Calibration and Simulation of Interest Rate Models

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Agenda

- Calibration to Market Data
- Calibration to Historical Data
- Simulation and Valuation
- Counterparty Credit Risk Analysis
- Questions and Answers

Interest Rate Models

Cox-Ingersoll-Ross

$$dr(t) = a(b - r)dt + \sigma\sqrt{r}dW(t)$$

Hull-White

$$dr(t) = (\theta(t) - ar)dt + \sigma dW(t)$$

G2++

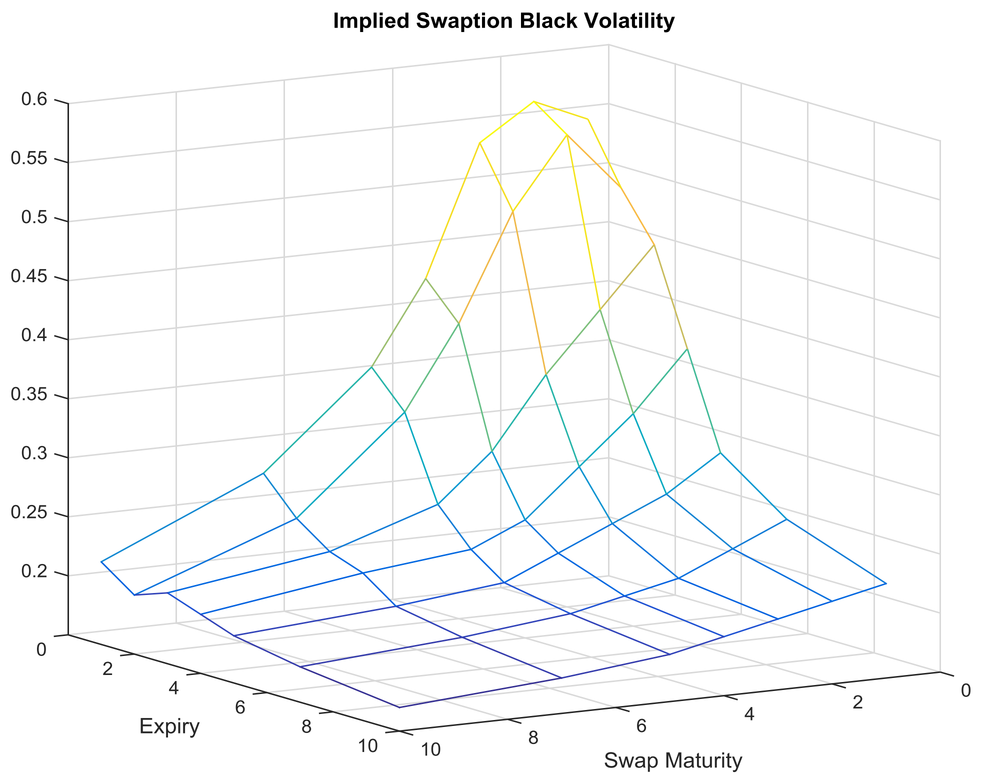
$$r(t) = x(t) + y(t) + \varphi(t)$$
$$dx(t) = -ax(t)dt + \sigma dW_1(t)$$
$$dy(t) = -by(t)dt + \eta dW_2(t)$$
$$dW_1(t)dW_2(t) = \rho dt$$

Calibrate to Market Data

- Choose a set of liquid calibration instruments – typically caps, floors, swaptions.
- Find the set of model parameters that matches as closely as possible the observed prices.

