Distributed MAP Inference for Undirected Graphical Models

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Workshop on Learning on Cores, Clusters and Clouds (LCCC)
Neural Information Processing Systems (NIPS) 2010

Motivation

- Graphical models are used in a number of information extraction tasks
- Recently, models are getting larger and denser
  - Coreference Resolution [Culotta et al. NAACL 2007]
  - Relation Extraction [Riedel et al. EMNLP 2010, Poon & Domingos EMNLP 2009]
  - Joint Inference [Finkel & Manning. NAACL 2009, Singh et al. ECML 2009]
- Inference is difficult, and approximations have been proposed
  - LP-Relaxations [Martins et al. EMNLP 2010]
  - Dual Decomposition [Rush et al. EMNLP 2010]
  - MCMC-Based [McCallum et al. NIPS 2009, Poon et al. AAAI 2008]

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Without parallelization, these approaches have restricted scalability

Motivation

Contributions:

1. Distribute MAP Inference for a large, dense factor graph
   - 1 million variables, 250 machines
2. Incorporate sharding as variables in the model

Outline

Factor Graphs

Represent distribution over variables Y using factors ψ.

$$p(Y = y) \propto \exp \sum_{y_c \subseteq y} \psi_c(y_c)$$

Note: Set of factors is different of every assignment $Y = y$ ($\{\psi\}_y$)

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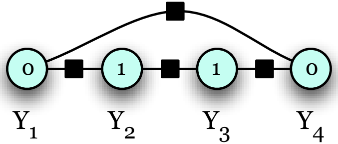

$$\{\psi\}_{0110} = \{\psi_{12}^{01}, \psi_{23}^{11}, \psi_{34}^{10}, \psi_{14}^{00}\}$$

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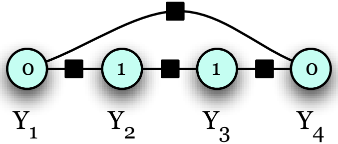

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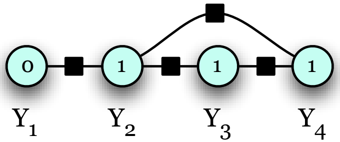

$\{\psi\}_{0111} = \{\psi_{12}^{01}, \psi_{23}^{11}, \psi_{34}^{11}, \psi_{24}^{11}\}$

MAP[1] Inference

We want to find the best configuration according to the model,

$$\begin{aligned}\hat{y} &= \arg\max_{y} p(Y = y) \\ &= \arg\max_{y} \exp \sum_{y_c \subseteq y} \psi_c(y_c)\end{aligned}$$

[1] MAP = maximum a posteriori

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Computational bottlenecks:

1. Space of Y is usually enormous (exponential)
2. Even evaluating $\sum_{y_c \subseteq y} \psi_c(y_c)$ for each y may be polynomial

[1] MAP = maximum a posteriori

MCMC for MAP Inference

Initial Configuration $y = y_0$

for (num_samples):

1. Propose a change to y to get configuration y'
   (Usually a *small* change)
2. Acceptance probability: $\alpha(y, y') = \min\left(1, \left(\frac{p(y')}{p(y)}\right)^{1/t}\right)$
   (Only involve computations local to the change)
3. if Toss(α): Accept the change, $y = y'$

return y

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$$\frac{p(y')}{p(y)} = \exp\left\{\sum_{y'_c \subseteq y'} \psi_c(y'_c) - \sum_{y_c \subseteq y} \psi_c(y_c)\right\}$$

Mutually Exclusive Proposals

Let $\{\psi\}_y^{y'}$ be the set of factors used to evaluate a proposal $y \rightarrow y'$

$$\text{i.e. } \{\psi\}_y^{y'} = \left(\{\psi\}_y \cup \{\psi\}_{y'}\right) - \left(\{\psi\}_y \cap \{\psi\}_{y'}\right)$$

Consider two proposals $y \rightarrow y_a$ and $y \rightarrow y_b$ such that,

$$\{\psi\}_y^{y_a} \cap \{\psi\}_y^{y_b} = \{\}$$

Completely different set of factors are required to evaluate these proposals.

