

SureFED: Robust Federated Learning via Uncertainty-Aware Inward and Outward Inspection

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Abstract—In this work, we introduce SureFED, a novel framework for byzantine robust federated learning. Unlike many existing defense methods that rely on statistically robust quantities, making them vulnerable to stealthy and colluding attacks, SureFED establishes trust using the local information of benign clients. SureFED utilizes an uncertainty aware model evaluation and introspection to safeguard against poisoning attacks. In particular, each client independently trains a clean local model exclusively using its local dataset, acting as the reference point for evaluating model updates. SureFED leverages Bayesian models that provide model uncertainties and play a crucial role in the model evaluation process. Our framework exhibits robustness even when the majority of clients are compromised, remains agnostic to the number of malicious clients, and is well-suited for non-IID settings. We theoretically prove the robustness of our algorithm against data and model poisoning attacks in a decentralized linear regression setting. Proof-of-Concept evaluations on benchmark image classification data demonstrate the superiority of SureFED over the state of the art defense methods under various colluding and non-colluding data and model poisoning attacks.

1. Introduction

In contemporary large networks, decentralized operations involve individual devices (clients) having access to their local datasets. However, these local datasets often lack the capacity to learn a global model with sufficient accuracy, and privacy concerns impede raw data sharing among clients. To address these challenges, federated learning algorithms [24] like Federated Averaging have been proposed. Furthermore, an array of extensions were introduced to enable improved scalability in a peer-to-peer setting or adaptation to non-IID data distributions [27], [22], [33], [35], [34], [1].

However, federated learning algorithms have been shown to be vulnerable to various byzantine and adversarial attacks, including data and model poisoning attacks [23], [30], [31], [16], [4], [40], [29]. State of the art defense methods in federated learning can be categorized into (1) statistics-based defenses [41], [5], [7], [10], [12], such as Trimmed Mean [41] and Clipping [2], (2) defenses based on model evaluation on a local dataset [38], [39], such as Zeno [38],

and (3) defenses based on model comparison with the server’s local model, such as FLTrust [9].

Statistics-based defense methods use a statistically robust quantity, such as median, instead of mean, in the model aggregation to eliminate the effect of byzantine updates on the aggregated value. Such methods rely on the assumption that the majority of clients are benign, aiming to protect statistical measures like median from being influenced by a minority of poisoned updates. The defense methods in the second category, such as Zeno [38], draw samples from a clean local dataset and evaluate a model update based the amount of improvement in the loss function. The defenses in the third category, such as FLTrust [9], are based on bootstrapping the trust using the server’s local model. FLTrust assumes that the server has access to a clean local dataset. The server computes a model update using its local dataset similar to what clients do and compares received updates with its own.

The existing defense methods have been shown to be vulnerable to specific attacks such as colluding attacks of "A Little is Enough" [3] and Trojan attacks [36]. In addition, the statistics-based defense methods require the majority of clients to be benign. Some methods, such as Trimmed Mean [41] and Zeno [38], necessitate knowledge of an upper bound on the number of compromised clients. Zeno is also inherently vulnerable to Trojan attacks due to their stealthy nature. FLTrust has also been shown to be vulnerable to specific attacks that target methods based on cosine similarity [18]. A potential drawback of FLTrust is that if the server’s model becomes poisoned early on in the training process, the future model evaluations become meaningless.

This paper introduces SureFED, which utilizes uncertainty quantification to address limitations in the existing literature. SureFED draws inspirations from the state of the art defense methods described earlier and further incorporates additional essential components to address the known vulnerabilities of the existing methods. SureFED employs a peer-to-peer federated learning setting in which each client has access to a local dataset but also obtains models from other clients. In contrast with prior work, in SureFED, the clients train two distinct models: a social model, designed to capture the benefits of federated operation by aggregating the models trained on diverse datasets across the network,

and a **Local Bayesian Model**, trained exclusively on each client's local dataset to be utilized as a model evaluation basis with explicit uncertainty quantification. We note that these local models are never aggregated with the received updates; Instead, they are used in an **Uncertainty Aware Model Evaluation** of the model updates to subsequently aggregate them with the social model based on a new secure aggregation rule. Additionally, SureFED employs an introspection procedure, in which each client evaluates its own social model using its clean local model. This ensures the system's ability to revert to a clean state after potential contamination.

The process of model evaluation and aggregation in SureFED draws inspiration from the concepts found in social science such as bounded confidence models of opinion dynamics [15], which aim to simulate how humans alter their opinions during interactions with others. In this regard, SureFED seeks to emulate the innate intelligence of humans within the complex domain of AI. Notably, in SureFED, each received model update is independently evaluated without the need for comparisons with other updates. This feature allows SureFED to function without requiring a majority of benign clients or knowledge of the number of compromised clients.

We investigate and demonstrate the superior performance of SureFED under a broad set of byzantine and adversarial attacks. In particular, we consider six colluding and non-colluding data and model poisoning attacks, namely, Label-Flipping [31], Trojan [36], Bit-Flip [37], General Random [37], A little is Enough [3], and Gaussian [26] Attacks. A little is Enough and Trojan attacks are colluding attacks, while Trojan attack is also a stealthy attack. We compare our results with the state of the art defense methods described above. Specifically, we compare SureFED with Trimmed Mean [41], and Clipping [2] from the first category, Zeno [38], which is the state of the art in the second category, and FLTrust [9], which is the state of the art defense belonging to the third category. We also compare with BayP2PFL [34], as a variational peer-to-peer federated learning baseline without a defense method to confirm that SureFED's robustness does not come at the cost of reliability and/or performance. In addition to experimental analysis, we provide theoretical guarantees for the robustness of SureFED in a decentralized linear regression setting under non-IID data distribution. Note that theoretical guarantees in non-IID settings are rarely found in the literature. In summary, our contributions are as follows:

- We introduce SureFED, a new peer-to-peer federated learning algorithm which is robust to a variety of data and model poisoning attacks.
- SureFED is shown to have the following desirable properties: (i) its robustness does not require the majority of the benign nodes over the compromised ones and is agnostic to the number of byzantine workers; (ii) SureFED effectively adapts to and derives benefits from the non-IID datasets of benign clients. (iii) SureFED is robust to numerous poisoning attacks and shows superior performance compared to the state of the art defense methods.

- We theoretically prove the robustness of SureFED in a decentralized linear regression setting under certain poisoning attacks. We prove that the benign nodes in SureFED learn the correct model parameter under Label-Flipping attacks. We then generalize our theoretical results to the General Random model poisoning attack.
- Comprehensive, Proof-of-Concept evaluations on benchmark data from image classification (MNIST, FEMNIST, and CIFAR10) demonstrate the superiority of SureFED over the existing defense methods, namely, Zeno, Trimmed Mean, Clipping, and FLTrust. The robustness is evaluated with respect to all six considered types of poisoning attacks.

2. Background

2.1. Variational Bayesian Learning

Variational Bayesian learning [6] is the learning method used in this paper. In the variational Bayesian learning, weight uncertainty is introduced in the neural network by learning a parameterized distribution over the weights (instead of just learning the weight values). More specifically, assume that θ is the weight vector of the neural network. We denote the parameterized posterior distribution over θ by $q(\theta|w)$, where w is the parameter vector of the posterior distribution. Also, assume that at round t of the training, we have a prior distribution over θ , denoted by $\mathcal{P}_{t-1}(\theta)$. After observing the data batch \mathcal{D}_t , the posterior distribution parameters, w , are updated by minimizing the variational free energy loss function:

$$\mathcal{F}(\mathcal{D}_t, w) = \text{KL}[q(\theta|w) || \mathcal{P}_{t-1}(\theta)] - \mathbb{E}_{q(\theta|w)}[\log \mathcal{P}(\mathcal{D}_t|\theta)], \quad (1)$$

where $\mathcal{P}(\mathcal{D}_t|\theta)$ is the conditional likelihood of observing \mathcal{D}_t given θ .

In the case of variational Gaussian posteriors [34], [1], $q(\theta|w)$ is a Gaussian distribution with mean $\hat{\theta}$ and covariance matrix Σ which is a diagonal matrix with diagonal entries of $(\log(1+\exp(\rho)))^2$. Therefore, we have $w = (\hat{\theta}, \rho)$.

2.2. Uncertainty Aware Model Averaging

In a variational Bayesian federated learning setting such as [34], [1], clients incorporate the model uncertainties in the aggregation process. In particular, each client aggregates the received model updates as follows.

$$(\Sigma_t'^i)^{-1} = \sum_{j \in \mathcal{N}(i)} T^{ij} (\Sigma_t^j)^{-1} \quad (2a)$$

$$\hat{\theta}_t^i = \Sigma_t'^i \sum_{j \in \mathcal{N}(i)} T^{ij} (\Sigma_t^j)^{-1} \hat{\theta}_t^j \quad (2b)$$

$$\Sigma_t^i = \Sigma_t'^i \quad (2c)$$

where $w_t^i = (\hat{\theta}_t^i, \Sigma_t^i)$ is the Gaussian posterior belief parameter, and T^{ij} is the predetermined trust of client i on client j . As can be seen in the above equation, the

model elements with higher uncertainty (variance) are given smaller weights in the aggregation process.

3. Problem Statement

3.1. System Model

We consider a peer-to-peer federated learning setting with N clients who are distributed over a time-varying directed graph $G_t = (\mathcal{N}, \mathcal{E}_t)$. \mathcal{N} is the set of clients, where each client corresponds to one node of graph G_t . An edge $(i, j) \in \mathcal{E}_t$ indicates that clients i can communicate with client j at time t . The adjacency matrix of the graph at time t is denoted by A_t . We denote the set of client i together with its in-neighbors at time t by $\mathcal{N}_t(i)$, and the set of out-neighbors of client i at time t is denoted by $\mathcal{N}^o_t(i)$. Each client i has access to a data set \mathcal{D}^i consisting of data samples (x_t^i, y_t^i) . We assume the dataset of clients are non-IID with $x_t^i \in \mathcal{X}^i$ distributed according to \mathcal{P}^i . Clients learn the model parameter, θ , according to a decentralized variational learning algorithm with Gaussian variational posteriors which will be explained in section 4.

