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Thème ***:***

Sliding mode direct current motor control

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ABSTRACT

The work presented in this thesis concerns the modeling and control of a DC motor using two control techniques, the first control using classical methods such as PI and the second using SMC which is a nonlinear control method. Through some illustrative examples, the simulation results show the difference between the classical PI and sliding mode method and the efficiency of the SMC technique.

ملخص

يتعلق العمل المقدم في هذه الاطروحة بنمذجة و التحكم في محرك ذو التيار المستمر باستخدام تقنيتي تحكم .

الاولى باستخدام طريقة كلاسيكية باستخدام المتحكم PI و الثانية باستخدام SMC وهي طريقة تحكم غير خطية .

من خلال بعض الامثلة التوضيحية اظهرت النتائج الفرق بين التقنيتين و كفاءة تقنية SMC عند تغير عوامل المحرك او التعرض لاضطراب خارجي.

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LIST DES SYMBOLES

LTI	**L**inear **T**ime **I**nvariant
PD	**P**roportional **D**erivative
PI	**P**roportional **I**ntegral
PID	**P**roportional **I**ntegral **D**erivative
PWM	**P**ulse **W**idth **M**odulator
SMC	**S**liding **m**ode **C**ontrol
DC	**D**irect **C**urrent
Jm	**M**otor **I**nertia
Jc	**L**oad **I**nertia
Cr	**L**oad **T**orque
r	**S**peed **R**educer **R**atio
ω	**S**peed
R	**R**esistance
E	**B**attery
I	**M**otor **C**urent
bc	**L**oad **S**ide **S**peed **R**educer **He**ight
bm	**M**otor **S**ide **S**peed **R**educer **He**ight
L	**I**nductance

C1,C2	**A**xis **F**riction
V	**I**nput **V**oltage
Cm	**M**achine **T**orque
V	**L**yapunov **F**unction
S	**S**liding **S**urface
ω_{ref}	**D**esired **S**peed
I_{ref}	**D**esired **C**urrent
DCM	**D**irect **C**urent **M**otor

GENERALE INTRODUCTION

DC motors are widely used in control applications.speed and position, which are often encountered in industry. They have certain characteristics, such as high efficiency, successful performance and easy controllability. Thus, they are widely preferred in robotics and other applications. In fact, the control of DC motors is one of the most challenging applications due to the growing interest in their use in control systems.

Many different techniques are offered for controlling DC motors. These approaches are PID control, fuzzy logic, linear quadratic regulator (LQR) and sliding mode control.

The sliding mode control (SMC) method is one of the nonlinear control methods and has attractive characteristics, such as fast response, successful adaptation to the effects of disturbances, and insensitivity to the change of system parameters.

In this thesis, we made the modeling and the control of a direct current motor

using two very famous control techniques in the field of systems control, the first using PI which is a classic method and the second using SMC which is a nonlinear control method.

This thesis is organized as follows.

Chapter 1: The first chapter presents the mathematical modeling of the DC machine and the open-closed loop study using classic PI regulator.

Chapter 2: In the second chapter, we will discuss the theory of sliding mode control and the advantages and disadvantages that the technique brings such as the phenomenon of chattering and how to solve them in different ways.

Chapter 3: In the third chapter, we will discuss the speed control of the DC motor by the SMC technique based all different control models (cascade, reduced and complete model) and the design of an observer for the estimation of the acceleration.

Chaptre 1

DESCRIPTION AND MODELISATION OF DIRECT CURRENT MOTOR

.

1.1 Introduction

In this chapter, we will present the modeling of the DC machine. Analysis of machine performance in terms of open and closed loop stability and response dynamics are presented at the end of this chapter.

1.2 Mathematical modeling of dc motor

The DC motor, with permanent magnets, inertia J_m drives an inertia load J_c through a speed reducer of ratio $r = \omega_m / \omega = C_2 / C_1$, with friction on the axes and a load torque C_r (figure 1.1). We want to find the transfer function of the load speed ω.

On the motor side we have:

$$L\dot{I} = -RI - e_b + v \tag{1.1}$$

$$J_m \dot{\omega}_m = C_m - b_m \omega_m - C_1 \tag{1.2}$$

With :

$$e_b = k_b w_m \tag{1.3}$$

$$C_m = k_m I \tag{1.4}$$

On the load side we have:

$$J_c \dot{\omega} = C_2 - b_c \omega - C_r \tag{1.5}$$

Using relations (1.3)-(1.4) and speed reducer relations, $\omega_m = r\omega$, $C_1 = C_2 / r$,

We get:

$$L\dot{I} = -RI - rk_b\omega + v \tag{1.6}$$
$$rJ_m\dot{\omega} = k_m I - rb_m\omega - C_2 / r \tag{1.7}$$

By multiplying (1.7) by r, we get:

$$r^2 J_m\dot{\omega} = rk_m I - r^2 b_m\omega - C_2 \tag{1.8}$$

So the sum of (1.5) and (1.8) produces:

$$(J_c + r^2 J_m)\dot{\omega} = rk_m I - (b_c + r^2 b_m)\omega - C_r \tag{1.9}$$

Using the notation $J = J_c + r^2 J_m$ et $b = b_c + r^2 b_m$, we can have the following machine + load model:

$$\begin{aligned} L\dot{I} &= -RI - rk_b\omega + v \\ J\dot{\omega} &= -b\omega + rk_m I - C_r \end{aligned} \tag{1.10}$$

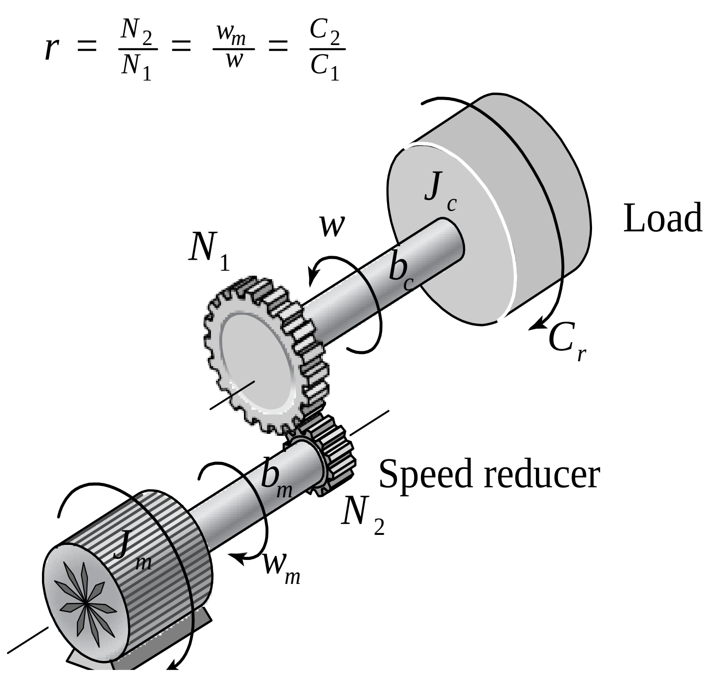


Figure 1.1: Motor driving a load through a speed reducer

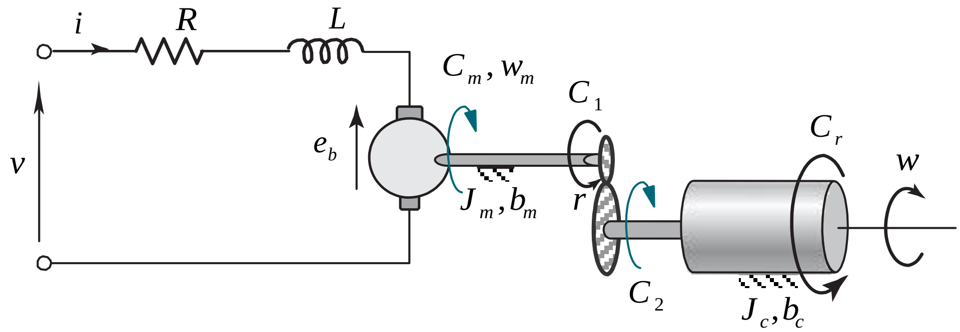


Figure 1.2: equivalent diagram

1.3 Open loop study

For simulation; we will use the DCM with the following settings:

$E = 20V$, $R = 1\Omega$, $L = 0.02H$, $b_m = 0$, $b_c = 0.0001N.m.s$, $J_m = 0.001Kg.m^2$, $J_c = 0.01Kg.m^2$

$k_m = 0.1$, $k_b = 0.1$, $r = 10$

In order to study the performance of the DC machine in open loop we will test the response of the machine for an input voltage V=15V and a load torque of 5Nm ,The simulink diagram of the open-loop DC machine is shown in Figure (1.3).

The simulation results for a voltage V=15V and a load torque Cr=5Nm introduced at time t=1.5 sec, are represented by figures (1.5). For an input voltage V , the speed of the machine is constant, this means that the system is stable.

The application of a load torque results in a drop in speed figure (1.5), and the machine Loses Its speed.

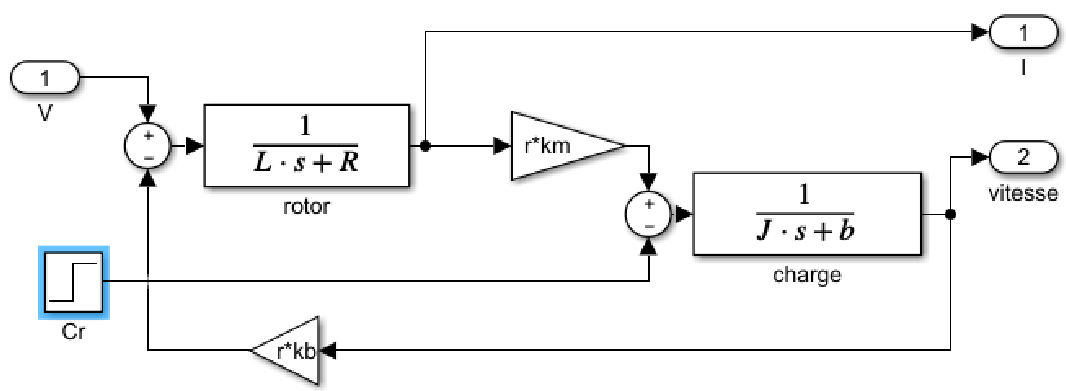


Figure 1.3 : DC motor's simulink diagram

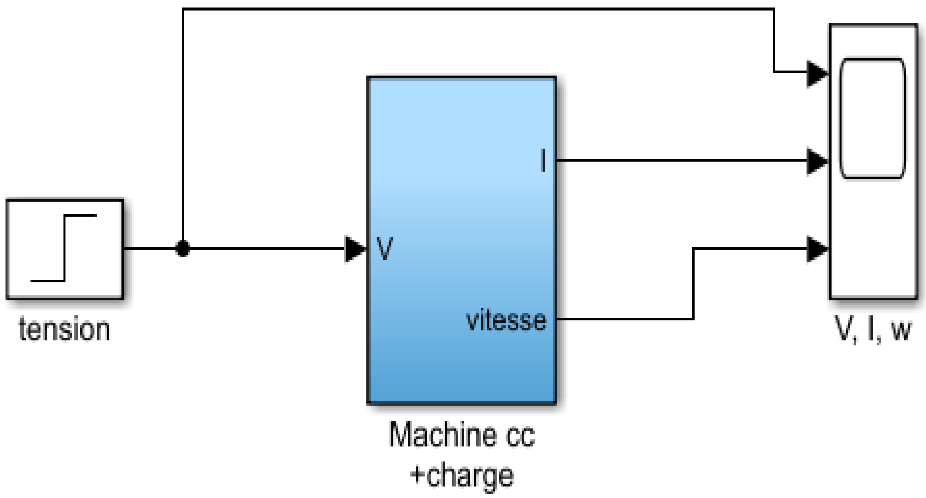


Figure 1.4 : simplified model of the used motor

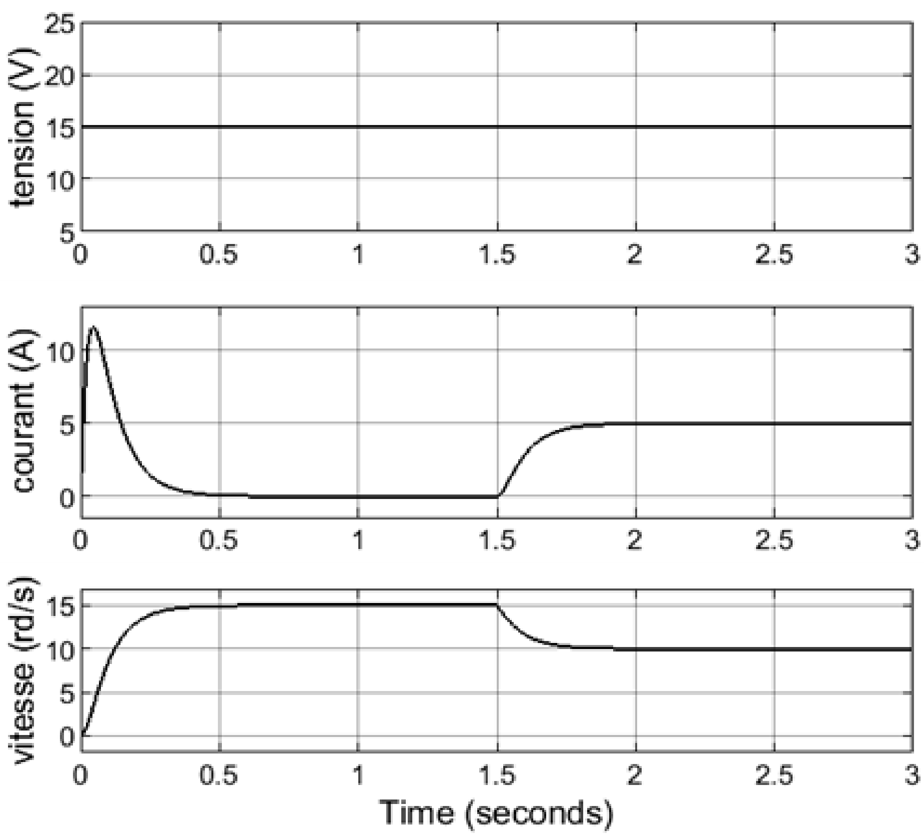


Figure 1.5 : DC motor open loop response.

