

Contents

Part I The Early Years

1	Solution of a Large-Scale Traveling-Salesman Problem	7
	George B. Dantzig, Delbert R. Fulkerson, and Selmer M. Johnson	
2	The Hungarian Method for the Assignment Problem	29
	Harold W. Kuhn	
3	Integral Boundary Points of Convex Polyhedra	49
	Alan J. Hoffman and Joseph B. Kruskal	
4	Outline of an Algorithm for Integer Solutions to Linear Programs and An Algorithm for the Mixed Integer Problem	77
	Ralph E. Gomory	
5	An Automatic Method for Solving Discrete Programming Problems .	105
	Ailsa H. Land and Alison G. Doig	
6	Integer Programming: Methods, Uses, Computation	133
	Michel Balinski	
7	Matroid Partition	199
	Jack Edmonds	
8	Reducibility Among Combinatorial Problems	219
	Richard M. Karp	
9	Lagrangian Relaxation for Integer Programming	243
	Arthur M. Geoffrion	
10	Disjunctive Programming	283
	Egon Balas	

Part II From the Beginnings to the State-of-the-Art

11 Polyhedral Approaches to Mixed Integer Linear Programming	343
Michele Conforti, Gérard Cornuéjols, and Giacomo Zambelli	
11.1 Introduction	343
11.1.1 Mixed integer linear programming	343
11.1.2 Historical perspective	344
11.1.3 Cutting plane methods	345
11.2 Polyhedra and the fundamental theorem of integer programming	348
11.2.1 Farkas' lemma and linear programming duality	349
11.2.2 Carathéodory's theorem	352
11.2.3 The theorem of Minkowski-Weyl	353
11.2.4 Projections	355
11.2.5 The fundamental theorem for MILP	356
11.2.6 Valid inequalities	357
11.2.7 Facets	357
11.3 Union of polyhedra	359
11.4 Split disjunctions	362
11.4.1 One-side splits, Chvátal inequalities	365
11.5 Gomory's mixed-integer inequalities	366
11.5.1 Equivalence of split closure and Gomory mixed integer closure	368
11.6 Polyhedrality of closures	369
11.6.1 The Chvátal closure of a pure integer set	370
11.6.2 The split closure of a mixed integer set	370
11.7 Lift-and-project	373
11.7.1 Lift-and-project cuts	374
11.7.2 Strengthened lift-and-project cuts	376
11.7.3 Improving mixed integer Gomory cuts by lift-and-project	377
11.7.4 Sequential convexification	378
11.8 Rank	380
11.8.1 Chvátal rank	380
11.8.2 Split rank	382
References	384
12 Fifty-Plus Years of Combinatorial Integer Programming	387
William Cook	
12.1 Combinatorial integer programming	387
12.2 The TSP in the 1950s	389
12.3 Proving theorems with linear-programming duality	397
12.4 Cutting-plane computation	399
12.5 Jack Edmonds, polynomial-time algorithms, and polyhedral combinatorics	404
12.6 Progress in the solution of the TSP	410
12.7 Widening the field of application in the 1980s	415

12.8	Optimization \equiv Separation	418
12.9	State of the art	420
	References	425
13	Reformulation and Decomposition of Integer Programs	431
	François Vanderbeck and Laurence A. Wolsey	
13.1	Introduction	431
13.2	Polyhedra, reformulation and decomposition	433
13.2.1	Introduction	433
13.2.2	Polyhedra and reformulation	434
13.2.3	Decomposition	440
13.3	Price or constraint decomposition	441
13.3.1	Lagrangian relaxation and the Lagrangian dual	443
13.3.2	Dantzig-Wolfe reformulations	445
13.3.3	Solving the Dantzig-Wolfe relaxation by column generation	448
13.3.4	Alternative methods for solving the Lagrangian dual	451
13.3.5	Optimal integer solutions: branch-and-price	456
13.3.6	Practical aspects	464
13.4	Resource or variable decomposition	464
13.4.1	Benders' reformulation	465
13.4.2	Benders with integer subproblems	468
13.4.3	Block diagonal structure	470
13.4.4	Computational aspects	471
13.5	Extended formulations: problem specific approaches	471
13.5.1	Using compact extended formulations	472
13.5.2	Variable splitting I: multi-commodity extended formulations	473
13.5.3	Variable splitting II	477
13.5.4	Reformulations based on dynamic programming	480
13.5.5	The union of polyhedra	483
13.5.6	From polyhedra and separation to extended formulations	485
13.5.7	Miscellaneous reformulations	487
13.5.8	Existence of polynomial size extended formulations	489
13.6	Hybrid algorithms and stronger dual bounds	490
13.6.1	Lagrangian decomposition or price-and-price	490
13.6.2	Cut-and-price	491
13.7	Notes	493
13.7.1	Polyhedra	494
13.7.2	Dantzig-Wolfe and price decomposition	494
13.7.3	Resource decomposition	496
13.7.4	Extended formulations	496
13.7.5	Hybrid algorithms and stronger dual bounds	498
	References	498