3.2. Threat Model

We consider an adversarial environment where a subset of the nodes, \mathcal{N}^c , are under poisoning attacks by attackers from outside the system. While our analysis can be applied to many types of poisoning attacks, we focus on the following attacks.

- Data Poisoning Attacks:
 - **Trojan:** A subset of the dataset of the compromised clients is added with a Trojan and labeled with a target label [36]. This is a stealthy and colluding attack.
 - **Label-Flipping:** The label of some classes are changed to a target class [31].
- Model Poisoning Attacks:
 - **Bit-Flip:** In this attack, some of the bits in the binary representations of the model weights are flipped [37].
 - **General Random:** The attackers randomly choose some of the model weight elements and multiply them by a large number [37].
 - **A Little is Enough:** In this attack, the adversaries place their model weights close to a number of benign clients that are far from the mean to gain the majority power [3]. This type of attack is categorized as a colluding attack.
 - **Gaussian:** The adversaries add Gaussian noise to the model updates [26].

4. SureFED: A Robust Federated Learning Framework

In this section, we describe SureFED, our novel peer-to-peer federated learning algorithm. SureFED uses the variational learning method with Gaussian posteriors explained

in Section 2. All of the steps of the SureFED algorithm are presented in Algorithm 1. SureFED distinguishes itself from other frameworks by incorporating several innovative components, which are explained in the following. The most important component of SureFED is the introduction of **Local and Social Models**. In SureFED, clients learn local likelihood functions (also referred to as models or beliefs) on the model parameters. In particular, clients train two models; a local model, and a social model. The local model is trained only using the local dataset based on the variational Bayesian learning method, while the social model is trained according to the federated learning algorithm that will be described later. The social and local models are Gaussian likelihood functions with parameters denoted by $\bar{w}_t^i = (\bar{\theta}_t^i, \bar{\Sigma}_t^i)$ and $\hat{w}_t^i = (\hat{\theta}_t^i, \hat{\Sigma}_t^i)$, respectively, where $\bar{\theta}_t^i$ and $\hat{\theta}_t^i$ are the means of the social and local Gaussian belief of client i on θ and $\bar{\Sigma}_t^i$ and $\hat{\Sigma}_t^i$ are their covariance matrices. The learning happens with the social model, while the local model has two critical applications: (1) Identifying Compromised Clients, and (2) Introspection and Model Overwriting:

(1) Identifying Compromised Clients: It is well established in the literature that if a number of clients are under attack in a peer-to-peer federated learning algorithm, the models of the benign clients can also become poisoned (due to aggregating their models with the malicious model updates). Therefore, these models can not be used as a ground truth to evaluate other clients' models and identify poisoned ones. However, local models, trained solely on local datasets of benign clients, remain entirely clean. Therefore, they can be utilized as a reliable ground truth to identify compromised clients. Nonetheless, the local models suffer from low accuracy due to the limited size of the local datasets. To address this limitation, Bayesian models prove advantageous by providing uncertainties in their model training. These uncertainties are incorporated into the process of evaluating received model updates from different clients. This enhances the evaluation process significantly, as clients primarily rely on elements of their models with higher certainty to flag malicious users. Consequently, this approach minimizes false positives, ensuring that benign clients are not erroneously flagged as malicious. This is facilitated by the novel **Uncertainty Aware Model Aggregation** method used in SureFED, which is another important component of our method and is described in the following.

Unlike BayP2PFL [34] that considers predetermined time-invariant trust weights in Eq.(2), we consider time-dependent trust weights, T_t^{ij} , which are defined to robustify the algorithm against poisoning attacks. We define the trust weights in SureFED, referred to as the bounded confidence trust weights, such that each client only aggregates the opinion of those with similar opinion to it, where two models are deemed similar if their element-wise distance is less than a confidence bound. The confidence bound is determined by the uncertainty of the client over its own model, which is provided by the Bayesian models. The trust weights are formally defined below.

Definition 4.1 (Bounded Confidence Trust Weights). For

each client i , the set $I(\bar{w}_t^{\mathcal{N}_t(i)}, \hat{w}_t^i)$, which is called the confidence set of client i at time t , is defined as follows.

$$I(\bar{w}_t^{\mathcal{N}_t(i)}, \hat{w}_t^i) = \{j \in \mathcal{N}_t(i) : |(\hat{\theta}_t^i)_k - (\bar{\theta}_t^i)_k| \leq \kappa \sqrt{(\hat{\Sigma}_t^i)_{k,k}}, \forall k \in \mathcal{K}\} \quad (3)$$

where $\mathcal{K} = \{1, \dots, K\}$ and K is the dimension of the model parameter, and κ is a hyper parameter. We define the trust weights as follows.

$$T_t^{ij} = \frac{1}{|I(\bar{w}_t^{\mathcal{N}_t(i)}, \hat{w}_t^i)|} \mathbf{1}(j \in I(\bar{w}_t^{\mathcal{N}_t(i)}, \hat{w}_t^i)) \quad (4)$$

The hyperparameter κ determines how strict the aggregation rule is. If κ is too large, poisoned updates might be aggregated and if κ is too small, both benign and poisoned updates might not be aggregated. We will see through our experiments that κ can be easily tuned to have a robust algorithm that also performs well in benign settings.

(2) Introspection and Model Overwriting: The local models are also used in an introspection process, in which a client will evaluate its own social model. This is done based on the bounded confidence measure described in definition 4.1 such that if a hypothetical client with the social belief of client i would not be in the bounded confidence set of client i , then it will overwrite its social belief with its local belief. Notice that the model overwriting is done in the beginning rounds of the algorithm where the accuracy of the local model is still improving (before overfitting happens).

Algorithm 1 SureFED Algorithm executed by client i

Input: Σ_0^i , Σ^{thr} , κ , and T_{max} .
Initialize $t = 1$, $\bar{w}_0^i = \hat{w}_0^i = (0, \Sigma_0^i)$.
while $\bar{\Sigma}_{t-1}^i > \Sigma^{thr}$ and $t < T_{max}$ **do**
 Receive \mathcal{D}_t^i and compute $\hat{w}_t^i = (\hat{\theta}_t^i, \hat{\Sigma}_t^i)$ and $\bar{w}_t^i = (\bar{\theta}_t^i, \bar{\Sigma}_t^i)$, by minimizing the variational free energy loss function (1), starting from \hat{w}_{t-1}^i and \bar{w}_{t-1}^i , respectively.
 Share \bar{w}_t^i with $\mathcal{N}_t^o(i)$ and receive \bar{w}_t^j for $j \in \mathcal{N}_t(i)$.
 Set

$$I(\bar{w}_t^{\mathcal{N}_t(i)}, \hat{w}_t^i) = \{j \in \mathcal{N}_t(i) : |(\hat{\theta}_t^i)_k - (\bar{\theta}_t^i)_k| \leq \kappa \sqrt{(\hat{\Sigma}_t^i)_{k,k}}, \forall k \in \mathcal{K}\}$$

 Set $T_t^{ij} = \frac{1}{|I(\bar{w}_t^{\mathcal{N}_t(i)}, \hat{w}_t^i)|} \mathbf{1}(j \in I(\bar{w}_t^{\mathcal{N}_t(i)}, \hat{w}_t^i))$.

$$(\bar{\Sigma}_t'^i)^{-1} = \sum_{j \in \mathcal{N}_t(i)} T_t^{ij} (\bar{\Sigma}_t^j)^{-1} \quad (5a)$$

$$\bar{\theta}_t^i = \bar{\Sigma}_t'^i \sum_{j \in \mathcal{N}_t(i)} T_t^{ij} (\bar{\Sigma}_t^j)^{-1} \bar{\theta}_t^j \quad (5b)$$

 Set $\bar{\Sigma}_t^i = \bar{\Sigma}_t'^i$.
 if $\exists k \in \mathcal{K} : |(\hat{\theta}_t^i)_k - (\bar{\theta}_t^i)_k| > \kappa \sqrt{(\hat{\Sigma}_t^i)_{k,k}}$ **then**
 Set $(\bar{\theta}_t^i)_k = (\hat{\theta}_t^i)_k$.
 end if
 $t = t + 1$
end while

5. Learning and Robustness Analysis

In order to provide a thorough theoretical analysis of the learning in SureFED, in this section, we consider a special case where the clients' models constitute of a single linear layer with parameter $\theta^* \in \mathbb{R}^K$ (decentralized linear regression). We assume that the labels $y_t^i = \langle \theta^*, x_t^i \rangle + \eta_t^i$, where $\langle \cdot \rangle$ denotes the inner product and we assume $\eta_t^i \sim N(0, \Sigma^i)$ is a Gaussian noise. In order to simulate the non-IID datasets of clients, we assume $x_t^i \in \mathcal{X}^i$ and $\mathcal{X}^i := \{x \in \mathbb{R}_+^K : x_k = 0, \text{for } k \notin \mathcal{K}^i\}$, where $\mathcal{K}^i \in \mathcal{K}$ is a subset of model parameter indices that user i makes local observations on. The dataset of each client can be deficient for learning θ^* . That is, we can have $\mathcal{K}^i \neq \mathcal{K}$, for all or some $i \in \mathcal{N}$. Note that this model can represent both vertical ($\mathcal{K}^i \neq \mathcal{K}$) and horizontal ($\mathcal{K}^i = \mathcal{K}$) federated learning settings. For decentralized linear regression problem, the variational Bayesian learning updates will be simplified to a Kalman filter update on the parameters of the beliefs as described in Appendix equations (6) and (7).

