From Algorithm to Generic, Parallel Code

Dietmar Kühl Bloomberg L.P.

Overview

- Introduce inclusive scan
- Explain the algorithm based on a version from a text book
- Implement versions using OpenMP and Threading Building Blocks (TBB)
- Implement a version using an executor

- Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:

1 2 3 4 5 6 7 8 9

Output:

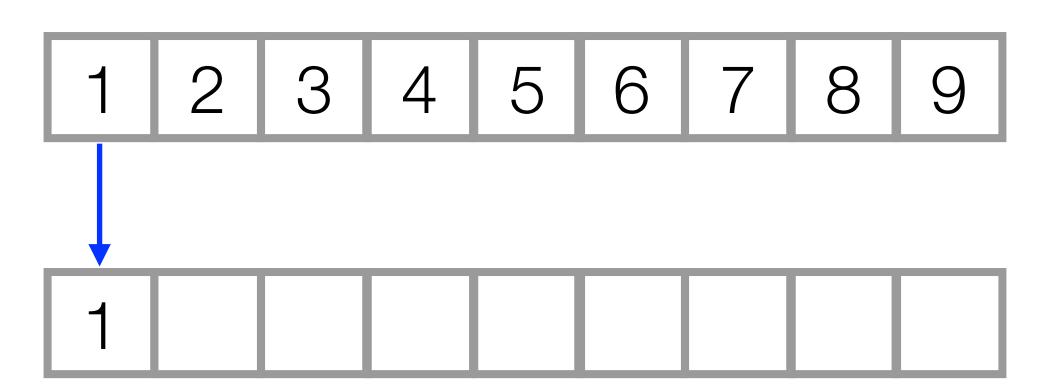
1 3 6 10 15 21 28 36 45

- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:

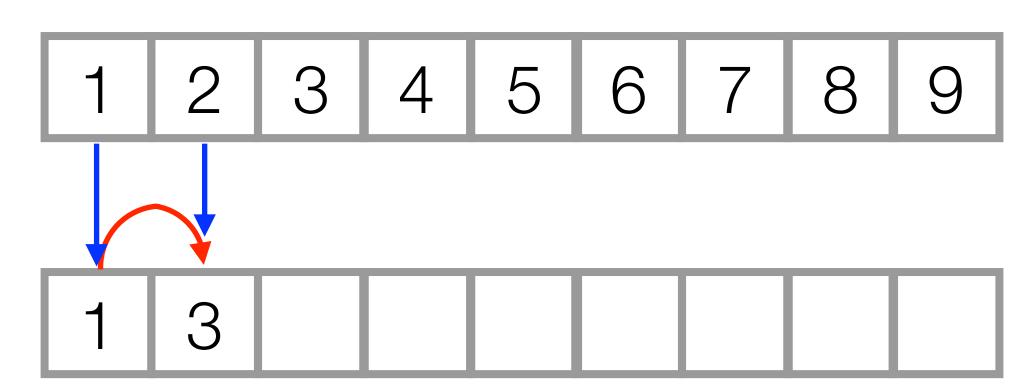




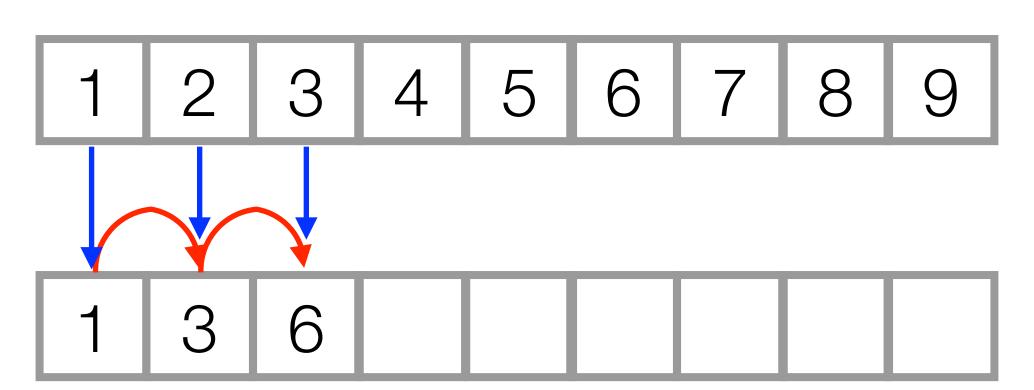
- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



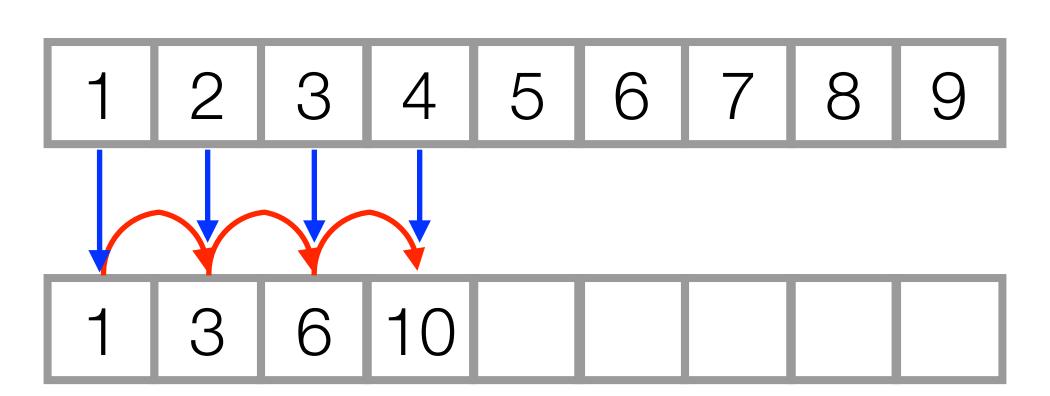
- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



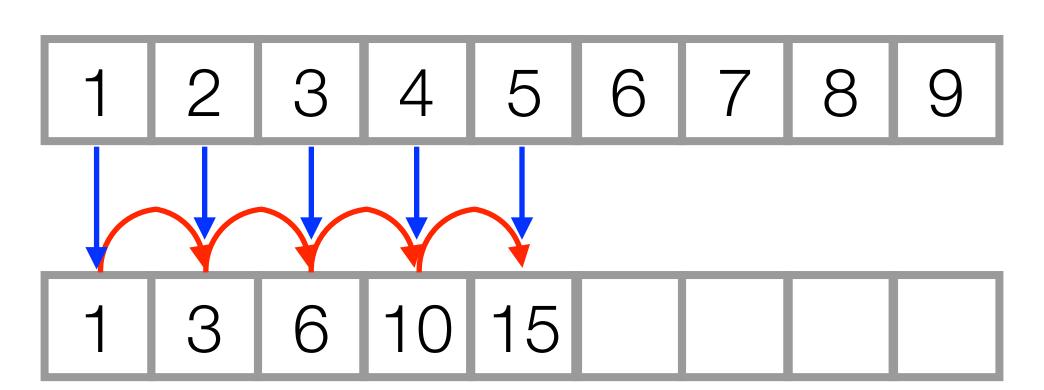
- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



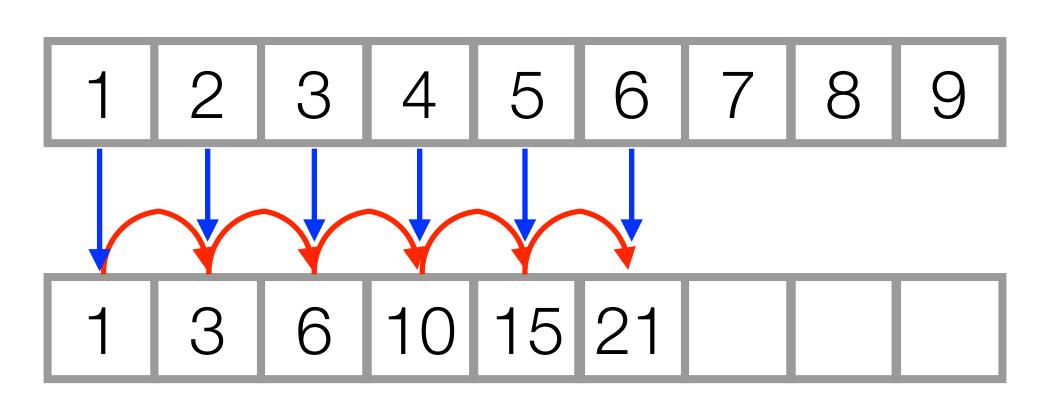
- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



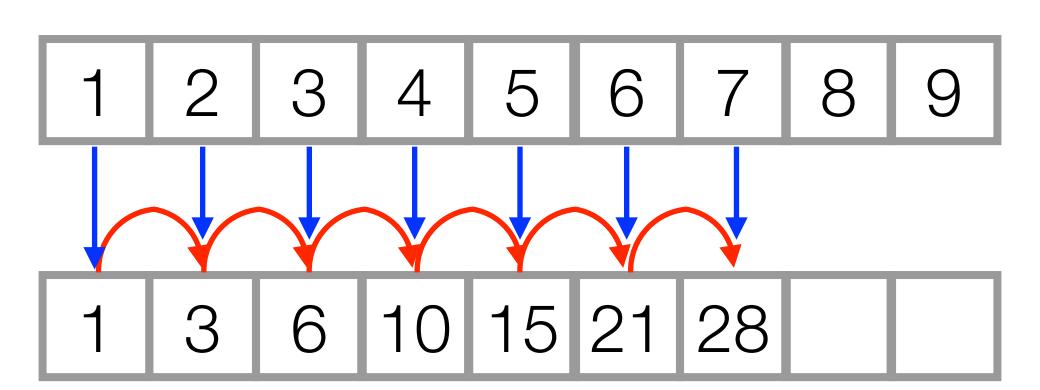
- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