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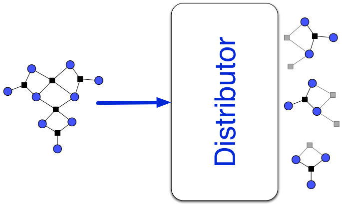
These two proposals can be evaluated (and accepted) in parallel.

Distributed Inference



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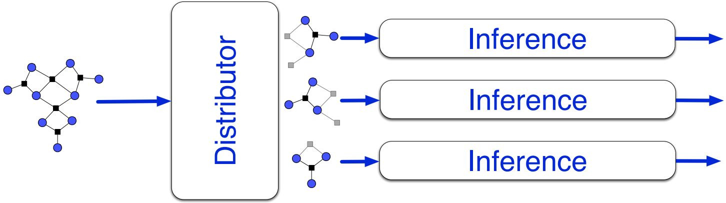

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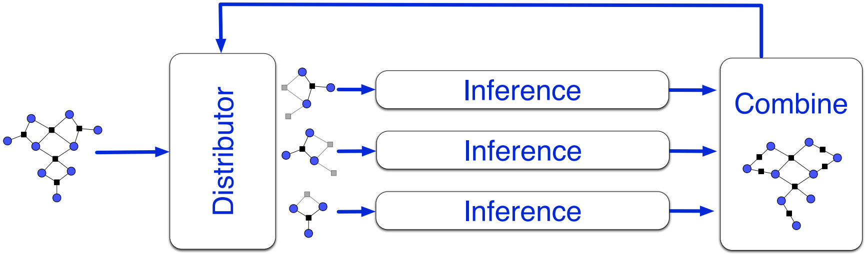


Outline

Coreference Problem

... The Physiological Basis of Politics," by **Kevin B. Smith**, Douglas Oxley, Matthew Hibbing...

...during the late 60's and early 70's, **Kevin Smith** worked with several local...

...the term hip-hop is attributed to **Lovebug Starski**. What does it actually mean...

The filmmaker **Kevin Smith** returns to the role of Silent Bob...

Nothing could be more irrelevant to **Kevin Smith**'s audacious "Dogma" than ticking off...

Firefighter **Kevin Smith** spent almost 20 years preparing for Sept. 11. When he...

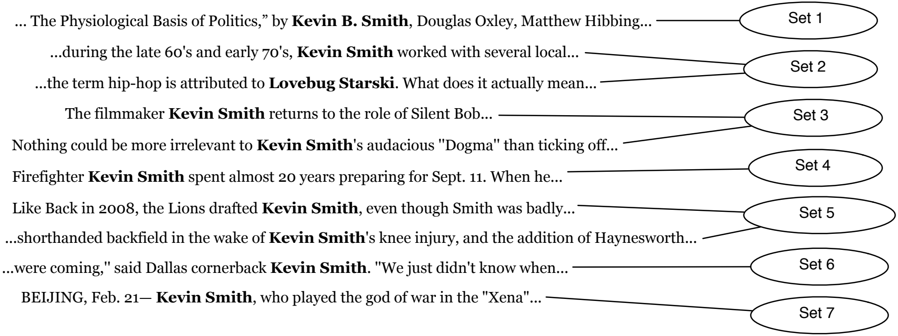
Like Back in 2008, the Lions drafted **Kevin Smith**, even though Smith was badly...

...shorthanded backfield in the wake of **Kevin Smith**'s knee injury, and the addition of Haynesworth...

...were coming," said Dallas cornerback **Kevin Smith**. "We just didn't know when...

BEIJING, Feb. 21— **Kevin Smith**, who played the god of war in the "Xena"...

Coreference Problem



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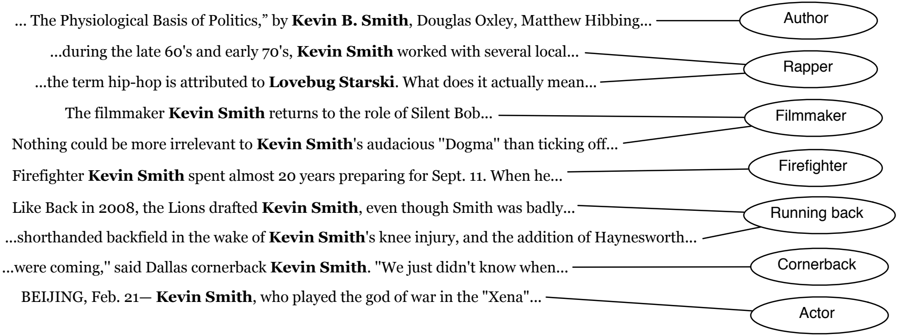
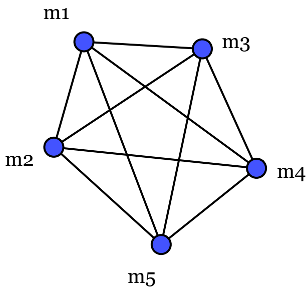


