Interval search tree

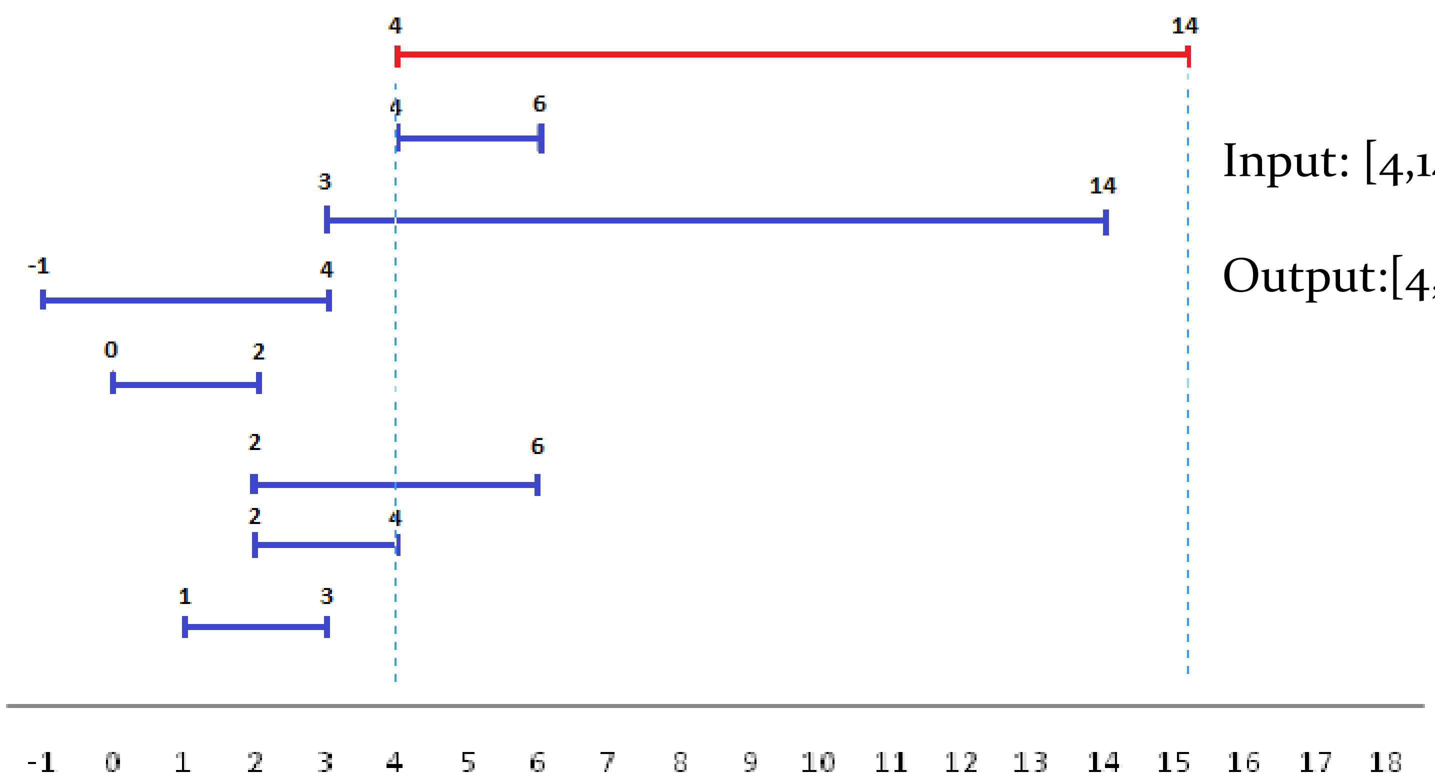
Considering a set of intervals,find all intervals that overlap with any given interval or point.

Supervisor:
Lecturer Ph.D. Cristian Mihăescu

Author:
Lucian Iordache

Basic ideea

We have set of intervals and we want to see if a given interval intersects any of these intervals.