1.4 closed-loop study

In order to study the performances of the machine in closed loop, we will introduce in this section a simple control structure, based on a PI controller, to track a reference speed.

1.4.1 PI regulator principle:

First, a transfer function PI regulator is used.

$$PI(s) = k_p + \frac{k_I}{s} = \frac{k_p s + k_I}{s} \tag{1.11}$$

The block diagram of the corrected system is given by figure (1.6). This corrector Thanks to its integral action, it cancels the difference between the setpoint and the measurement, as well as the effect of constant disturbances.

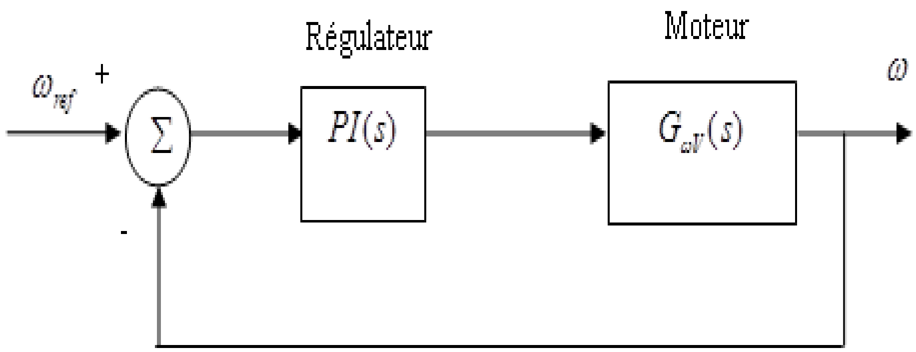


Figure 1.6: block diagram of a speed PI regulator

The closed-loop transfer function is given by:

$$G_{w-wref} = \frac{PI(s)G_{wv}(s)}{1 + PI(s)G_{wv}(s)} \tag{1.12}$$

With $G_{wv}(s)$ is the speed-voltage transfer function given by:

$$G_{wv} = \frac{K}{(Ls+R)(Js+b)+K^2} \tag{1.13}$$

Substituting (1.11) and (1.13) in equation (1.12) we find the close-loop transfer function:

$$G_{w-wref} = \frac{K(k_P s + k_I)}{s((Ls+R)(Js+b)+K^2)+K(k_P s + k_I)} \tag{1.14}$$

If we neglect the inductance La , we find:

$$G_{w-wref} = \frac{1}{RJ}\frac{K(k_p\,\mathrm{s}+k_I)}{s^2+\frac{(\mathrm{Rb}+Kk_p+K^2)}{RJ}s+\frac{Kk_I}{RJ}} \tag{1.15}$$

We can write the denominator of equation (1.15) by a 2 order characteristic equation as:

$$s^2+\frac{(\mathrm{Rb}+Kk_p+K^2)}{RJ}s+\frac{Kk_I}{RJ} = s^2+2\zeta\omega_n s+\omega_n^2 \tag{1.16}$$

Which give:

$$k_P = \frac{2\zeta RJ-(\mathrm{Rb}+\mathrm{K}^2)}{K}$$

$$k_I = \frac{RJ\omega_n^2}{K}$$

For the choices of $\zeta=1$, $\omega_n=50$ We obtain:

$$k_P = 9.999$$

$$k_I = 275$$

1.4.2 Simulation results:

Let's test the effectiveness of this regulation by simulating the response of the closed-loop system.The simulink diagram of the regulation is given by the figure (1.7)

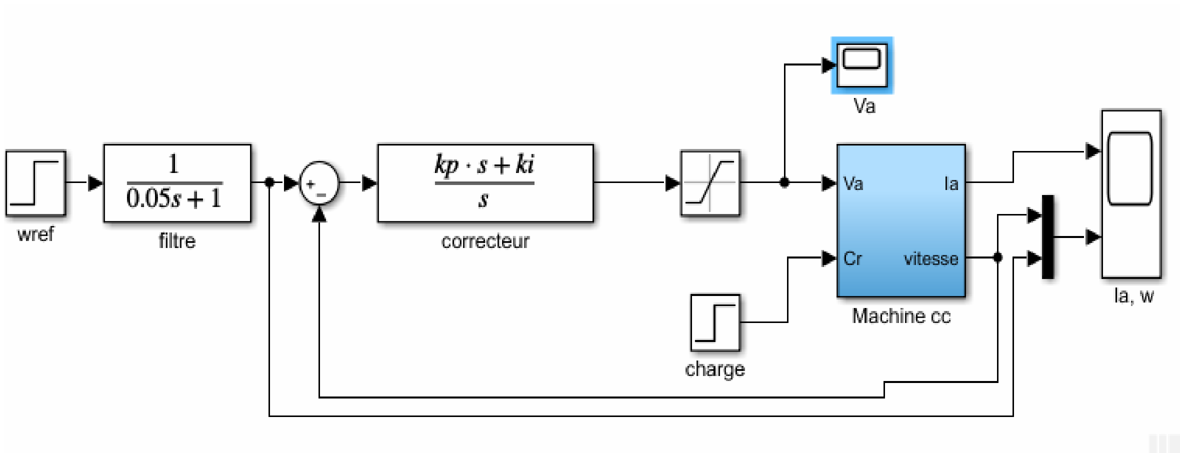


Figure 1.7: speed regulation simulink diagram.

Figure (1.8) gives the response of the system with the presence of a load torque Cr=5Nm at t=1.5sec. The simulation results show that the system is able to regulate the speed despite such a variation in torque. For this it provides electrical energy, which translates into a stabilization of the response of the machine.

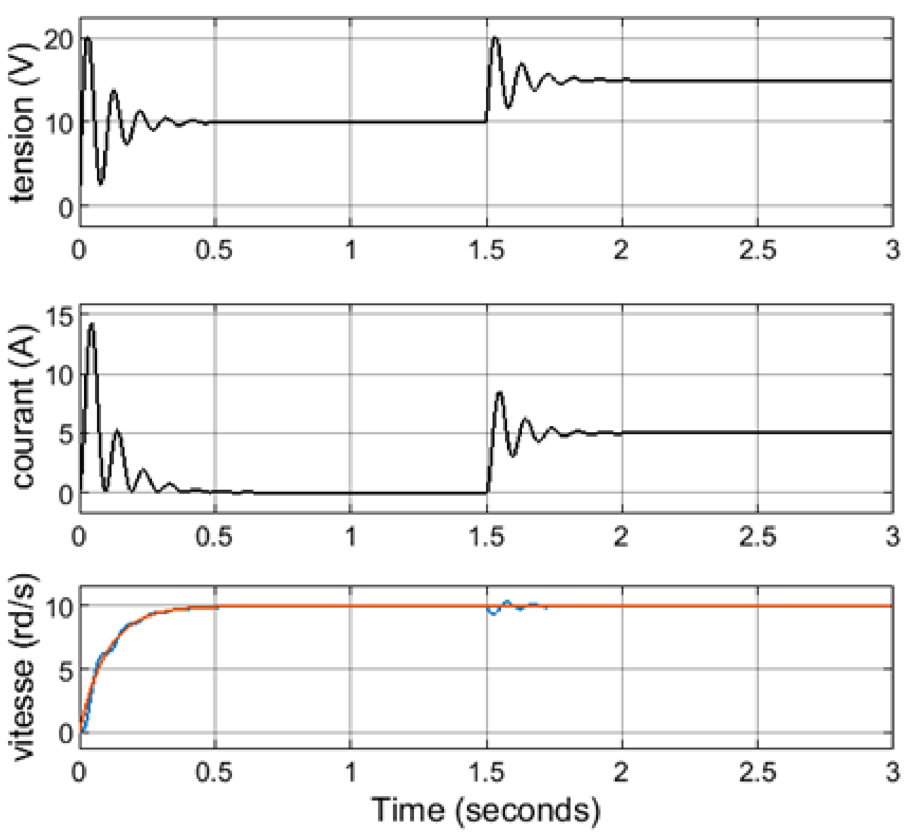


Figure 1.8: closed-loop DC motor response with a single PI speed control.

1.4.3 Cascade based speed and current regulation.:

1.4.3.1 Principle of cascade regulation:

We now integrate the current loop into the speed loop, as illustrated by figure(1.9).

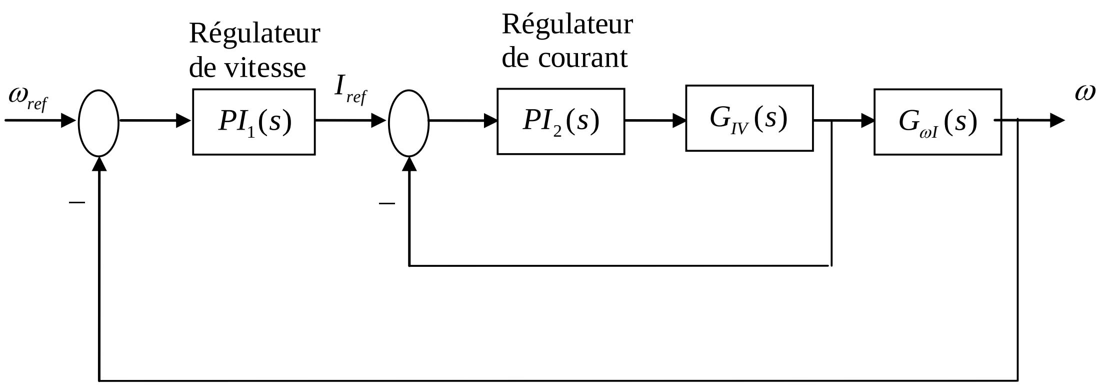


Figure 1.9: diagram of cascade regulation Principle.

The advantages of this cascade regulation are:

- the Transient state is faster.

- the current loop allows the regulation of the current, and we can therefore:

- limiting the torque, i.e. working in a transient state at constant torque.
- regulate the torque (or force) transmitted to the load.
- limit the current for reasons of safety of the motor and the associated converter.

1.4.3.2 Calculation of the regulator:

1.4.3.2.1 Pole placement method:

Speed loop

The closed loop transfer function of the velocity loop is defined as:

$$\frac{PI_1(s)G_{wI}(s)}{1+PI_1(s)G_{wI}(s)} = \frac{1}{J}\frac{K(k_{P1}s+k_{I1})}{s^2+\frac{(b+k_{P1}K)}{J}s+\frac{k_{I1}K}{J}} \qquad (1.17)$$

With: $G_{wI}(s) = \dfrac{w(s)}{I(s)} = \dfrac{k}{(Js + b)}$ and $PI_1(s) = \dfrac{k_{p1}s + k_{I1}}{s}$

We can write the denominator of equation (1.17) by a 2 order characteristic equation as:

$$s^2 + \tfrac{(b + k_{P1}K)}{J}s + \tfrac{k_{I1}K}{J} = s^2 + 2\zeta\omega_v s + \omega_v^2 \tag{1.18}$$

By the equation of the two conditions (1.18), we get:

$$k_{P1} = \frac{2\zeta\omega_v J - b}{K}$$
$$k_{I1} = \frac{J\omega_v^2}{K}$$

Current loop

The closed loop transfer function of the current loop is defined as::

$$\frac{PI_2(s)G_{IV}(s)}{1 + PI_2(s)G_{IV}(s)} = \frac{1}{L}\frac{k_{P2}s + k_{I2}}{s^2 + \frac{(R + k_{P2})}{L}s + \frac{k_{I2}}{L}} \tag{1.19}$$

With : $G_{IV}(s) = \dfrac{I(s)}{V(s)} = \dfrac{1}{(Ls + R)}$ and $PI_2(s) = \dfrac{k_{p2}s + k_{I2}}{s}$

We can write the denominator of equation (1.19) by a 2 order characteristic equation as:

$$s^2 + \tfrac{(R + k_{P2})}{L}s + \tfrac{k_{I2}}{L} = s^2 + 2\zeta\omega_i s + \omega_i^2 \tag{1.20}$$

By the equation of the two conditions (1.20), we get:

$$k_{P2} = 2\zeta\omega_i L - R$$
$$k_{I2} = L\omega_i^2$$

For Choices of $z = 1$, $\omega_i = 500$ et $\omega_v = 0.1\omega_i$

We have

$k_{p1} = 10.9999$

$k_{I1} = 275$

and :

$k_{p2} = 19$

$k_{I2} = 5000$

1.4.3.3 Simulation results:

The response of the cascade regulation is simulated thanks to the simulink diagram of figure (1.10).

We observe the absence of oscillations in the transient state in the case of the Cascade regulation.

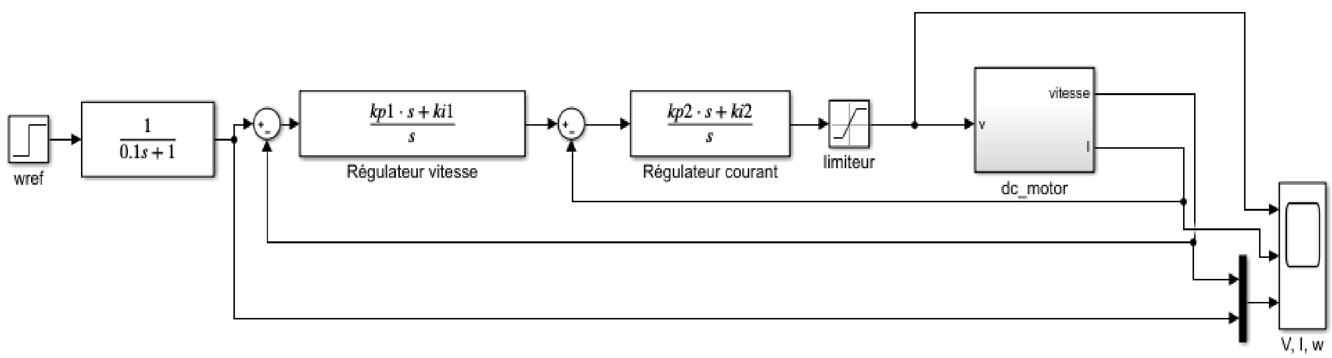


Figure 1.10: simulink diagram of cascade control.