In order to analyze the learning in SureFED, we need to make the following connectivity assumption on the communication graph.

Definition 5.1 (Relaxed Connectivity Constraint). If there exists a set $\{s_1, s_2, \dots, s_t, \dots\}$ with integers $s_t > 0$, such that a graph with adjacency matrix $\bar{A}_t = \prod_{l=s_t}^{s_{t-1}} A_l$ is strongly connected, then we say that the communication graph A_t satisfies the relaxed connectivity constraint.

Note that under the relaxed connectivity constraint, the communication graph over a cycle does not have to be connected and at some times it can be a disconnected graph. The intuitive explanation is that in order for the learning to happen correctly, it is sufficient that the message of a client is received by others after finite cycles and although there might not be a path between two clients at each cycle t , it is sufficient that a path is formed between them in near future. We also note that the relaxed connectivity constraint is similar to the B-strong connectivity condition introduced in [25].

In the next theorem, we show that in SureFED, clients learn the true model parameter in a benign setting.

Theorem 5.2 (Learning by SureFED). *If agents learn according to SureFED algorithm and the communication graph satisfies the relaxed connectivity constraint, and if no agent is compromised, i.e., $\mathcal{N}^c = \emptyset$, then each agent i learns the model parameter θ^* with mean square error that is decreasing proportional to $\frac{1}{t}$.*

We note that the learning rate in SureFED algorithm is of the same order, $\frac{1}{t}$, as the learning rate in BayP2PFL. It means that modifications made to add robustness to the learning process have not compromised its performance in a benign setting. We also need to mention that the actual learning rate in SureFED is even higher than BayP2PFL as is observed through our experiments.

We also note that the mean square error of the model parameter estimation decreasing proportional to $\frac{1}{t}$ indicates that the mean square error of the label predictions will also decrease proportional to $\frac{1}{t}$. Thus, if $\frac{1}{t} < \delta$, for some small and arbitrarily chosen δ , we have $\mathbb{E}[(y_t^i - \hat{y}_t^i)^2] < \xi\delta$, where \hat{y}_t^i is the predicted label and ξ is a constant.

We analyze the robustness of SureFED under a special case of the Label-Flipping data poisoning attack described in Section 3.2, in which the attackers poison the dataset of the compromised nodes by adding a bias b to their data sample labels, $(y_t^i)' = y_t^i + b$, $\forall i \in \mathcal{N}^c$. Without loss of generality, we assume the attackers to different clients agree on a single bias b to add to the data labels. The second attack that we consider is the General Random model poisoning attack described in Section 3.2. All of the proofs of the theorems in this section can be found in Appendix 7.6.

In order to study the robustness of SureFED, we need to state the following assumptions.

Assumption 5.3 (Sufficiency). The collection of the datasets of benign clients is sufficient for learning the model parameter θ^* . That is, $\cup_{i \in \mathcal{N}/\mathcal{N}^c} \mathcal{K}^i = \mathcal{K}$.

Assumption 5.4 (Relaxed Connectivity). The communication graph of the benign nodes satisfies the relaxed connectivity constraint.

Assumption 5.5 (Joint Learning). For every pair of benign node i and compromised node j that can communicate with each other at some time, we have $\mathcal{K}^i \cap \mathcal{K}^j \neq \emptyset$.

The Sufficiency and Relaxed Connectivity assumptions are common assumptions needed for learning. The Joint Learning assumption is made to ensure benign users can detect the compromised ones. In order for this detection to happen, a benign client has to be learning at least one common element of the model parameter with a compromised client to be able to evaluate its updates and detect it.

Theorem 5.6 (Robustness of SureFED, Label-Flipping Attack). *In SureFED, if nodes \mathcal{N}^c are compromised by Label-Flipping attack, and if assumptions 5.3, 5.4, and 5.5 hold, then the estimation of the benign users converge to θ^* with mean square error that is decreasing proportional to $\frac{1}{t}$.*

We need to note here that in order for the benign users to learn the true model parameter under Label-Flipping attack, we do not need to make any assumptions on the number of compromised nodes in each neighborhood nor do we need the majority of the benign nodes over the compromised ones. As long as the above three assumptions hold, the benign nodes can learn the true model parameter even if all but one of their neighborhood are compromised.

In the next theorem, we will generalize our results to the General Random model poisoning attacks. This generalization is done by taking into account the fact that the models that are widely used in practice are huge, and this will make it almost impossible for the attackers to remain undetected in SureFED.

Theorem 5.7 (Robustness of SureFED, General Random

Attack). *In SureFED, if nodes \mathcal{N}^c are compromised by the General Random model poisoning attack, and if assumptions 5.3 and 5.4 hold, then with probability of almost 1, the estimations of the benign users converge to θ^* with mean square error that is decreasing proportional to $\frac{1}{t}$.*

Note that we do not need the Joint Learning assumption in the above theorem. This theorem offers a probabilistic guarantee on the robustness of SureFED with a probability that is almost 1 when the size of the model is large (see the proof in Appendix for more detail).

6. Experiments

In this section, we present our experimental results for a decentralized image classification setting. We consider a network of 50 clients where 40% of them are under data and model poisoning attacks.

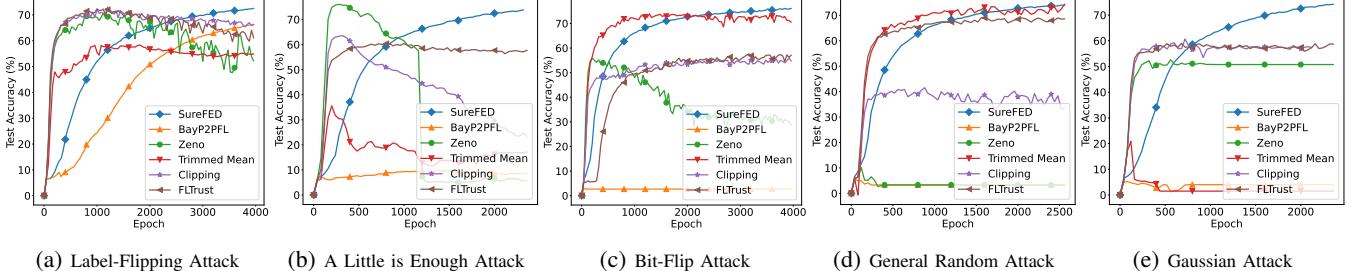
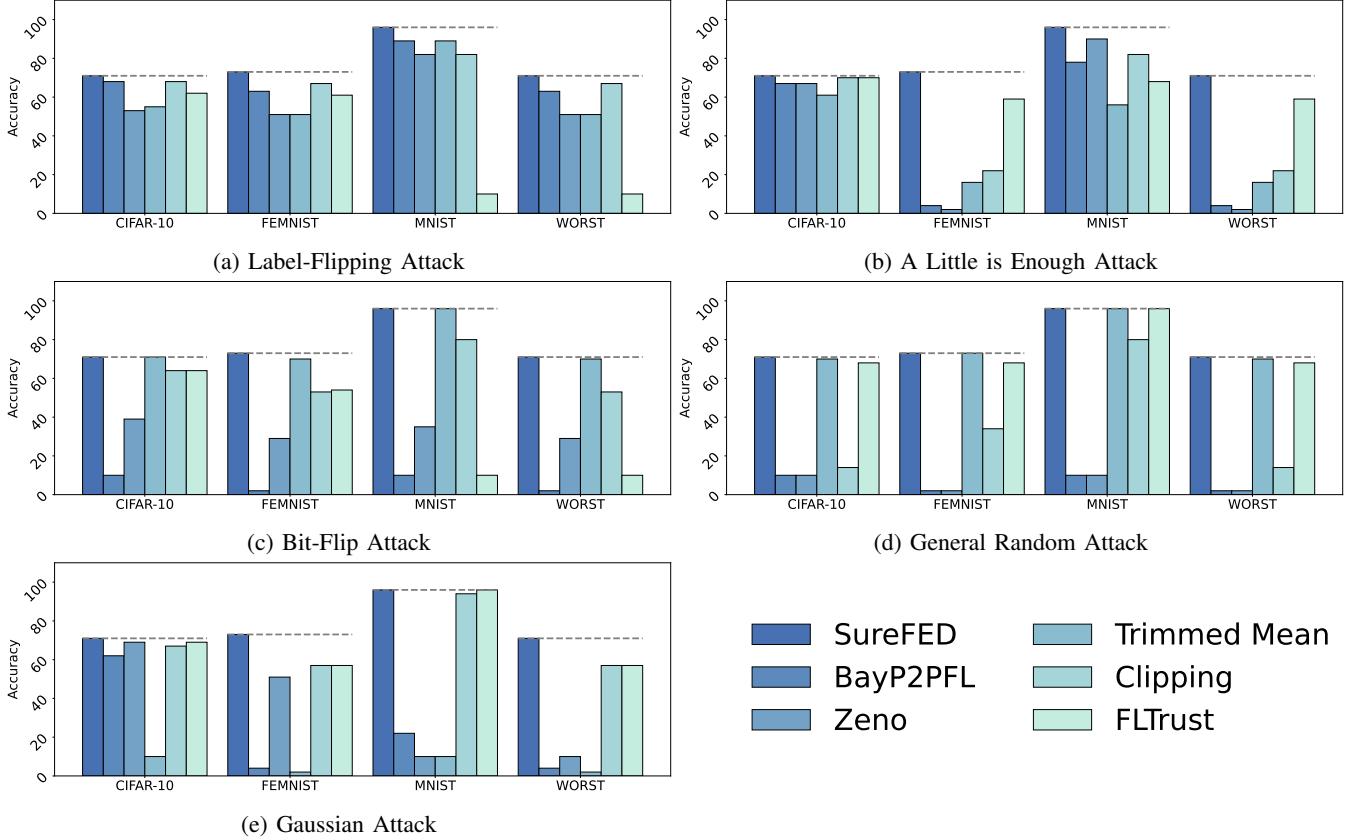
Datasets: We evaluate the performance of SureFED on three image classification datasets, namely, MNIST and FEMNIST from LEAF [8], and CIFAR10 [21]. The details of these datasets are summarized in Tab. 3 in the Appendix.