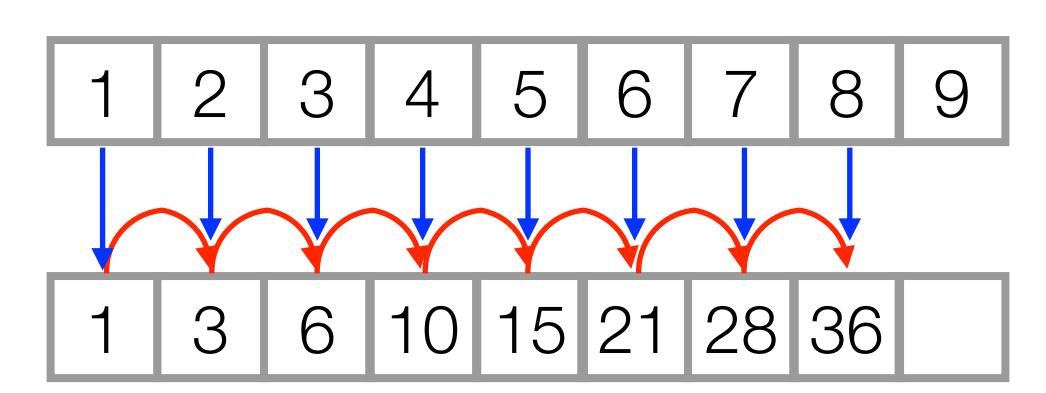
- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



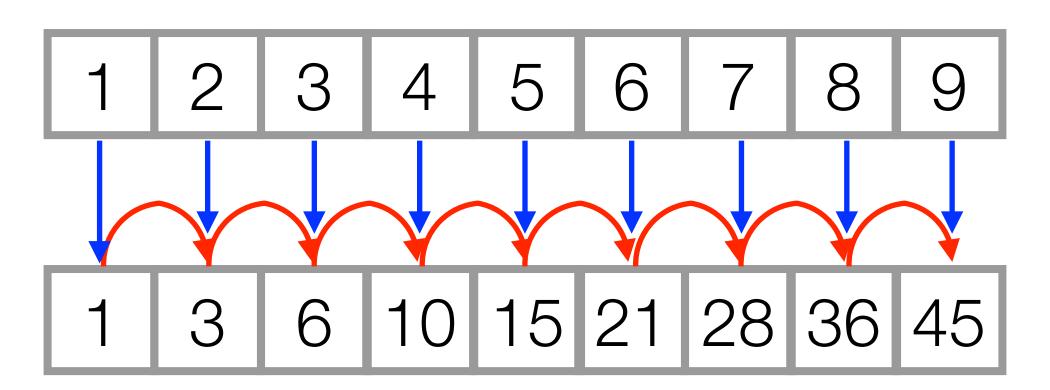
- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



- · Compute all of the sums of initial subsequences:
- For all i compute $y_i = \sum_{j=0}^{i} x_j$
- Trivial to do sequentially
- Example:
 - Input:



```
template <typename In, typename Out, typename Op = std::plus<>>
Out inclusive_scan(In it, In end, Out to, Op op) {
  if (it != end) {
     auto value(*it);
     *to++ = value:
     for (; ++it != end; ++to)
        *to = value = op(value, *it);
  return to;
```

```
template <typename In, typename Out, typename Op = std::plus<>>
Out inclusive_scan(In it, In end, Out to, Op op) {
  return it != end? inclusive_scan(it + 1, end, to + 1, op, *to = *it): to;
template <typename In, typename Out, typename Op, typename Value>
Out inclusive_scan(In it, In end, Out to, Op op, Value value) {
  for (; it != end; ++it, ++to)
     *to = value = op(value, *it);
  return to;
```

```
template <typename In, typename Out, typename Op = std::plus<>>
Out inclusive_scan(In it, In end, Out to, Op op) {
  return it != end? inclusive_scan(it + 1, end, to + 1, op, *to = *it): to;
template <typename In, typename Out, typename Op, typename Value>
Out inclusive_scan(In it, In end, Out to, Op op, Value value) {
  for (; it != end; ++it, ++to)
     *to = value = op(value, *it);
  return to:
```

```
template <typename In, typename Out, typename Op = std::plus<>>
Out inclusive_scan(In it, In end, Out to, Op op) {
  return it != end? inclusive_scan(it + 1, end, to + 1, op, *to = *it): to;
template <typename In, typename Out, typename Op, typename Value>
Out inclusive_scan(In it, In end, Out to, Op op, Value value) {
  for (; it != end; ++it, ++to)
     *to = value = op(value, *it);
  return to:
```

```
template <typename In, typename Out, typename Op = std::plus<>>
Out inclusive_scan(In it, In end, Out to, Op op) {
  return it != end? inclusive_scan(it + 1, end, to + 1, op, *to = *it): to;
template <typename In, typename Out, typename Op, typename Value>
Out inclusive_scan(In it, In end, Out to, Op op, Value value) {
  for (; it != end; ++it, ++to)
     *to = value = op(value, *it);
  return to;
```

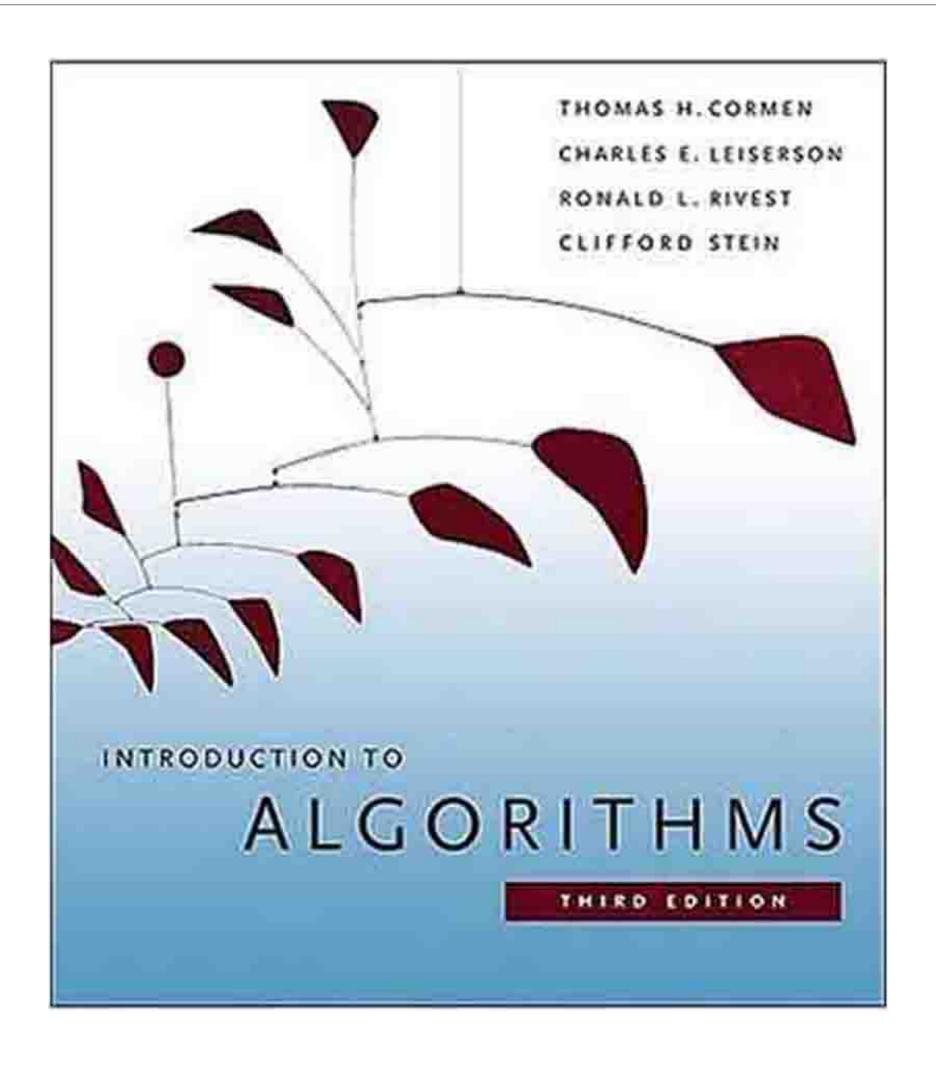
Parallelization

- It doesn't look suitable to parallelize inclusive scan: Later values in the sequence depend on all earlier ones
- · By doing more work the expected time to completion (span) can be reduced:
 - Recursively decompose the sequence into first/second halves
 - Compute the sums of the halves
 - Compute the partial sums based on the halves
 - Twice as many operations (work) but only log(n) of the time (span)

Basic Idea of the Algorithm

- With the middle sum of each half, these can be populated concurrently
 - Recursively decomposing the populating yields an O(log N) span for this
- Computing the sum of a range can be recursively decomposed:
 - Compute the sum of each half and just add them
 - Doing so actually yields the middle sums needed for populating
 - · Recursively decomposing this computation also has an O(log N) span

Parallel Algorithm (from "Introduction to Algorithms", Cormen et al)



Parallel Algorithm (from "Introduction to Algorithms", Cormen et al)

- P-Scan-Up computes the sum of halves
- P-Scan-Down computes the partial sums based on the halves

```
P-Scan-3(x)

n = x.length

let y[1..n] and t[1..n] be new arrays

y[1] = x[1]

if n > 1

P-Scan-Up(x, t, 2, n)

P-Scan-Down(x[1], x, t, y, 2, n)

return y
```

Parallel Algorithm: STL interface version

```
template <typename In, typename Out, typename Op>
Out p_scan(In begin, In end, Out to, Op op) {
  auto n = std::distance(begin, end);
  if (0 < n) {
     to[0] = begin[0];
    if (1 < n) {
       std::vector<std::decay_t<decltype(*begin)>> t(n);
       p_scan_up(begin + 1, end, t.begin(), op);
       p_scan_down(begin[0], begin + 1, end, t.begin(), to + 1, op);
  return to + n;
```

P-Scan-Up: Compute Auxiliary Sums of Sub-Ranges

```
P-Scan-Up(x, t, i, j)
  if i == i
     return x[i]
 else
     k = \lfloor (i + i) / 2 \rfloor
     t[k] = spawn P-Scan-Up(x, t, i, k)
     right = P-Scan-Up(x, t, k + 1, j)
     sync
     return t[k] ⊗ right
```

P-Scan-Up: STL interface version

```
template <typename In, typename Tmp, typename Op>
auto p_scan_up(In b, In end, Tmp tmp, Op op) {
  auto n = std::distance(b, end);
  if (1 == n) { return *b; }
  else {
     auto k = n / 2:
     auto fut = std::async([\&]{tmp[k]} = p_scan_up(b, b + k, tmp, op); });
     auto right = p_scan_up(b + k, end, tmp + k, op);
     fut.wait();
     return op(tmp[k], right);
```