Input Features



Define similarity between mentions, $\phi : \mathcal{M}^2 \to \mathcal{R}$

- $\phi(m_i, m_j) > 0$: m_i, m_j are similar
- $\phi(m_i, m_j) < 0$: m_i, m_j are dissimilar

We use cosine similarity of the context bag of words:

$$\phi(m_i, m_j) = cosSim(\{c\}_i, \{c\}_j) - b$$

Graphical Model

The random variables in our model are entities (E) and mentions (M)

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For any assignment to these entities ($E = e$), we define the model score:

$$p(E = e) \propto \exp\left\{\sum_{m_i \sim m_j} \psi_a(m_i, m_j) + \sum_{m_i \nsim m_j} \psi_r(m_i, m_j)\right\}$$

where $\psi_a(m_i, m_j) = w_a \phi(m_i, m_j)$, and
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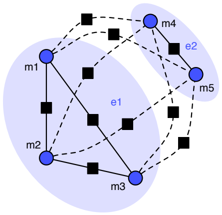

For the following configuration,

$$p(e_1, e_2) \propto \exp \Big\{ \; w_a \left(\phi_{12} + \phi_{13} + \phi_{23} + \phi_{45}\right) - w_r\big(\phi_{15} + \phi_{25} + \phi_{35} + \phi_{14} + \phi_{24} + \phi_{34}\big)\Big\}$$

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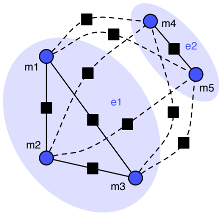

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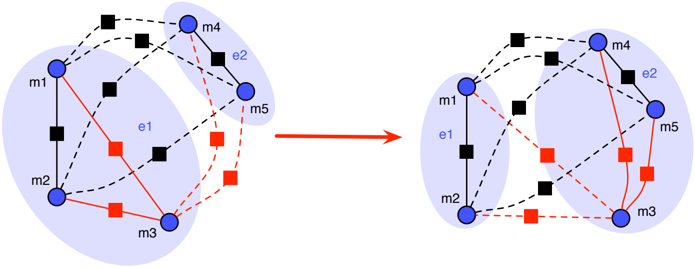
1. Space of E is Bell Number(n) in number of mentions
2. Evaluating model score for each $E = e$ is $O(n^2)$

MCMC for MAP Inference

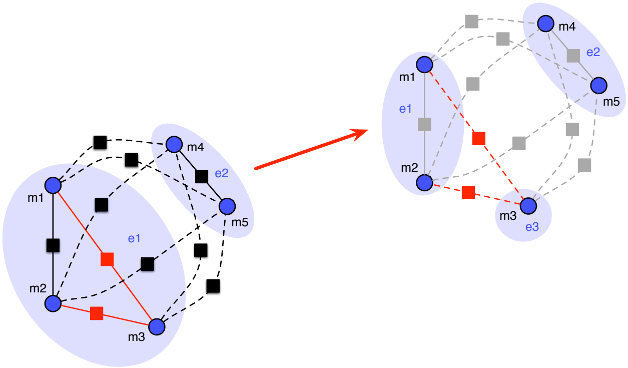


$$p(e) \propto \exp\{w_a\left(\phi_{12} + \phi_{13} + \phi_{23} + \phi_{45}\right) - w_r\left(\phi_{15} + \phi_{25} + \phi_{35} + \phi_{14} + \phi_{24} + \phi_{34}\right)\}$$

$$p(\acute{e}) \propto \exp\{w_a\left(\phi_{12} + \phi_{34} + \phi_{35} + \phi_{45}\right) - w_r\left(\phi_{15} + \phi_{25} + \phi_{13} + \phi_{14} + \phi_{24} + \phi_{23}\right)$$

$$\log \frac{p(\acute{e})}{p(e)} = w_a\left(\phi_{34} + \phi_{35} - \phi_{13} - \phi_{23}\right) - w_r\left(\phi_{13} + \phi_{23} - \phi_{34} - \phi_{35}\right)$$