Baselines: We compare the performance of SureFED with the state of the art defense methods for federated learning in all of the three categories that were described in the introduction. Specifically, we compare with Trimmed Mean [41], and Clipping [2] from the first, Zeno [42] form the second, and FLTrust [9] from the third category. We also included BayP2PFL as a baseline to show the performance of a peer-to-peer federated learning using Bayesian models without any defense mehtods.

Attack Details: We consider six types of colluding and non-colluding data and model poisoning attacks that were described in Section 3.2. For the Label-Flipping, Bit-Flip, and A Little is Enough attacks, we followed the experimental settings from Karimireddy *et.al.* [17]. For the Trojan attack, we followed the settings from DBA [36]. For the General Random attack, we followed the settings from Xie *et.al.* [37]. The Gaussian attack is based on the settings of [26].

Evaluation Metrics: For the non-stealthy attacks of Label-Flipping, Bit-Flip, General Random, Gaussian, and A Little is Enough attacks that reduce the model accuracy, we use **Test Accuracy** measured on the benign test dataset to measure the robustness of different frameworks. The higher the test accuracy, the more robust the framework is against the considered attacks.

Trojan attack is a stealthy attack that should not affect the test accuracy to remain stealthy. For Trojan attack, we evaluate **Main Task Accuracy (MA)** (which is the same as test accuracy on clean test dataset) to measure the stealthiness of the attack. The main task accuracy needs to remain high under Trojan attack to ensure the stealthiness of the attack. We also evaluate **Backdoor Accuracy (BA)**, which is the percentage of Trojaned test samples that are successfully labeled with the Trojan target label (it is also referred to as attack success rate). The lower the backdoor accuracy, the more robust the framework is to Trojan attack.



6.1. Results

Performance under non-stealthy attacks: In Fig.1, we show the final model accuracy of SureFED and other baselines under Label-Flipping, A Little is Enough, Bit-Flip, General Random, and Gaussian poisoning attacks for all of the three datasets of MNIST, FEMNIST, and CIFAR10. Note that these attacks are non-stealthy and a poisoned model will have a lower test accuracy. Tables with the final model accuracy numbers are provided in the Appendix

(tables 5, 6, and 7). In Fig.2, we also show the plot of test accuracy w.r.t. the training epochs for FEMNIST dataset. The test accuracy plots of MNIST and CIFAR10 can be found in Appendix figures 2 and 7. As can be seen from the plots in Fig.1, SureFED is consistently robust against all of the five poisoning attacks. Its accuracy surpasses that of the other five baselines, matching the benign final model accuracy of 96% for MNIST, 73% for FEMNIST, and 71% for CIFAR10 (benign model accuracy is computed by training SureFED with no attackers). In contrast,

the other baselines exhibit varying degrees of robustness against certain attacks, with lower accuracy than SureFED, and all of the baselines experience significant failures for particular attacks and datasets. In Fig.3, we show the lowest model accuracy observed for each method across different datasets and attacks, highlighting the worst-case scenario for each framework. Notably, SureFED demonstrates robustness across all examined datasets and attacks, whereas the other methods exhibit poor performance in at least one dataset and attack, with an average accuracy of 6%.

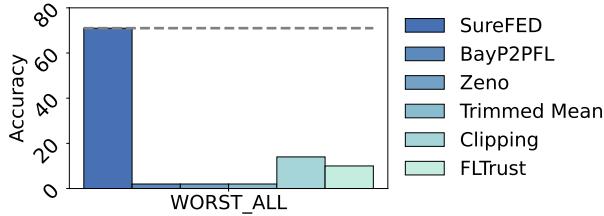


Figure 3: The worst final model accuracy of SureFED and other baselines across different poisoning attacks and datasets of CIFAR10, FEMNIST and MNIST. SureFED is the only method that shows consistent robustness against all of the attacks and with all of the three datasets.

Performance under Trojan attack (stealthy attack): For Trojan attack, we report the final Main Task Accuracy and Backdoor Accuracy of SureFED and the other frameworks in Fig.4. As is expected from a stealthy Trojan attack, the main task accuracy needs to remain high for all of the frameworks and this is confirmed in Fig.4. The backdoor accuracy, however, shows the attack success rate and the lower this number is for a framework, the more robust it is to Trojan attacks. As can be seen in Fig.4, the backdoor accuracy of SureFED is 24% and 23% for MNIST and FEMNIST datasets, respectively, while the other frameworks show 100% backdoor accuracy for MNIST and around 80% for FEMNIST dataset.

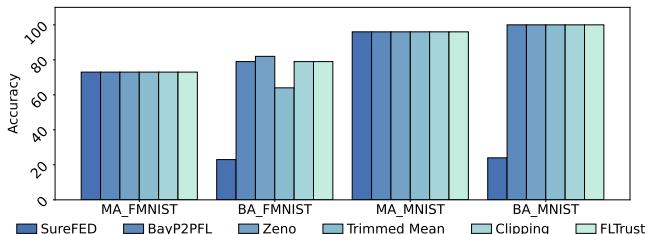


Figure 4: Main Task Accuracy (MA) and Backdoor Accuracy (BA) of SureFED and the other baselines under Trojan attack. **High MA** indicates the success of the Trojan attack in maintaining its stealthiness (MA of all baselines should be high). **Low BA** indicates success in defending against the Trojan attack. SureFED is the only method with low BA, indicating its robustness against Trojan attacks.

6.2. Ablation Studies

We have done three types of ablation studies on SureFED. The first study is on the percentage of compromised clients in the network. As explained before, SureFED does not require the majority of benign clients over the compromised ones to be robust against data and model poisoning attacks. This result is confirmed in Fig.5 where we show the plot of final model accuracy of SureFED and the other frameworks w.r.t. the percentage of compromised clients. It can be seen that SureFED shows a robust behavior for all of the adversary percentages. Also note that SureFED is agnostic to the number of adversaries.

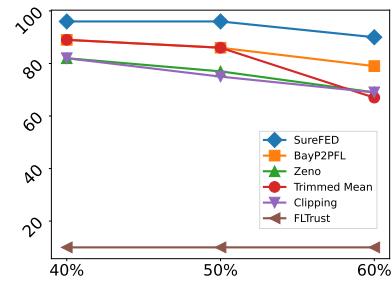


Figure 5: Final Model Accuracy of SureFED and the other baselines for MNIST dataset under Label-Flipping attack and varying number of compromised clients percentages.

The second ablation study is on the communication network between the clients. It was stated in Section 5 that for the learning and robustness in SureFED to happen, we need the communication graph to satisfy the relaxed connectivity constraint defined in Def.5.1. Such graphs could be incomplete and time-varying. In Table 1, we provide the results of the final model accuracy with incomplete and time varying graphs and under Label-Flipping attack. This result confirms that SureFED is robust against the considered attack even when the communication graph is incomplete and time-varying.

Graph	Complete	Incomplete	Time Varying
SureFED' Accuracy	73%	73%	73%

TABLE 1: SureFED’s Model Accuracy under Label-Flipping attack with FEMNIST dataset and different communication graphs. The incomplete graph is constructed by randomly dropping 20% of edges. The dropped edges are renewed every 100 epochs to construct the time-varying graph.

Our third ablation study is on the choice of clients that are under attack. We consider two extremes, where in one, the clients with the best dataset quality are under attack and in the other, the worst ones are under attack. The quality of the clients’ datasets is evaluated by their best local model accuracy. We compare the two extremes with a case where we randomly choose clients to be under attack. For these experiments, we consider Label-Flipping attack and FEMNIST dataset. In Table 2 we see that the performance of SureFED is not affected by how the clients are chosen to be under attack.

Compromised Clients	Best	Worst	Random
SureFED' Accuracy	73%	73%	73%

TABLE 2: SureFED’s Model Accuracy under Label-Flipping attack and FEMNIST dataset, where different sets of clients are under attack. Best and worst refer to the clients with the best and worst local model accuracy, respectively.

7. Conclusion

In this work, we presented SureFED, which is a novel robust federated learning framework based on uncertainty quantification. SureFED was designed to address the vulnerabilities of the existing defense methods by effectively using the local information of clients to concurrently train clean local models alongside social models. The local models can be used as a ground truth to evaluate the received model updates in an uncertainty-aware manner, and also to conduct introspection, making sure the learning is done correctly. These crucial components empower SureFED to withstand various data and model poisoning attacks, including colluding A Little is Enough attack and stealthy Trojan attacks. SureFED demonstrates superior performance compared to the state of the art defense methods for federated learning, achieving model accuracies that match the benign training accuracy, while being agnostic to the number of adversaries and not requiring the majority of benign clients over the compromised ones.

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Appendix

7.1. Experiment Setup

Our experiments are done with Python 3.9.7 and benchmarked on Linux Ubuntu 20.04.5 platform. Our workflow is built upon PyTorch [32] version 1.9.0 and trained on four NVIDIA TITAN Xp GPUs each with 12 GB RAM, and 48 Intel(R) Xeon(R) CPUs with 128 GB RAM.

7.2. Dataset Details

In Table 3, from left to right, we show the label number, average training and test dataset size per client.

Name	Label	Avg. Training Data	Avg. Test Data
MNIST	10	101	12
FEMNIST	62	197	22
CIFAR-10	10	2560	400

TABLE 3: Dataset Statistics

MNIST dataset consists of images of handwritten digits, and FEMNIST consists of handwritten digits and lower/upper case alphabets by different writers. The datasets are divided between clients such that each client only observes one writer's images to create a non-IID setup. CIFAR-10 is the biggest benchmark studied in the secure FL literature [11]. It consists of 60k 32x32 color images in 10 classes (50k for training and 10k for testing), with 6000 images per class. The dataset is divided randomly and evenly, where each client only observes part of the dataset.