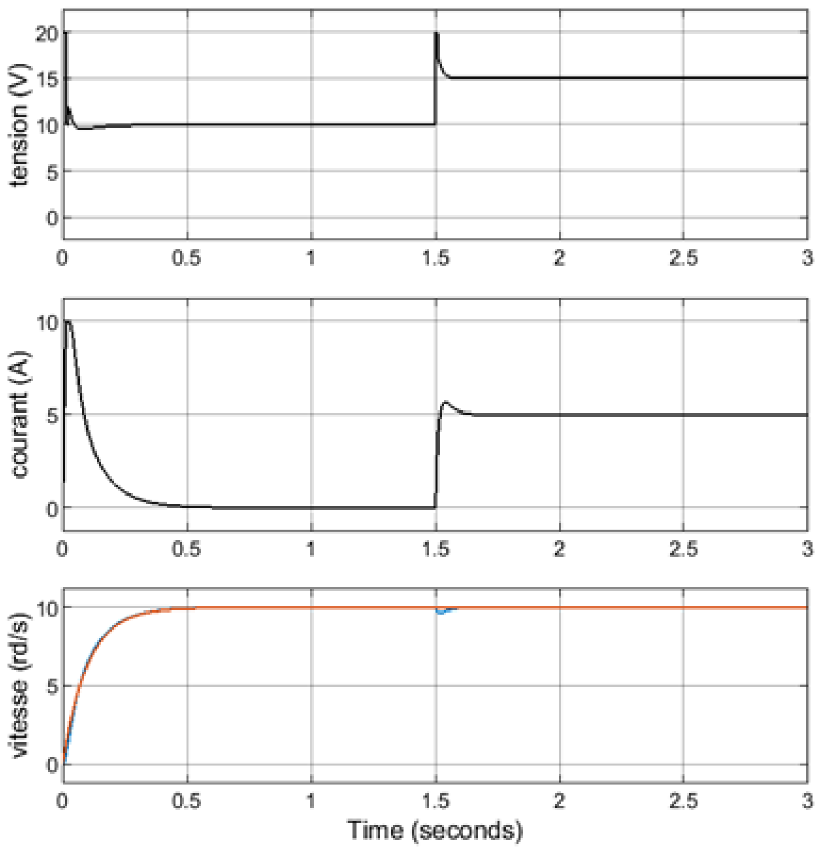


Figure 1.11: closed loop DC motor response with two PI cascade regulators.

1.5 DCM implementation with converter.

Figures (1.12) and (1.13) represent the implementation of the DC motor with a converter And the model of the converter used, respectively. The role of the converter is to supply the machine with voltage between E and -E. Figure (1.14) shows that the control follow the setpoint even in the presence of a load torque.

The performances of the PI regulator in the case of a parametric change of resistance and inertia of 50% are represented by the figure (1.15), we can notice that the change of the parameters influences the PI regulator.

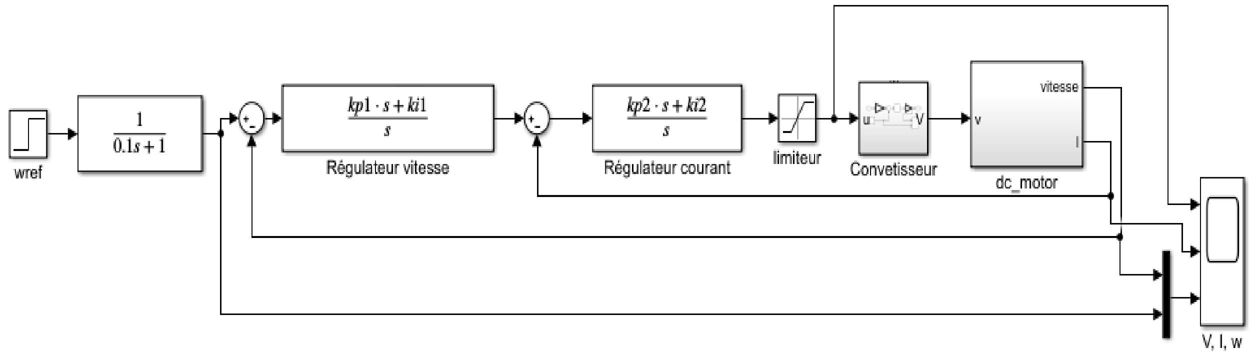


Figure 1.12: simulink diagram of DCM implementation with converter.

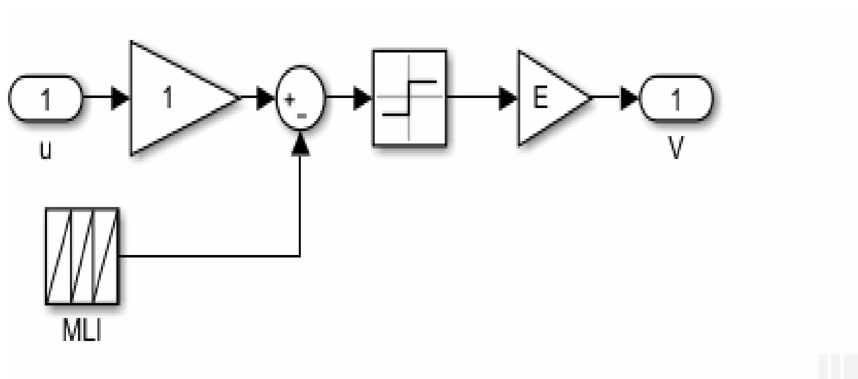


Figure 1.13: Converter model.

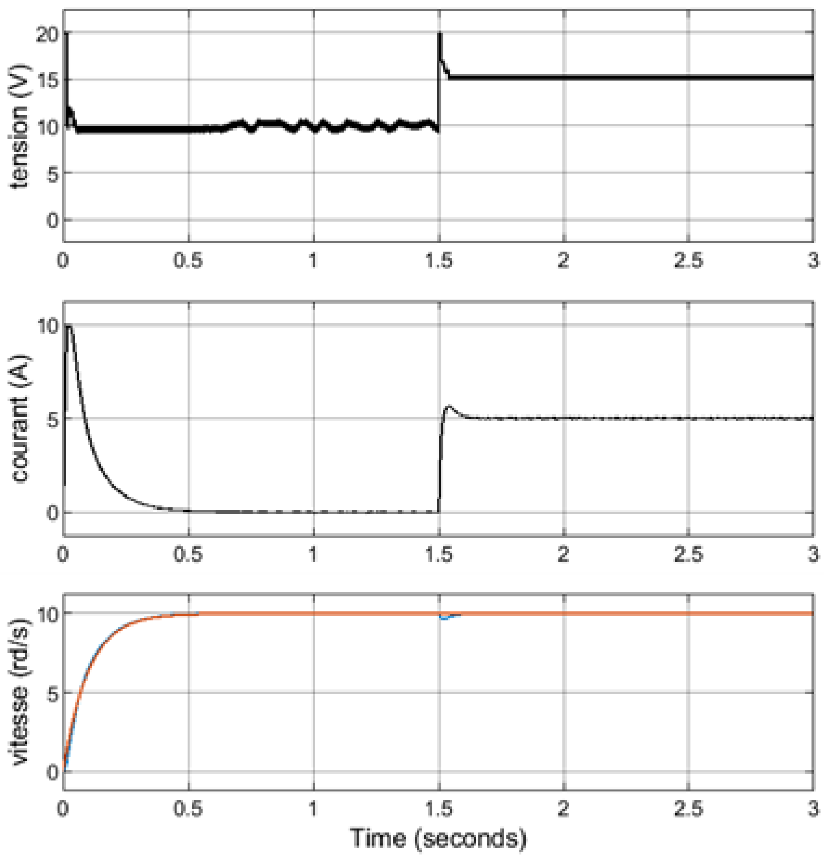


Figure 1.14 : DCM response with converter.

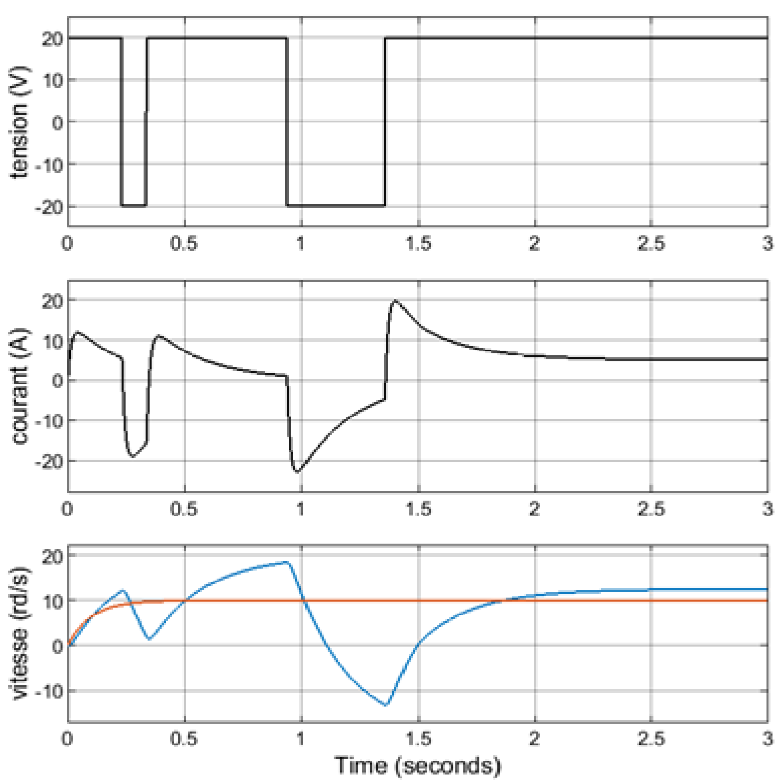


Figure 1.15 : DCM response with a change of resistance R and inertia J 50%

1.5 Conclusion

In this chapter, we presented the modeling of the DC motor . The open-loop simulation shows that under the effect of the load torque, the machine loses its speed. The introduction of a PI regulator in the control loop and Thanks to the integral action, it has been possible to eliminate the effects of the load torque and ensure the following of a desired speed. The application of cascade regulation (two PI regulators) eliminates unwanted oscillations in the transient state. This allows a better control of the machine and performance. On another side,The PI regulator is sensitive to parameter changes.

Chapter 2

SLIDING MODE CONTROL THEORY

2.1 Introduction

In this chapter, we introduce Sliding Mode Control (SMC) theory with clear examples, chattering phenomenon has been discussed and solutions how to solve it in different ways have been given.

2.2 Sliding Mode Control (SMC)

2.2.1 Introduction

Sliding mode control (SMC) is a discontinuous control technique introduced by the russian in the years 70 . The SMC method is designed to conduct the states of the system on a particular surface in the state space, called the sliding surface. Once the sliding surface is reached, the sliding mode control maintains the states of systems in the near of the sliding surface.

The sliding mode controller design is composed of two parts. The first part involves designing a sliding surface so that sliding motion meets the design specifications. The second is about the choice of a control law.

The SMC makes it possible to control the uncertainty of the systems whose parameters and perturbations are known ,approximately and bounded [4].

2.2.2 simple description

To approach the SMC in a more general framework, consider the nonlinear system of order n:

$$\dot{x} = f(\mathrm{x},\mathrm{t}) + g(\mathrm{x},\mathrm{t}).u + d(\mathrm{x},\mathrm{t})$$
$$y(\mathrm{x}) = \mathrm{h}(\mathrm{x},\mathrm{t}) \tag{2.1}$$

where y and u are the output and the input variable, and $x \in R^n$ is the state vector .

The purpose of the control is to make the output variable y follow a desired output y_{des} , it is necessary that the output error variable $e = y - y_{des}$ tends to zero.

Often the sliding surface $s = f(\mathrm{e}, \dot{e}, \ldots\ldots, \overset{r-1}{e})$ depends on the tracking error with a number of its derivative [4].

The function must be chosen so that its disappearance s=0, gives rise to a "stable" function. differential equation of which any solution e will eventually tend towards zero.

The most typical choice for the sliding surface is a linear combination of the type:

$$s = e + ce. \tag{2.2}$$

from the geometric point of view, the equation s=0 defines a surface in the errors's space, This is called the "sliding surface". System trajectories controlled are forced onto the sliding surface, along which the behavior of the system meets design specifications.

A typical shape for the sliding surface is as follows, which depends on a single scalar "p" parameter [4].

$$s = \left(\frac{d}{dt} + p\right)^{r-1} e \tag{2.3}$$

The choice of the positive parameter p is almost arbitrary, r being the relative degree of the output y of the system .

where: $e = y - y_d, \dot{e} = \dot{y} - \dot{y}_d, \ldots, \overset{r}{e} = \overset{r}{y} - \overset{r}{y}_d$

After the derivation of the sliding surface, we get:

$$\dot{s}(\mathrm{x}) = \frac{\partial s}{\partial x}\dot{x} = \frac{\partial s}{\partial x}\left[f(\mathrm{x}) + \mathrm{g}(\mathrm{x})\,\mathrm{u} + \mathrm{d}(\mathrm{x},\mathrm{t})\right] \\ = L_f s(\mathrm{x}) + \mathrm{L}_g\, s(\mathrm{x})\,\mathrm{u} + L_d s(\mathrm{x}) \tag{2.4}$$

Note: “L“ is the derivative of Lie

The desired output and its derivatives $y_d, \dot{y}_d, \ldots, y_d^{r}$ are given and they are finite.

