Liquid Democracy and Perceptional Competence

Lauren H. Cooke, Andrew Holmes, Patrick Thornton

April 26, 2023

Abstract

In social choice theory, the conventional liquid democracy model requires all voters to agree upon a competency level for each voter. Our work removes that assumption by introducing *perceptional liquid democracy* (PLD): a delegation-based voting rule wherein each voter is afforded their own separate evaluation of other voters' competence. Through simulations on a social network, we consider *meta-competence*, or voter accuracy in predicting other voters' competence. After establishing a number of theoretical results, we find that PLD's performance is generally weaker than LD's, with higher meta-competence correlating with higher performance. Our findings suggest that imposing realistic assumptions on LD diminishes its practical effectiveness, although there is need for further investigation into delegation rules and their implications.

1 Introduction

In seeking to produce the perfect voting rule for binary decision making, social choice experts often struggle to balance voter interest with voter engagement when polling the general populace. At one extreme we have *direct democracy* (DD), a voting rule where every eligible voter votes directly on every issue. DD has much to recommend it, but it requires an equal level of engagement across all voters regardless of expertise or political savvy; this can be quite burdensome, since arriving at an informed decision can be a time-consuming task [4].

To tackle the issue of unequal desire toward engagement among a populace, some social choice scholars have placed their confidence in a new model, *liquid democracy* (LD) [1]. In LD, voters are allowed either (1) to vote directly on an issue, or (2) to delegate their vote to another voter whom they feel can make a sensible decision on their behalf. This way, voters can engage in either a representative model of governance or a direct one, depending on their desired engagement level [1].

Canonical work on LD considers a simple model where an election is represented as a binary decision problem; there is one correct alternative and one incorrect alternative, and voters are given arbitrary competence levels which designate their objective probability to pick the correct alternative. In addition, each voter is given an oracle understanding of all voters' objective competencies, and will only delegate to a voter with a greater objective competency than their own [1]. We submit that an 'oracle understanding' is a highly unrealistic assumption; in the real world, do you believe everyone is precisely as competent as they say they are?

This work looks to address this mismatch by introducing the *perceptional liquid democracy* (PLD) model, where rather than using concrete competence levels, we introduce *meta-competence*, where each voter perceives the competence of other voters according to a competence perception mechanism. After perceiving all of their neighbors' competence levels subject to their own meta-competence, the voter would then make a delegation decision based on these perceived competence levels. In this work, we investigate PLD's performance in relation to LD and DD empirically, running a series of simulations on different social networks with varying degrees of meta-competence afforded to each voter. As general meta-competence decreases, what happens to accuracy? Is there a meta-competence threshold at which we see a dramatic change in accuracy? If so, is this meta-competence level at all reasonable or realistic to achieve?

2 Models

As stated in the introduction, this work analyzes the relative performance of direct democracy, liquid democracy, and perceptional liquid democracy. In each model, we maintain the original problem: elections will be modeled as a binary decision problem with a correct alternative a_1 and an incorrect alternative a_2, and before delegation, voters will vote for a_1 with probability equal to their competence. If a majority of voters vote for a_1, the correct candidate is elected, which we consider a successful outcome.

2.1 Direct Democracy

In direct democracy (DD), we model a social network as a directed graph $G = (V, E, \mathbf{c})$, where V is the set of voters, E the set of possible delegations (although we will see that in DD these never occur, so E is meaningless here), and $\mathbf{c}$ the set of true competencies for each voter, where c_v gives the probability that voter v will vote for a_1.

Delegation is not a feature of DD, so the election is decided by querying every voter $v \in V$ for a vote (i.e. sampling from a $\text{Bern}(c_v)$ distribution); thus the election is entirely contingent on each voters' intrinsic competence c_v.

2.2 Liquid Democracy

In the conventional model of liquid democracy (LD), we start with the same basic setup as DD: the undirected graph $G = (V, E, \mathbf{c})$. We also take a constant $\alpha \in [0, 1)$, which represents a threshold for delegation; a voter will only choose to delegate if they find a voter whose competence is at least α above their own. In terms of delegation mechanisms, our instances of LD (including PLD later on) use a mechanism that forces delegation if delegation is possible, and direct voting only as a last resort.