P-Scan-Down: Compute the Actual Result

```
P-Scan-Down(v, x, t, y, i, j)

if i == j

y[i] = v ⊗ x[i]

else

k = [(i + j) / 2]

spawn P-Scan-Down(v, x, t, y, i, k)

P-Scan-Down(v + t[k], x, t, y, k + 1, j)

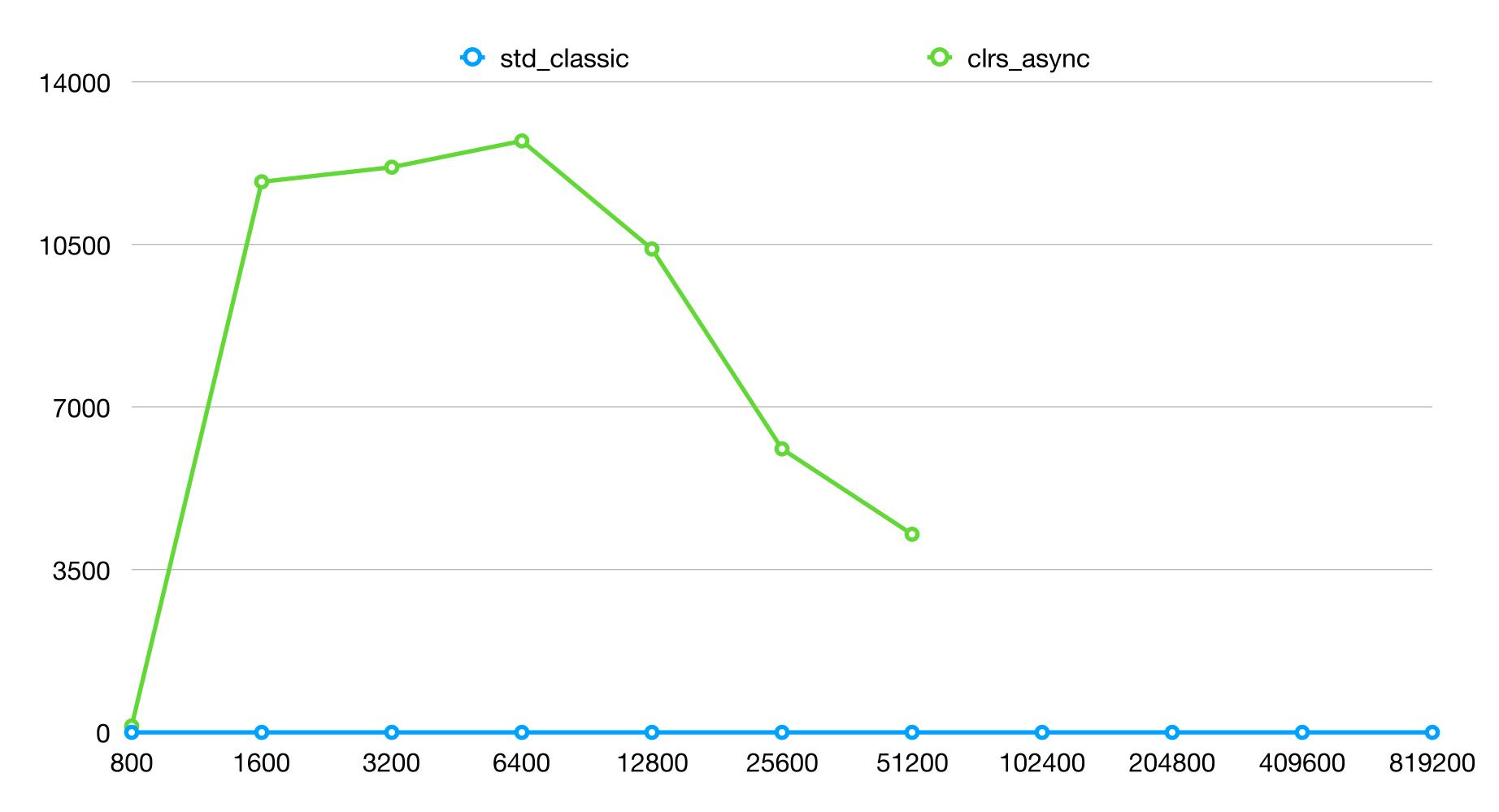
sync
```