7.3. Model Training Details

The architecture of the Bayesian neural network (BNN) and deep neural network (DNN) models used in this paper are summarized in Tab. 4. For fair comparison, both models consist of two convolution layers and two linear layers. The output dimension, label_num, in layer Linear2 is set to 10 in MNIST dataset and 62 in FEMNIST dataset. The BNN is implemented according to the variational Bayesian learning model in [6] and the federated version in [34]. For CIFAR-10, We use the ResNet-20 [14] as the backbone model for classification. The ResNet-20 is converted to the Bayesian neural network using Bayesian-Torch [20].

We set the maximum number of epochs for training to 8000. The batch size is set to 5, and each client trains on 5 batches in one epoch and then sends its model update to its peers for aggregation. For BNN models, we use an AdaM optimizer with learning rate set to 0.001. Other BNN implementation details are the same as BayP2PFL [34]. For Trimmed Mean and Clipping, we followed similar settings as Karimireddy *et.al.* [17] and set the learning rate to 0.01 with momentum set to 0.9. For Zeno, we followed similar settings as Xie *et.al.* [38] and set the learning rate to 0.1. Note that, we choose SGD over AdaM optimizer to train DNN models due to its better convergence and accuracy.

For SureFED, we set the hyperparameters, κ , to 2, and stop training the local model when its accuracy starts to drop in order to avoid overfitting local training data. For Zeno and Trimmed Mean, by default, we exclude the number of clients that have been compromised from the aggregation. SureFED, Clipping, and FLTrust are agnostic to the number of compromised clients in the network, whereas Zeno and Trimmed Mean require this information during the training.

Layers	Patch Size/Stride	Output	Activation
BNN Conv1 / DNN Conv1	$5 \times 5 / 1$	$6 \times 28 \times 28$	ReLU
Max Pooling1	$2 \times 2 / 2$	$64 \times 1 \times 1$	-
BNN Conv2 / DNN Conv2	$5 \times 5 / 1$	$16 \times 14 \times 14$	ReLU
Max Pooling2	$2 \times 2 / 2$	$64 \times 1 \times 1$	-
BNN Linear1 / DNN Linear1	784	120	ReLU
BNN Linear2 / DNN Linear2	120	label_num	Softmax

TABLE 4: Model Architecture

7.4. Experiment Results

In tables 5, 6, and 7 we present the final model accuracy of SureFED and the other baselines under the five considered non-stealthy attacks and with the three datasets of MNIST, FEMNIST, and CIFAR10. The main task accuracy and backdoor accuracy of SureFED and the other baselines under stealthy Trojan attack are provided in Table 8. The plot of test accuracy w.r.t. the epochs of all methods is provided in figures 6 and 7.

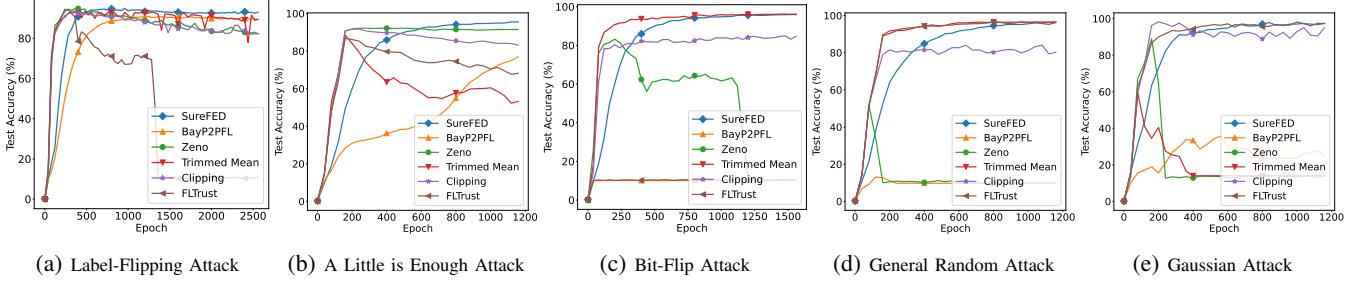


Figure 6: Accuracy plot of SureFED compared with BayP2PFL, Zeno, Trimmed Mean, Clipping, and FLTrust defense methods under different data and model poisoning attacks evaluated on MNIST dataset.

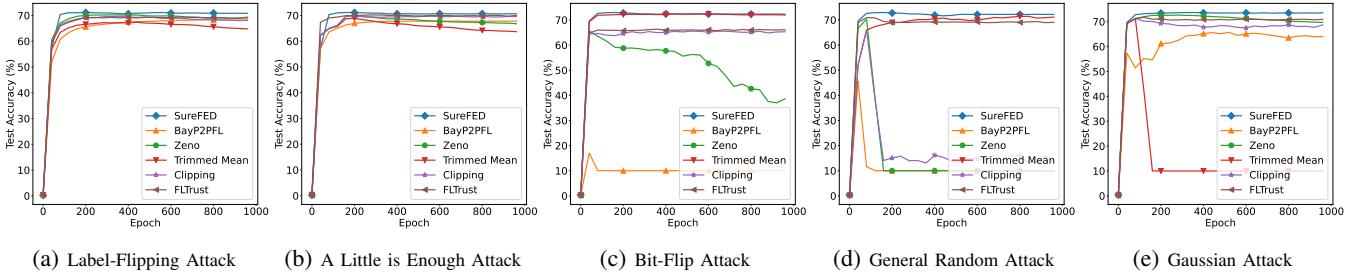


Figure 7: Accuracy plot of SureFED compared with BayP2PFL, Zeno, Trimmed Mean, Clipping, and FLTrust defense methods under different data and model poisoning attacks evaluated on CIFAR10 dataset.

Algorithm \ Attack	Label-Flipping	A Little is Enough	Bit-Flip	General Random	Gaussian
SureFED	96%	96%	96%	96%	96%
BayP2PFL	89%	78%	10%	10%	22%
Zeno	82%	90%	35%	10%	10%
Trimmed Mean	89%	56%	96%	96%	10%
Clipping	82%	82%	80%	80%	94%
FLTrust	10%	68%	10%	96 %	96%
Clean	96%	96%	96%	96%	96%

TABLE 5: Final Model Accuracy for SureFED and the other baselines under different attacks for MNIST dataset.

Algorithm \ Attack	Label-Flipping	A Little is Enough	Bit-Flip	General Random	Gaussian
SureFED	73%	73%	73%	73%	73%
BayP2PFL	63%	4%	2%	2%	4%
Zeno	51%	2%	29%	2%	51%
Trimmed Mean	51%	16%	70%	73%	2%
Clipping	67%	22%	53%	34%	57%
FLTrust	61%	59%	54%	68%	57%
Clean	73%	73%	73%	73%	73%

TABLE 6: Final Model Accuracy for SureFED and the other baselines under different attacks for FEMNIST dataset.

7.5. SureFED for Decentralized Linear Regression

In this section, we describe SureFED for the case where the models have a single linear layer. We also assume that the labels are generated according to the linear equation of $y_t^i = \langle \theta^*, x_t^i \rangle + \eta_t^i$, where $\langle \cdot \rangle$ denotes the inner product and we assume $\eta_t^i \sim N(0, \Sigma^i)$ is a Gaussian noise. In this setting, the local belief updates are done according to Bayes rule and there is no need to fit the posteriors to a Gaussian distribution due to the posteriors themselves being Gaussian distributions. Therefore, the belief updates that are done locally are simplified to Kalman filter updates on the parameters of the Gaussian

Algorithm \ Attack	Label-Flipping	A Little is Enough	Bit-Flip	General Random	Gaussian
SureFED	71%	71%	71%	71%	71%
BayP2PFL	68%	67%	10%	10%	62%
Zeno	53%	67%	39%	10%	69%
Trimmed Mean	55%	61%	71%	70%	10%
Clipping	68%	70%	64%	14%	67%
FLTrust	62%	70%	64%	68 %	69%
Clean	71%	71%	71%	71%	71%

TABLE 7: Final Model Accuracy for SureFED and the other baselines under different attacks for CIFAR10 dataset.

Algorithm	Main Task Accuracy	Backdoor Accuracy
SureFED	96%	24%
BayP2PFL	96%	100%
Zeno	96%	100%
Trimmed Mean	96%	100%
Clipping	96%	100%
FLTrust	96%	100%

(a) MNIST Dataset

Algorithm	Main Task Accuracy	Backdoor Accuracy
SureFED	73%	23%
BayP2PFL	73%	79%
Zeno	73%	82%
Trimmed Mean	60%	64%
Clipping	73%	79%
FLTrust	73%	79%

(b) FEMNIST

TABLE 8: Main Task and Backdoor Accuracy of SureFED and the other baselines under Trojan attack. **High Main Task Accuracy** indicates the success of the Trojan attack in maintaining its stealthiness (main task accuracy of all baselines should be high). **Low Backdoor Accuracy** indicates the success of the algorithm in defending against the Trojan attack.

distributions. In particular, after receiving a data sample (x_t^i, y_t^i) , the belief parameters are updated as follows.

$$\hat{\theta}_t^i = \hat{\theta}_{t-1}^i + \frac{\hat{\Sigma}_{t-1}^i x_t^i}{x_t^{iT} \hat{\Sigma}_t^i x_t^i + \Sigma^i} (y_t^i - x_t^{iT} \hat{\theta}_{t-1}^i) \quad (6a)$$

$$\hat{\Sigma}_t^i = \hat{\Sigma}_{t-1}^i - \frac{\hat{\Sigma}_{t-1}^i x_t^i x_t^{iT} \hat{\Sigma}_{t-1}^i}{x_t^{iT} \hat{\Sigma}_{t-1}^i x_t^i + \Sigma^i} \quad (6b)$$

$$\bar{\theta}_t^i = \bar{\theta}_{t-1}^i + \frac{\bar{\Sigma}_{t-1}^i x_t^i}{x_t^{iT} \bar{\Sigma}_{t-1}^i x_t^i + \Sigma^i} (y_t^i - x_t^{iT} \bar{\theta}_{t-1}^i) \quad (7a)$$

$$\bar{\Sigma}_t^i = \bar{\Sigma}_{t-1}^i - \frac{\bar{\Sigma}_{t-1}^i x_t^i x_t^{iT} \bar{\Sigma}_{t-1}^i}{x_t^{iT} \bar{\Sigma}_{t-1}^i x_t^i + \Sigma^i} \quad (7b)$$

7.6. Proofs

Proof of Theorem 5.2. In order to prove the theorem, we show that the social estimations of agents will converge to θ and the social covariance matrices converge to 0. We first assume that the trust weights are fixed throughout the algorithm (similar to BayP2PFL [34]) and we will later incorporate the bounded confidence trust weights used in SureFED.