LD demands that delegation happens before voting, so each voter i first considers the set of their immediate neighbors in the graph, i.e all j for which $(i, j) \in E$. (In social terms, this is the set of voters i is familiar with.) Then i will delegate to j if and only if $c_j > c_i + \alpha$. If no neighbor of i fulfills this criterion, i will vote directly as in DD; otherwise, i will vote exactly as their choice for delegation does.

There are many standard results on DD and LD which we do not replicate here. However, there is something to note about LD which we will soon contrast with PLD. Consider the graph of delegations $G_D = (V, E_D, \mathbf{c})$ formed after the delegation procedure in an arbitrary LD election, which is identical to G except that E is replaced with the set of delegations E_D, where $(i, j) \in E_D$ if and only if i has delegated to j. Then:

Theorem 1. *For any arbitrary LD election, G_D is acyclic; there can be no series of delegations such that a voter delegates to themselves.*

Proof. Recall that voter i only delegates to voter j when $c_j > c_i + \alpha$. Because $\alpha \in [0,1)$ by definition, voters can only delegate to those who have a strictly greater competence then them, meaning that any chain of delegations forms a strictly increasing sequence of competencies.

Suppose, for contradiction, that G_D contains a cycle; namely, a path $[v_1, \ldots, v_n]$ where each $v_i \in V$ and $v_1 = v_n$. Then we know that the sequence of their competencies is strictly increasing, so

$$c_1 < \ldots < c_n.$$

But since $v_1 = v_n$, we must have $c_1 = c_n$, which contradicts that $c_1 < c_n$, so G_D must be acyclic. □

Certain publicly available implementations of Liquid Democracy such as LiquidFeedback can have cycles, as real agents may not adhere to this approval restriction, which is part of the motivation for our model. Examining LiquidFeedback's documentation, their solution to cyclic delegation graphs is to ignore all votes of all voters in a delegation cycle, which seems against the spirit of liquid democracy, which is supposed to maximize voter engagement and representation [3]. In the next section, we will detail how our new model tackles the problem of cycles.

2.3 Perceptional Liquid Democracy

In our model of perceptional liquid democracy (PLD), we start with the same setup as DD and LD, but we add a new element; $G = (V, E, \mathbf{c}, \mathbf{m})$, with constant $\alpha \in [0,1)$. V, E, $\mathbf{c}$, and α have retained their meanings from LD, but the set of meta-competencies $\mathbf{m}$ is new to PLD's particular means of delegation.

PLD elections can be computed in roughly three phases: competence evaluation, delegation, and voting. Competence evaluation is new to PLD; delegation and voting are roughly the same as they were for LD. For competence evaluation, each voter i will consider themself and each of their neighbors j, where j is a neighbor of i if $(i,j) \in E$. Then, using some competence evaluation mechanism $M(c_j, m_i)$, which takes voter i's meta-competence $m_i \in \mathbf{m}$ and voter j's true competence $c_j \in \mathbf{c}$ as arguments, voter j is given a *perceived competence* to voter i, equal to $p_{ij} = M(c_j, m_i)$. It is important to note: M can be stochastic; it needn't return the same perceived competence given the same pair of arguments each time. We also permit mechanisms that allow a voter to have uncertainty over their own competence, such that $p_{ii} \neq c_i$, although in this paper all mechanisms discussed will use $p_{ii} = c_i$. Simulations of PLD will require that we specify exactly what M is; we will do so in our discussions later on. Regardless of what M is, we require that its outputs keep with the restrictions of competencies thus far, so all $p_{ij} \in [0,1]$.

Then follows delegation, in which each voter i considers each of its neighbors j in G. Then i will delegate to j if and only if $p_{ij} > p_{ii} + \alpha$ (noting again that in this paper we generally have $p_{ii} = c_i$, although this doesn't have to be the case). If no neighbor of i fulfills this criterion, i will vote directly as in DD; otherwise, i will vote exactly as their choice for delegation does.

Now we can draw a theoretical distinction between LD and PLD. Consider the graph of delegations $G_D = (V, E_D, \mathbf{c}, \mathbf{m})$ formed after the delegation procedure in an arbitrary PLD election, which is identical to G except that E is replaced with the set of delegations E_D, where $(i,j) \in E_D$ if and only if i has delegated to j. Then:

Theorem 2. *In a PLD election, G_D may be cyclic; there can be a series of delegations such that a voter delegates to themselves.*

Proof. The existence of perceived competence within PLD is what makes the acyclicality of G_D no longer hold in PLD's framework. Consider an instance of PLD with the following graph G; there are two voters, $V = \{v_1, v_2\}$; they both know each other, $E = \{(v_1, v_2), (v_2, v_1)\}$; and each has a true competence of 0.5, $\mathbf{c} = [.5, .5]$. Let M, the competence evaluation mechanism, give the true competence of the voter if perceiving their own confidence, else ignore its arguments and simply be a random uniform draw from the interval $[0, 1]$. (We don't need to define $\mathbf{m}$, then, since M ignores it.)