$$\sum_{k=0}^{n} (P_i - \hat{P}_i(\theta))^2$$

P_i: Market Price
$\hat{P}_i$: Model Price
θ: Model Parameters



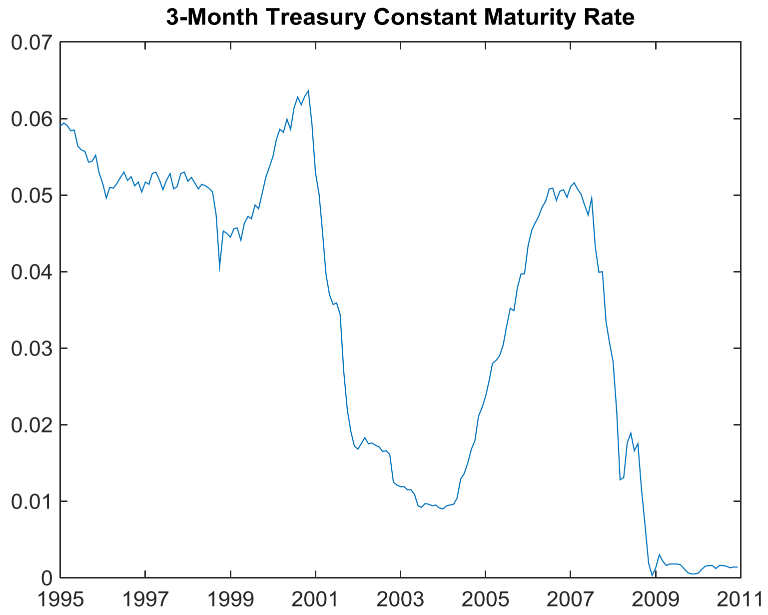
Calibrate CIR Model using MLE of Transition Density

$$dr(t) = a(b - r)dt + \sigma\sqrt{r}dW(t)$$

a: mean reversion speed

σ: volatility of the short rate

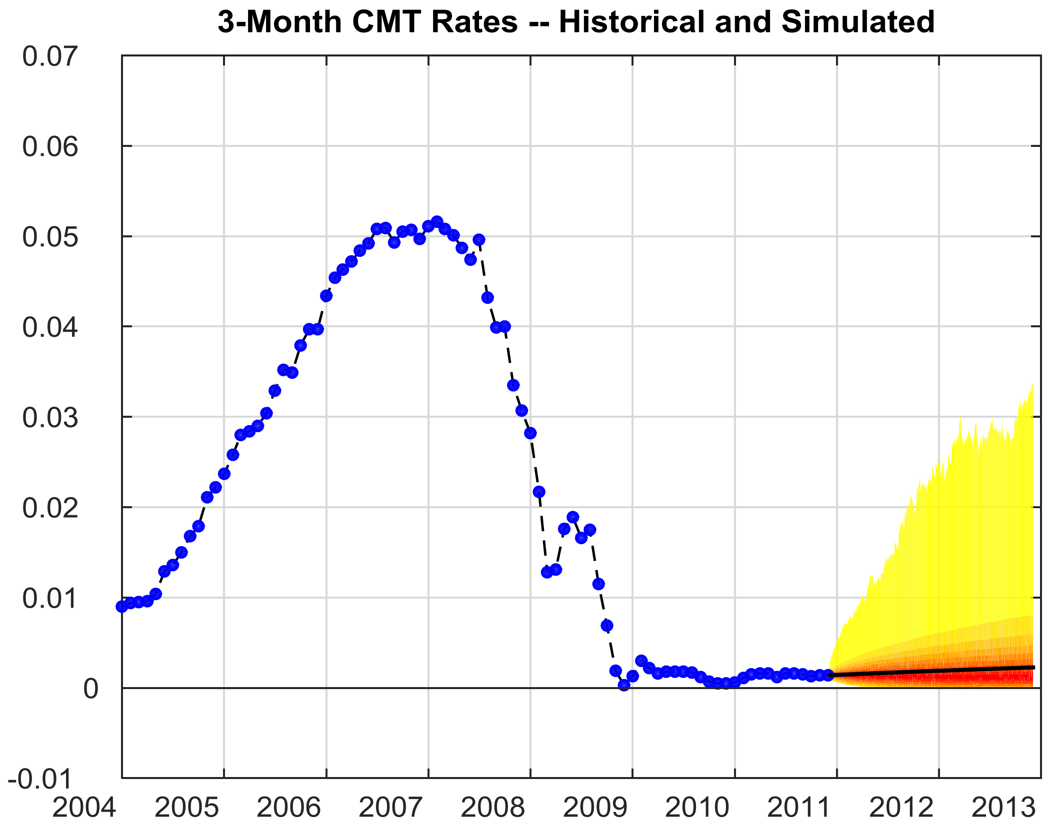
b : level



Aït-Sahalia, Y. (1999). Transition densities for interest rate and other nonlinear diffusions. *The Journal of Finance*, *54*(4), 1361-1395.