Mutually Exclusive Proposals



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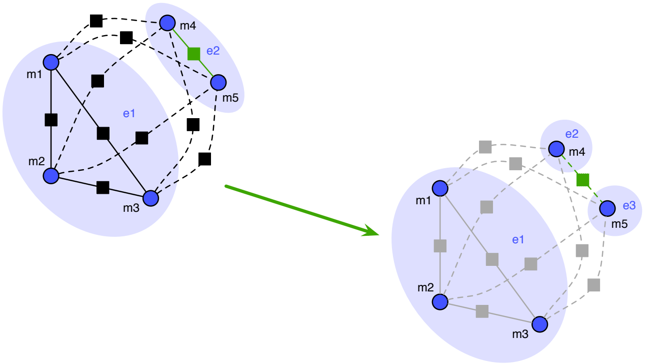

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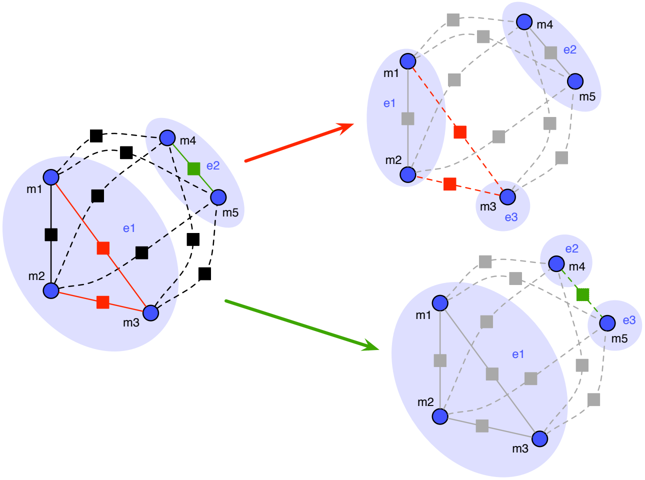
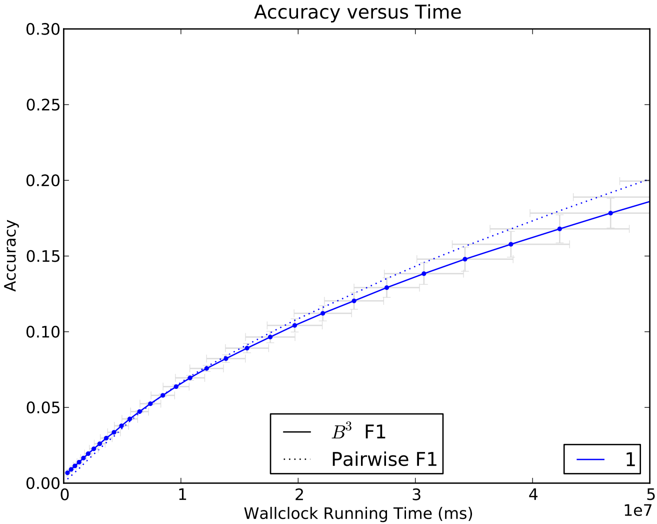
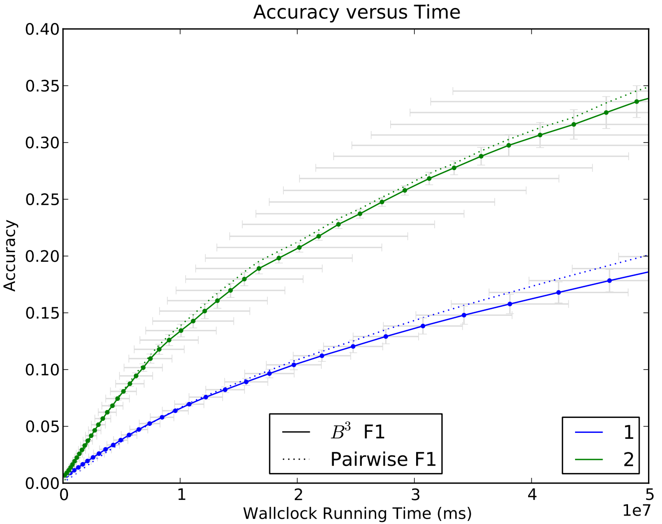