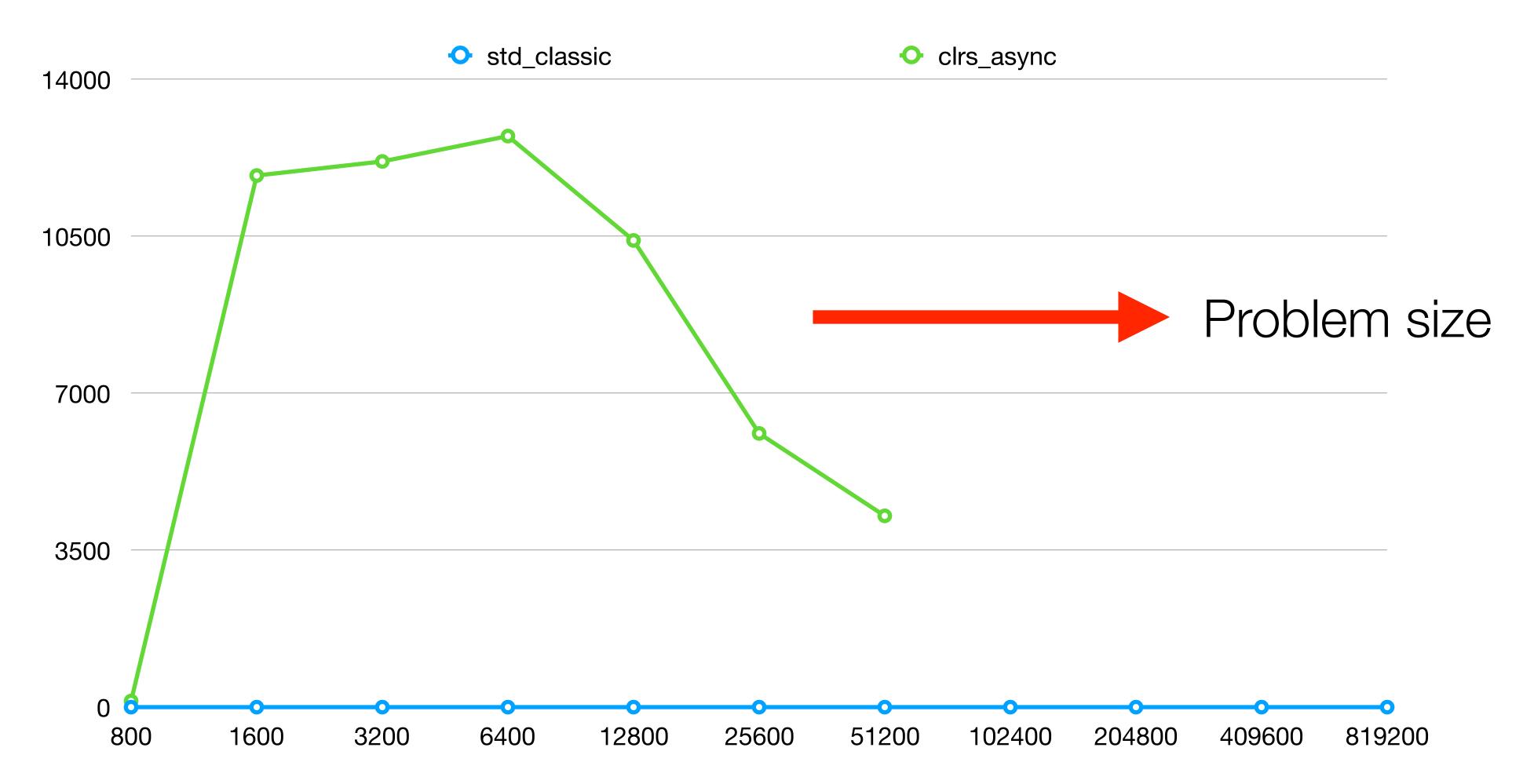
P-Scan-Down: STL interface version

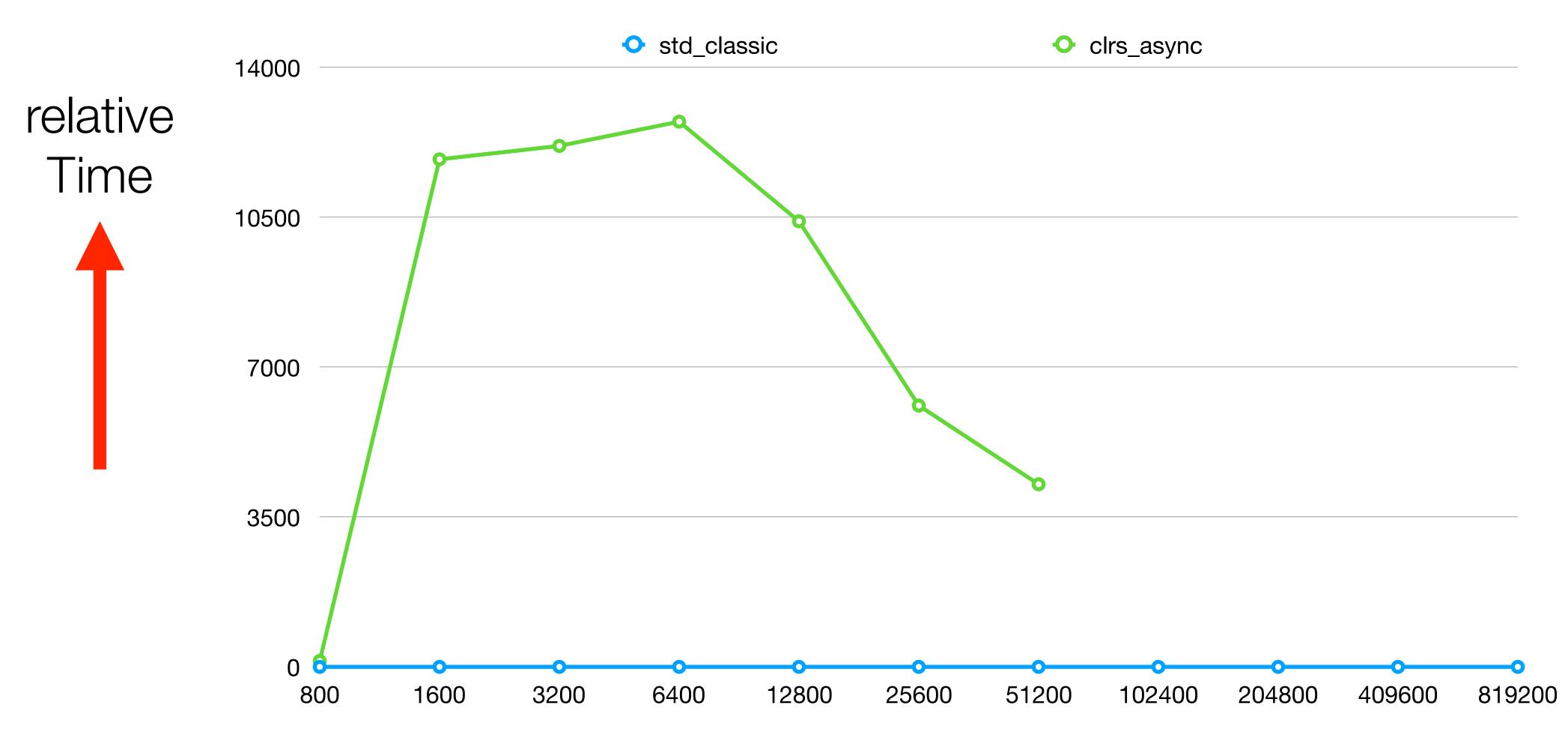
```
template <typename V, typename In, typename T, typename To, typename Op>
void p_scan_down(V v, In b, In end, T tmp, To to, Op op) {
  auto n = std::distance(b, end);
  if (1 == n) \{ *to = op(v, *b); \}
  else {
     auto k = n / 2:
     auto fut = std::async([&]{p_scan_down(v, b, b + k, tmp, to, op); });
     p_scan_down(op(v, tmp[k]), b + k, end, tmp + k, to + k, op);
     fut.wait();
```

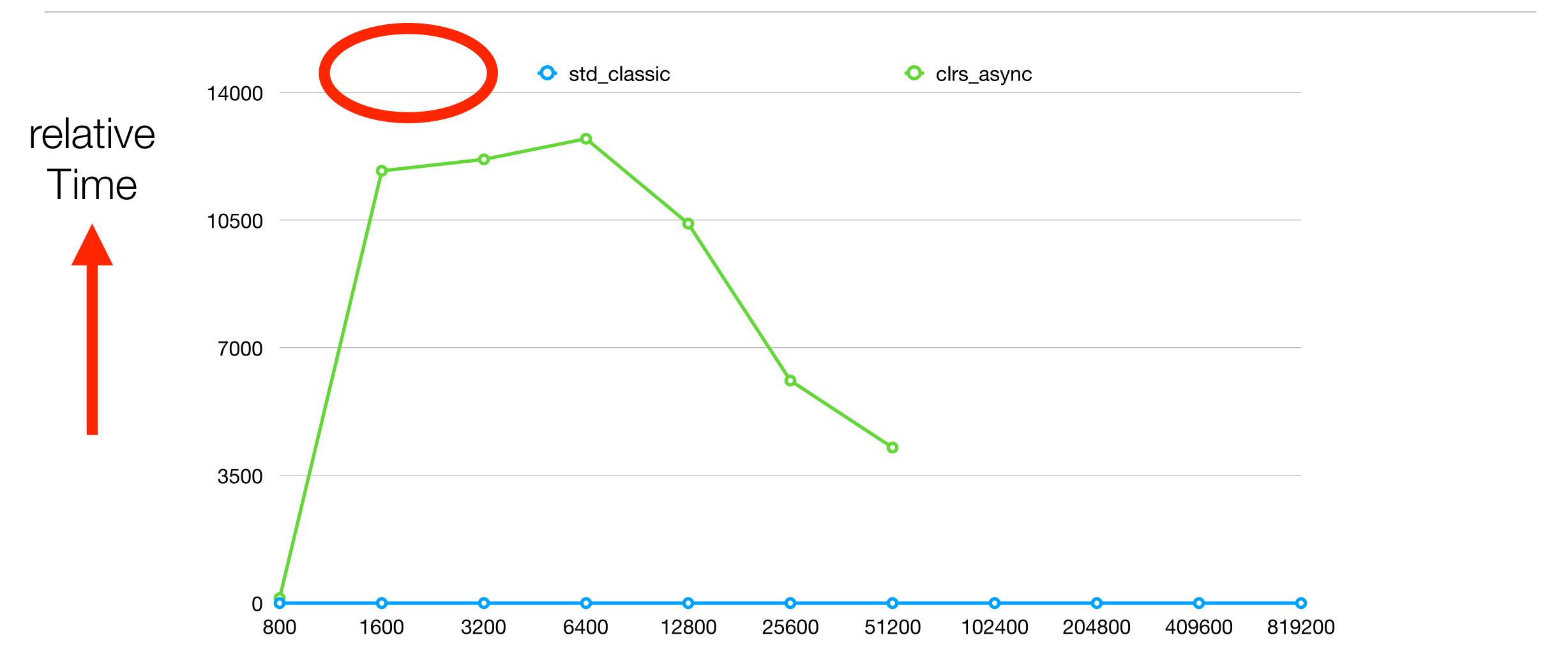
The Benchmarks

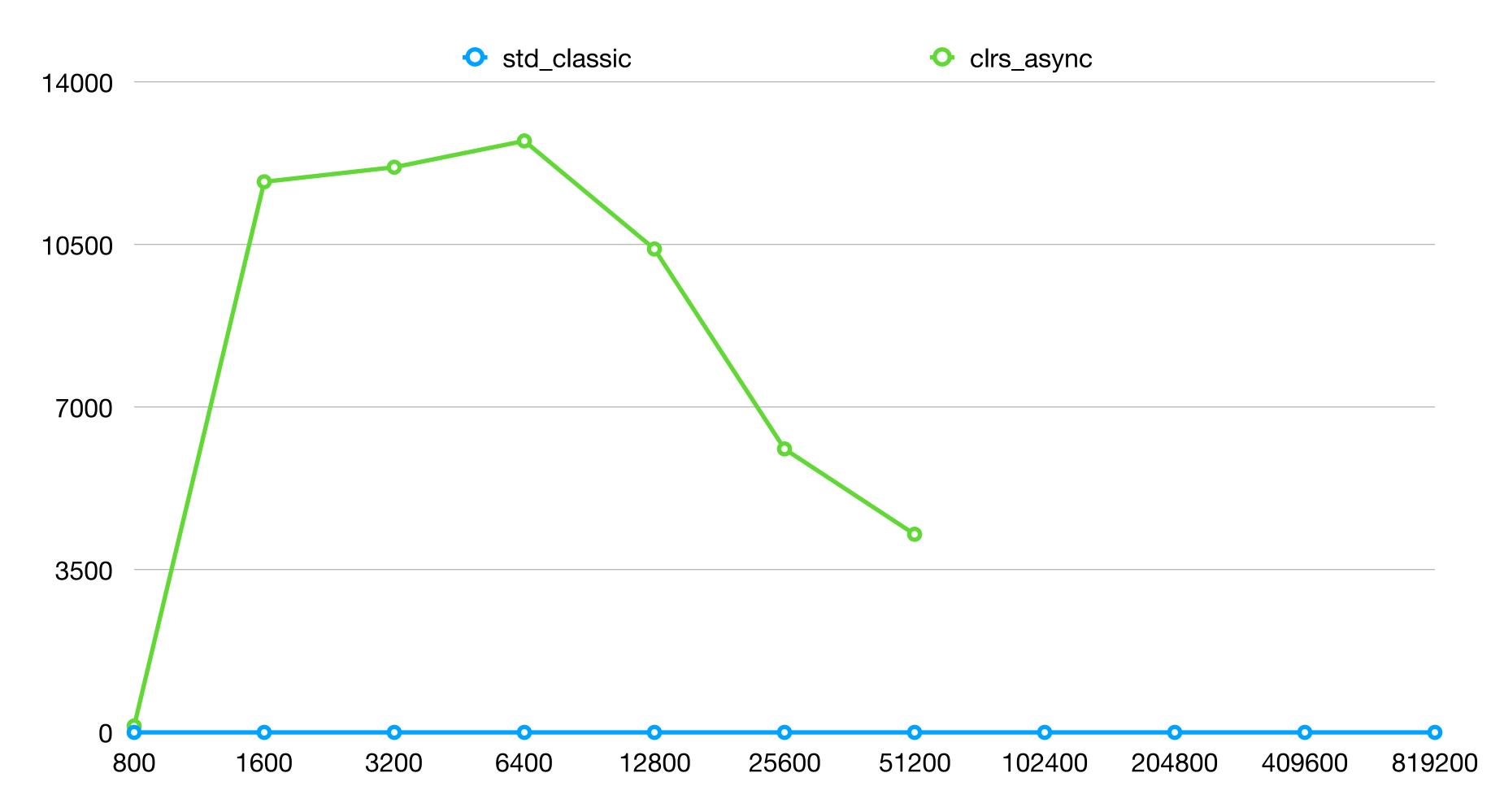
- · All benchmarks are run on a Knights Landing/Xeon Phi machine
 - 64 Intel Atom cores at 1.2GHz, 4 times hyper-threaded
 - 16GB fast access memory, 96GB memory
- The work load is multiplying random generated 4x4 matrices
 - Use the same data a few times to make it not entirely memory bound
- The curves time taken relative to a sequential implementation: $low \Rightarrow good$











No Thread Pool in std::async()

- There is no promise that a task started with std::async() doesn't block.
- The example work-load actually does block:

```
auto fut = std::async([&]{ p_scan_down(v, b, b + k, tmp, to, op); }); p_scan_down(op(v, tmp[k]), b + k, end, tmp + k, to + k, op); fut.wait();
```

Thus, each task is run on its own thread.