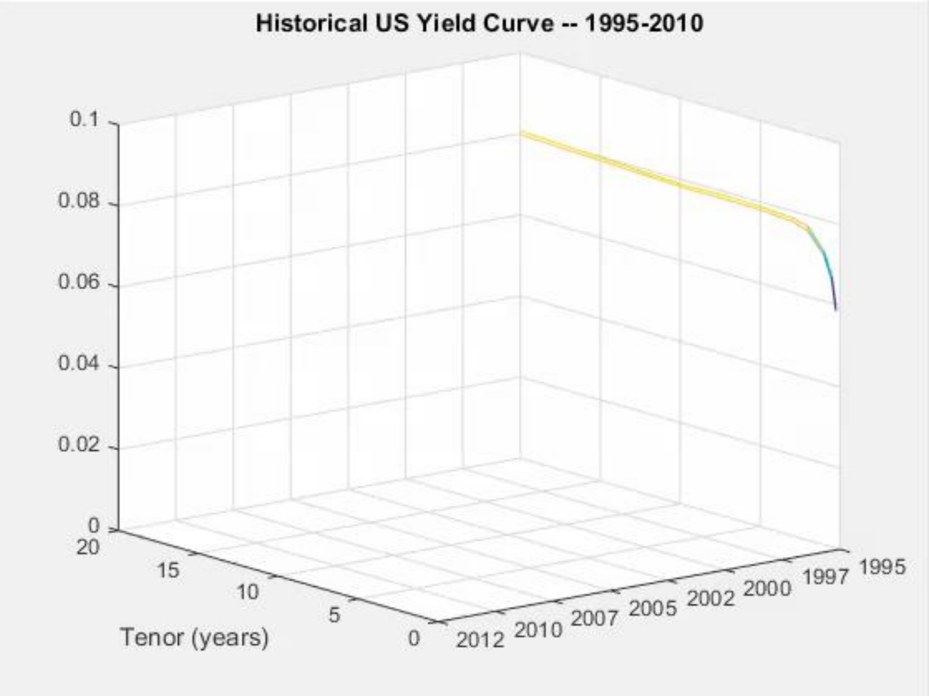
Stochastic Differential Equation Models

- Suite of models including :
  `bm, gbm, cir, hwv, heston, cev`
- Simulate methods
- Framework for creating custom models



Calibrate using Kalman Filter

- Formulate models as state space systems.
- Use Kalman filter to estimate parameters.
- Estimate parameters from historical yield curves.



Park, F.C., “Implementing Interest Rate Models: A Practical Guide.” Capital Markets & Portfolio Research, Inc. white paper, 2004