Lemma 7.1. *The covariance matrix of the social beliefs $\bar{\Sigma}_t^i$ decrease with rate proportional to $\frac{1}{t}$, where t is the number of local data observations.*

Proof. According to equation (7), and using Sherman–Morrison formula [13], we can write

$$\begin{aligned} (\bar{\Sigma}_t^i)^{-1} &= (\bar{\Sigma}_{t-1}^i - \frac{\bar{\Sigma}_{t-1}^i x_t^i x_t^{iT} \bar{\Sigma}_{t-1}^i}{x_t^{iT} \bar{\Sigma}_{t-1}^i x_t^i + \Sigma^i})^{-1} \\ &= (\bar{\Sigma}_{t-1}^i)^{-1} + \frac{(\bar{\Sigma}_{t-1}^i)^{-1} \frac{\bar{\Sigma}_{t-1}^i x_t^i x_t^{iT} \bar{\Sigma}_{t-1}^i}{x_t^{iT} \bar{\Sigma}_{t-1}^i x_t^i + \Sigma^i} (\bar{\Sigma}_{t-1}^i)^{-1}}{1 - \frac{x_t^{iT} \bar{\Sigma}_{t-1}^i (\bar{\Sigma}_{t-1}^i)^{-1} \bar{\Sigma}_{t-1}^i x_t^i}{x_t^{iT} \bar{\Sigma}_{t-1}^i x_t^i + \Sigma^i}} \\ &= (\bar{\Sigma}_{t-1}^i)^{-1} + \frac{x_t^i x_t^{iT}}{\Sigma^i} \end{aligned} \quad (8)$$

Notice that the above equation indicates that if the covariance matrix $\bar{\Sigma}_{t-1}^i$ is invertible, then $\bar{\Sigma}_t^i$ is also invertible. Since we start with an invertible covariance matrix, all of the covariance matrices are invertible. We denote $Z_t^i = (\bar{\Sigma}_t^i)^{-1}$. Then using equation (5) and (8), we have

$$Z_t^i = \sum_{j \in \mathcal{N}_t(i)} T_t^{ij} (Z_{t-1}^j + \frac{x_t^j x_t^{jT}}{\Sigma^j}) \quad (9)$$

If we denote $T_t = (T_t^{ij})_{i \in \mathcal{N}, j \in \mathcal{N}_t(i)}$ (the ij th element of T_t is T_t^{ij} if $j \in \mathcal{N}_t(i)$), $(Z_t)_{sl} = ((Z_t^i)_{sl})_{i \in \mathcal{N}}$, and $(X_t)_{sl} = (\frac{(x_t^i x_t^{iT})_{sl}}{\Sigma^i})_{i \in \mathcal{N}}$, we can write

$$(Z_t)_{sl} = T_t(Z_{t-1})_{sl} + T_t(X_t)_{sl} \quad (10)$$

Therefore, we can write

$$(Z_t)_{sl} = T_{t:t-\tau(t)}(Z_{\tau(t)-1})_{sl} + \sum_{\tau=t-\tau(t)}^t T_{t:\tau}(X_\tau)_{sl} \quad (11)$$

where $\tau(t)$ is chosen such that $T_{t:t-\tau(t)} > 0$ (all elements are positive). We know such $\tau(t)$ exists due to Assumption 5.4. Since the collective dataset of all agents is sufficient to learn the true model parameter, for each s , there exists at least one agent j with $(x_\tau^j x_\tau^{jT})_{ss} > 0$ with probability 1. Therefore, $\sum_{\tau=t-\tau(t)}^t T_{t:\tau}(X_\tau)_{ss} > 0$. Hence, on average, at each time step, $\frac{1}{\tau(t)} \sum_{\tau=t-\tau(t)}^t T_{t:\tau}(X_\tau)_{ss} > 0$ is added to $(Z_t)_{ss}$. Therefore, $\text{tr}(Z_t)$ is increasing with rate proportional to t . Since $Z_t = (\bar{\Sigma}_t^i)^{-1}$ and $\bar{\Sigma}_t^i$ is a positive semi-definite matrix, we know that $\text{tr}(Z_t) = \text{tr}(\bar{\Sigma}_t^i)^{-1}$. Therefore, $\text{tr}(\bar{\Sigma}_t^i)$ decreases with rate proportional to $\frac{1}{t}$. Hence, $\bar{\Sigma}_t^i$ is decreasing with rate proportional to $\frac{1}{t}$. \square

Lemma 7.2. *The social estimations, $\bar{\theta}_t^i$, converge to θ for all $i \in \mathcal{N}$.*

Proof. According to equation (7), we have the following.

$$\begin{aligned} \bar{\theta}_1^i &= \frac{\Sigma_0^i x_1^i}{x_1^i T \Sigma_0^i x_1^i + \Sigma^i} (x_1^i \theta + \eta_1^i) \\ &= \frac{\Sigma_0^i x_1^i x_1^{iT}}{x_1^i T \Sigma_0^i x_1^i + \Sigma^i} \theta + \frac{\Sigma_0^i x_1^i}{x_1^i T \Sigma_0^i x_1^i + \Sigma^i} \eta_1^i \end{aligned} \quad (12)$$

We denote

$$G_t^i = I - \frac{\bar{\Sigma}_{t-1}^i x_t^i x_t^{iT}}{x_t^i T \bar{\Sigma}_{t-1}^i x_t^i + \Sigma^i} \quad (13)$$

$$f_t^i = \frac{\bar{\Sigma}_{t-1}^i x_t^i}{x_t^i T \bar{\Sigma}_{t-1}^i x_t^i + \Sigma^i} \quad (14)$$

Based on equation (5), we can write

$$\bar{\theta}_1^i = \bar{\Sigma}_1^{i'} \left(\sum_{j \in \mathcal{N}_1(i)} T_1^{ij} (\bar{\Sigma}_1^j)^{-1} ((I - G_1^j) \theta + f_1^j \eta_1^j) \right) \quad (15)$$

Similarly, we can write

$$\bar{\theta}_1 = D(\bar{\Sigma}'_1) \tilde{D}(T_1) D((\bar{\Sigma}_1)^{-1}) ((I - D(G_1)) \tilde{I} \theta + D(f_1) \eta_1) \quad (16)$$

where we define $\tilde{I} = \begin{bmatrix} I_K \\ I_K \\ \vdots \end{bmatrix}$, and

$$\tilde{D}(T_1) = \begin{bmatrix} T_1^{11} & 0 & \cdots & T_1^{12} & 0 & \cdots & 0 & \cdots \\ 0 & T_1^{11} & \cdots & 0 & T_1^{12} & \cdots & 0 & \cdots \\ \vdots & & & & & & & \\ T_1^{21} & 0 & \cdots & T_1^{22} & 0 & \cdots & 0 & \cdots \\ 0 & T_1^{21} & \cdots & 0 & T_1^{22} & \cdots & 0 & \cdots \\ \vdots & & & & & & & \end{bmatrix} \quad (17)$$

We define

$$\bar{T}_t = D(\bar{\Sigma}'_t) \tilde{D}(T_t) D((\bar{\Sigma}_t)^{-1}) \quad (18)$$

Note that we have $\bar{T}_t \tilde{I} = \tilde{I}$ and therefore, \bar{T}_t is a row stochastic matrix. We can write

$$\bar{\theta}_1 = \bar{T}_1((I - D(G_1))\tilde{I}\theta + D(f_1)\eta_1) \quad (19)$$

Similarly, we have

$$\begin{aligned} \bar{\theta}_2 &= \bar{T}_2(D(G_2)\bar{\theta}_1 + (I - D(G_2))\tilde{I}\theta + D(f_2)\eta_2) \\ &= \bar{T}_2(D(G_2)\bar{T}_1((I - D(G_1))\tilde{I}\theta + D(f_1)\eta_1) + (I - D(G_2))\tilde{I}\theta + D(f_2)\eta_2) \\ &= \bar{T}_2\tilde{I}\theta - \bar{T}_2D(G_2)\tilde{I}\theta + \bar{T}_2D(G_2)\bar{T}_1\tilde{I}\theta - \bar{T}_2D(G_2)\bar{T}_1D(G_1)\tilde{I}\theta + \bar{T}_2D(f_2)\eta_2 + \bar{T}_2D(G_2)\bar{T}_1D(f_1)\eta_1 \\ &= \tilde{I}\theta - \bar{T}_2D(G_2)\bar{T}_1D(G_1)\tilde{I}\theta + \bar{W}_2D(f_2)\eta_2 + \bar{T}_2D(G_2)\bar{T}_1D(f_1)\eta_1 \end{aligned} \quad (20)$$

where the last equality is due to the fact that $\bar{T}_t \tilde{I} = \tilde{I}$. By generalizing the above equations to time t , we can write

$$\bar{\theta}_t = \tilde{I}\theta - \bar{T}_tD(G_t) \cdots \bar{T}_1D(G_1)\tilde{I}\theta + \bar{\eta}_t \quad (21)$$

where $\bar{\eta}_t$ is the cumulative noise terms up to time t ,

$$\bar{\eta}_t = \bar{T}_tD(f_t)\eta_t + \bar{T}_tD(G_t)\bar{T}_{t-1}D(f_{t-1})\eta_{t-1} + \cdots + \bar{T}_tD(G_t) \cdots \bar{T}_1D(f_1)\eta_1 \quad (22)$$