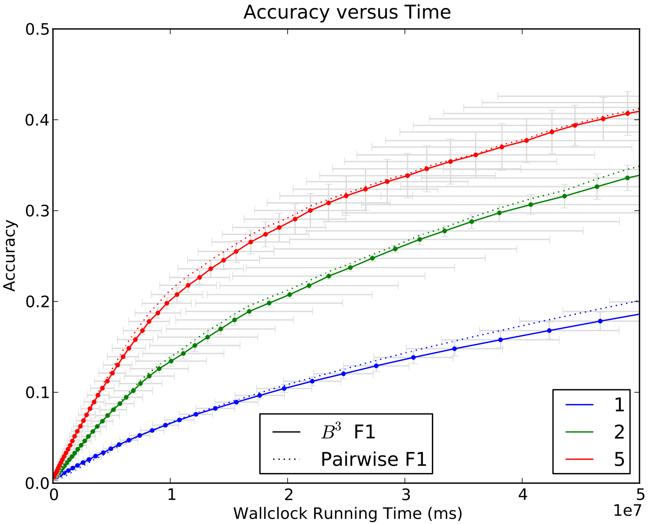
Results


Accuracy versus Time
Accuracy
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
1e7
Wallclock Running Time (ms)
B^3 F1
Pairwise F1
1

Results


Accuracy versus Time
Accuracy
Wallclock Running Time (ms)
1e7
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0
1
2
3
4
5
B^3 F1
Pairwise F1
1
2

Results


Accuracy versus Time
Accuracy
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1e7
0.0
0.1
0.2
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2
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4
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B^3 F1
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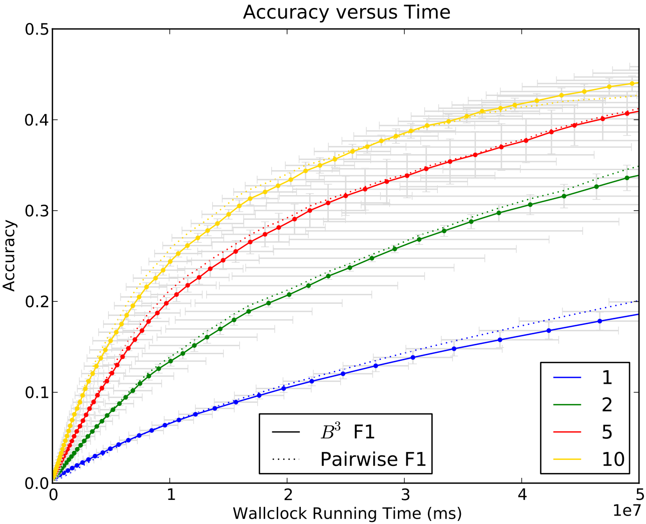
1
2
5

Results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2
3
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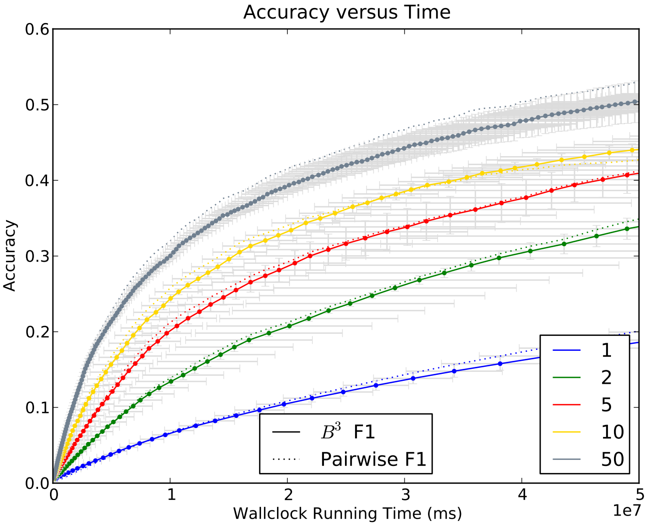
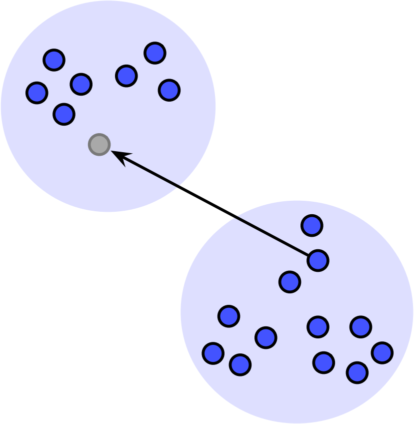
1
2
5
10