State Space formulation for G2++ Model

Transition Equation $x_t = Ax_{t-1} + B\mu$

Measurement Equation $y_t = Cx_t + D\epsilon$

$$A = \begin{bmatrix} e^{-a\Delta t} & 0 \\ 0 & e^{-b\Delta t} \end{bmatrix}$$

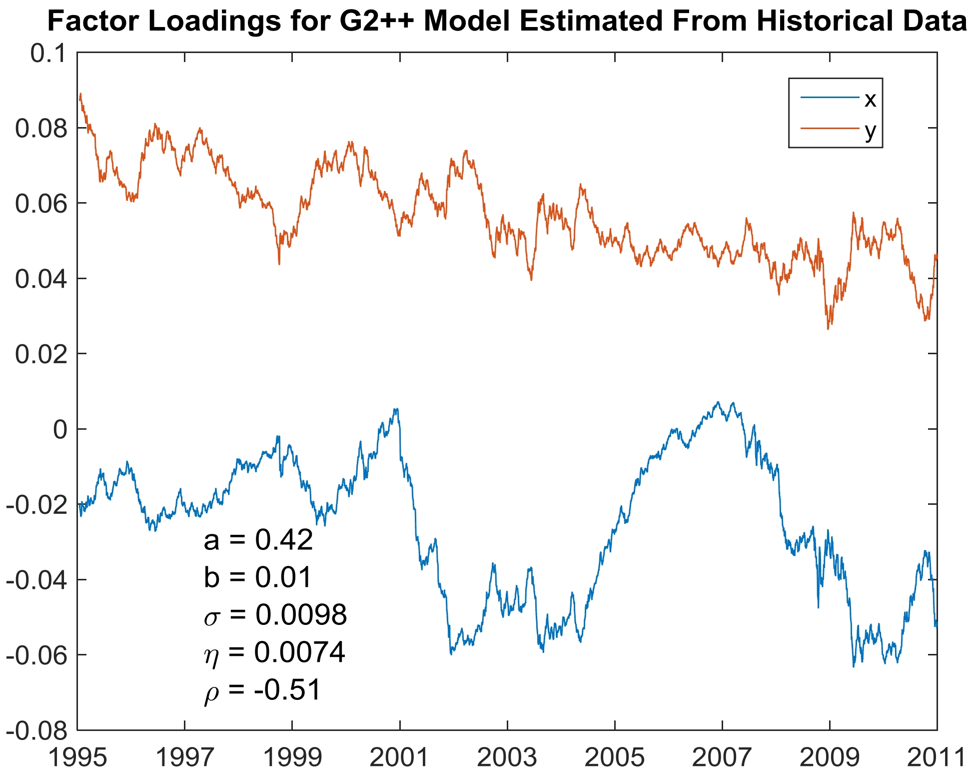
$$B = \begin{bmatrix} \sigma\sqrt{\dfrac{1-e^{-2a\Delta t}}{2a}} & 0 \\ 0 & \eta\sqrt{\dfrac{1-e^{-2b\Delta t}}{2b}} \end{bmatrix}$$

Park, F.C., “Implementing Interest Rate Models: A Practical Guide.” Capital Markets & Portfolio Research, Inc. white paper, 2004

State Space Model

New state space model, `ssm` in Econometrics Toolbox™.

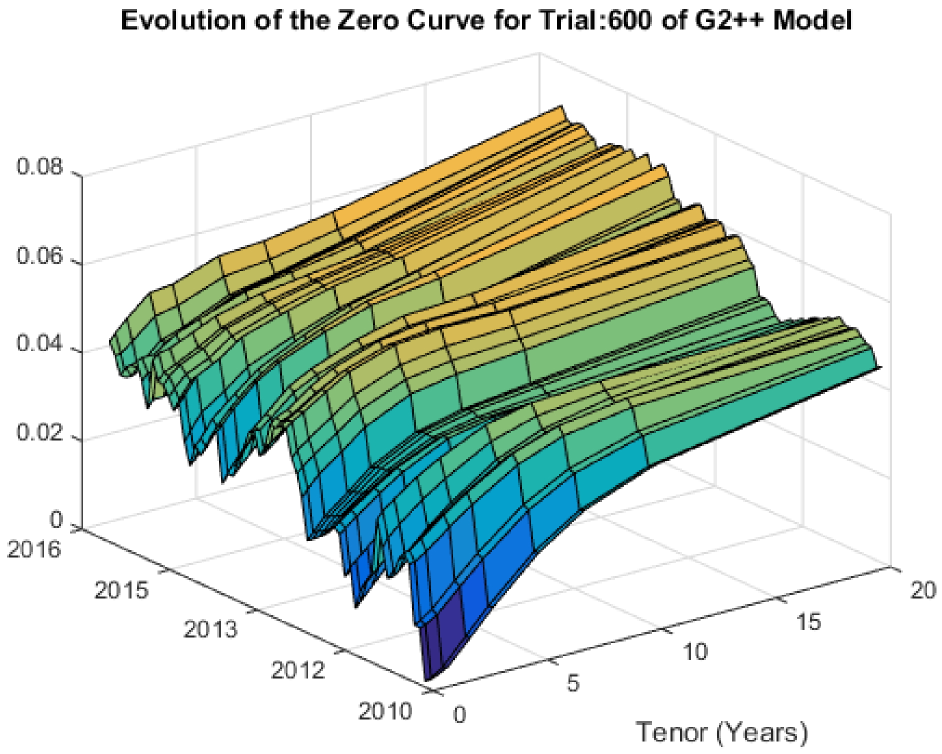
- Supports time-invariant and time-varying, linear state-space models.
- Perform univariate and multivariate time-series data analysis.
- Functionality to: `estimate, filter, smooth, simulate, forecast`