Then during the competence evaluation process, suppose v_1 assigns $p_{v_1 v_2} \geq 0.8$ via M, and v_2 assigns $p_{v_2 v_1} \geq 0.8$ via M, so both voters perceive each other to have a much higher competence than their own. Supposing $\alpha = .1$, both voters will then delegate to each other during the delegation procedure, since $p_{v_1 v_2} > p_{v_1 v_1} + \alpha$ (as $0.8 > 0.6$) and $p_{v_2 v_1} > p_{v_2 v_2} + \alpha$ (as $0.8 > 0.6$). This forms a cycle of delegations in G_D, so we are done; both a and b have delegated to themselves. $\square$

Hopefully it is clear why this poses a threat to the coherency of our model; if there is a series of voters, each of whom have delegated to themselves, who votes? Rather than remove these voters from the election as is common in real-world Liquid Democracy implementations, we would instead like for our model to prevent cyclic delegation graphs altogether [3].

To avoid getting a cycle in practice, we assign a random voting order among all voters $v \in V$ before the delegation procedure. Voters will then delegate in that order. If at any point a voter i decides to delegate to a voter j in a way that forms a cycle in G_D, we remove (i, j) from E and ask i to choose again. Then either i delegates to a new voter j in an acyclic fashion, or i is forced to vote directly for want of options. This random ordering of delegations has the potential to affect election results. To mitigate these effects in our simulations, we run multiple trials of each election and average the results to avoid negative consequences due to random chance.

2.3.1 Some Canonical Results for PLD

Since PLD is novel to this paper, it is worth establishing a few standard results. Let $\text{gain}(M, G) = P_{PLD+M}(G) - P_{DD}(G)$, where $P_V(G)$ is the average proportion of voters in G who vote for the correct alternative under the instance of voting method V using graph G. Since we are comparing delegation mechanisms, we must recognize that PLD forms a new delegation mechanism for each different competence evaluation mechanism M it uses; for the purposes of this section, PLD refers to a family of voting rules, one for each possible M. Then we establish two results, one in the negative and one in the positive.

Theorem 3. *For some M, PLD does not satisfy the do-no-harm property (DNH); i.e., there exists $\varepsilon > 0$ such that for all $n_0 \in \mathbb{N}$, there exists some graph G_n on $n \geq n_0$ vertices such that $\text{gain}(M, G_n) < -\varepsilon$. Intuitively, there exists an instance of PLD wherein DD outperforms it.*

Proof. Let $\varepsilon = .1 > 0$, and let $n_0 \in \mathbb{N}$ be arbitrary. WLOG, suppose n_0 is odd (it is trivial to modify this proof in the case of even n_0). Then suppose the graph $G_n = (V, E, \mathbf{c}, \mathbf{m})$ on $n = n_0 + 1$ vertices, wherein $V = \{v_1, \ldots, v_{n_0}, v_n\}$; E contains an edge from each of $v_1, \ldots, v_{n_0}$ to v_n and nothing else (so $E = \{(i, j) \mid i \in V,\ j \in V,\ i \neq v_n,\ j = v_n\}$); and $\mathbf{c} = [.5, \ldots, .5, 0]$. We set $\alpha = .1$, and the competence evaluation mechanism M to be the constant one function when evaluating other voters, but returns the true competence of the voter if perceiving the competence of themselves (so defining $\mathbf{m}$ is unnecessary). We represent G_n graphically below in Figure 1.