Scheduling Tasks Isn't Free

- The algorithm uses lots of tiny tasks (unless the operation is huge)
- In theory that is fine, in practice it is too expensive:
 - The overhead of managing and scheduling tasks matters
 - There aren't infinite processors to take advantage of many tasks
- Processing the input in blocks improves the performance

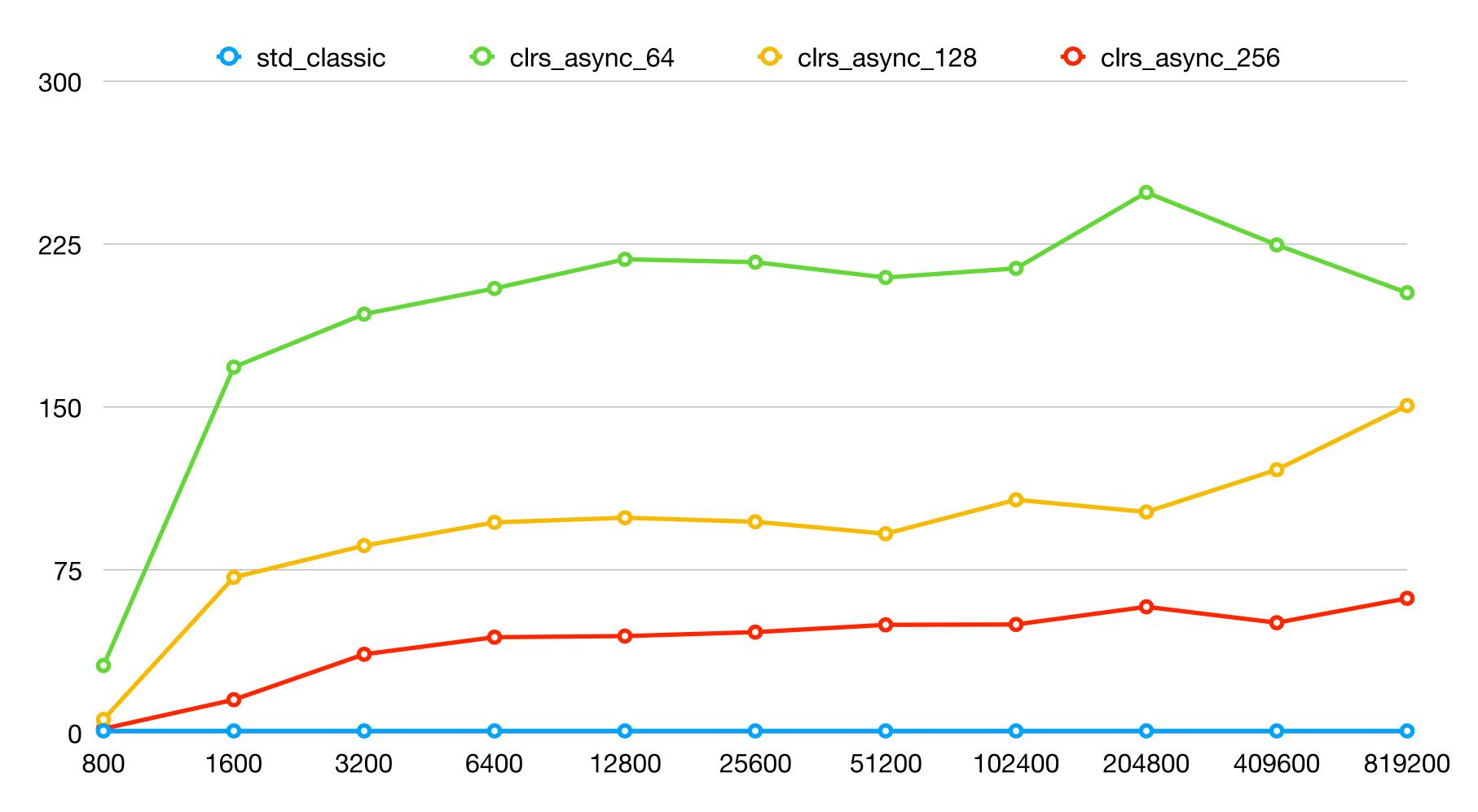
P-Scan-Up: processing data in blocks

```
template <typename In, typename Tmp, typename Op>
auto p_scan_up(In b, In end, Tmp tmp, Op op) {
  auto n = std::distance(b, end);
  if (n < MinSize) { return std::accumulate(begin + 1, end, *begin, op); }
  else {
     auto k = n / 2:
     auto fut = std::async([\&]{ tmp[k] = p_scan_up(b, b + k, tmp, op); });
     auto right = p_scan_up(b + k, end, tmp + k, op);
     fut.wait();
     return op(tmp[k], right);
```

P-Scan-Down: processing data in blocks

```
template <typename V, typename In, typename T, typename To, typename Op>
void p_scan_down(V v, In b, In end, T tmp, To to, Op op) {
  auto n = std::distance(b, end);
  if (n < MinSize) { inclusive_scan(b, end, to, op, v); }
  else {
     auto k = n / 2;
     auto fut = std::async([\&]{ p_scan_down(v, b, b + k, tmp, to, op); });
     p_scan_down(op(v, tmp[k]), b + k, end, tmp + k, to + k, op);
     fut.wait();
```

P-Scan: Performance using std::async()



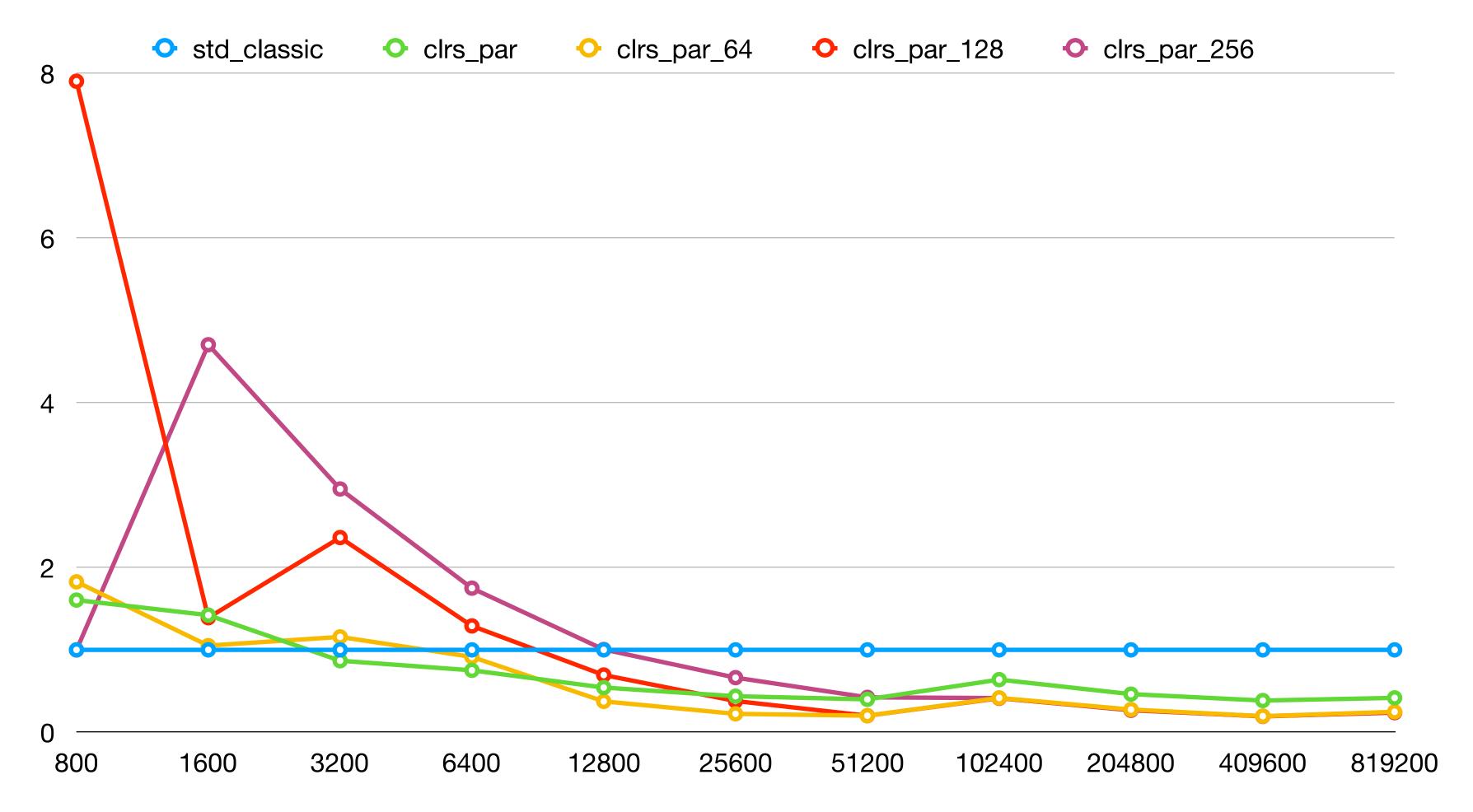
P-Scan-Up: processing data in blocks using tab::task_group

```
template <typename In, typename Tmp, typename Op>
auto p_scan_up(In b, In end, Tmp tmp, Op op) {
  auto n = std::distance(b, end);
  if (n < MinSize) { return std::accumulate(begin + 1, end, *begin, op); }
  else {
     auto k = n / 2;
     tbb::task_group g; g.run([&]{ tmp[k] = p_scan_up(b, b + k, tmp, op); });
     auto right = p_scan_up(b + k, end, tmp + k, op);
     g.wait();
     return op(tmp[k], right);
```

P-Scan-Down: processing data in blocks using tab::task_group

```
template <typename V, typename In, typename T, typename To, typename Op>
void p_scan_down(V v, In b, In end, T tmp, To to, Op op) {
  auto n = std::distance(b, end);
  if (n < MinSize) { inclusive_scan(b, end, to, op, v); }
  else {
     auto k = n / 2;
     tbb::task_group g; g.run([\&]{ p_scan_down(v, b, b + k, tmp, to, op); });
     p_scan_down(op(v, tmp[k]), b + k, end, tmp + k, to + k, op);
     g.wait();
```

P-Scan: Performance using tbb::task_group



Parallel Algorithm: reformulate to be more friendly to parallelization

- Recursive decomposition is a natural match for divide and conquer
- Using an iteration makes the problem accessible to parallel tools:
 - OpenMP
 - Thread Building Blocks (TBB)
 - · Standard library algorithms to some extent

