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Abstract. The capacitated vehicle routing problem with fuzzy demand is considered. Since customers' demand is not precisely known in advance, but is given as uncertain quantities, i.e. as fuzzy numbers, a recommended route may not meet each demand for capacity reasons. Route failure will result in losing the part of demand which could not be satisfied. Consequently, the possibility, or indeed necessity, to satisfy all customers' demand has to be high. Optimization should additionally consider several conflicting objectives, e.g. minimizing total travel costs as well as maximizing sales.

In this article, a fuzzy multi-criteria modeling approach, based on a mixed integer linear mathematical programming model, is presented. In order to solve even larger problems, the established savings heuristic for the classical vehicle routing problem is modified appropriately with regard to fuzzy demand and multi-criteria optimization. A compromise solution is determined interactively by the decision maker, who adjusts the degree of satisfaction with the different goals. The solution method is demonstrated in a 75-customer example.

1 Introduction

The capacitated vehicle routing problem (CVRP) assumes certainty for customers' demand, which has to be satisfied by a homogeneous fleet of delivery vehicles of fixed capacity from a single depot at minimal cost. A wide range of real-world problems has been modeled by the CVRP. For an overview, see Crainic and Laporte [8]. In particular, vehicle routing problems (VRP) with various modifications and different approaches are considered by Laporte [21], Desrosiers et al. [9], and Fisher [14]. Bibliographies in VRP can be found in Laporte and Osman [23] and in Laporte [22]. The VRP with or without side-constraints is an $\mathcal{NP}$-hard problem [24], which makes it difficult to optimally solve larger problems in a reasonable amount of computing time. For problems with more than 50 customers, solution procedures generally utilize heuristics. Stochastic vehicle routing problems (SVRP) are introduced by Tillman [31] in 1969, who suggests a savings approach for the multi-depot VRP with stochastic demand. Not until two decades later, research focuses

J.-L. Verdegay (ed.), *Fuzzy Sets Based Heuristics for Optimization*

on stochastic models of SVRP. For an overview, see Powell et al. [26], Bertsimas and Simchi-Levi [4], and Gendreau et al. [16]. In particular, models for vehicle routing problems with stochastic demand (VRPSD) are considered by Dror et al. [12], Bertsimas [2], and Dror [10]. Recourse versions of VRPSD are solved to optimality by Dror et al. [11], who restrict the number of route failures, in 1993 and by Gendreau et al. [15] in 1995. A real world application is presented by Cheung and Powell [5] in 1996. Heuristic procedures to solve the VRPSD are just as rarely considered: See Stewart and Golden [28] and Dror and Trudeau [13] for modified savings algorithms, Bertsimas et al. [3] for a priori heuristics, Gendreau et al. [17] for a tabu search algorithm, Secomandi [27] for neuro-dynamic programming algorithms, and Yang et al. [37] for two-phase methods.

This article deals with the CVRP in a fuzzy environment: The customers' demand is not precisely known in advance, but can be modeled as uncertain quantities, i.e. as fuzzy numbers. Because stochastic models assume stochastic quantities, i.e. random numbers, precise information about the probability functions of demand is needed. To handle imprecise and vague information, an approach which is based on fuzzy sets theory is appropriate. Nevertheless, the capacitated vehicle routing problem with fuzzy demand (CVRPFD) has not yet been published except for Teodorović and Kikuchi [29], who developed a savings algorithm for the CVRP with fuzzy travel times, Teodorović and Pavković [30], who suggest an approximate reasoning algorithm for routing one vehicle with regard to fuzzy demand, and Werners and Kondratenko [36], who present a fuzzy multi-criteria model for tanker routing problems with fuzzy demand.

In the following section the CVRPFD is described in detail, and a fuzzy multi-criteria modeling approach, based on a mixed integer linear mathematical programming model, is presented. An interactive approach is suggested to find a compromise solution, which is suitable for small problems because optimal solutions of the crisp equivalent can be found by using standard mixed integer linear programming software. In Sect. 3, a heuristic, which is adequate for larger problems, is developed and demonstrated in a 75-customer problem. Some conclusions are given in the last section.

2 A Fuzzy Multi-Criteria Modeling Approach

2.1 General Description of the Problem

At a single depot 0, a homogeneous fleet of m delivery vehicles is available: The vehicles are identical and have the same fixed capacity Q. A set of n customers has to be served, who know their respective demand approximately. There are enough vehicles to satisfy any actual total demand. All distances between depot and customers are crisp and given. Each vehicle starts and finishes its route at the depot. We assume late information policy: No customer knows his or her demand precisely until a vehicle has completely served it