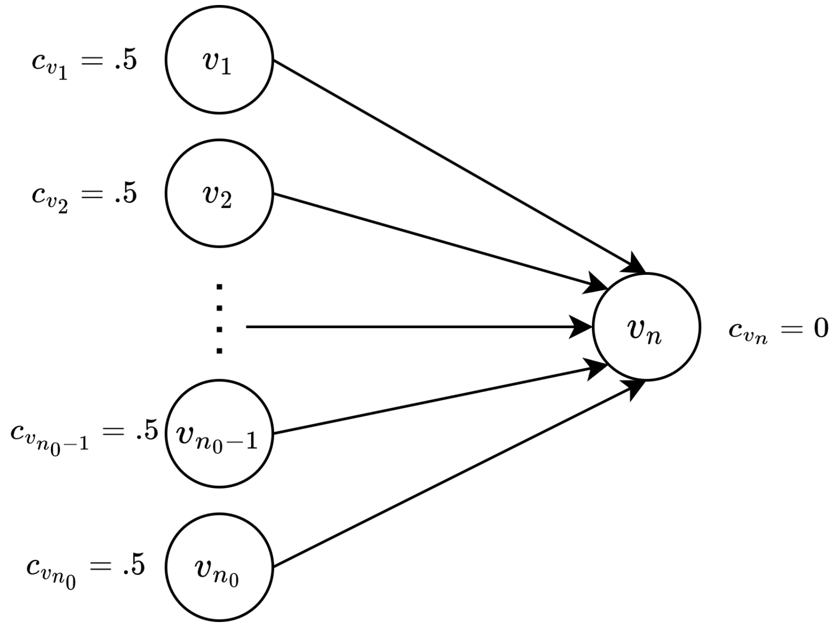

Figure 1: Diagram of G_n.

We first evaluate $P_{DD}(G_n)$; suppose ties are broken in favor of the correct candidate. Then a victory is achieved whenever a majority of $v_1, \dots, v_{n_0}$ votes correctly. Since each of these voters has a competence of .5, this is the probability that an odd number of flipped coins yields a majority of heads; namely, this probability is .5, so $P_{DD}(G_n) = .5$, since P represents an average.

Then we consider $P_{PLD+M}(G_n)$. Since M is the constant one function, each v_i in $v_1, \dots, v_{n_0}$ will assign $p_{v_i v_n} = 1$, and thus delegate to v_n since v_n is v_i's only neighbor and the condition $p_{v_i v_n} > p_{v_i v_i} + \alpha$ is met as $1 > .6$. So v_n will vote unilaterally for everyone, and always vote incorrectly as $c_{v_n} = 0$, so $P_{PLD+M}(G_n) = 0$.

Thus

$$\text{gain}(M, G_n) = P_{PLD+M}(G_n) - P_{DD}(G_n) = 0 - .5 = -.5 < -.1 = -\varepsilon,$$

so we're done. $\square$

Theorem 4. *For some M, PLD satisfies the positive gain property (PG); i.e., there exists $\varepsilon > 0$ and graph G such that* $\text{gain}(M, G) \geq \varepsilon$. *Intuitively, there exists an instance of PLD wherein it outperforms DD.*

Proof. Let $\varepsilon = .1 > 0$. Then suppose the graph $G = (V, E, \mathbf{c}, \mathbf{m})$, wherein $V = \{v_1, v_2, v_3\}$; $E = \{(v_1, v_2)\}$; and $\mathbf{c} = [0, 1, 0]$. We set $\alpha = .1$, and the competence evaluation mechanism M to be the constant one function when evaluating other voters, but returns the true competence of the voter if perceiving the competence of themselves (so defining $\mathbf{m}$ is unnecessary). We represent G_n graphically below in Figure 2.

We first evaluate $P_{DD}(G)$; suppose ties are broken in favor of the correct candidate. Then a victory is achieved whenever a majority of v_1, v_2, v_3 vote correctly, which will never happen, since $c_{v_1} = c_{v_3} = 0$. So $P_{DD}(G_n) = 0$.

Then we consider $P_{PLD+M}(G_n)$. Since M is the constant one function, v_1 will perceive v_2 as having a competence $p_{v_1 v_2} = 1$. Then v_1 will delegate to v_2 as $p_{v_1 v_2} > p_{v_1 v_1} + \alpha$, for $1 > .1$. This is

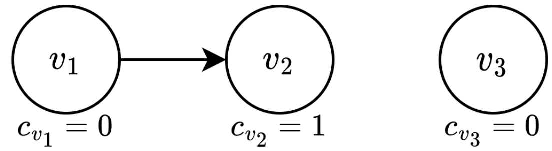

Figure 2: Diagram of G.

the only delegation that could possibly occur, so v_1 and v_2 will vote as a collective and v_3 will vote alone. Since $c_{v_2} = 1$, this means that at least two voters will always vote correctly, which ensures that the election will always go to the correct candidate, so $P_{PLD+M}(G) = 1$.