Parallel Algorithm (Overview)

Parallel Algorithm (Overview)

Parallel Algorithm (Setup)

```
template <class Inlt, class Outlt, class Op>
Outlt inclusive_scan(Inlt begin, Inlt end, Outlt to, Op op) {

■ Auxiliary set up (types and variables)

using ptrdiff_t = typename std::iterator_traits<Inlt>::difference_type;

using value_type = std::decay_t<decltype(op(*begin, *begin))>;

ptrdiff_t const size = std::distance(begin, end);

ptrdiff_t const count = (size + chunk - 1) / chunk;

std::vector<std::optional<value_type>> tmp(count);
```

- ☐ Compute sums of subsequences (parallel)
- □ Compute results for elements on boundaries
- ☐ Compute the complete values (parallel)

Parallel Algorithm (Setup)

```
template < class InIt, class OutIt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {
  □ Auxiliary set up (types and variables)
  using ptrdiff_t = typename std::iterator_traits<Inlt>::difference_type;
  using value_type = std::decay_t<decltype(op(*begin, *begin))>;
  ptrdiff_t const size = std::distance(begin, end);
  ptrdiff_t const count = (size + chunk - 1) / chunk;
  std::vector<std::optional<value_type>> tmp(count);
```

- ⊞ Compute sums of subsequences (parallel)
- □ Compute the complete values (parallel)

Parallel Algorithm (Overview)

Parallel Algorithm (Overview)

Parallel Algorithm (Pre Processing, Sequential)

```
template < class InIt, class OutIt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {
  Auxiliary set up (types and variables)
  ☐ Compute sums of subsequences (parallel)
  for (ptrdiff_t i = 0; i < count - \mathbf{1}; ++i) {
     tmp[i] = std::accumulate(begin + chunk * i + 1, begin + chunk * (i + 1),
                                begin[chunk * i], op);

    ■ Compute results for elements on boundaries

    ⊞ Compute the complete values (parallel)
```

Parallel Algorithm (Pre Processing, OpenMP)

```
template < class InIt, class OutIt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {
  Auxiliary set up (types and variables)
  ☐ Compute sums of subsequences (parallel)
  #pragma omp parallel for
  for (ptrdiff_t i = 0; i < count - 1; ++i) {
     tmp[i] = std::accumulate(begin + chunk * i + 1, begin + chunk * (i + 1),
                               begin[chunk * i], op);

    ■ Compute results for elements on boundaries

    ⊞ Compute the complete values (parallel)
```

Parallel Algorithm (Pre Processing, tbb::task_group)

□ Compute the complete values (parallel)

```
template < class InIt, class OutIt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {
  ☐ Compute sums of subsequences (parallel)
  tbb::task_group group;
  for (ptrdiff_t i = 0; i < count - 1; ++i) group.run([&, i]{
    tmp[i] = std::accumulate(begin + chunk * i + 1, begin + chunk * (i + 1),
                            begin[chunk * i], op);
  }); group.wait();

    ■ Compute results for elements on boundaries
```

Parallel Algorithm (Pre Processing, tbb::parallel_for)

```
template < class InIt, class OutIt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {

    Auxiliary set up (types and variables)

  ☐ Compute sums of subsequences (parallel)
  tbb::parallel_for(ptrdiff_t(), count - 1, [&](auto i){
     tmp[i] = std::accumulate(begin + chunk * i + 1, begin + chunk * (i + 1),
                                begin[chunk * i], op);
  });

    ■ Compute results for elements on boundaries

    ⊞ Compute the complete values (parallel)
```

Parallel Algorithm (Pre Processing, use Executor)

```
template < class InIt, class OutIt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {
  ☐ Compute sums of subsequences (parallel)
  for_each_subrange(exec, chunk, begin, end,
    [&](auto i, auto b, auto e){
       tmp[i + 1] = std::accumulate(b + 1, e, *b, op);
  });

    ■ Compute results for elements on boundaries

☐ Compute the complete values (parallel)
```

Parallel Algorithm (Overview)

Parallel Algorithm (Overview)

Parallel Algorithm (Intermediate Procesing)

```
template < class InIt, class OutIt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {
  Auxiliary set up (types and variables)

    □ Compute sums of subsequences (parallel)

  □ Compute results for elements on boundaries
  partial_sum(tmp.begin(), tmp.end(), tmp.begin(),
               [op](auto& a, auto& b){ return op(*a, *b); });

    □ Compute the complete values (parallel)
```

Parallel Algorithm (Overview)

Parallel Algorithm (Overview)

Parallel Algorithm (Post Processing)

```
template < class Inlt, class Outlt, class Op>
OutIt inclusive_scan(InIt begin, InIt end, OutIt to, Op op) {

    □ Compute sums of subsequences (parallel)

    □ Compute results for elements on boundaries

  □ Compute the complete values (parallel)
  for (ptrdiff_t i = 0; i < count; ++i) {
     auto it = begin + chunk * i, ce = it + std::min(end - it, chunk);
     auto value = i? op(*tmp[i - 1], *it): *it;
     inclusive_scan(it + 1, ce, to + (it - begin) + 1, op, to[it - begin] = value);
```

Parallel Algorithm (Post Processing, OpenMP)

```
template < class Inlt, class Outlt, class Op>
Outlt inclusive_scan(InIt begin, InIt end, Outlt to, Op op) {

    ⊞ Compute sums of subsequences (parallel)

    □ Compute results for elements on boundaries

  □ Compute the complete values (parallel)
  #pragma omp parallel for
  for (ptrdiff_t i = 0; i < count; ++i) {
     auto it = begin + chunk * i, ce = it + std::min(end - it, chunk);
    auto value = i? op(*tmp[i - 1], *it): *it;
     inclusive_scan(it + 1, ce, to + (it - begin) + 1, op, to[it - begin] = value);
```

Parallel Algorithm (Post Processing, tbb::task_group)

```
template < class Inlt, class Outlt, class Op>
Outlt inclusive_scan(InIt begin, InIt end, Outlt to, Op op) {

    □ Compute sums of subsequences (parallel)

    □ Compute results for elements on boundaries

  □ Compute the complete values (parallel)
  tbb::task_group group;
  for (ptrdiff_t i = 0; i < count; ++i) group.run([&, i]{
     auto it = begin + chunk * i, ce = it + std::min(end - it, chunk);
     auto value = i? op(*tmp[i - 1], *it): *it;
     inclusive_scan(it + 1, ce, to + (it - begin) + 1, op, to[it - begin] = value);
  }); group.wait();
```

Parallel Algorithm (Post Processing, tbb::parallel_for)

```
template < class Inlt, class Outlt, class Op>
Outlt inclusive_scan(InIt begin, InIt end, Outlt to, Op op) {

    □ Compute sums of subsequences (parallel)

    □ Compute results for elements on boundaries

  □ Compute the complete values (parallel)
  tbb::parallel_for(ptrdiff_t(), count, [&](auto i){
     auto it = begin + chunk * i, ce = it + std::min(end - it, chunk);
     auto value = i? op(*tmp[i - 1], *it): *it;
     inclusive_scan(it + 1, ce, to + (it - begin) + 1, op, to[it - begin] = value);
  });
```

Parallel Algorithm (Post Processing, use Executor)