Results


Accuracy versus Time
Accuracy
Wallclock Running Time (ms)
1e7
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
B^3 F1
Pairwise F1
1
2
5
10
50

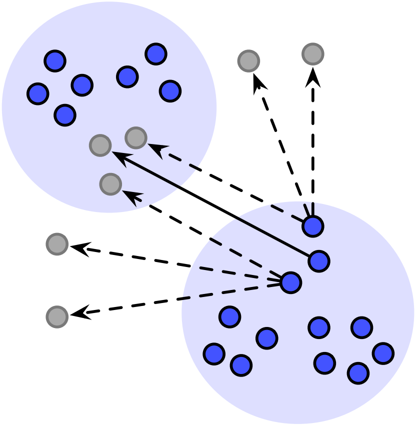
Outline

Sub-Entities



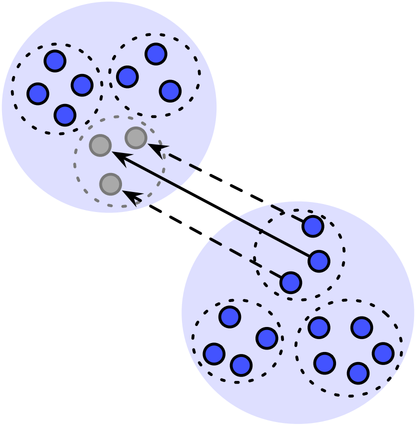
- Consider an accepted move for a mention

Sub-Entities



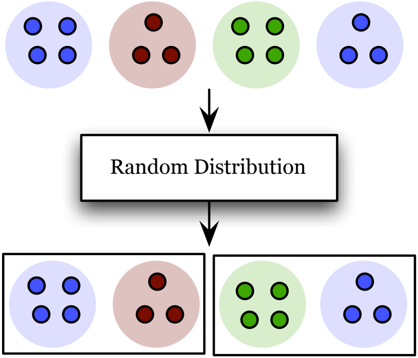
- Ideally, *similar* mentions should also move to the same entity
- Default proposal function does not utilize this
- *Good* proposals become more rare with larger datasets

Sub-Entities



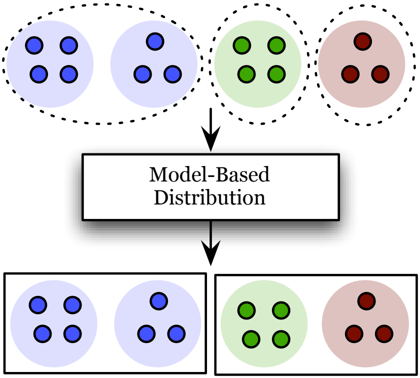
- Include Sub-Entity variables
- Model score is used to sample sub-entity variables
- Propose moves of mentions in a sub-entity simultaneously

Super-Entities



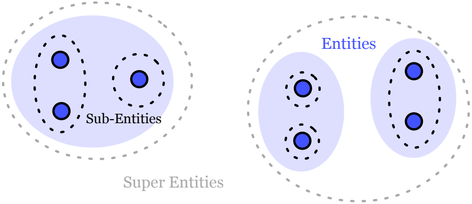
- Random distribution may not assign *similar* entities to the same machine
- Probability that similar entities will be assigned to the same machine is small

Super-Entities



- Augment model with Super-Entities variables
- Entities in the same super-entity are assigned the same machine
- Model score is used to sample super-entity variables