Thus

$$\text{gain}(M, G) = P_{PLD+M}(G) - P_{DD}(G) = 1 - 0 = 1 \geq .1 = \varepsilon,$$

so we're done. $\square$

Since M is allowed to be any deterministic or nondeterministic mechanism we want, these previous two results are fairly straightforward to show. But we used the same 'blind faith' constant one function for M in both proofs, showing that this choice of M cannot fulfill both DNH and PG. This invites the question of whether there exists an M that fulfills both DNH and PG, which we explore in the following two results:

Theorem 5. *There exists a competence evaluation mechanism M for which PLD+M fulfills both DNH and PG.*

Proof. Let $M(c_j, m_i) = 1$ if $c_j = 1$ and be 0 otherwise. This means that any voter i's set of approved voters in G, $A_G(i)$, can only include voters with true competence 1. It's trivial to see this fulfills both DNH and PG; for DNH, we note that a delegation to a voter with competence 1 can only possibly help, as it does nothing but turn one certain votes for the correct candidate into two; and for PG, we may simply consider the graph found in Figure 2 in light of this choice for M. $\square$

This seems surprising, as the nonexistence of such a mechanism is a major result of LD. It turns out that the above is the rough equivalent of setting $\alpha = 1$ within LD, something the conventional LD model explicitly forbids. If we impose a similar restriction on PLD, we achieve the expected result:

Theorem 6. *Let M be a competence evaluation mechanism such that, for some graph G and voter i, we have that M has a nonzero probability of approving a voter j with competence $c_j \neq 1$ or that M never approves voters. Then PLD+M does not fulfill both DNH and PG.*

Proof. We consider two cases.

First, suppose that M has a nonzero probability of approving a voter j with competence $c_j \neq 1$. Let $c_j \neq 1$ be the true competence of voter j and m_i be the true meta-competence of voter i; we WLOG assume $c_i = .5$.

Let $\varepsilon = .1 > 0$, and let $n_0 \in \mathbb{N}$ be arbitrary. Then suppose the graph $G' = (V, E, \mathbf{c}, \mathbf{m})$ on $n = 2n_0 + 1$ vertices, wherein $V = \{i, j\} \cup T \cup F$; $E = \{(i, j)\}$; and $\mathbf{c} = [.5, c_j] + \mathbf{c}_T + \mathbf{c}_F$. We let $T = [t_1, \ldots, t_{n_0}]$ and $F = [y_1, \ldots, y_{n_0-1}]$, with $\mathbf{c}_T$ being a vector of all ones and $\mathbf{c}_F$ being a vector of all zeroes. All voters are given a meta-competence of 0 except for i, who is given m_i. We can represent this setup graphically in Figure below.

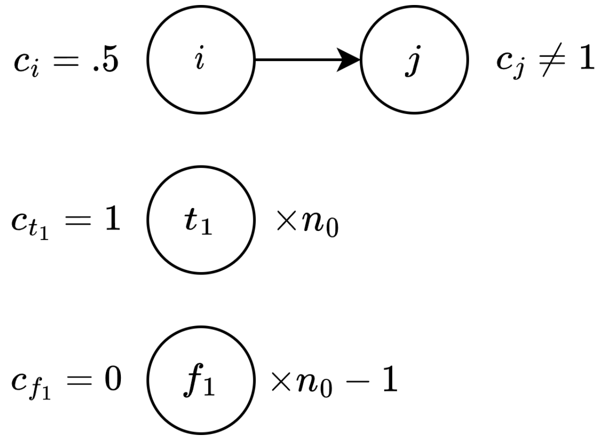

Figure 3: Diagram of G'.