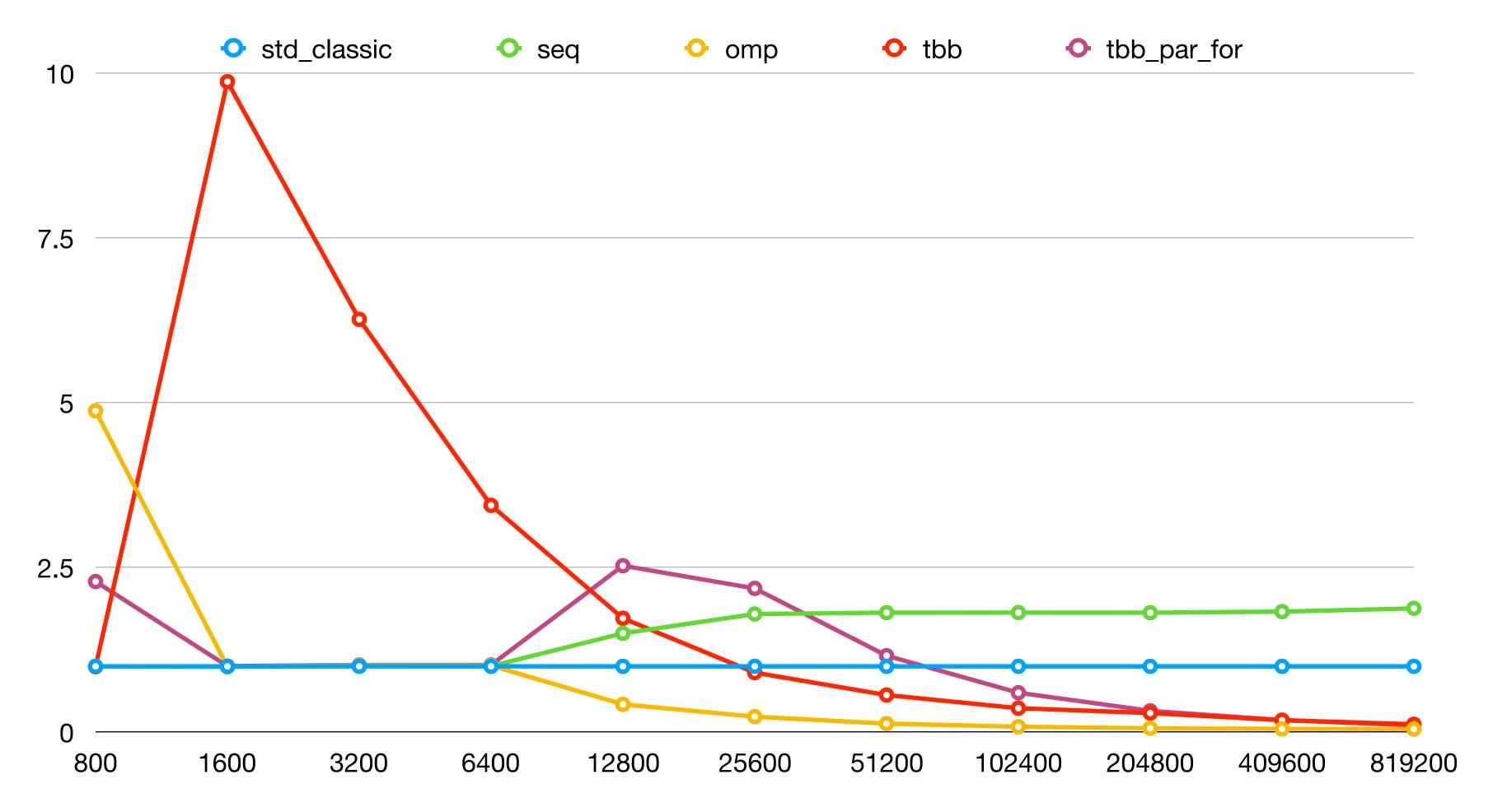
```
template < class Inlt, class Outlt, class Op>
Outlt inclusive_scan(InIt begin, InIt end, Outlt to, Op op) {

    ⊞ Compute sums of subsequences (parallel)

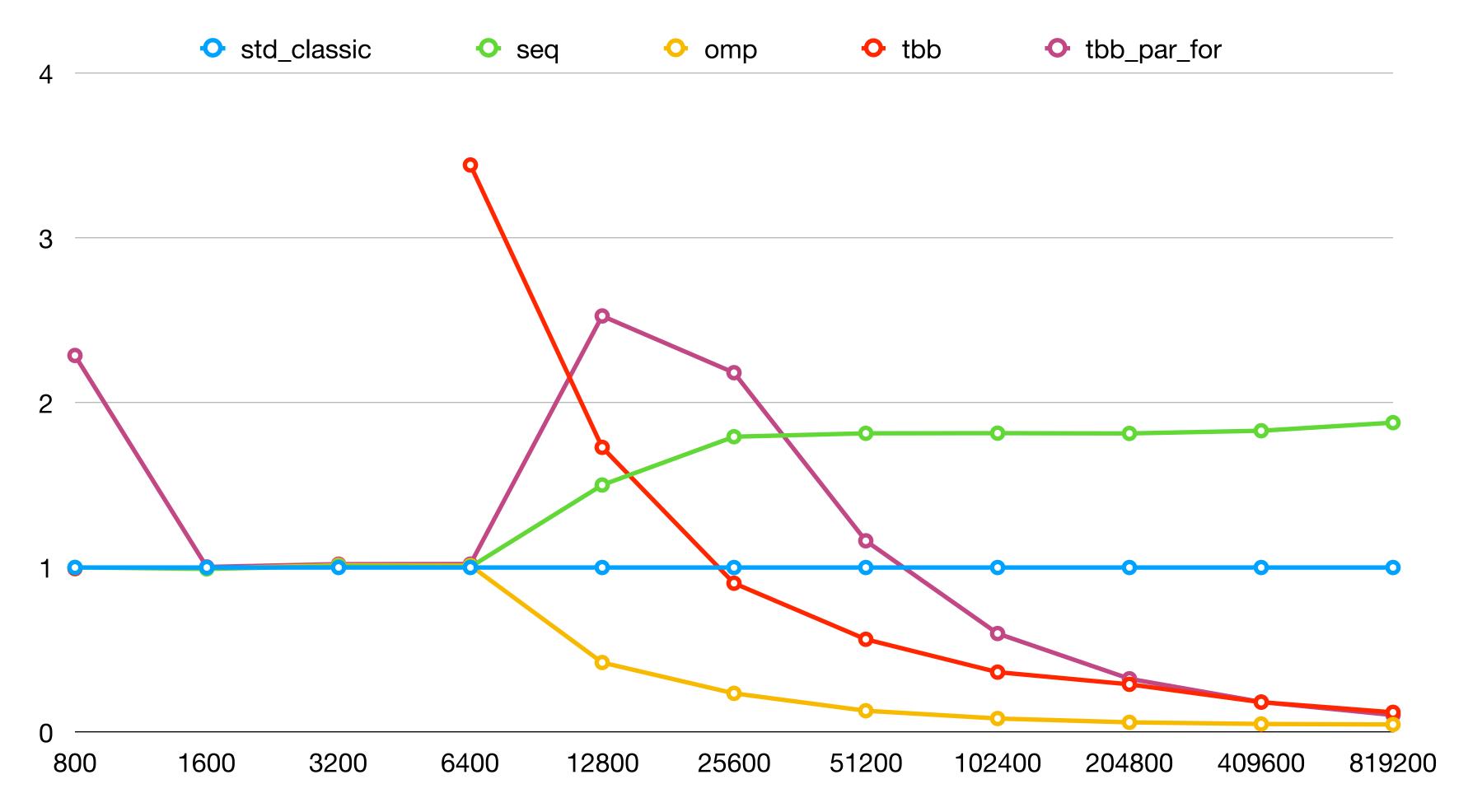
    ■ Compute results for elements on boundaries

  □ Compute the complete values (parallel)
  for_each_subrange(exec, chunk, begin, end,
    [&](auto i, auto sbegin, auto send) {
         auto value = i? op(*tmp[i], *sbegin): *sbegin;
         auto off(std::distance(begin, sbegin));
         inclusive_scan(std::next(sbegin), send, to + off + 1, op, to[off] = value);
  });
```

Performance using iterative algorithms



Performance using iterative algorithms



for_each_range()

```
template <typename Executor, typename Size, typename Inlt, typename Fun>
void for_each_subrange(Executor exec, Size chunk, Inlt it, Inlt end, Fun fun) {
  if (it == end) { return; }
  ptrdiff_t const s = std::distance(it, end), count = (s + chunk - 1) / chunk;
  latch (count);
  for (ptrdiff_t i(0); i != count - 1; ++i) {
     auto cend(std::next(it, chunk));
     exec.execute([&, i, it, cend, a = latch_arriver(l)]{ fun(i, it, cend); });
     it = cend:
  exec.execute([&, it, a = latch_arriver(l)]{ fun(count - 1, it, end); });
  l.wait();
```

The Missing Bits

- OpenMP and TBB use a task scheduler
- The task schedule is backed by a thread pool:
 - Multiple threads processing queued jobs
 - TBB uses a job stealing thread pool

Executor: on the same thread

```
class immediate_executor {
  public:
    explicit immediate_executor(int = 0) {}
    template <typename Fun>
    void execute(Fun&& fun) {
       fun();
    }
};
```

Executor: thread pool (interface)

```
class thread_pool {
                                  d_flag{true};
  bool
  std::mutex
                                  d mutex;
  std::condition_variable
                                  d condition;
  std::deque<function<void()>> d_queue;
  std::deque<jthread>
                                  d pool;
  void work();
public:
  thread_pool(int count);
  ~thread_pool() { this->stop(); } void stop();
  template <typename Fun> void execute(Fun&& fun);
```

Executor: thread pool (starting threads)

```
thread_pool::thread_pool(int count) {
    std::lock_guard cerberus(this->d_mutex);
    count = std::max(1, count);
    for (int i{0}; i != count; ++i) {
        this->d_pool.emplace_back(&thread_pool::work, this);
    }
}
```

Executor: thread pool (enqueue a job)

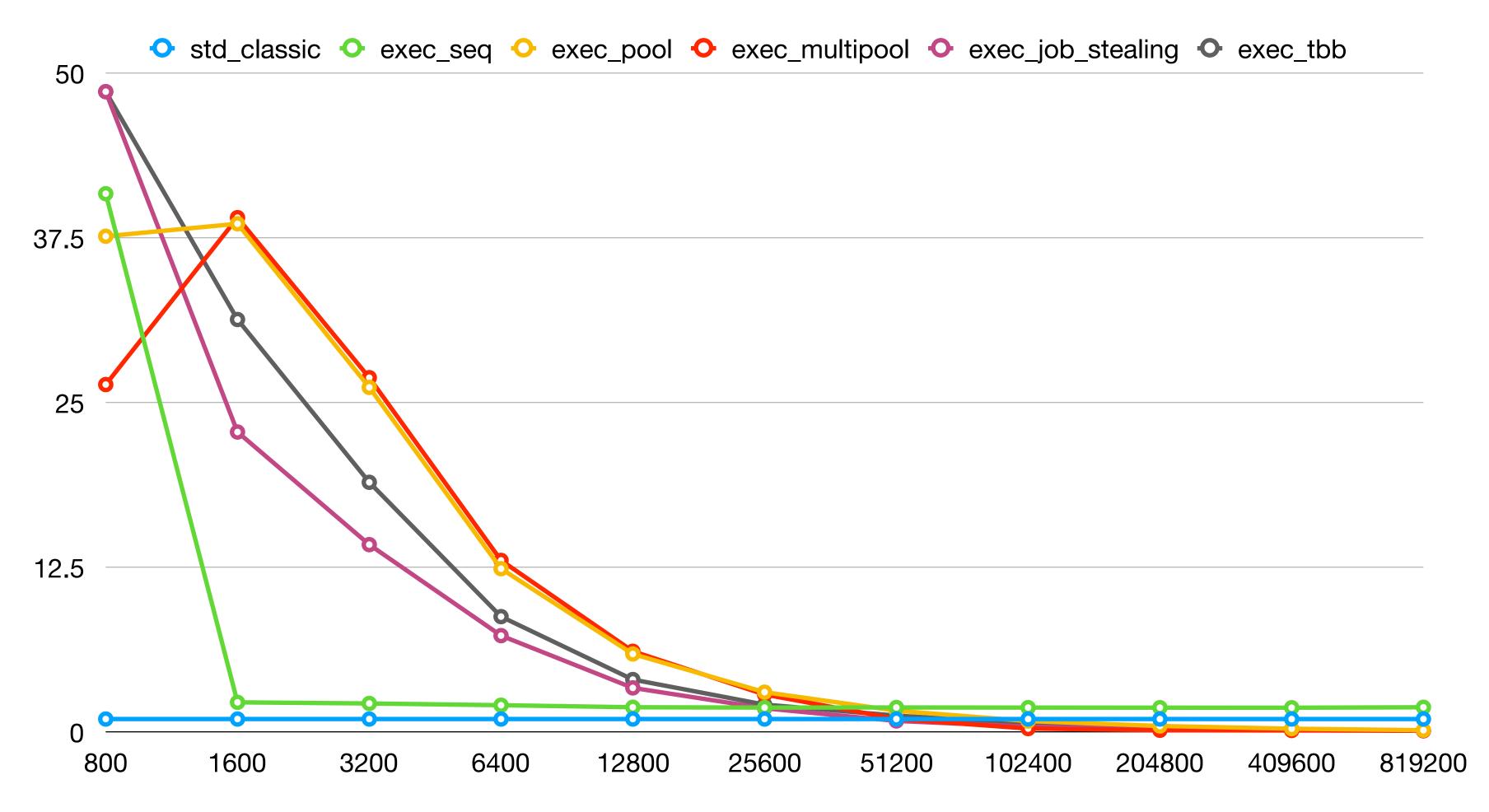
Executor: thread pool (execute work)

```
void thread_pool::work() {
  while (true) {
     std::unique_lock cerberus(this->d_mutex);
     this->d_condition.wait(cerberus,
        [this]{ return !this->d_queue.empty() | !this->d_flag; });
     if (this->d_queue.empty()) { break; }
     auto fun(std::move(this->d_queue.front()));
     this->d_queue.pop_front();
     cerberus.unlock();
     fun();
```