Interest Rate Model Simulation

Specify models and simulate entire term structure

- Support for Hull-White, G2++ and LIBOR Market Model.
- `simTermStructs` simulates entire term structure.



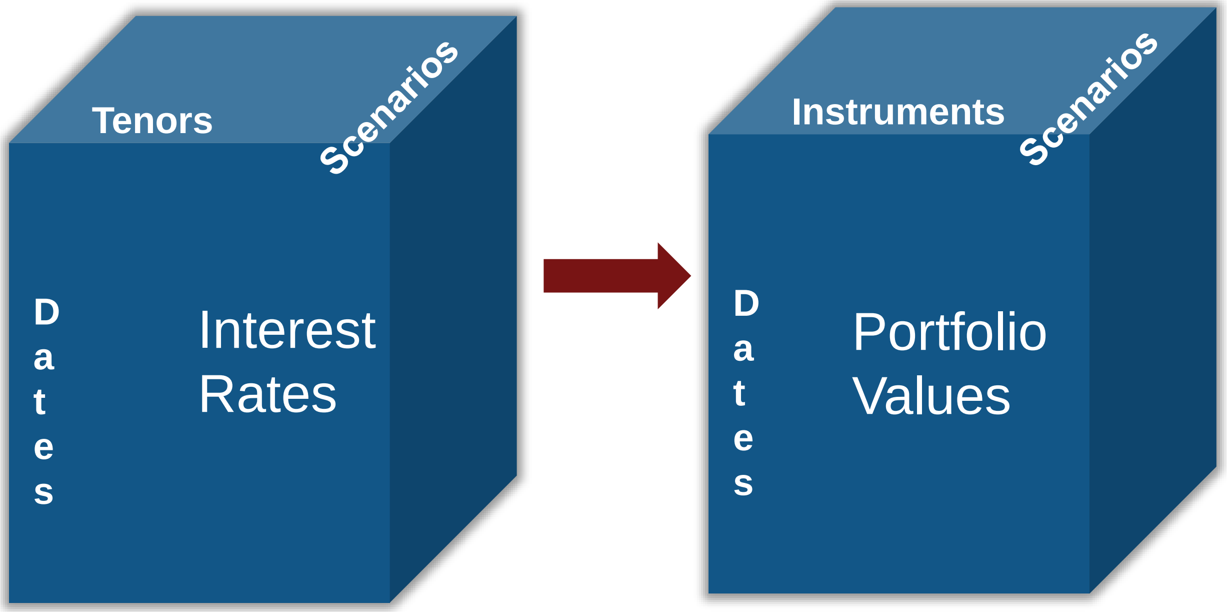
Swap Portfolio

- Store data in a MATLAB Table.
- Easy to read in data.
- Tabular display.

```
>> SwapPort = readtable('SwapPortfolio.xlsx')

SwapPort = 

    Notional      Maturity       RecType    PayType    RecRate    PayRate    RecReset    PayReset
    ________    _____________    _______    _______    _______    _______    ________    ________

      1e+07     '1/15/2018'      1          0           0.031         10     12          12      
      5e+06     '2/15/2018'      0          1              20      0.032     12          12      
      1e+06     '3/15/2019'      1          0           0.033         30     12          12      
      2e+06     '4/15/2019'      0          1              40      0.034     12          12      
      1e+07     '5/15/2020'      1          0           0.036         50     12          12      
      7e+06     '6/15/2020'      0          1              65      0.036     12          12      
    7.5e+06     '7/15/2021'      1          0          0.0385         70     12          12      
      8e+06     '8/15/2021'      0          1              75       0.04     12          12      
      3e+06     '9/15/2022'      1          0           0.039         85     12          12      
    3.5e+06     '10/15/2022'     0          1              95       0.04     12          12      

>>
```

Valuing the Portfolio

- Value portfolio using `swapbyzero`
- Use `parfor` to loop over simulation dates.