Hierarchical Representation



- **Factors**
  - Affinity factors between mentions / sub-entities / entities in the same sub-entities / entities / super-entities
  - Repulsion factors are similarly symmetric across levels

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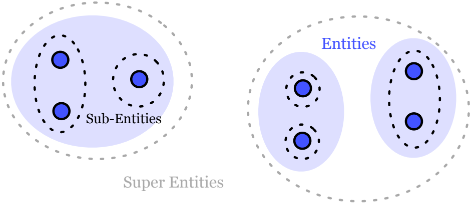
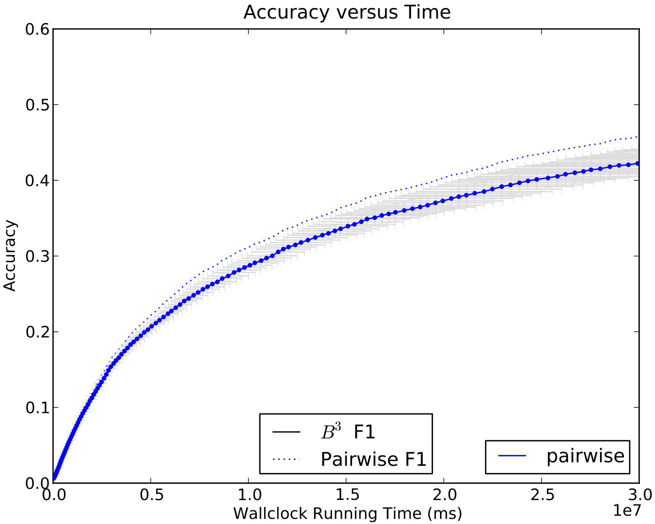


- **Factors**
  - Affinity factors between {mentions / sub-entities / entities} in the same {sub-entities / entities / super-entities}
  - Repulsion factors are similarly symmetric across levels
- **Sampling:** Fix variables of two levels, sample the remaining level

Evaluation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B^3 F1
Pairwise F1
pairwise

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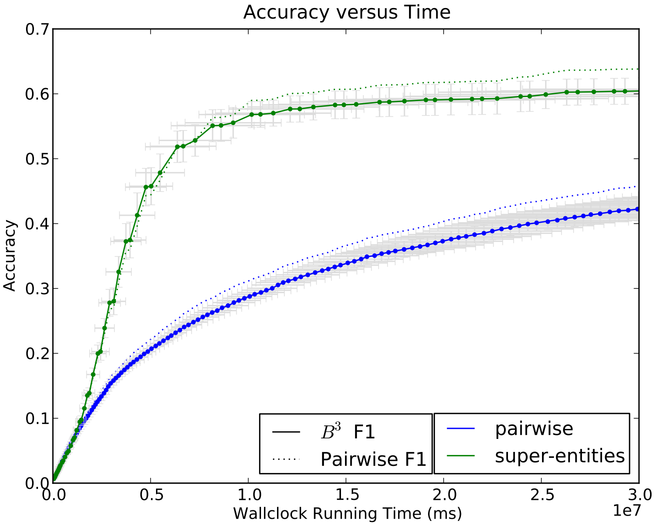
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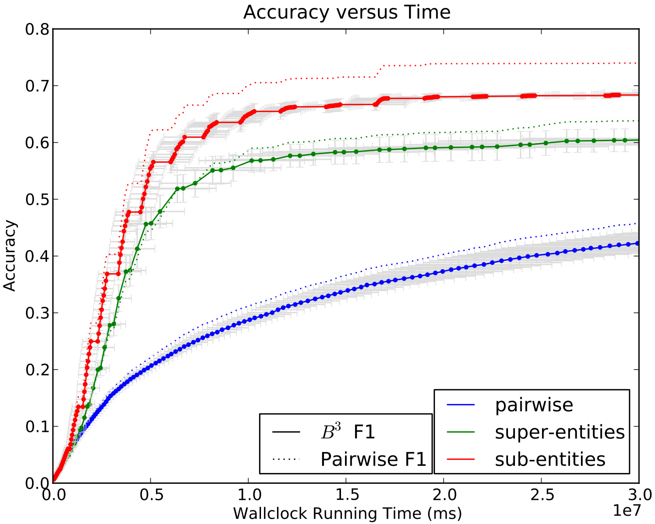
pairwise
super-entities