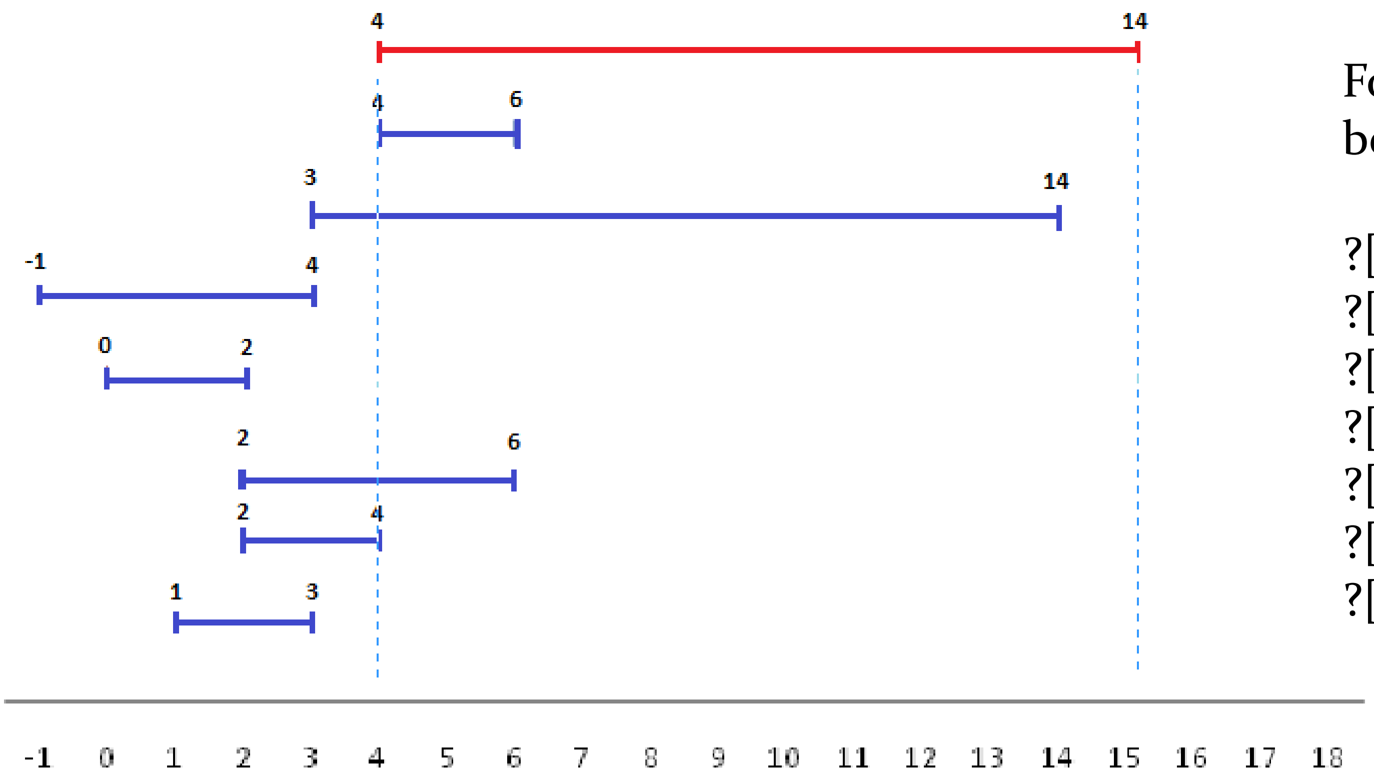


Input: [4,14]

Output:[4,6],[3,14],[2,6],[2,4].

Trivial solution

The trivial solution is to visit each interval and test whether it intersects the given point or interval.