According to the weak law of large numbers, we have $\bar{\eta}_t \xrightarrow{P} 0$. Furthermore, we can show that all eigenvalues of the G_t^i matrices are less than one. We had

$$G_t^i = I - \frac{\bar{\Sigma}_{t-1}^i x_1^i x_1^{i T}}{x_1^{i T} \bar{\Sigma}_{t-1}^i x_1^i + \Sigma^i} = I - \frac{\bar{\Sigma}_{t-1}^i x_1^i x_1^{i T}}{\text{tr}(\bar{\Sigma}_{t-1}^i x_1^i x_1^{i T}) + \Sigma^i} \quad (23)$$

Since $\text{tr}(\bar{\Sigma}_{t-1}^i x_1^i x_1^{i T}) = \sum_{s=1}^K \lambda^s$, where λ^s is the s th eigenvalue of $\bar{\Sigma}_{t-1}^i x_1^i x_1^{i T}$, all eigenvalues of G_t^i are less than or equal to one. Notice that $\bar{\Sigma}_t^i$ is a covariance matrix and therefore, is positive semi-definite. But since our covariance matrices are invertible, they are positive definite. Hence, if all elements of x_t^i are positive, then all eigenvalues of G_t^i would be less than one. The eigenvalues of G_t^i that are one are due to some elements of x_t^i being zero, thus making some rows of $\bar{\Sigma}_{t-1}^i x_1^i x_1^{i T}$ to be all zeros. Therefore, contraction would not happen on those rows. However, for each element of θ , e.g., $(\theta)_k$, there is at least one agent with non-zero $(x_t^i)_k > 0$. Based on assumption 5.4, for some set $\{s_1, \dots, s_t\}$, we have $\prod_{s=s_{t-1}}^{s_t} \bar{T}_s D(G_s)$ to have eigenvalues that are less than one (each block of \bar{T}_t is a row stochastic matrix and therefore, all its eigenvalues are less than or equal to one, and we also assume that if $T_t^{ij} > 0$, then $T_t^{ij} > \delta$ for some $\delta > 0$) and thus, $\prod_{s=s_{t-1}}^{s_t} \bar{T}_s D(G_s)$ is a contraction. Therefore, we have $\lim_{t \rightarrow \infty} \|\bar{T}_t D(G_t) \cdots \bar{T}_1 D(G_1) \tilde{I}\theta\| = 0$. Consequently, we have $\lim_{t \rightarrow \infty} \bar{\theta}_t = \tilde{I}\theta$. \square

Using lemmas 7.1 and 7.2, the estimation of clients converges to the true model parameter with their uncertainty (variance) converging to zero. Hence, clients can learn the true model parameter with sufficient data observations.

The difference that SureFED has with the setting of the above proof above is that T_t changes by time according to our robust aggregation rule. Therefore, in order to prove the theorem, it suffices to show that T_t will satisfy the relaxed connectivity constraint. Since A_t satisfies the relaxed connectivity constraint, we need to show that if there is an edge between users i and j , then $T_t^{ij} > \delta$ with high probability for some $\delta > 0$.

Since the beliefs are Gaussian, for agent i we have $|(\hat{\theta}_t^i)_k - (\theta)_k| < 2\sqrt{(\hat{\Sigma}_t^i)_{k,k}}$ for all $k \in \mathcal{K}^i$ with high probability (≈ 0.97). The same is true for the social belief of agent j . Therefore, we have $|(\hat{\theta}_t^i)_k - (\bar{\theta}_t^j)_k| < 2\sqrt{(\hat{\Sigma}_t^i)_{k,k}} + 2\sqrt{(\bar{\Sigma}_t^j)_{k,k}}$ with high probability (≈ 0.999). Hence, if $(\hat{\Sigma}_t^i)_{k,k} \simeq (\bar{\Sigma}_t^j)_{k,k}$, and for $\kappa \approx 4$, agent i will aggregate the updates of agent j with high probability. Notice that there is a possibility for agent i to not aggregate the updates of agent j when $(\bar{\Sigma}_t^j)_{k,k} \gg (\hat{\Sigma}_t^i)_{k,k}$. This indicates that if the model of agent j is too bad, agent i will not aggregate it. \square

Proof of Theorem 5.6. Before proving this theorem, we need to state and prove the following theorem.

Theorem 7.3. *If the trust weight matrix T_t is fixed through time (similar to BayP2PFL), and if nodes \mathcal{N}^c are compromised by Label-Flipping attack and the communication graph satisfies the relaxed connectivity constraint, the estimation of all users converge to $\theta^* + cb$ for some vector c with positive elements.*

Proof of Theorem 7.3. We denote the benign nodes (the nodes that are not compromised) by \mathcal{N}^b . Also, the benign neighbors of node i at time t are denoted by $\mathcal{N}_t^b(i)$. Similarly, we denote the compromised neighbors of node i at time t by $\mathcal{N}_t^c(i)$. According to equation (5) and (7), and similar to equation (15), we can write

$$\bar{\theta}_1^i = \bar{\Sigma}_1^{i'} \left(\sum_{j \in \mathcal{N}_1(i)} T_1^{ij} (\bar{\Sigma}_1^j)^{-1} ((I - G_1^j) \theta + f_1^j \eta_1^j) \right) + \bar{\Sigma}_1^i \sum_{j \in \mathcal{N}_1^c(i)} T_1^{ij} (\bar{\Sigma}_1^j)^{-1} f_1^j b \quad (24)$$

Therefore, we have

$$\bar{\theta}_1 = \bar{T}_1 ((I - D(G_1)) \tilde{I} \theta + D(f_1) \eta_1) + \bar{T}_1 D(f_1) \mathbf{1}^c b \quad (25)$$

where \tilde{I} and \bar{T}_1 are defined in the proof of Theorem 5.2. We also define $(\mathbf{1}^c)_{(j-1)K:jK} = 1$ if $j \in \mathcal{N}^c$ and $(\mathbf{1}^c)_{(j-1)K:jK} = 0$, otherwise. Similarly, we can write

$$\bar{\theta}_t = \tilde{I} \theta - \bar{T}_t D(G_t) \cdots \bar{T}_1 D(G_1) \tilde{I} \theta + \bar{\eta}_t + \mathbf{c}_t b \quad (26)$$

where

$$\mathbf{c}_t = \bar{T}_t D(f_t) \mathbf{1}^c + \bar{T}_t D(G_t) \bar{T}_{t-1} D(f_{t-1}) \mathbf{1}^c + \cdots + \bar{T}_t D(G_t) \cdots \bar{T}_1 D(f_1) \mathbf{1}^c \quad (27)$$

In the proof of Theorem 5.2 we showed that the sum of the first two terms in equation (26) will converge to θ . In the following, we will show that \mathbf{c}_t will converge to $\tilde{I}\mathbf{c}$, where \mathbf{c} is a vector of size K with positive elements. We can write

$$\mathbf{c}_t = \bar{T}_t D(f_t) \mathbf{1}^c + \bar{T}_t D(G_t) \mathbf{c}_{t-1} \quad (28)$$

We know that $f_t^i \rightarrow 0$ and $G_t^i \rightarrow I$. Therefore, for large enough t , we have

$$\mathbf{c}_t \simeq \bar{T}_t \mathbf{c}_{t-1} \quad (29)$$

We show the convergence of \mathbf{c}_t based on the convergence analysis in the opinion dynamics literature [15], [19]. In this literature, there are a network of agents, each of which has an opinion over a specific matter. The opinions of the agent i at time t is denoted by \mathbf{x}_t^i , and the vector of agent's opinions is denoted by \mathbf{x}_t . The opinion dynamics is usually assumed to be $\mathbf{x}_{t+1} = A_t \mathbf{x}_t$, where A_t is a row stochastic matrix. One can see that \mathbf{c}_t evolves according to a similar model. We first state the next well known lemma from [28] (Theorem 3.1).

Lemma 7.4. *If A is a row stochastic matrix, then we have*

$$v(A\mathbf{x}) \leq (1 - \min_{1 \leq i, j \leq n} \sum_{k=1}^n \min\{a_{ik}, a_{jk}\}) v(\mathbf{x}), \quad (30)$$

where $\mathbf{x} \in \mathbb{R}^n$ and

$$v(\mathbf{x}) = \max_{1 \leq i \leq n} \mathbf{x}^i - \min_{1 \leq i \leq n} \mathbf{x}^i = \min_{1 \leq i, j \leq n} (\mathbf{x}^i - \mathbf{x}^j) \quad (31)$$

The proof of the above lemma can be found in [28].

We also state the following condition on a row stochastic matrix A_t based on [19]. Suppose there exists numbers $0 \leq \delta_t \leq 1$ such that $\sum_{t=0}^{\infty} \delta_t = \infty$ and $\sum_{k=1}^n \min\{a_t^{ik}, a_t^{jk}\} \geq \delta_t$. We refer to this condition as the joint connectivity condition. Using this condition and according to [19], for the model of $\mathbf{x}_{t+1} = A_t \mathbf{x}_t$, we have $v(\mathbf{x}_t) \leq e^{-\sum_{s=0}^t \delta_s} v(\mathbf{x}_0)$. Therefore, we have $v(\mathbf{x}_t) \rightarrow 0$ and thus, \mathbf{x}_t converges to a vector with the same elements, which we denote as $\mathbf{x} = x\mathbf{1}$. We note that one can relax the condition on the matrix A_t , by defining a set $\{s_1, s_2, \dots, s_t, \dots\}$ and $\bar{A}_t = \prod_{l=s_t}^{s_{t+1}-1} A_l$. If the joint connectivity condition holds for \bar{A}_t , then the result still holds. We refer to this relaxed condition as the relaxed joint connectivity condition.