We first evaluate $P_{DD}(G')$; only one delegation can possibly happen in this setup, that from i to j. (We know it can happen given M as i and j in this setup provide precisely the same pair of arguments to M as were guaranteed in the theorem's assumption; namely, j's true competence and i's meta-competence.) The incorrect candidate will be elected if and only if both i and j vote for the incorrect candidate, as we need two more incorrect votes to reverse the lead the correct candidate currently has. So the probability of the correct candidate being voted in is the inverse:

$$P_{DD}(G') = 1 - (1 - c_i)(1 - c_j) = 1 - .5(1 - c_j) = .5 + .5c_j.$$

Then we consider $P_{PLD+M}(G')$. In the instances where the delegation from i to j occurs, j must now vote on behalf of both i, j. Then the incorrect candidate is elected if j votes incorrectly, so the probability of the incorrect candidate being voted in is the inverse:

$$1 - (1 - c_j) = c_j.$$

(Note the above is not $P_{PLD+M}(G')$ exactly, since M may be nondeterministic and this delegation only happen sometimes.) But if $c_j \neq 1$ as we assumed, $c_j < .5 + .5c_j$ in all cases, so the probability of the correct candidate being chosen has dropped in at least one instance from DD, violating DNH.

Second, suppose that M never approves any voters. Then PLD+M can never delegate at all, collapsing it down all the way to DD in all instances; this means it fulfills DNH, but clearly fails to fulfill PG.

Thus in all cases, PLD+M fails to achieve both DNH and PG. □

Lastly, it is worth noting a way in which PLD can collapse into LD in a special case. Consider the competence evaluation mechanism $M_{LD}(c_j, m_i) = c_j$, where the meta-competence is essentially ignored and the true competence of the other voter is returned unaltered. Then:

Theorem 7. *PLD with competence evaluation mechanism M_{LD} is equivalent to LD.*

Proof. With this competence evaluation mechanism, $p_{ij} = c_j$ for all perceived competencies p_{ij}, so the criterion for delegation $p_{ij} > c_i + \alpha$ becomes $c_j > c_i + \alpha$, the exact criterion for delegation we find in LD. From there, elections proceed identically for LD and PLD, so the two are equivalent in this case. □

3 Simulating Social Networks

In addition to tackling a number of theoretical results, we will be simulating the effectiveness of PLD on a social network graph; here we outline how we construct this graph. Our graph will be undirected, under the assumption that we will be dealing with a network of mutual relationships. (In terms of our model, for the set of possible delegations E, we will always have a $(j, i) \in E$ for every $(i, j) \in E$.) We take a *preferential attachment* graph with 1000 nodes. With preferential attachment, the more connected a node is (i.e. the higher its degree), the more likely it is to receive a new edge. There is precedent for simulating a social network with this technique [2], as it captures a sort of rich-get-richer aspect to social networking; popular people are more likely to make new friends and connections than unpopular people. This procedure is stochastic, so we can produce many different social networks this way. One such network we generated is pictured below in Figure 4; edges represent relationships and nodes represent voters, where a node is green if they voted for the correct candidate and red otherwise.

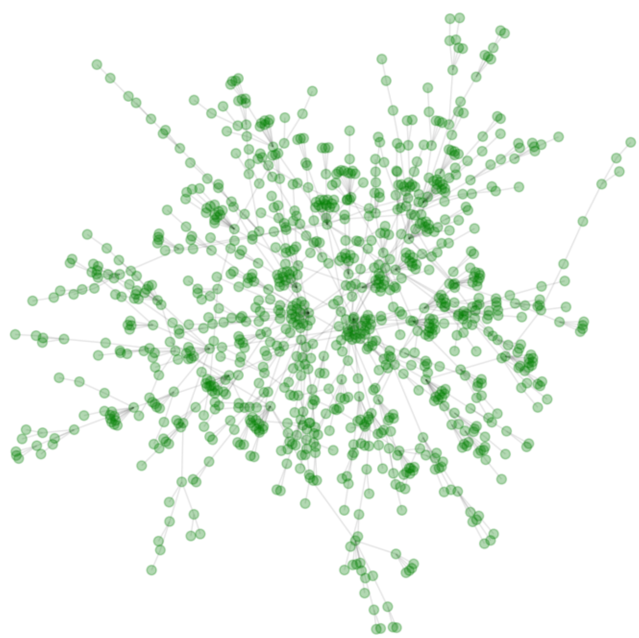

Figure 4: Social Network 0 constructed using preferential attachment.

The effects of preferential attachment are clear to see; popular nodes clump together, while others are more removed from the herd. The assumption of preferential attachment marks a limitation in our research. It needn't necessarily be the case that a person's ability to make connections is directly proportional to their number of connections, but that is not captured in our particular methodology for social network generation.