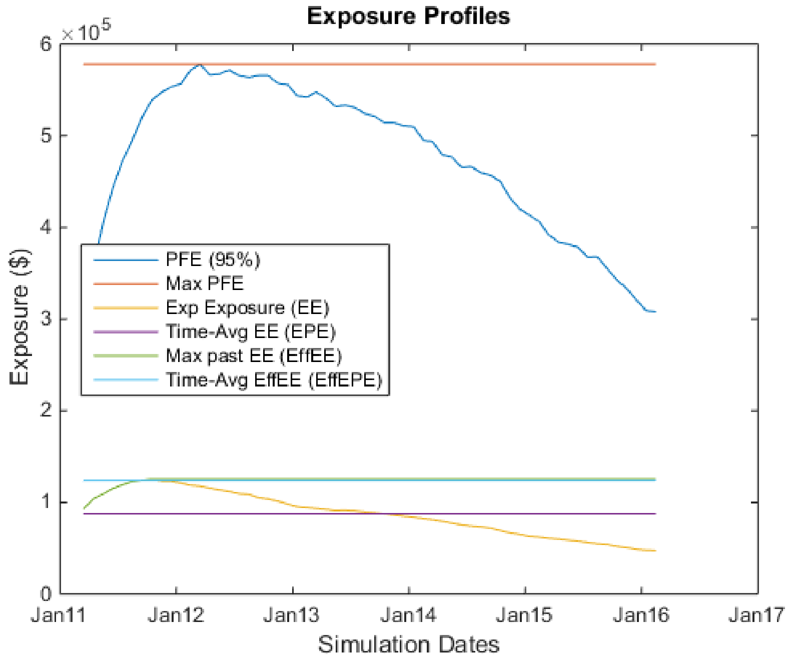



```
>> Values = zeros(nDates,nSwaps,nTrials);
>> parfor dateidx=1:nDates
       Values(dateidx,:,:) = swapbyzero(...)
   end
```

Counterparty Credit Risk Functions

Compute exposures and CCR profiles

- Support for computing credit exposures.
- Support for computing various credit exposure profiles, including potential future exposure and expected exposure.

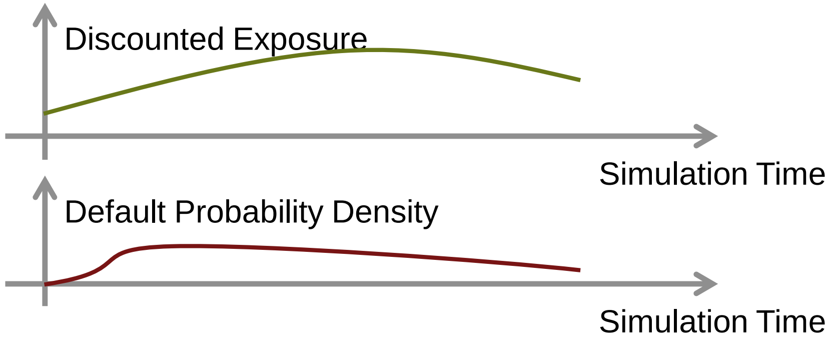



```
>> Exposures = creditexposures(Values);
>> Profiles = exposureprofiles(SimDates,Exposures);
```

Computing Credit Valuation Adjustment

- Compute exposure from `exposureprofiles`

- Compute default probabilities from `cdsbootstrap`



$$CVA = (1 - R)\int_0^T DiscExp(t)dPD(t)$$

Summary

- Calibration Approaches
  - Market Data: `lsqnonlin, simulannealbnd`
  - Historical Data: `mle, ssm`
- Monte Carlo Simulation in MATLAB
  - `cir`
  - `HullWhite1F, LinearGaussian2F`
- Counterparty Credit Risk
  - `creditexposures, exposureprofiles`
  - `cdsbootstrap`

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