models

Results across HF strategies: median adjusted R^2 of conditional forecast

		Rolling Window (OOS)				Kalman Filter (OOS)				Static Linear Regression (IS)				Kalman Smoother (IS)			
Hedge Fund Strategy	# funds	Eq.	Eq.+AR	Eq.+HFF	All	Eq.	Eq.+AR	Eq.+HFF	All	Eq.	Eq.+AR	Eq.+HFF	All	Eq.	Eq.+AR	Eq.+HFF	All
Arbitrage	301	11.2%	9.4%	12.9%	11.9%	12.0%	17.8%	20.1%	23.5%	14.5%	18.1%	21.2%	25.6%	32.2%	39.3%	43.7%	48.5%
Bottom-Up	268	13.0%	11.2%	16.8%	15.5%	15.2%	22.0%	22.6%	27.8%	15.2%	19.3%	22.2%	26.5%	34.9%	40.2%	46.8%	51.6%
CTA/Managed Futures	1014	11.7%	11.2%	14.3%	13.2%	14.5%	19.1%	20.9%	25.1%	15.2%	20.5%	22.3%	26.4%	35.1%	40.7%	45.9%	49.7%
Distressed Debt	112	15.1%	13.2%	16.6%	14.2%	16.6%	21.2%	23.5%	28.8%	16.9%	22.3%	22.8%	28.1%	33.9%	41.0%	42.9%	47.4%
Diversified Debt	24	6.4%	7.6%	13.6%	14.0%	10.7%	15.9%	20.8%	24.2%	15.8%	21.0%	18.2%	24.3%	24.7%	35.1%	41.6%	42.1%
Dual Approach	127	14.2%	12.8%	17.2%	14.1%	16.2%	20.7%	23.6%	25.6%	16.5%	22.8%	24.8%	28.0%	32.1%	39.2%	45.9%	49.3%
Event Driven	275	12.5%	10.6%	16.1%	13.9%	16.3%	18.0%	23.9%	26.7%	14.0%	18.3%	21.7%	24.7%	35.9%	41.5%	48.7%	49.4%
Fixed Income	504	10.6%	9.9%	12.2%	10.8%	13.1%	17.3%	20.5%	24.3%	12.5%	17.1%	19.0%	24.4%	35.2%	39.6%	45.5%	50.1%
Long Short Equities	2429	11.7%	10.1%	13.0%	12.1%	14.1%	19.4%	21.2%	25.7%	14.0%	17.8%	20.1%	23.9%	35.4%	40.6%	45.6%	49.1%
Macro	451	9.8%	9.4%	11.1%	10.4%	13.6%	17.4%	20.1%	23.5%	15.3%	17.9%	20.4%	24.0%	29.8%	36.9%	42.9%	45.1%
Multi-Strategy	573	12.6%	9.8%	14.4%	12.1%	16.9%	19.3%	24.0%	27.5%	15.5%	19.0%	22.2%	25.4%	32.2%	38.8%	45.2%	48.6%
Others	165	13.2%	12.0%	16.7%	15.0%	14.0%	20.1%	26.3%	28.9%	15.6%	19.9%	24.8%	25.3%	38.5%	44.1%	45.3%	49.0%
Relative Value	141	11.2%	10.4%	12.0%	12.5%	14.2%	18.8%	20.4%	26.0%	12.9%	18.7%	20.2%	24.8%	35.5%	38.3%	41.7%	45.1%
Top-Down	35	13.6%	8.4%	20.3%	16.7%	16.3%	24.5%	22.8%	28.5%	24.0%	24.9%	23.4%	29.6%	28.9%	47.8%	52.0%	56.9%
Value	219	11.4%	10.6%	13.1%	12.3%	15.2%	20.4%	23.1%	25.9%	13.4%	20.2%	20.6%	26.0%	35.6%	39.8%	46.8%	46.8%

Conclusion

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Some of the questions we wanted to investigate

- Does formally allowing for time-varying parameters improve the performance of the factor models?
- Does it result in more stable parameters?
- Does inclusion of an autoregressive term provide a better fit on average?
- How does that compare with adding additional factors?
- How well do out-of-sample models perform relative to in-sample ones?
- Do the results vary across different hedge fund strategies?
- How much harder are these models to implement than a rolling window regression?

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- Does it result in more stable parameters? YES
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- How does that compare with adding additional factors? Depends
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- How well do out-of-sample models perform relative to in-sample ones? So-so
- Do the results vary across different hedge fund strategies? Not really
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- Does formally allowing for time-varying parameters improve the performance of the factor models? YES
- Does it result in more stable parameters? YES
- Does inclusion of an autoregressive term provide a better fit on average? YES
- How does that compare with adding additional factors? Depends
- How well do out-of-sample models perform relative to in-sample ones? So-so
- Do the results vary across different hedge fund strategies? Not really
- How much harder are these models to implement than a rolling window regression? Not much

Conclusion

Advantages of using a State Space Modelling framework

- Extremely flexible modelling framework.
- Easily incorporates time-varying parameters.
- Dynamic factor estimates are smoother and more reliable than those from rolling windows (at least for HF returns).
- Both in-sample and out-of-sample testing can be implemented without resource-intensive loops.
- Linear Gaussian models can be efficiently estimated using fast, efficient recursive closed-form techniques such as the Kalman filter and smoother.
- Less restrictive models (non-linear/non-Gaussian) can be estimated using modified filtering techniques or with numerical methods.
- Recursive estimation makes it easy to simulate and forecast from estimated models as well as to deal with missing data points.
- Intuitive link with Bayesian statistical methods and econometrics.
- MathWorks® and third-party SSM toolboxes readily available.