Using the Lyapunov method provides a more accurate result. Either the function:

$$V = \frac{1}{2}s^2 \tag{2.5}$$

After the derivation of lyaponov function (2.5), we get the final expression of $\dot{V}(x)$ as:

$$\dot{V}(x) = s(x)\dot{s}(x) \\ = s(x)(L_f s(x) + L_g\, s(x)u + L_d\, s(x))$$

And the final linearizing control law is:

$$u = \frac{v - L_f s(\mathrm{x})}{L_g s(\mathrm{x})} \tag{2.6}$$

Which implies

$$\dot{V}(\mathrm{x}) = s(x)(v + L_d s(x))$$

If the disturbance is assumed to be bounded by:

$$\left|L_d s(\mathrm{x})\right| \le p(x)$$

It follows this:

$$\dot{V}(\mathrm{x}) \le s(x)v + \left|s(x)\right| p(x)$$

If the auxiliary command is chosen as: $v = -(p(x) + 2)sign(s(x))$

we get: $\dot{V}(\mathrm{x}) \le -2\left|s(x)\right|$, this implies a global stability.

An illustrative example:

Consider the system:

$$\ddot{x} = u + d(x,t)$$

where d(x ,t) is the perturbation bounded by $|d(x,t)| \leq p(x)$ where p(x) is a known function

with state variables $x = x_1, \dot{x} = x_2$ we have

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u + d(x,t) \end{aligned} \tag{2.7}$$

If we seek to force the states of the system (2.7) to converge towards the equilibrium point (0,0),we define the sliding function as:

$$s(x) = x_2 + cx_1 \tag{2.8}$$

with the constant c>0

The sliding function dynamics (2.8) is then :

$$\dot{s}(x) = \dot{x}_2 + c\dot{x}_1 = cx_2 + u + d(x,t)$$

To determine the SMC law that ensures global stability, we define

Lyapunov's function V as:

$$V(x) = \frac{1}{2}s^2(x) \tag{2.9}$$

Note that V(x) is globally positive definite and has no radial bounds.

The derivative of the Lyapunov function (2.9) is given by:

$$\dot{V}(x) = s(x)\dot{s}(x) = s(x)(cx_2 + u + d(x,t))$$

We define the control law:

$$u = -cx_2 + v \tag{2.10}$$

So we have:

$$\dot{V}(x) = s(x)(v + d(x,t)) \leq s(x)v + |s(x)|\, p(x)$$

The control law term v is defined as:

$$v = -k\,sign(s(x)), k > 0 \qquad (2.11)$$

We have:

$$\dot{V}(x) \leq -|s(x)|\,(k - p(x))$$

We then choose $k = p(x) + \frac{2}{\sqrt{2}}$, which produces the result: $\dot{V}(x) \leq -\frac{2}{\sqrt{2}}|s(x)|$

That is : $\dot{V}(x) \leq -2\sqrt{V}$

Which gives after integration: $\sqrt{V} \leq -\frac{\alpha}{2}t + \sqrt{V(0)}$

The convergence time is bounded by:

$$t_r \leq \frac{2\sqrt{V(0)}}{\alpha} = \frac{\sqrt{2}}{\alpha}|s(0)|$$

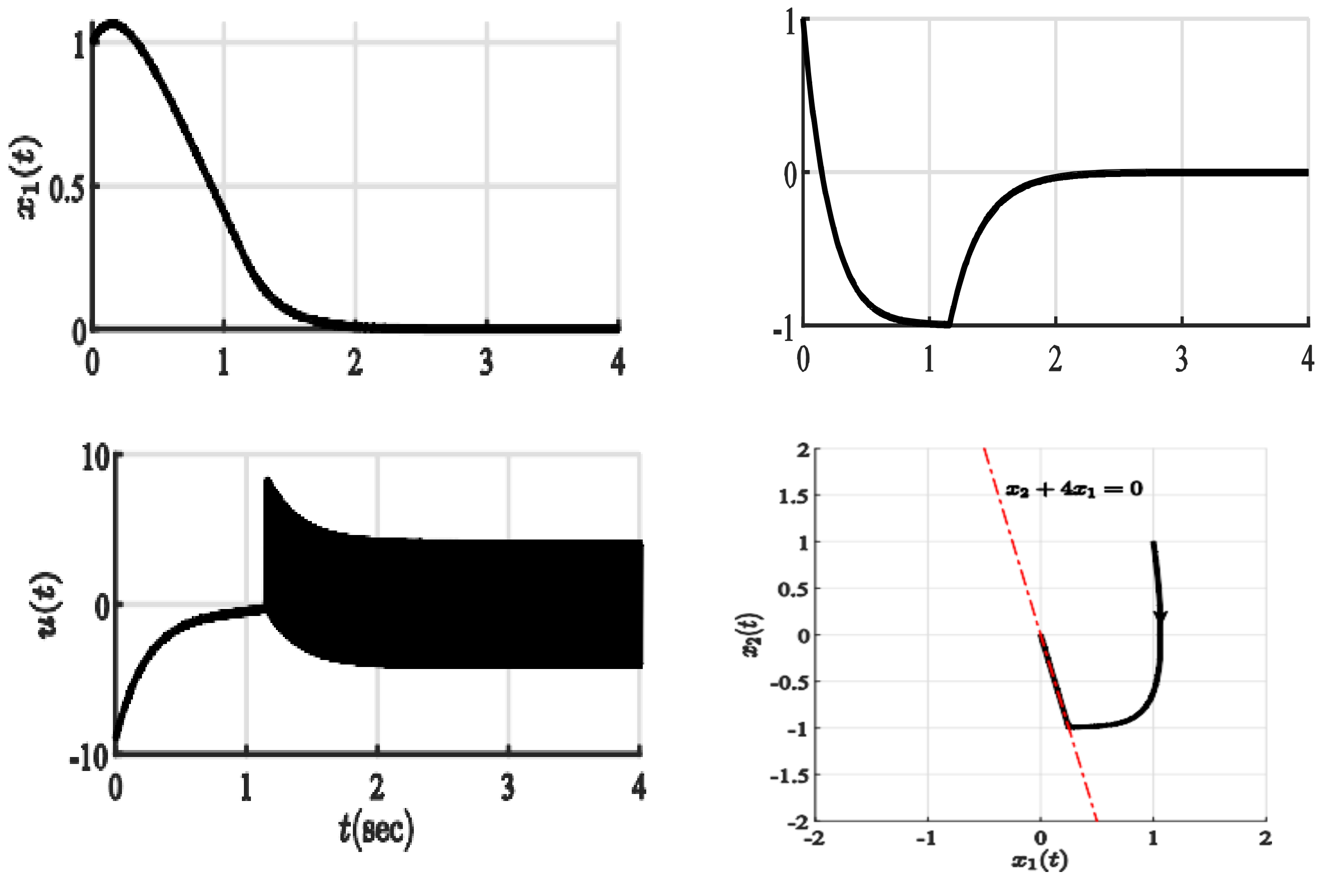


Figure 2.1: a) response temporal , b) sliding mode

2.2.3 Chattering

In reality, the switching frequency of the SMC is not infinite as suppose in the ideal sliding mode of figure (2.1). as the switching frequency is over, a switching phenomenon (called chattering) appears on the response of the system figure (2.2). This phenomenon Is undesirable in practice since it excites the modes in high frequencies of the system and generate a big oscillation in the control action . To reduce or eliminate the chattering phenomenon, several approaches are propose [5].

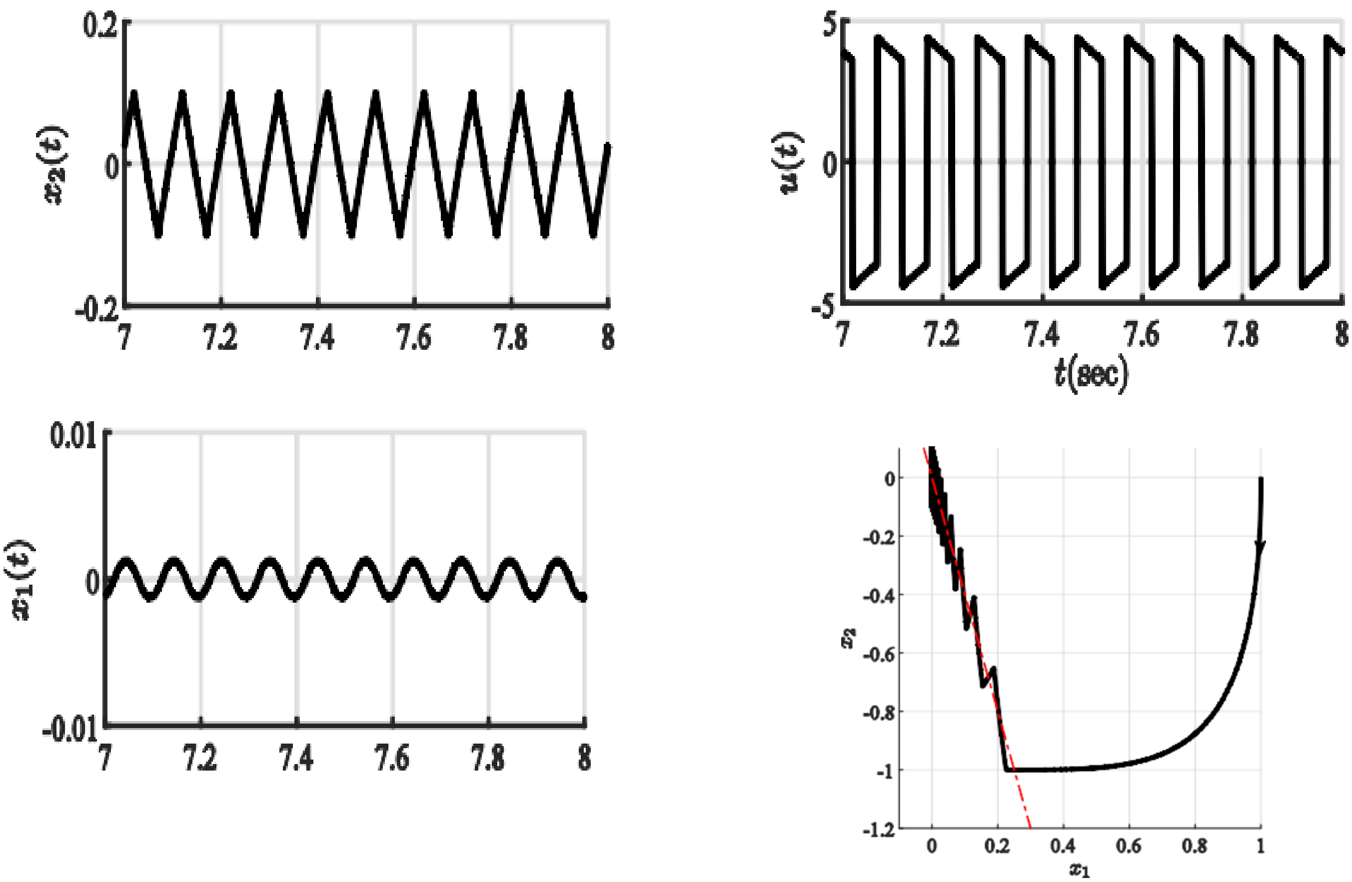


Figure 2.2: chattering phenomenon. a) temporal response, b) sliding mode.

In order to solve the above problem (called "chattering phenomenon"), there are two proposed methods.

2.2.3.1 Quasi-sliding mode

In this approach, the sign function is replaced by a smoother approximation (continue), such that $s(x)/(\varepsilon+s(x)), \varepsilon>0$, or the saturation $sat(s(x)/\varepsilon), \varepsilon>0$ We notice that the limit of the two functions, for $\varepsilon \rightarrow 0$ tends to the function $sign(\mathrm{s(x)})$

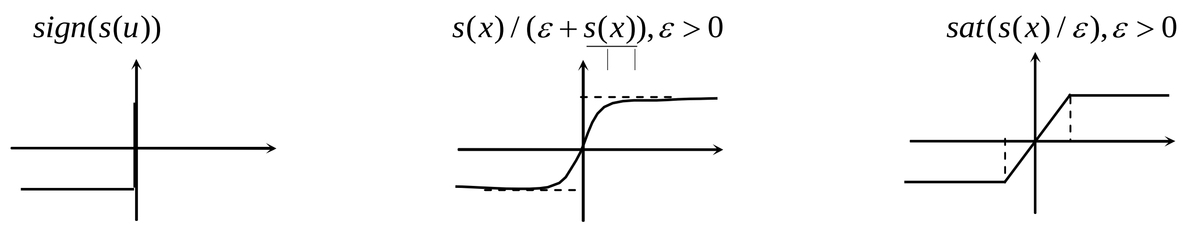


Figure 2.3: Functions of smoothing from sign .