4 Simulating Elections

We aim to generate a series of simulations to compare the performance of traditional liquid democracy (LD), direct democracy (DD), and perceptional liquid democracy (PLD) using different meta-competence levels. Specifically, we will consider DD; LD; and PLD with global meta-competencies 0.0, 0.2, 0.4, 0.5, 0.6, 0.8, 0.9, and 1. By 'PLD with meta-competence x', we mean an instance of PLD where the set of meta-competencies $\mathbf{m}$ is set to an array of all values x, so every voter operates with meta-competency x; we will call this 'PLD x' for short.

Elections for DD and LD will proceed precisely as they are described in our description of each model, with $\alpha = .1$ and $\mathbf{c}$ consisting of a vector of random real numbers drawn from the unit interval $[0, 1]$. For our simulations we used PLD with competence evaluation mechanism M:

$$M(c_j, m_i) = \begin{cases} c_i & i = j \\ \text{Uniform}\left(\max\left(0, c_j - \frac{m_i}{2}\right), \min\left(1, c_j + \frac{m_i}{2}\right)\right) & \text{else,} \end{cases}$$

where $\text{Uniform}(a, b)$ draws a random real number in the interval $[a, b]$.

This choice for a competence evaluation mechanism M is meant to capture many aspects of the kinds of elections we hope to simulate; namely, that perceived competence should fall roughly around true competence, and the range of that spread should scale inversely with meta-competence, so a highly meta-competent voter will typically perceive a competence very close to the voters' true competence. At the extreme end of this proportionality, we find that if a voter has meta-competence 1, $M(c_j, 1) = M_{LD}(c_j, 1)$ for all c_j, so a PLD instance with all voters having meta-competence 1 collapses into an LD instance by Theorem 7.

With our procedure in store, we elected to generate a social network using the procedure outlined above, and then run 100 simulated elections using each voting rule we're considering on that social network. We can visualize the effect of various voting rules on a typical 1000-voter social network in Figure 9, and observe some statistics drawn from those 100 election results in Table 1.

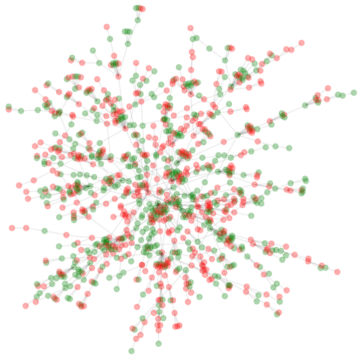

Figure 5: DD

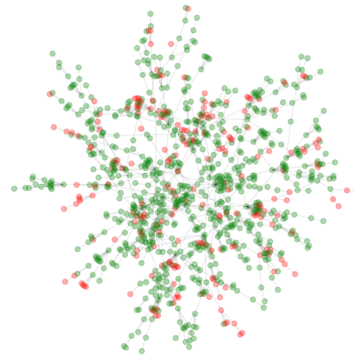

Figure 6: LD

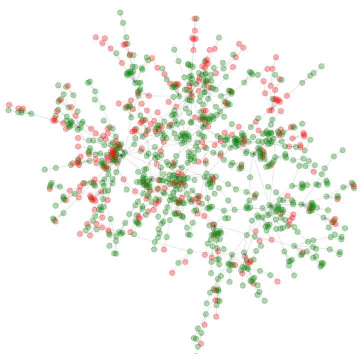

Figure 7: PLD .2

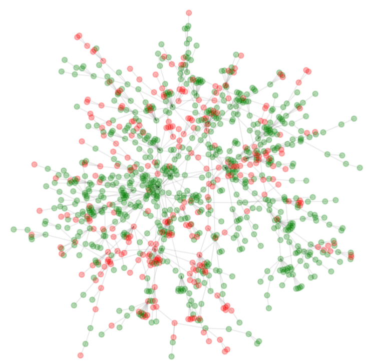

Figure 8: PLD .8

Figure 9: Result of elections given various voting rules; red nodes indicate votes for the incorrect candidate, and green nodes indicate votes for the correct candidate.

5 Results

From our results shown in Table 1, we can get a sense of the practical effectiveness of PLD with respect to DD and LD. DD returns an average accuracy of $\approx .5$, which is to be expected given that voters' competencies were sampled uniformly from the unit interval $[0, 1]$. This is a significantly worse average accuracy than any other voting method in the bunch; it would appear that, even when factoring in the imprecision brought about by meta-competencies lower than 1, delegation is still an effective strategy to improve voter accuracy in our simulation.