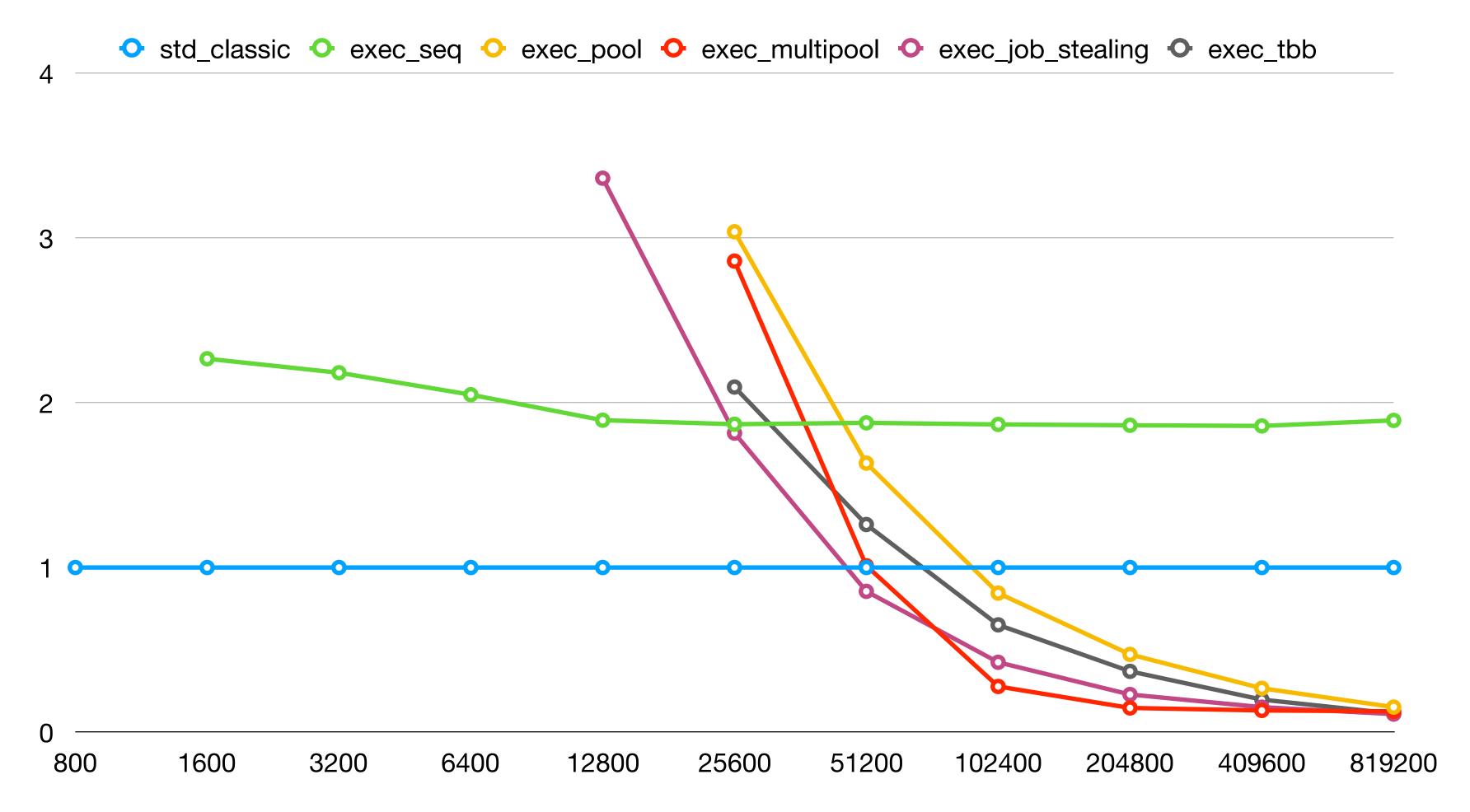
Executor: pool executor

```
class pool_executor {
 std::shared_ptr<thread_pool> d_pool;
public:
  explicit pool executor(int count = std::thread::hardware concurrency())
     : d_pool(std::make_shared<thread_pool>(count)) {
  void stop() { this->d_pool->stop(); }
  template <typename Fun>
  void execute(Fun&& fun) {
    this->d_pool->execute(std::forward<Fun>(fun)); }
```

Performance using iterative algorithms with executors



Performance using iterative algorithms with executors



Proper Way to Do Inclusive Scan

• The algorithm can be implemented, but it is already in the standard C++ library

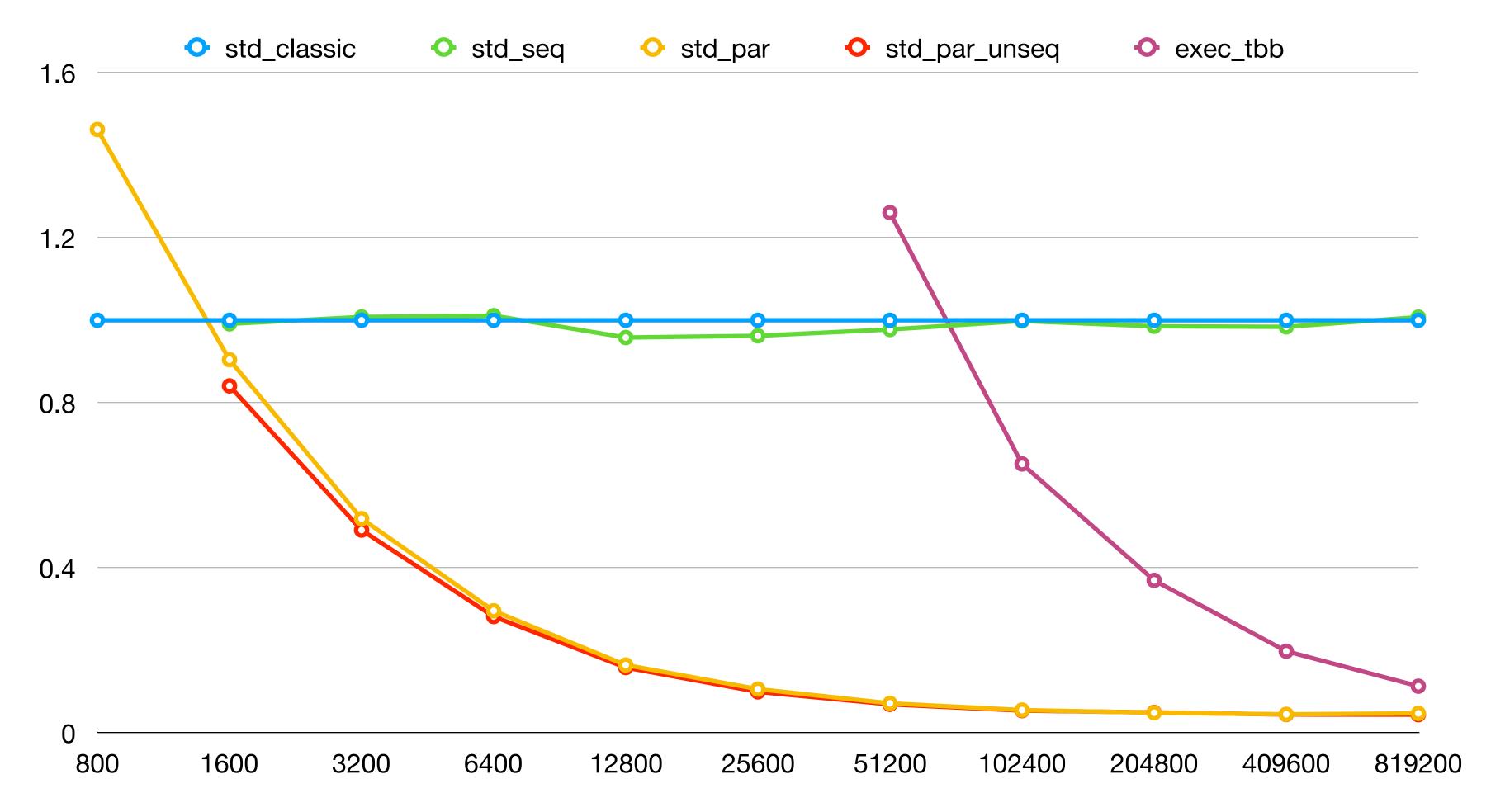
```
to_end = std::inclusive_scan(std::execution::par, begin, end, to, op);

to_end = std::inclusive_scan(std::execution::par_unseq, begin, to, op);

to_end = std::inclusive_scan(std::execution::seq, begin, end, to, op);
```

- Most algorithms in the standard C++ library have a parallel version
- These provide a benchmark to compare against

Performance of Standard Library Algorithms



Conclusion

- · Parallel algorithms can speed up processing of larger data sets
- · ... even if the processing isn't massively parallel
- Implementing parallel algorithms isn't magic
 - The algorithm from the book needs quite a bit of work to make it practical
 - Doing so would be easier with standard thread pools (executors)
- The standard C++ library has parallel versions and does this better

Thank you!