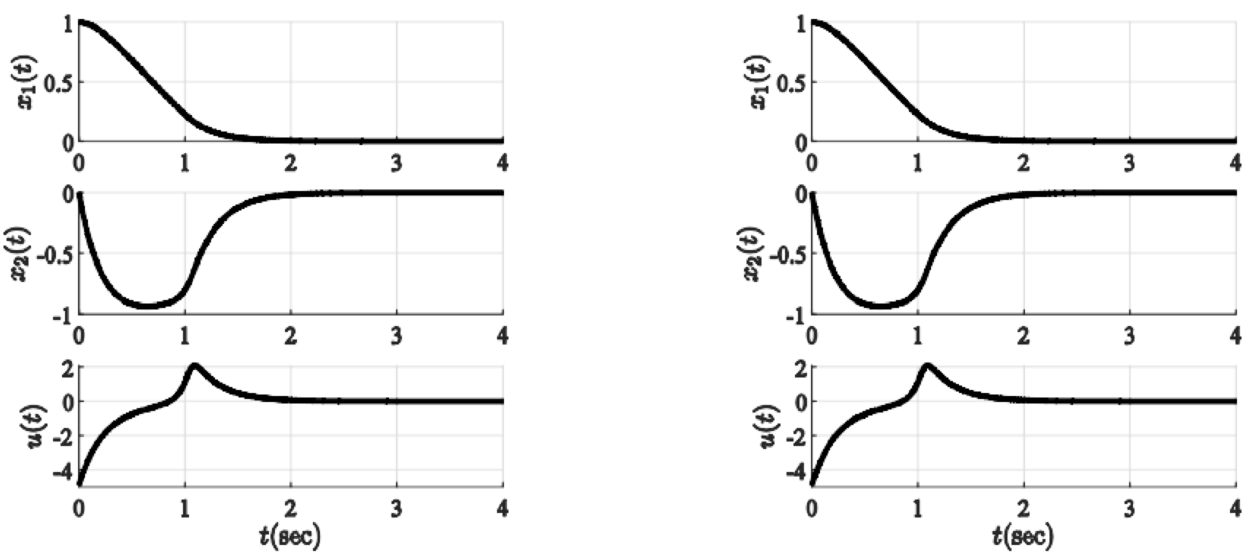


Figure 2.4: smoothing from sign: has) $s(x)/(s(x)+0.1)$, b) sat $(s(x)/0.1)$.

2.2.3.2 Asymptotic sliding mode

In this method the proposed control law is the integral of the discontinuous control law (high frequency). This produces an attenuation of the effects of switching and chattering reduction.

An illustrative example:

The system in this configuration is rewritten as :

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u + d(x,t)$$
$$\dot{u} = v$$

We will assume that $|d| \le p(x), |\dot{d}| \le \sigma_0(x)$ are given.

The switching variable is : $s(x) = x_2 + cx_1$

we defines the auxiliary switching variable: $\sigma(x) = \dot{s}(x) + c_2 s(x), c2 > 0$

Whether we design a law of order v who allow having $\sigma(x) \to 0$ then a sliding mode is established, such that $\dot{s}(x) + c_2 s(x) = 0$

Hence it follows that asymptotically $s(x), \dot{s}(x) \to 0$ $x_1, x_2 \to 0$ In this case, we have a Asymptotic $\sigma(x)$ sliding mode.

the derivation of $\sigma(x)$ product:

$$\dot{\sigma}(x) = \ddot{s}(x) + c_2 \dot{s}(x) = x_2 + c_1 \ddot{x}_1 + c_2 \left(\dot{x}_2 + c_1 \dot{x}_1 \right)$$
$$= \dot{u} + \dot{d} + c_1(u + d) + c_2(u + d + c_1 x_2)$$
$$= v + (c_1 + c_2)u + (c_1 + c_2)d + c_2 c_1 x_1 + d$$

the derivative of the Lyapunov function $V(x) = \frac{1}{2} s^2(x)$ is:

$$\dot{V}(x) = \sigma(x)\dot{\sigma}(x) = \sigma(x)(v + (c_1 + c_2)u + (c_1 + c_2)d + c_2 c_1 x_2 + \dot{d})$$

We defines the auxiliary action like :

$$v = -(c_1 + c_2)u - c_2 c_1 x_2 + v_1 \qquad (2.12)$$

Which give:

$$\dot{V}(x) = \sigma(x)(v_1 + (c_1 + c_2)d + \dot{d} \le \sigma(x)v_1 + |\sigma(x)|((c_1 + c_2)p(x) + p_0(x))$$

We selected the discontinuous term as :

$$v_1 = -k(x) sign(\sigma(x))$$

Which will give:

$$\dot{V}(x) \leq -k(x)|\sigma(x)| + |\sigma(x)|((c_1 + c_2\ )p(x) + p_0\ (x)) = -|\sigma(x)|(k(x) - ((c_1 + c_2\ )p(x) + p0\ (x)))$$

If we choose :

$$k(x) = (c_1 + c_2\ )p(x) + p_0(x) + \frac{\alpha}{\sqrt{2}}$$

We get :

$$\dot{V}(x) \leq -\frac{2}{\sqrt{2}}|\sigma(\mathrm{x})|$$

Finally the law control is defined by :

$$v = -(c_1 + c_2)u - c_2 c_1 x_2 - k(x) sign(\sigma(x)) \tag{2.13}$$

$$u = \int v \tag{2.14}$$

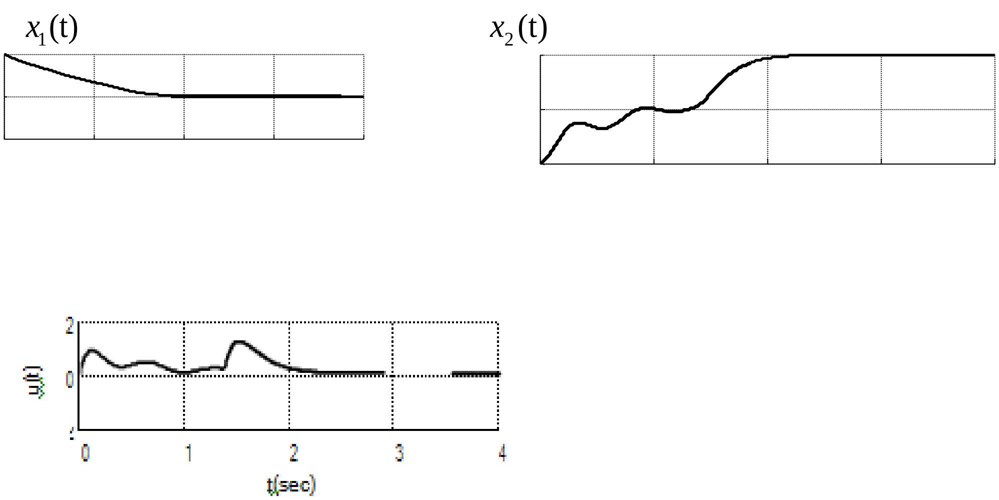


Figure 2.5: chattering phenomenon. a) temporal response, b) sliding mode.

2.3 Conclusion

In the second chapter, we discussed the sliding mode control theory deeply ,and the advantages and disadvantages that the technique brings like the chattering phenomena and how to solve it in different ways.

The next chapter will be the study of the application of the smc to DC motors.

Chapter 3

DC MOTOR SPEED CONTROL BY SLIDING MODE

3.1 introduction :

In this chapter, we will introduce the principle of sliding mode control of a DC motor based On three different models like cascade, reduced and complete model. An acceleration observer is designed to estimate the acceleration of the DC motor.

3.2 control scheme

In real applications, the DC motor is not directly controlled using the u input but it is controlled using a converter which uses PWM as a generator.

The diagram in Figures 3.1 and 3.2 shows how the DC machine is practically controlled And equivalent diagram of the power converter with PWM control.

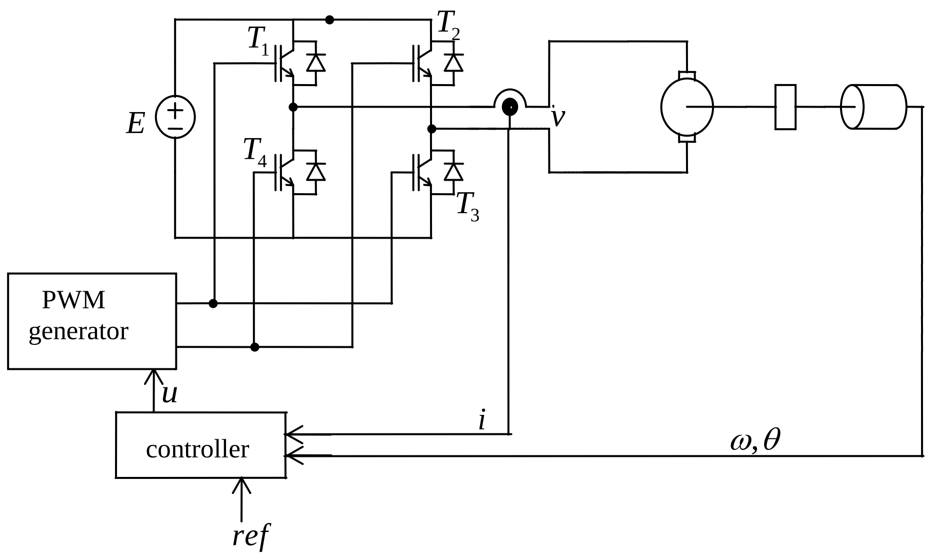


Figure 3.1: Control diagram.

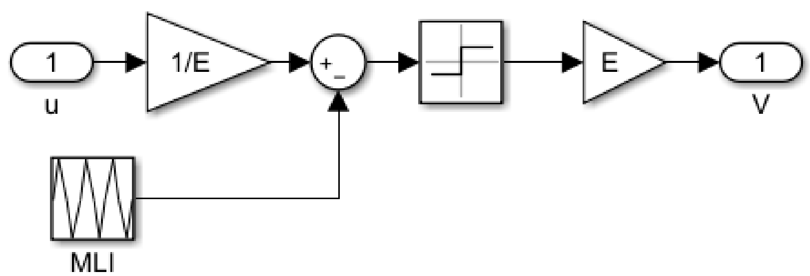


Figure 3.2: Equivalent diagram of the power converter with PWM.

3.3 Speed control.

For speed control of a DC motor, we have a multi-diagram control that uses multiple loops as a cascade, and a single loop as reduced and complete model.

3.3.1 cascade SMC

For controlling the speed of a DC motor, a control structure in cascade is generally preferred, with an internal current control loop and an external speed control loop. The control input u can be continuous or discontinuous, depending on the output power of the DC motor.

For a low power system, continuous control can be selected. For a high power system, discontinuous regulation (in the form of PWM) should be used [13].

In this chapter, we will focus on discontinuous control, because control discontinuous DC motors are universal in that they can be used for both low and high power systems.

Figure 3.3 shows the typical control structure of a drive system based on a DC motor[13].

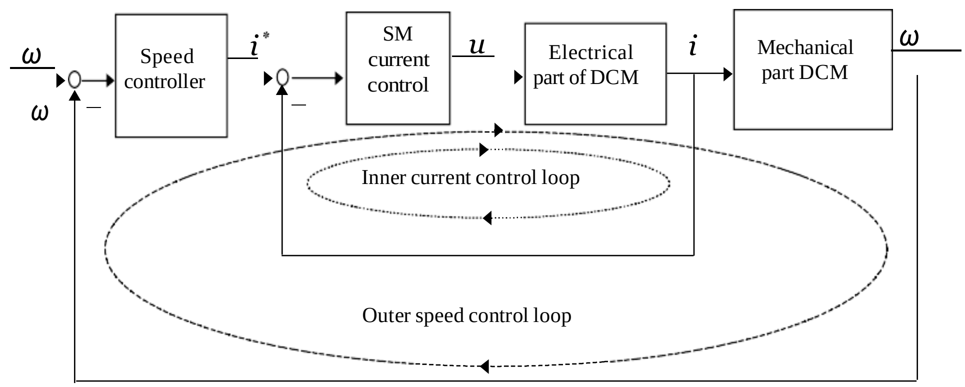


Figure 3.3 : Cascade control structure of DC motors.

Let's define the two sliding functions for speed and current as::

$$\begin{aligned} s_\omega &= \omega - \omega_{ref} \\ s_I &= I - I_{ref} \end{aligned} \tag{3.1}$$

And the Lyapunov function as:

$$V = \frac{1}{2} J s_\omega^2 + \frac{1}{2} L s_I^2 \tag{3.2}$$

Note that V(x) is globally positive definite and has no radial bounds.

the derivative of (3.2) is:

$$\begin{aligned} \dot{V} &= s_\omega J \dot{s}_\omega + s_I L \dot{s}_I \\ &= s_\omega \left(J\dot{\omega} - J\dot{\omega}_{ref} \right) + s_I \left(L\dot{I} - L\dot{I}_{ref} \right) \\ &= s_\omega \left(-b\omega + rk_m I - C_r - J\dot{\omega}_{ref} \right) + s_I \left(-RI - rk_b\omega + v - L\dot{I}_{ref} \right) \end{aligned} \tag{3.3}$$

Let's define the machine's current as: $I = s_I + I_{ref}$ is substitutions in $\dot{V}$,

then the final expression of $\dot{V}$ is :

$$\begin{aligned} \dot{V} &= s_\omega \left(-b\omega + rk_m I_{ref} + k_m s_I - C_r - J\dot{\omega}_{ref} \right) + s_I \left(-RI - rk_b\omega + v - L\dot{I}_{ref} \right) \\ &= s_\omega \left(-b\omega + rk_m I_{ref} - C_r - J\dot{\omega}_{ref} \right) + s_I \left(-RI - -rk_b\omega + rk_m s_\omega + v - L\dot{I}_{ref} \right) \end{aligned} \tag{3.4}$$

To get $s_\omega, s_I \rightarrow 0$, It's necessary that $\dot{V} \le -\alpha_\omega s_\omega^2 - \alpha_I s_I^2$ being " negative semi-definite ", and to achieve this, we must choose the control input v and the reference current I_{ref} as:

$$I_{ref} = \frac{1}{rk_m}\left(-\alpha_\omega s_\omega - k_1 sign\left(s_\omega\right) + b\omega\right) \tag{3.5}$$

$$v = -\alpha_I s_I - k_2 sign\left(s_I\right) + RI + rk_b\omega - rk_m s_\omega \tag{3.6}$$

With :

$k_1 \geq \max\left|C_r + J\,\dot{\omega}_{ref}\right| \quad k_2 \geq \max\left|L\,\dot{I}_{ref}\right|$

Figure 3.4 shows the Simulink diagram of the cascade SMC. The blocks of the speed loop and current loop are shown in Figures 3.5 and 3.6 respectively.