Part III Current Topics

14	Integer Programming and Algorithmic Geometry of Numbers	505
	Friedrich Eisenbrand	
14.1	Lattices, integer programming and the geometry of numbers	505
14.2	Informal introduction to basis reduction	507
14.3	The Hermite normal form	509
14.4	Minkowski's theorem	515
14.5	The LLL algorithm	518
14.6	Kannan's shortest vector algorithm	525
14.7	A randomized simply exponential algorithm for shortest vector . . .	529
14.8	Integer programming in fixed dimension	535
14.9	The integer linear optimization problem	542
14.10	Diophantine approximation and strongly polynomial algorithms . .	545
14.11	Parametric integer programming	550
	References	556
15	Nonlinear Integer Programming	561
	Raymond Hemmecke, Matthias Köppe, Jon Lee, and Robert Weismantel	
15.1	Overview	562
15.2	Convex integer maximization	564
15.2.1	Fixed dimension	564
15.2.2	Boundary cases of complexity	565
15.2.3	Reduction to linear integer programming	568
15.3	Convex integer minimization	573
15.3.1	Fixed dimension	573
15.3.2	Boundary cases of complexity	577
15.3.3	Practical algorithms	580
15.4	Polynomial optimization	586
15.4.1	Fixed dimension and linear constraints: An FPTAS	587
15.4.2	Semi-algebraic sets and SOS programming	597
15.4.3	Quadratic functions	601
15.5	Global optimization	604
15.5.1	Spatial Branch-and-Bound	605
15.5.2	Boundary cases of complexity	607
15.6	Conclusions	611
	References	612
16	Mixed Integer Programming Computation	619
	Andrea Lodi	
16.1	Introduction	619
16.2	MIP evolution	621
16.2.1	A performance perspective	624
16.2.2	A modeling/application perspective	631
16.3	MIP challenges	632

16.3.1	A performance perspective	634
16.3.2	A modeling perspective	639
16.4	Conclusions	641
	References	642
17	Symmetry in Integer Linear Programming	647
	François Margot	
17.1	Introduction	647
17.2	Preliminaries	649
17.3	Detecting symmetries	651
17.4	Perturbation	653
17.5	Fixing variables	653
17.6	Symmetric polyhedra and related topics	656
17.7	Partitioning problems	658
17.7.1	Dantzig-Wolfe decomposition	659
17.7.2	Partitioning orbitope	660
17.7.3	Asymmetric representatives	662
17.8	Symmetry breaking inequalities	663
17.8.1	Dynamic symmetry breaking inequalities	664
17.8.2	Static symmetry breaking inequalities	664
17.9	Pruning the enumeration tree	668
17.9.1	Pruning with a fixed order on the variables	670
17.9.2	Pruning without a fixed order of the variables	673
17.10	Group representation and operations	674
17.11	Enumerating all non-isomorphic solutions	678
17.12	Furthering the reach of isomorphism pruning	679
17.13	Choice of formulation	679
17.14	Exploiting additional symmetries	681
	References	681
18	Semidefinite Relaxations for Integer Programming	687
	Franz Rendl	
18.1	Introduction	687
18.2	Basics on semidefinite optimization	690
18.3	Modeling with semidefinite programs	692
18.3.1	Quadratic 0/1 optimization	692
18.3.2	Max-Cut and graph bisection	693
18.3.3	Stable sets, cliques and the Lovász theta function	694
18.3.4	Chromatic number	696
18.3.5	General graph partition	698
18.3.6	Generic cutting planes	700
18.3.7	SDP, eigenvalues and the Hoffman-Wielandt inequality ..	702
18.4	The theoretical power of SDP	705
18.4.1	Hyperplane rounding for Max-Cut	705
18.4.2	Coloring	708
18.5	Solving SDP in practice	711

18.5.1	Interior point algorithms	711
18.5.2	Partial Lagrangian and the bundle method	715
18.5.3	The spectral bundle method	718
18.6	SDP and beyond	721
18.6.1	Copositive and completely positive matrices	721
18.6.2	Copositive relaxations	722
	References	723
19	The Group-Theoretic Approach in Mixed Integer Programming	727
	Jean-Philippe P. Richard and Santanu S. Dey	
19.1	Introduction	727
19.2	The corner relaxation	730
19.2.1	Linear programming relaxations	730
19.2.2	Motivating example	731
19.2.3	Gomory's corner relaxation	737
19.3	Group relaxations: optimal solutions and structure	739
19.3.1	Optimizing linear functions over the corner relaxation . . .	739
19.3.2	Using corner relaxations to solve MIPs	744
19.3.3	Extended group relaxations	749
19.4	Master group relaxations: definitions and inequalities	754
19.4.1	Groups	754
19.4.2	Master group relaxations of mixed integer programs	756
19.4.3	A hierarchy of inequalities for master group problems . . .	759
19.5	Extreme inequalities	766
19.5.1	Extreme inequalities of finite master group problems	766
19.5.2	Extreme inequalities for infinite group problems	769
19.5.3	A compendium of known extreme inequalities for finite and infinite group problems	784
19.6	On the strength of group cuts and the group approach	785
19.6.1	Absolute strength of group relaxation	785
19.6.2	Relative strength of different families of group cuts	788
19.6.3	Summary on strength of group cuts	795
19.7	Conclusion and perspectives	795
	References	797

Part IV DVD-Video / DVD-ROM

50 Years of Integer Programming 1958-2008

From the Early Years to the State-of-the-Art

Jünger, M.; Liebling, Th.M.; Naddef, D.; Nemhauser, G.L.;

Pulleyblank, W.R.; Reinelt, G.; Rinaldi, G.; Wolsey, L.A.

(Eds.)

2010, XX, 804 p. With DVD., Hardcover

ISBN: 978-3-540-68274-5