As Theorem 7 would suggest, LD and PLD 1 have extremely similar summary statistics, as they represent the same process. But as the global meta-competence for PLD declines, so too does its average, maximum, and minimum voter accuracy, to the point where PLD 0 performs markedly worse than PLD 1 (still much better than DD). The dropoff of voter accuracy is more pronounced for lower global meta-competencies, so the difference between the voter accuracy of PLD 0 and PLD .2 is far greater than the difference between the voter accuracy of PLD .8 and PLD 1.

Trial	Direct	Liquid	PLD 0	PLD .2	PLD .4	PLD .5	PLD .6	PLD .8	PLD .9	PLD 1
0	0.496	0.788	0.724	0.736	0.725	0.762	0.777	0.707	0.724	0.76
1	0.482	0.743	0.678	0.737	0.664	0.739	0.723	0.668	0.704	0.755
2	0.528	0.718	0.759	0.746	0.799	0.766	0.794	0.762	0.779	0.785
3	0.5	0.751	0.706	0.664	0.743	0.771	0.751	0.735	0.761	0.782
4	0.518	0.731	0.753	0.68	0.71	0.722	0.732	0.764	0.735	0.753
5	0.476	0.735	0.726	0.756	0.715	0.707	0.718	0.743	0.75	0.702
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
96	0.484	0.719	0.757	0.736	0.718	0.773	0.778	0.76	0.757	0.733
97	0.512	0.744	0.563	0.74	0.742	0.737	0.694	0.784	0.759	0.728
98	0.481	0.775	0.735	0.725	0.717	0.689	0.753	0.755	0.764	0.775
99	0.539	0.804	0.748	0.775	0.747	0.802	0.788	0.782	0.754	0.754
average	0.502	0.7566	0.7234	0.7310	0.7354	0.7438	0.7469	0.74987	0.7526	0.7522
max	0.539	0.818	0.791	0.793	0.81	0.802	0.813	0.818	0.812	0.83
min	0.47	0.684	0.563	0.612	0.647	0.655	0.654	0.668	0.682	0.643

Table 1: Election outcomes on our social network under various voting rules. For the first several rows, the numbers in each data entry represent voting accuracy, or the proportion of voters in that election who voted for the correct candidate. The bottom three rows give summary statistics of the rows above.

6 Discussion

The philosophical basis for our work with perceptional liquid democracy was to remedy what we perceived to be an unrealistic aspect of the original LD model, namely that all voters had an oracle understanding of their peers, knowing exactly how competent each one was. Since no one in reality possesses such an understanding, we wanted to see if some of the practical gains promised by LD persisted under what we felt to be a realistic adjustment. While the PLD model is quite open-ended and preserves many of the theoretical properties of LD (besides acyclicality), our particular simulation of it outfitted with our choice of competence evaluation mechanism specifically wished to evoke a relatively realistic election scenario compared to plain LD.

In general, we found that the imposition of realism had the intended effect: voter accuracy dropped in comparison to unadulterated LD. However, it did not drop off too steeply, and PLD even with very low meta-competence proved much better than DD. This leaves much future work ahead of us, including, but not limited to; study into assumptions we could impose on LD to stress-test its capabilities better; discovering how could different choices of M affect PLD, and the extent to which different choices of M mirror real liquid democracy elections; further simulation work on more wide-ranging election scenarios, such as the *influencer* scenario wherein many people are able and willing to delegate to an influencer (politician, celebrity, etc.) but that influencer is not able to delegate back to any of them, and so on. Our paper represents only the initial stages of PLD as an area of study.

References

[1] S. Mackenzie A. Kahng and A. D. Procaccia. "Liquid democracy: An Algorithmic Perspective". In: *Journal of Artificial Intelligence Research* 70 (2021), pp. 1223–1252.

[2] A. Barabasi and R. Albert. "Emergence of Scaling in Random Networks". In: *Science* 286.5439 (Oct. 1999), pp. 509–512. DOI: 10.1126/science.286.5439.509. URL: https://doi.org/10.1126%2Fscience.286.5439.509.

[3] Jan Behrens et al. *The Principles of LiquidFeedback*. Berlin: Interaktive Demokratie e. V., 2014.

[4] J. Green-Armytage. "Direct voting and proxy voting". In: *Constitutional Political Economy* 26 (2015), pp. 190–220.