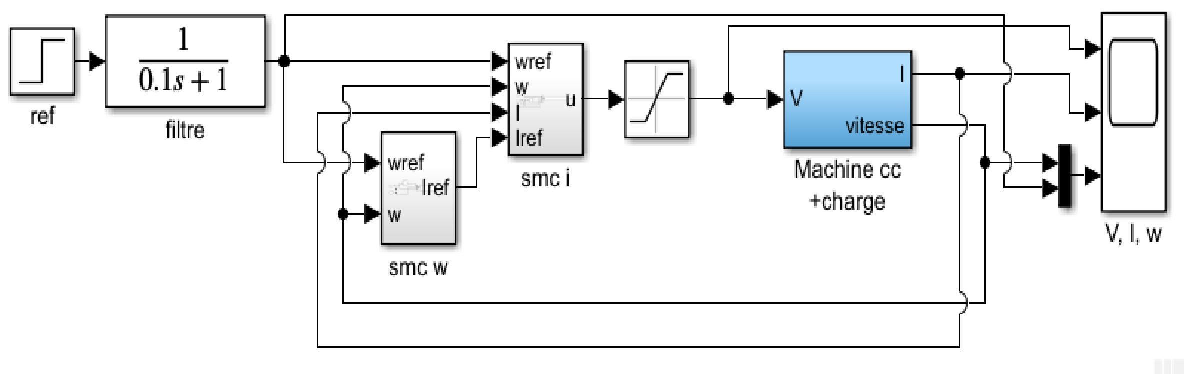


Figure 3.4 : cascade control Simulink diagram.

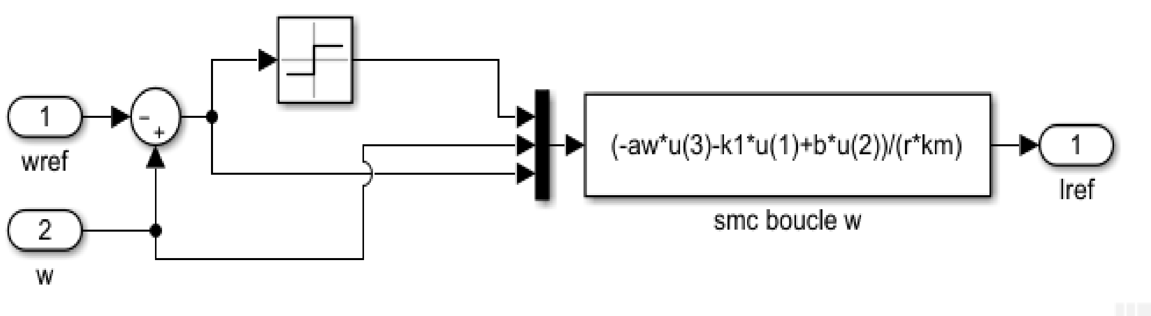


Figure 3.5 : Simulink diagram of reference current system I_{ref}

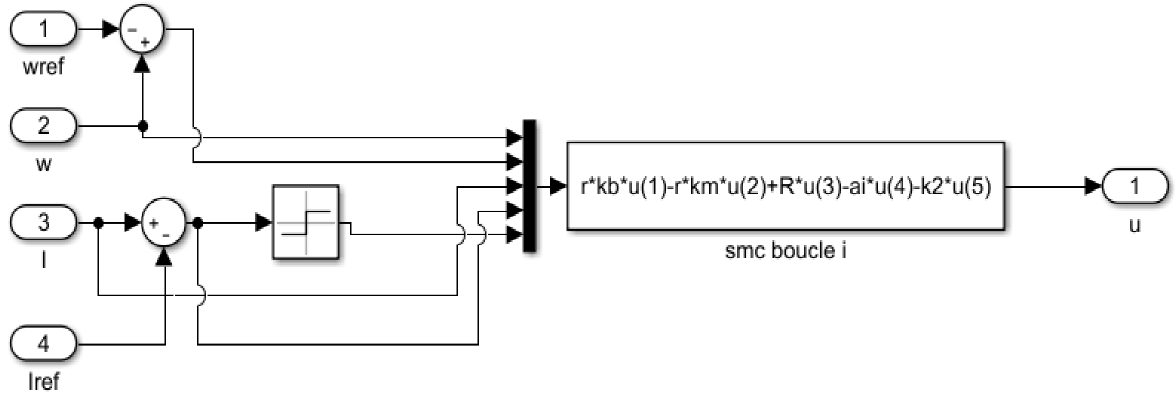


Figure 3.6 : Simulink control system diagram u

3.3.2 Simulation results

SMC control is calculated with parameters $k_1 = 5,\ \alpha_w = 0.5,\ k_2 = 0.5,\ \alpha_I = 5$,fc=25000

The performance of SMC without and with converter is shown in Figures 3.7- 3.8- 3.9 with nominal parameters. The effect of parameter variation on the SMC is shown in Figure-3.10. The use of the saturation function in the calculation of the SMC instead of the sign function has the role of eliminating the chattering problem as shown in Figures 3.7 and 3.8. The simulations show that the SMC perfectly follow the setpoint and eliminates the effect of parametric variations of the system (figure 3.10).

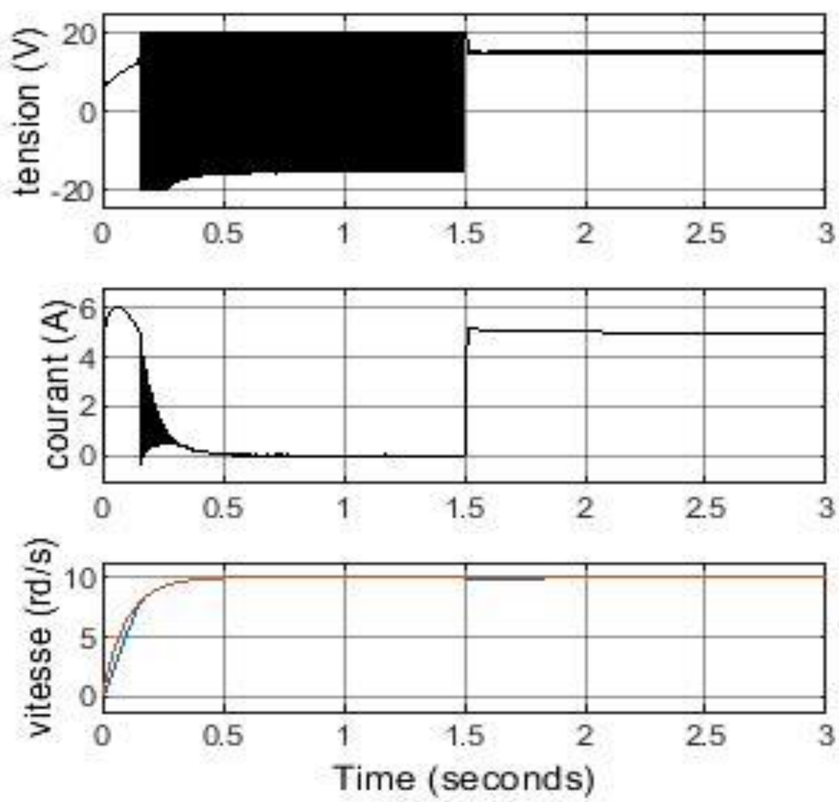


Figure 3.7 : system’s response with a sign function.

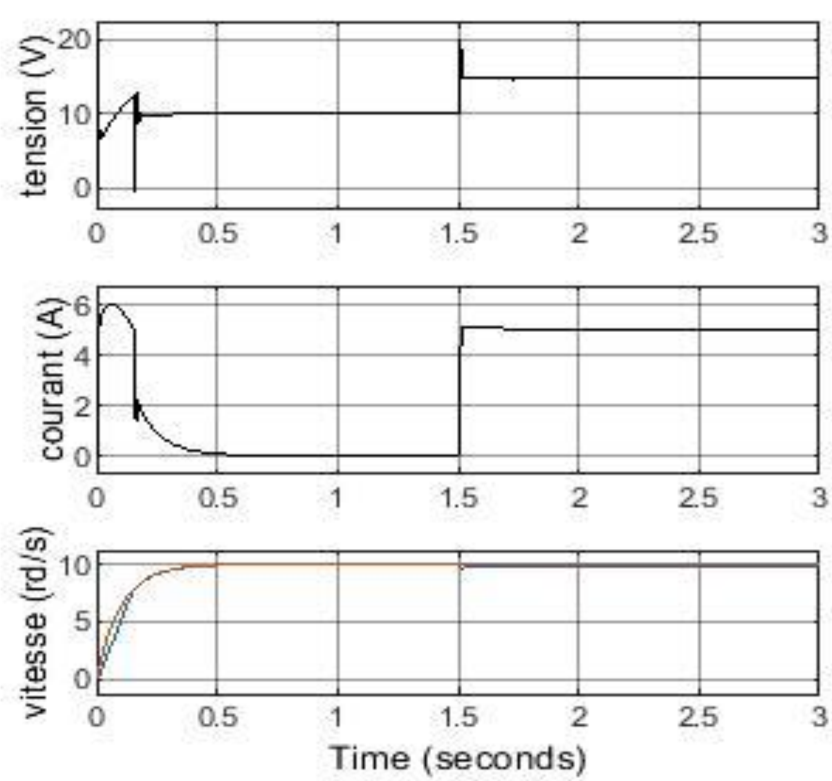


Figure 3.8 : system’s response with a saturation function.

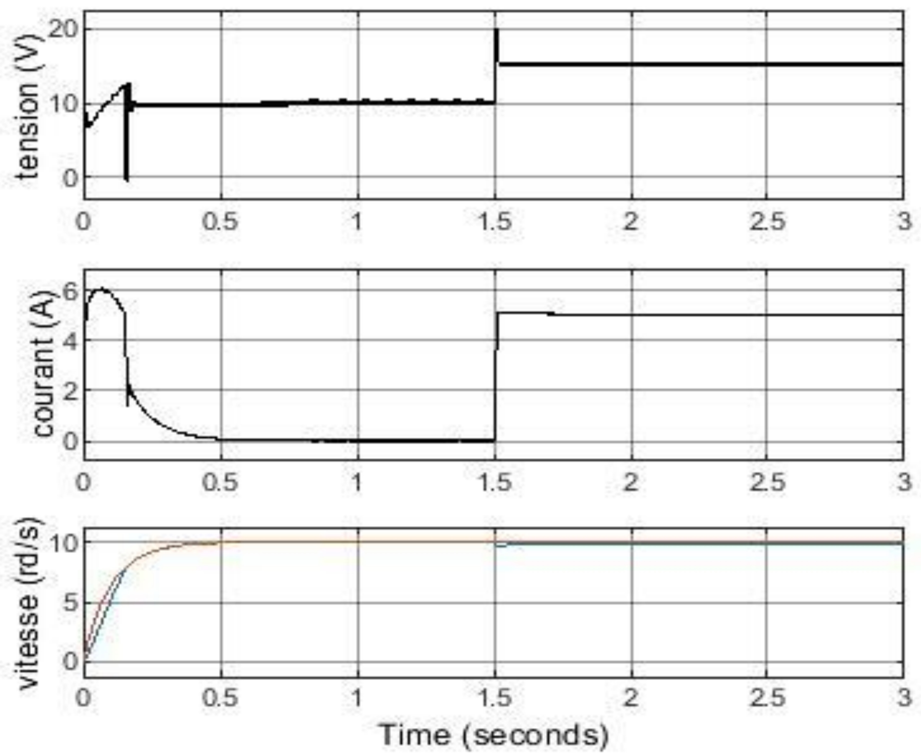


Figure 3.9 : system’s response with a converter.

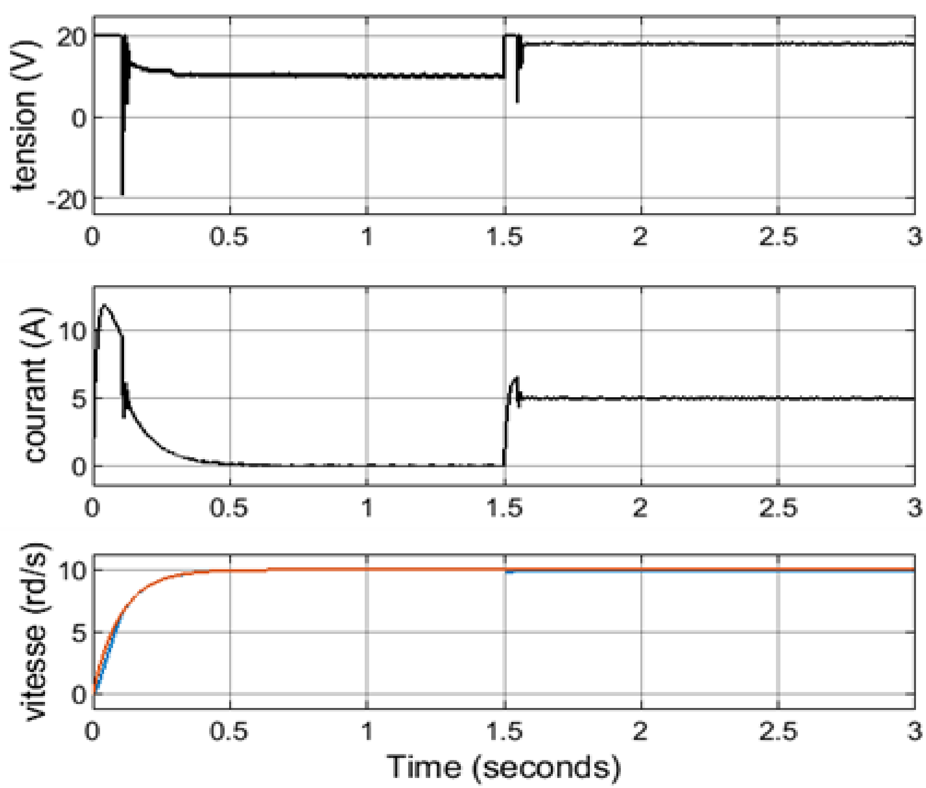


Figure 3.10 : system’s response with a converter and change of resistance R and inertia J with 50%.