Conclusion

Advantages of using MATLAB

- MATLAB Desktop environment allows for fast and easy interactive data analysis and model research and development.
- Distinct advantages of having integrated computational and application development environments.
- Wide availability of toolboxes and third-party libraries.
- Extensive plotting and visualisation tools.
- Thorough documentation and extensive/flexible support.
- Ability to integrate with many other environments (Excel, Access, SQL, etc.).
- Ability to compile and distribute packaged applications.
- One-off product license with reasonably priced maintenance costs.

Modelling Hedge Fund Returns Using State Space Models

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Other:

- Commandeur, Jacques JF, Siem Jan Koopman, and Marius Ooms. "Statistical software for state space methods." *Journal of Statistical Software* 41.1 (2011): 1-18.
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- MATLAB Econometrics Toolbox™ User's Guide (R2014a).

Appendix – Extra Slides

Traditional models for hedge fund returns

Modelling serial correlation and illiquidity in HF returns (Getmansky, Lo and Makarov, 2004)

$$y_t = \alpha + \sum_{j=1}^{k} \beta_j x_{j,t} + \varepsilon_t, \qquad E\left[x_{j,t}\right], E[\varepsilon_t] = 0, \qquad \varepsilon_t, x_{j,t} \sim IID$$

$$y_t^o = \theta_0 y_t + \theta_1 y_{t-1} + \cdots + \theta_p y_{t-p}$$

$$\theta_j \in [0,1], \quad j = 0, \ldots, p$$

$$\sum \theta_j = 1 \Longrightarrow E[y_t^o] = E[y_t]$$

Introduction to State Space Models

Example 2: a local level model with a single static market factor

$$y_t = Z_t' a_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$a_{t+1} = T_t a_t + R_t \eta_t, \qquad \eta_t \sim NID(0, Q_t)$$

$$a_t = \begin{pmatrix} \alpha_t \\ \beta_t \end{pmatrix}, \quad \eta_t = \begin{pmatrix} \xi_t \\ 0 \end{pmatrix}, \quad T_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Z_t = \begin{pmatrix} 1 \\ x_t \end{pmatrix},$$

$$R_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad Q_t = \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$y_t = \alpha_t + \beta_t x_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$\alpha_{t+1} = \alpha_t + \xi_t, \qquad \xi_t \sim NID(0, \sigma_\xi^2)$$

$$\beta_{t+1} = \beta_t$$

Introduction to State Space Models

Example 3: a local level model with a single time-varying market factor

$$y_t = Z_t^{'} a_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$a_{t+1} = T_t a_t + R_t \eta_t, \qquad \eta_t \sim NID(0, Q_t)$$

$$a_t = \begin{pmatrix} \alpha_t \\ \beta_t \end{pmatrix}, \quad \eta_t = \begin{pmatrix} \xi_t \\ \zeta_t \end{pmatrix}, \quad T_t = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad Z_t = \begin{pmatrix} 1 \\ x_t \end{pmatrix},$$

$$R_t = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad Q_t = \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & \sigma_\zeta^2 \end{bmatrix}$$

$$y_t = \alpha_t + \beta_t x_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$\alpha_{t+1} = \alpha_t + \xi_t, \qquad \xi_t \sim NID(0, \sigma_\xi^2)$$

$$\beta_{t+1} = \beta_t + \zeta_t, \qquad \zeta_t \sim NID(0, \sigma_\zeta^2)$$

Introduction to State Space Models

Example 4: an AR(2) model

$$y_t = Z_t' a_t + \varepsilon_t, \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$$

$$a_{t+1} = T_t a_t + R_t \eta_t, \qquad \eta_t \sim NID(0, Q_t)$$

$$a_t = \begin{pmatrix} y_t \\ \phi_2 y_{t-1} \end{pmatrix}, \quad Z_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$T_t = \begin{bmatrix} \phi_1 & 1 \\ \phi_2 & 0 \end{bmatrix}, \quad R_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$\eta_t = \begin{pmatrix} \xi_{t+1} \\ 0 \end{pmatrix}, \quad Q_t = \begin{bmatrix} \sigma_\xi^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \sigma_\varepsilon^2 = 0$$

$$y_{t+1} = \phi_1 y_t + \phi_2 y_{t-1} + \xi_{t+1}$$

$$\xi_t \sim NID(0, \sigma_\xi^2)$$