In order to apply the above argument to prove the convergence of \mathbf{c}_t , we assume that the non-diagonal entries of $\bar{\Sigma}_t^i$ are 0 for all $i \in \mathcal{N}$. That is, we assume that the belief parameters are shared separately for each element of the model parameter. This is what happens in the general variational learning version in Section 4. Notice that this modification does not affect the learning analysis of the algorithm done in Theorem 5.2. We denote the vector $\mathbf{c}_t^k = (\mathbf{c}_t^{(i-1)*K+k})_{i \in \mathcal{N}}$ to be the vector of elements of \mathbf{c}_t corresponding to the k_{th} element of the model parameter, then \mathbf{c}_t^k evolves according to the opinion dynamics model of $\mathbf{c}_t^k = \bar{T}_t^k \mathbf{c}_{t-1}^k$ for all $k \in \mathcal{K}$, where \bar{T}_t^k is the matrix consisting of the elements of \bar{T}_t corresponding to \mathbf{c}_t^k and \mathbf{c}_{t-1}^k (the elements that are multiplied to \mathbf{c}_{t-1}^k to generate \mathbf{c}_t^k in equation (29)). \bar{T}_t^k is a row stochastic matrix. We further mention that the relaxed connectivity condition in Assumption 5.4 will imply that \bar{T}_t^k satisfies the relaxed joint connectivity condition described earlier. Therefore, \mathbf{c}_t^k will converge to a vector of the same elements, $\mathbf{c}^k \mathbf{1}$. Consequently, \mathbf{c}_t converges to a vector $\tilde{I}\mathbf{c}$. Note that $c > 0$ because according to equation (29), \mathbf{c}_t is a summation of all positive terms. \square

In order to show the robustness of SureFED, we will show that the social estimations of the benign clients converge to θ and the social covariance matrices converge to 0.

In the proof of Theorem 5.2, we showed that the covariance matrices of the beliefs converge to 0. Furthermore, we can see in the proof of Lemma 7.1 that the covariance matrices of the beliefs are independent of the labels y_t^i and therefore, any bias in the labels will not affect the covariance matrices. Therefore, the covariance matrices will converge to 0 whether or not some users are compromised.

Similar to the proof of Theorem 7.3, this proof is also based on the convergence analysis in the opinion dynamics literature [15], [19]. In this proof, the opinion of users are their social estimations of θ . That is, we have $x_t^i = \bar{\theta}_t^i$. We will model the data acquisition of agents by adding two benign and malicious nodes to the network that play the role of stubborn opinion leaders [42] whose opinions are always θ and $\theta + b$, respectively, for the benign and malicious opinion leaders. The opinion of these two nodes will not change. The benign nodes directly hear the opinion of benign opinion leader with environment noise (data observation noise) and the compromised nodes, directly hear the opinion of malicious opinion leader. According to this model, the opinion dynamic of agents in the network is given by linear equations (7) and (7).

In order to prove that SureFED is robust, we show that during the dynamics of the estimations of users, we will see an opinion fragmentation (see [15]) between the benign and the compromised agents. If an opinion fragmentation happens for a benign user and a compromised one, the benign agent will not aggregate the belief of the compromised agent. In order to show that an opinion fragmentation happens, we show that a compromised user will be removed from the confidence set of agent i at some time t .

Similar to Lemma 7.1, one can show that the variances of the local belief of user i for the elements $k \in \mathcal{K}^i$, decrease with rate proportional to $\frac{1}{t}$, where t is the number of local observations. Based on the proof of Theorem 5.2, one can easily show that the local estimation of the benign user i on $(\theta)_k$ converge to $(\theta)_k$ for $k \in \mathcal{K}^i$, by setting the trust weights T_t^{ij} to 0 except for $j \neq i$.

Assume that agent j is compromised and agent i is benign and they can communicate with each other at some time. According to Assumption 5.5, there exists a $k \in \mathcal{K}^i \cap \mathcal{K}^j$. Since the local estimation of agent i on $(\theta)_k$ is converging to $(\theta)_k$ and the social estimation of agent j is converging to $(\theta + cb)_k$ for some $c > 0$, there is a time t at which we have $|(\hat{\theta}_t^i)_k - (\bar{\theta}_t^j)_k| > \kappa \sqrt{(\hat{\Sigma}_t^i)_{k,k}}$. It can be easily proved by contradiction. That is, assume that it does not happen. Since $(\hat{\Sigma}_t^i)_{k,k}$ is converging to 0, we must have $(\hat{\theta}_t^i)_k$ and $(\bar{\theta}_t^j)_k$ converge to the same point, which we know is a contradiction. Next, we will investigate how many observations are needed from agent i to make an opinion fragmentation with a compromised agent j . We note that since the beliefs are Gaussian, $|(\hat{\theta}_t^i)_k - (\theta)_k| < 2\sqrt{(\hat{\Sigma}_t^i)_{k,k}}$ with high probability (≈ 0.97) and the same can be said about other beliefs as well. Assume we have $\sqrt{(\hat{\Sigma}_t^j)_k} \leq a\sqrt{(\hat{\Sigma}_t^i)_k}$. If we have $|(\theta)_k - (\theta + cb)_k| > (2a + 2 + \kappa)\sqrt{(\hat{\Sigma}_t^i)_k}$, we know with high probability that $|(\hat{\theta}_t^i)_k - (\bar{\theta}_t^j)_k| > \kappa\sqrt{(\hat{\Sigma}_t^i)_{k,k}}$. Note that the multiplier $2a + 2 + \kappa$ is due to the uncertainty of $(\hat{\theta}_t^i)_k$ around $(\theta)_k$ (which is $2\sqrt{(\hat{\Sigma}_t^i)_{k,k}}$), the uncertainty of $(\bar{\theta}_t^j)_k$ around $(\theta + cb)_k$ (which is $2\sqrt{(\hat{\Sigma}_t^j)_k}, k \leq 2\kappa\sqrt{(\hat{\Sigma}_t^i)_{k,k}}$) and the allowed deviation of $(\hat{\theta}_t^i)_k$ from $(\bar{\theta}_t^j)_k$ (which is $\kappa\sqrt{(\hat{\Sigma}_t^i)_{k,k}}$). Therefore, if we have $(\hat{\Sigma}_t^i)_{k,k} < (\frac{c_k b}{2a+2+\kappa})^2$, then we have an opinion fragmentation with high probability. Since $(\hat{\Sigma}_t^i)_{k,k}$ decreases with rate proportional to $\frac{1}{t}$, we should have

$$t > l((\frac{2a + 2 + \kappa}{c_k b})^2) \quad (32)$$

for the opinion fragmentation to happen. The multiplier l is derived from the average decrease in $(\hat{\Sigma}_t^i)_{k,k}$ by each local data observation.

Notice that the last step of SureFED was to make sure that the social model of clients stay close to their local models for those elements that a client has a local model. This step will ensure that even if at the beginning of the algorithm (before the opinion fragmentation with the compromised clients happens), the social models get poisoned, the clients can correct them with their clean local models. Therefore, after the compromised clients are removed from the system, everything will become the same as the learning in a benign setting and according to Theorem 5.2, clients will learn the true model parameter if the conditions of the learning hold.

In conclusion, we can say that SureFED algorithm is robust against the considered Label-Flipping data poisoning attacks. \square

Proof of Theorem 5.7. Assume that a given attacker j is tampering with the model parameter weights in set \mathcal{K}^c with size $|\mathcal{K}^c| = C|\mathcal{K}|$. Also, assume that for a given client i , we have $|\mathcal{K}^i| = L|\mathcal{K}|$. In order for client i to detect the poisoned client

j , we need to have $\mathcal{K}^i \cap \mathcal{K}^c \neq \emptyset$. Since the model parameter elements in \mathcal{K}^c are chosen randomly, the probability of the above condition holding is as follows.

$$\mathbb{P}(\mathcal{K}^i \cap \mathcal{K}^c \neq \emptyset) = 1 - \frac{\binom{|\mathcal{K}| - |\mathcal{K}^i|}{|\mathcal{K}^c|}}{\binom{|\mathcal{K}|}{|\mathcal{K}^c|}} = 1 - \frac{\binom{(1-L)|\mathcal{K}|}{C|\mathcal{K}|}}{\binom{|\mathcal{K}|}{C|\mathcal{K}|}} \quad (33)$$

Fig. 8a shows the plot of the above probability w.r.t. the model size for different values of C and L , and in Fig. 8b, we see the probability plot for a fixed model size ($1e6$) and different values of C and L . We see that the probability is almost always equal to 1 for different ranges of L and C and model sizes.

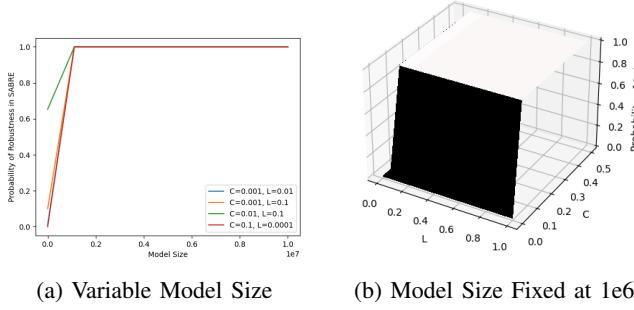


Figure 8: The probability of SureFED being robust against General Random attack. □