3.4 SMC with a single loop

In this new control structure, we propose another control approach based on the sliding mode control principle to follow a given reference speed.

3.4.1 Control based on reduced model

Consider the following complete model

$$L\dot{I} = -RI - rk_b\omega + v$$
$$J\dot{\omega} = -b\omega + rk_m I - C_r \tag{3.7}$$

If we neglect and b, we get:

$$I = \frac{v - k_b\omega}{R} \tag{3.8}$$

$$J\dot{\omega} = rk_m I - C_r \tag{3.9}$$

the reduced model of the DC motor is defined as:

$$JR\dot{\omega} + r^2 k_m k_b \omega = rk_m v - RC_r \tag{3.10}$$

Equation (3.10) is a reduced (first order) model of the DC motor system and can be used to control speed without involving knowledge of current and acceleration.

ω_{ref} is reference speed and $e = \omega - \omega_{ref}$ speed tracking error.

the sliding function is designed as:

$$s = e + \lambda\int e \tag{3.11}$$

and lyapunov's function as:

$$V = \frac{1}{2}JRs^2 \tag{3.12}$$

The derivative of Lyapunov's function $\dot{V}$ is :

$$\begin{aligned} \dot{V} &= sJR\dot{s} = sJR(\dot{e} + \lambda e) \\ &= s\left(JR\dot{\omega} - JR\dot{\omega}_{ref} + JR\lambda e\right) \\ &= s\left(-r^2 k_m k_b \omega + rk_m v + JR\lambda e - RC_r - JR\dot{\omega}_{ref}\right) \end{aligned} \tag{3.13}$$

so that the error $e \to 0$ it's necessary that $\dot{V} \le -\alpha s^2$ being " negative semi-definite ", and to achieve this, we must choose the control input v as:

$$v = \frac{1}{rk_m}\left(-\alpha s - ksign(s) + r^2 k_m k_b \omega - JR\lambda e + JR\dot{\omega}_{ref}\right) \tag{3.14}$$

with $k \ge \max|RC_r|$

Figures 3.11-3.12 show the Simulink diagram of the SMC based on reduced model and the control block v .

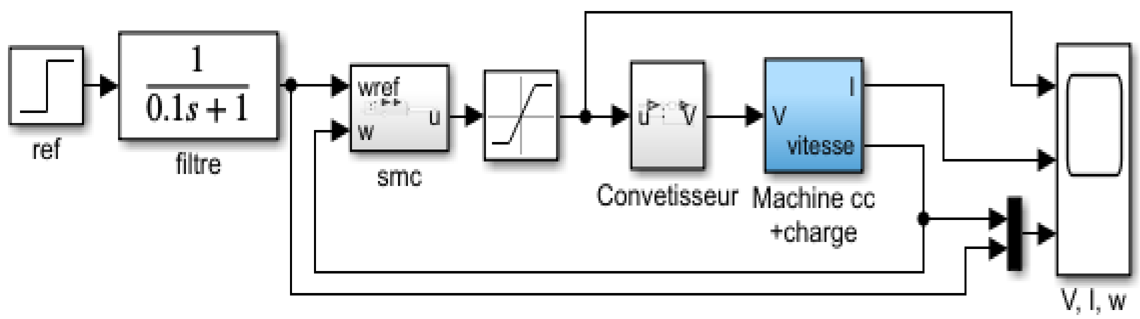


Figure 3.11: simulink diagram of reduced model method .

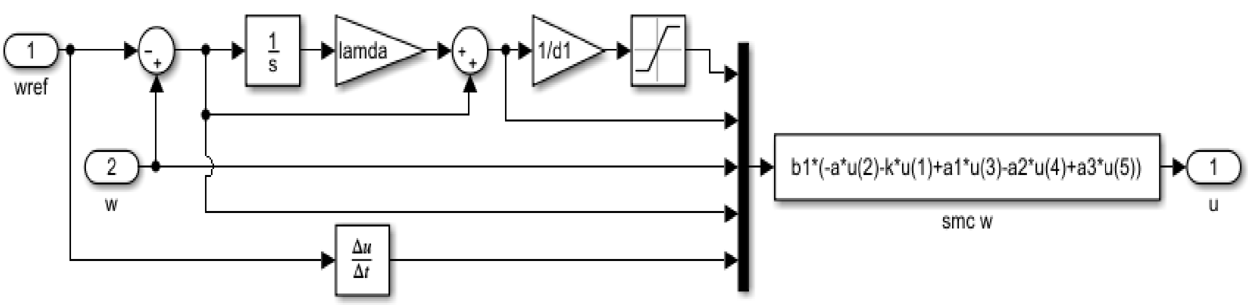


Figure 3.12: Simulink diagram of control system u .

3.4.1.1. Simulation results

The SMC is calculated with the parameters $k = 5,\ \alpha = 2,\ \lambda = 0.5$, Figure 3.13 shows the simulation result of the speed control with a single loop and a reduced model.

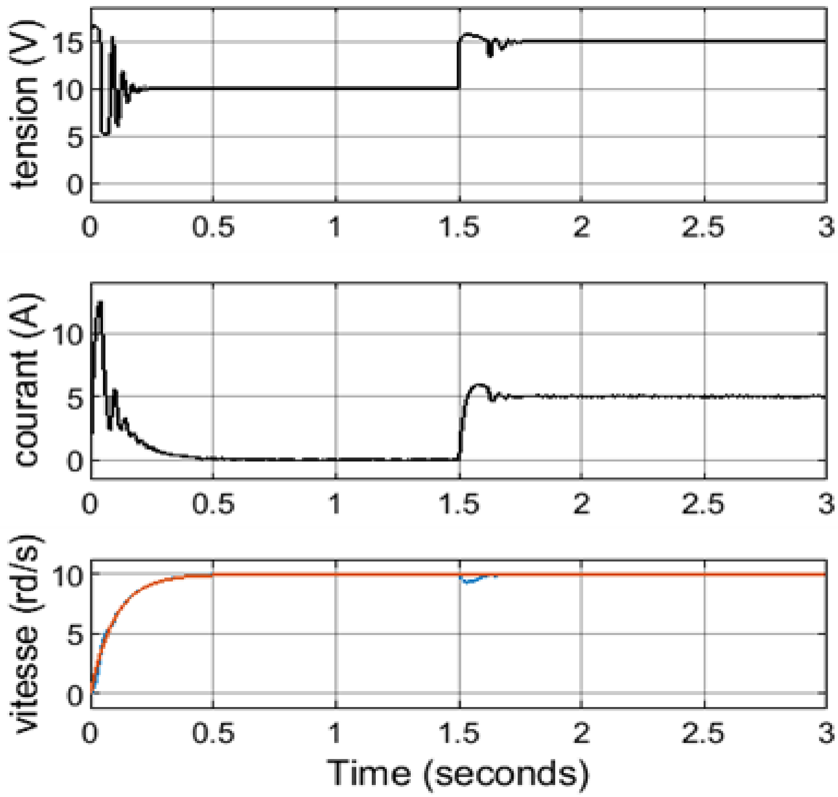


Figure 3.13 : single-loop dc motor smc based on reduced model.

3.4.2 Control based on complete model

Consider the following complete model

$$LsI(s) = -RI(s) - rk_b\omega(s) + v(s)$$
$$Js\omega(s) = -b\omega(s) + rk_m I(s) - C_r \tag{3.15}$$

Solving I from equation (3.15) gives:

$$I(s) = \frac{v(s) - rk_b\omega(s)}{(Ls+R)} \tag{3.16}$$

Substituting Equation (3.16) into Equation (3.15) gives the complete model of dc motor :

$$\omega(s) = \frac{rk_m v(s) - (Ls+R)C_r}{(Js+b)(Ls+R) + r^2 k_b k_m}$$

$$\omega(s) = \frac{rk_m v(s) - (Ls+R)C_r}{J\,L\,s^2 + (J\,R + Lb)s + r^2 k_b k_m + Rb} \tag{3.17}$$

We can write equation (3.17) as:

$$\omega(s)=\frac{\frac{rk_m}{JL}v(s)-\frac{(Ls+R)}{JL}C_r}{s^2+a_1 s+a_2} \tag{3.18}$$

With $a_1=\frac{R}{L}+\frac{b}{J}$, $a_2=\frac{r^2k_bk_m+Rb}{JL}$

Similarly, we rewrite equation (3.18) as:

$$s^2\,\omega(s)+a_1\,s\,\omega(s)+a_2\omega(s)=\frac{rk_m}{JL}v(s)-\frac{(Ls+R)}{JL}C_r$$

Switching to time gives the following differential equation:

$$\ddot{\omega}+a_1\dot{\omega}+a_2\omega=u-d \tag{3.19}$$

With $u=\frac{rk_m}{JL}v,\ d=\frac{L\dot{C}_r+RC_r}{JL}$

We define $x_1=\omega,\ x_2=\dot{\omega}$

We get the following state model:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=-a_2x_1-a_1x_2+u-d\end{aligned} \tag{3.20}$$

the same here let's define sliding function and speed tracking error as:

$$e=\omega-\omega_{ref}$$

$$s=\dot{e}+\lambda e=\left(\dot{\omega}-\dot{\omega}_{ref}\right)+\lambda\left(\omega-\omega_{ref}\right) \tag{3.21}$$

and lyapunov's function as:

$$V=\frac{1}{2}s^2 \tag{3.22}$$

The derivative of Lyapunov's function gives:

$$\begin{aligned}\dot{V}=s\dot{s}&=s\left(\dot{x}_2-\ddot{\omega}_{ref}+\lambda\left(x_2-\dot{\omega}_{ref}\right)\right)\\ &=s\left(-a_2x_1+(\lambda-a_1)x_2-\ddot{\omega}_{ref}+\lambda\,\dot{\omega}_{ref}+u-d\right)\end{aligned}$$

so that the error $e\to 0$ it's neccesary that $\dot{V}\le-\alpha s^2$ being " negative semi-definite ", and to achieve this, we must choose the control input u as:

$$u=-\alpha s-ksign(s)+a_2x_1-(\lambda-a_1)x_2+\ddot{\omega}_{ref}+\lambda\,\dot{\omega}_{ref} \tag{3.23}$$

with $k \geq \max\left| L\dot{C}_r + RC_r \right|$

Then the final control is represented as:

$$v = \frac{JL}{rk_m}u \tag{3.24}$$

Figures 3.14-3.15 show the Simulink diagram of the SMC based on complete model and the controller block v.

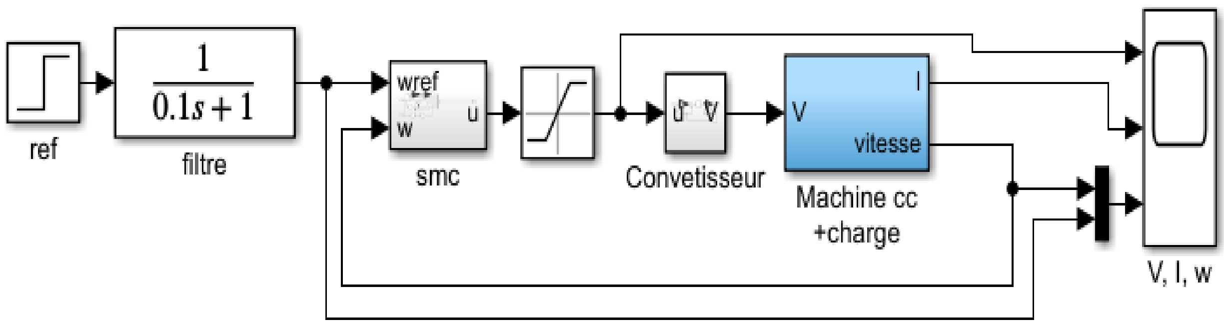


Figure 3.14: dc motor control based on complete model.

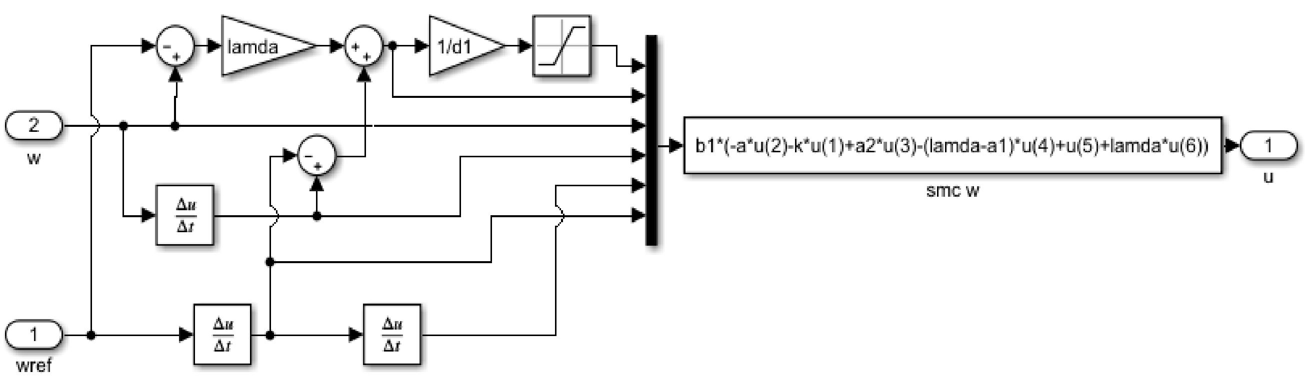


Figure 3.15: Simulink diagram of control system u.

3.4.2.1 Simulation results

The SMC is calculated with the parameters $k = 100$, $\alpha = 150$, $\lambda = 150$, Figure 3.16 shows the simulation result of speed control with single loop and based on complete model. This control method has eliminated the oscillations in the transient state.