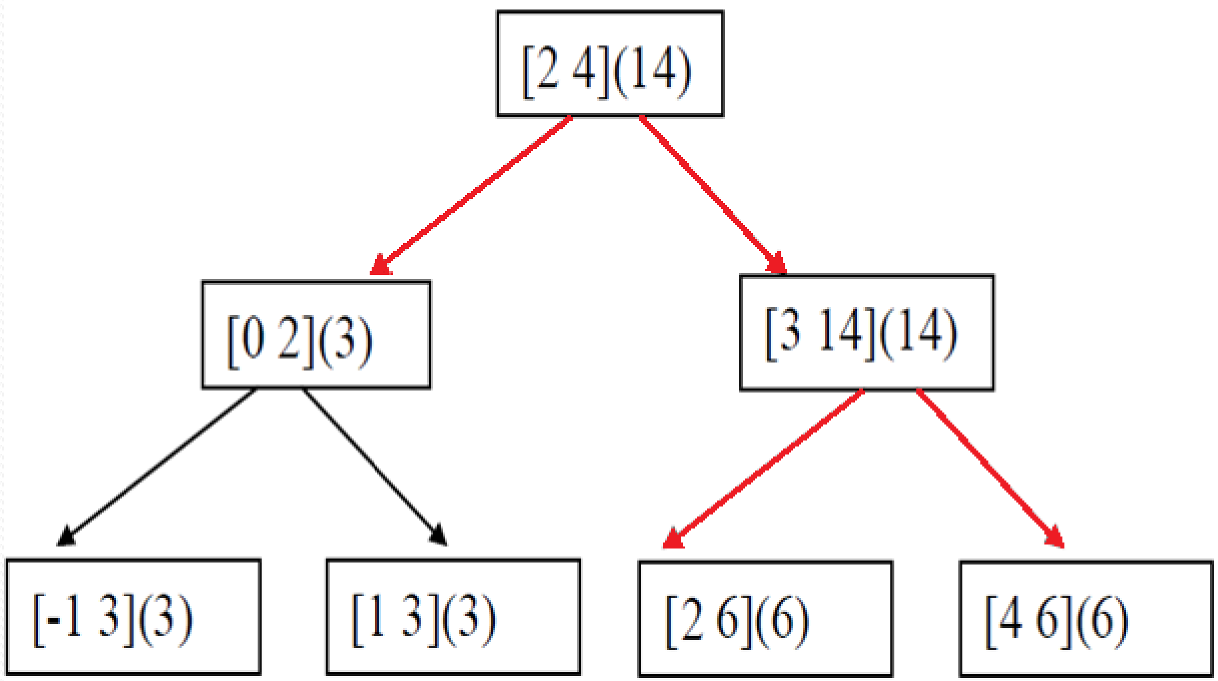


For this example will be 7 comparations:

?[4,14]∩[4,6]
?[4,14]∩[3,14]
?[4,14]∩[-1,4]
?[4,14]∩[0,2]
?[4,14]∩[2,6]
?[4,14]∩[2,4]
?[4,14]∩[1,3]

Interval search tree

Interval search tree implentation use a simple ordered tree,ordered by the 'low' values of the intervals, and an extra annotation is added to every node recording the maximum high value of both its subtrees,so we know that two intervals A and B overlap only when both A.low $\leq$ B.high and A.high $\geq$ B.low.



For this example will be 5 comparations:

?[4,14]∩[2,4]
?[4,14]∩[0,2]
?[4,14]∩[3,14]
?[4,14]∩[2,6]
?[4,14]∩[4,6]

Structure

```
struct BSTNode
{
BSTNode *left;
BSTNode *right;
Interval nodeInfo;
int maxRight;
};
```

Where:
BSTNode *left-is a pointer to the left node of the tree
BSTNode *right- is a pointer to the right node of the tree
Interval nodeInfo keep the left and the right end of the interval
int maxRight-keep the maximum end of the intervals added

Running time (computed)

- Trivial solution: Θ(n) time, where *n* is the number of intervals in the collection.
- Interval search tree: Θ(*nlog n*) avarage time, Θ(n) worst case(when all intervals intersects), where *n* is the number of intervals in the collection.

Demonstration -Master Theorem-

The master theorem concerns recurrence relations of the form:

- $T(n) = aT(n/b) + f(n)$, where:

- a is the number of subproblems in the recursion.
- n/b is the size of each subproblem
- n is the size of the problem.
- $f(n)$ is the cost of the work done outside the recursive calls

Demonstration -Master Theorem-

In our case : a = 2 , b = 2 , f(n) = Θ(1),so recurence relation is:

$$T(n) = 2T(\frac{n}{2}) + \theta(1)$$

$$c = \log_b^a = 1$$

$$=> T(n) = \theta(n^{\log_b^a} \log n) = \theta(n \log n)$$

Running time (measured)