Evaluation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0.2
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0.6
0.7
0.8
0.0
0.5
1.0
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2.5
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B^3 F1
Pairwise F1
pairwise
super-entities
sub-entities

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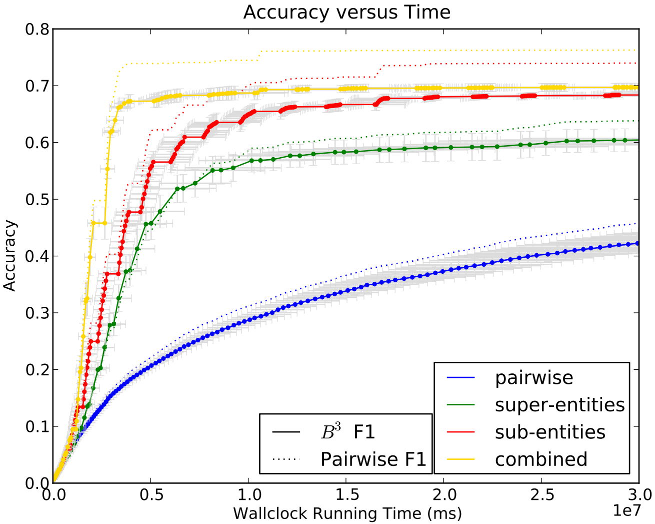
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1.5
2.0
2.5
3.0
B^3 F1
Pairwise F1
pairwise
super-entities
sub-entities
combined

Outline

Preliminary Large-Scale Experiments

Data

- *New York Times Annotated Corpus* [Sandhous LDC 2008]
  20 years of articles (1987-2007)
- prune rare names ($<$1000): $\sim$1 million person name mentions

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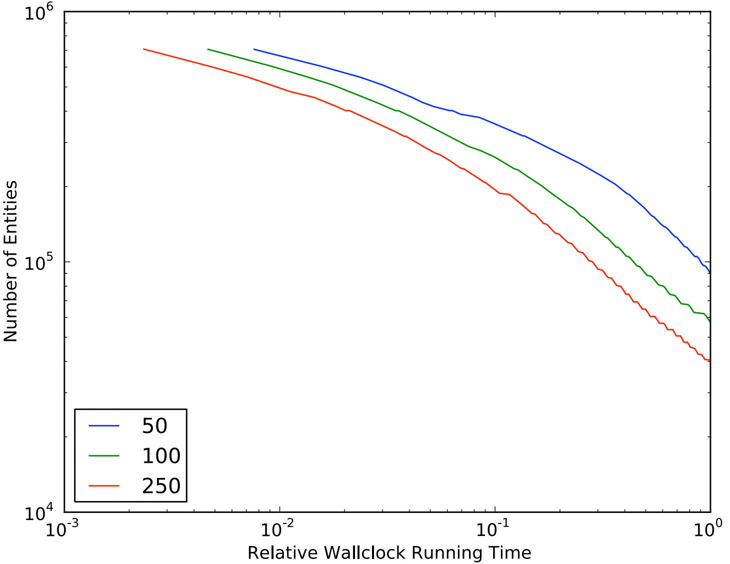
Evaluation

- Automated labels are too noisy for evaluation
- Instead, we estimate the speed of inference
  - trust the model to accept good proposals
  - observe the number of predicted entities

Speed of Inference



Related Work

- GraphLab [Low et al. UAI 2010]
  - how do we represent dynamic graphs
  - how do we represent hierarchical models

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    - how do we represent hierarchical models
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Related Work

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  - how do we represent dynamic graphs
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  - graph structure changes with every configuration
  - BP messages are enormous for exponential-domain variables
- Topic Models [Smola & Narayanmurthy. VLDB 2010, Asuncion et al. NIPS 2009]
  - restrictions since they are calculating probabilities
  - we allow non-random distribution and customized proposals

Conclusions

1. propose distributed inference for graphical models
2. enable distributed cross-document coreference
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4. demonstrate utility on large datasets

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Future Work:

- more scalability experiments
- study mixing and convergence properties
- add more expressive factors
- supervision: labeled data, noisy evidences

Thanks!

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