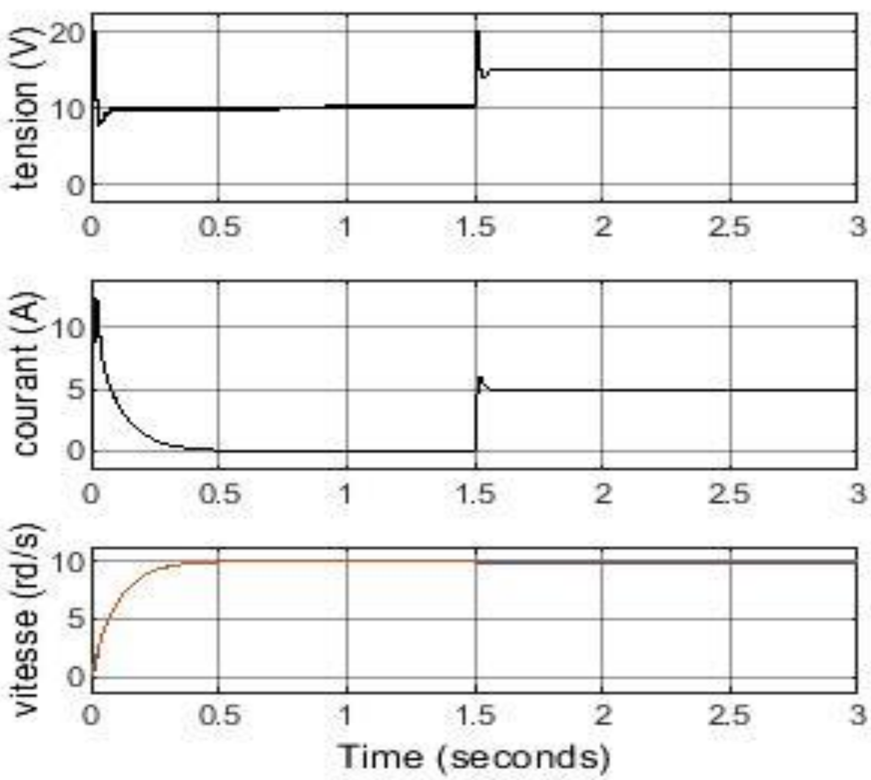


Figure 3.16 : dc-motor’s response based on complete model.

3.5 Speed control with acceleration observer

For the implementation of the command given in by equations (3.23) and (3.24) ,angular acceleration and speed are needed to calculate the control action u in Equation (3.23) . However, in practice, the angular acceleration is not measured.

Instead, motor current i and speed ω are normally available. For the measure of acceleration, an observer had to estimate the true value of the DC motor’s acceleration.

3.5.1 Observer design

Consider the following DC motor model:

$$\dot{I} = -\frac{R}{L}I - \frac{rk_b}{L}\omega + \frac{1}{L}v$$
$$\dot{\omega} = -\frac{b}{J}\omega + \frac{rk_m}{J}I - \frac{C_r}{J} \tag{3.25}$$

assume that the load torque is constant $C_r = cst$ and $b \approx 0$ so the model (3.25) becomes :

$$\dot{\omega} = \frac{rk_m}{J}I - \frac{C_r}{J}$$

Let's define

$$x_1 = \omega$$

$$x_2 = \dot{\omega}$$

We can rewrite the DC motor model in terms of x_1 and x_2 :

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \ddot{\omega} = \frac{rk_m}{J}\dot{I} = \frac{rk_m}{J}\left(-\frac{R}{L}I - \frac{rk_b}{L}\omega + \frac{1}{L}v\right)$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{r^2k_mk_b}{JL}x_1 - \frac{Rrk_m}{JL}I + \frac{rk_m}{JL}v \tag{3.26}$$

Finally, the mathematical model in the state space of the DC motor (3.25)

in terms of x_1 and x_2 is represented as:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -a_1x_1 - a_2I + a_3v \tag{3.27}$$

With $a_1 = \frac{r^2k_mk_b}{JL}$, $a_2 = \frac{Rrk_m}{JL}$,and $a_3 = \frac{rk_m}{JL}$

The model of the observer in state space is represented by:

$$\dot{\hat{x}}_1 = \hat{x}_2 + l_1\left(x_1 - \hat{x}_1\right)$$
$$\dot{\hat{x}}_2 = -a_1\hat{x}_1 - a_2I + a_3v + l_2\left(x_1 - \hat{x}_1\right) \tag{3.28}$$

where l_1 and l_2 are the observer's gains.

The errors between the actual value of speed and acceleration and their estimated values are :

$$e_1 = x_1 - \hat{x}_1$$
$$e_2 = x_2 - \hat{x}_2$$

where $\hat{x}_1$ is the estimated speed and $\hat{x}_2$ is the estimated acceleration .

The error's state space model is represented by:

$$\begin{aligned} \dot{e}_1 &= e_2 - l_1 e_1 \\ \dot{e}_2 &= -(a_1 + l_2) e_1 \end{aligned} \tag{3.29}$$

Equation (3.29) can be written as

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -l_1 & 1 \\ -(a_1 + l_2) & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{3.30}$$

We can write the characteristic polynomial of the system (3.30) by a 2 order characteristic equation as:

$$s^2 + l_1 s + (a_1 + l_2) = s^2 + 2\zeta_0 \omega_0 s + \omega_0^2 \tag{3.31}$$

By equating the two terms of (3.31), we obtain:

$$\begin{aligned} l_1 &= 2\zeta_0 \omega_0 \\ l_2 &= \omega_0^2 - a_1 \end{aligned} \tag{3.32}$$

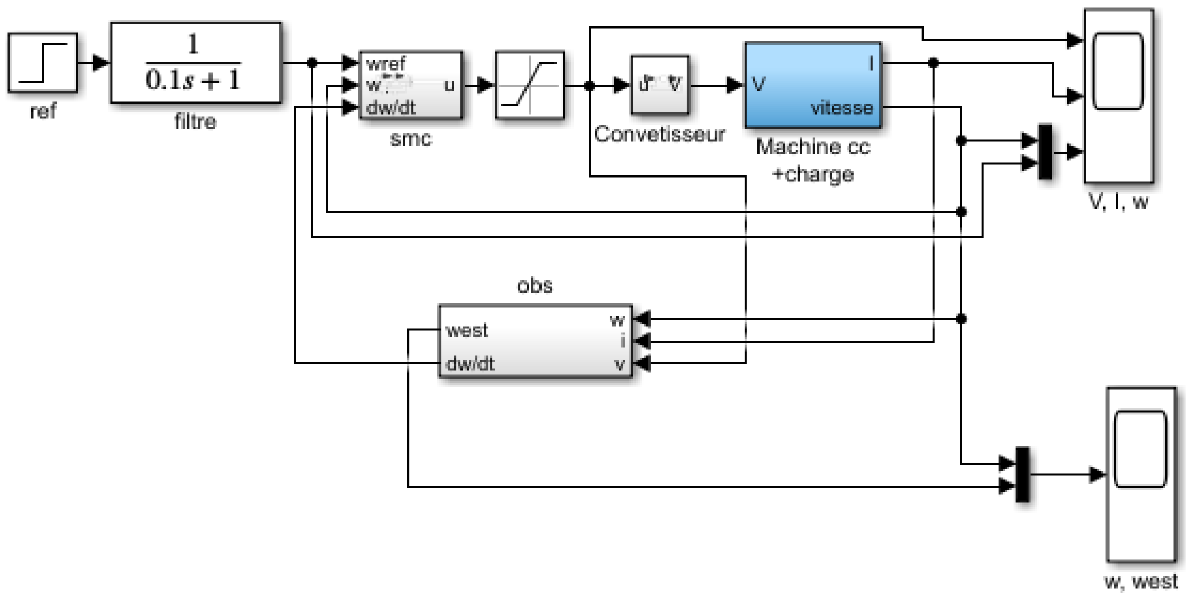


Figure 3.17: Simulink diagram of SMC with observer

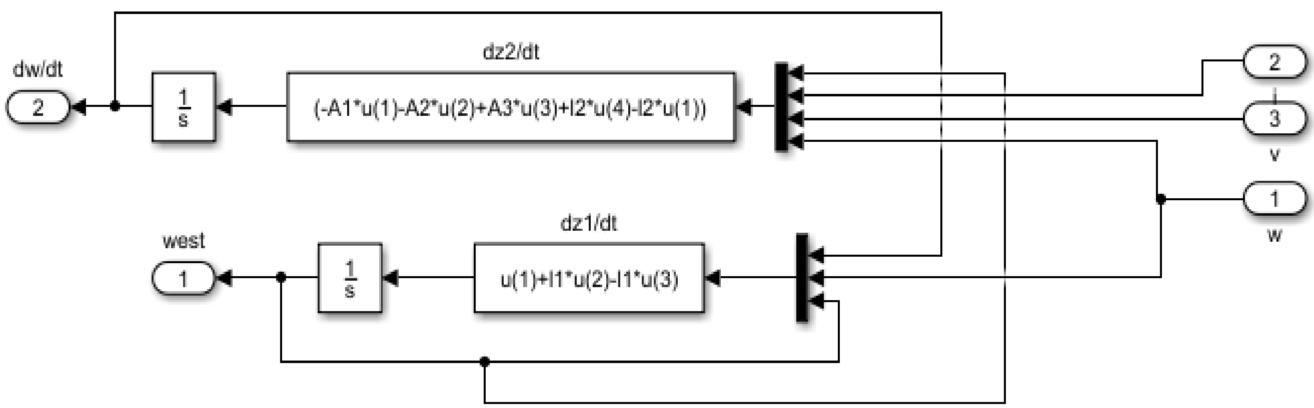


Figure 3.18: Observer block diagram .

3.5.2 Simulation results:

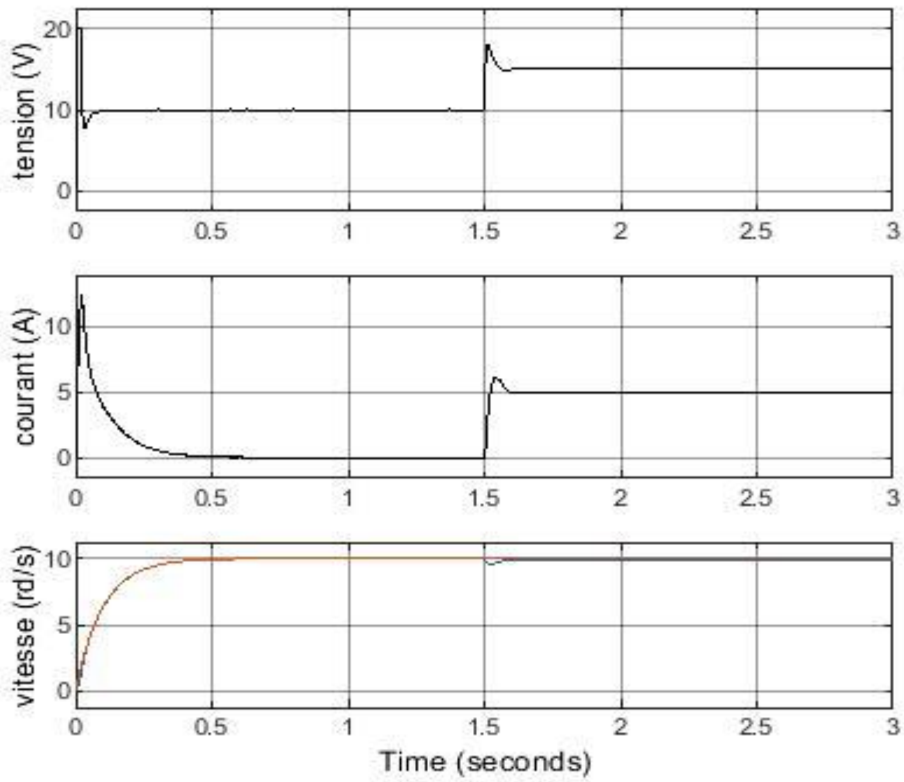
For choices of $z = 0.7$, $\omega_0 = 50$, We have:

$l_1 = 70$

$l_2 = 2.0455e+03$

Figure 3. 18 shows the simulation result of the speed control by introducing a

Observer which makes it possible to estimate the acceleration not accessible to measurement .



Figures 3.19: DC machine's response using SMC with observer.

3.6 Conclusion

In the third chapter , we discussed the speed control of the motor by the SMC technique Based on three different models (cascade-reduced-complete) and the design of an Observer for estimating machine's acceleration . This study has shown that the SMC perfectly follow the desired reference speed and eliminates the effect of disturbances and systemes parameter changes .

GENERAL CONCLUSION

Conclusion

The objective of this thesis is the study of the speed control of the DC motor using the sliding mode control. Next, the implementation of the command using different control models like cascade, reduced and complete model are presented.
The main conclusions are:

- Modeling of the DC machine , and its control in terms of regulation, the aim of which is to make the speed of the DC machine follow a desired speed using the classic PI regulator and the control by sliding mode (SMC) .

- The construction of the SMC is based on the use of the stability theory and particularly the second method of Lyapunov and makes it easy to find the control law to ensure convergence of the error.

- The application of PI control on the DC machine made it possible to follow its speed to a desired speed and to eliminate the effect of the load torque. the PI regulator, thanks to its integral action, made it possible to eliminate the static error. The control using a PI regulator has shown its effectiveness in the implementation of the DC machine with converter. Then a cascade-PI regulator showed its effectiveness because it allows to adjust the speed and the current at the same time. On the other hand, the PI control is influenced by the parametric variations of the system

- The SMC has shown its effectiveness in controlling the DC motor. On the other hand, the SMC made it possible to eliminate the effects of disturbances

  and parameter changes by ensuring perfect tracking of the setpoint . The use of the observer it’s been able possible to estimate the accelerarion .

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