Composition of Semantic Relations: Theoretical Framework and Case Study

EDUARDO BLANCO and DAN MOLDOVAN, The University of Texas at Dallas

Extracting semantic relations from text is a preliminary step towards understanding the meaning of text. The more semantic relations are extracted from a sentence, the better the representation of the knowledge encoded into that sentence. This article introduces a framework for the Composition of Semantic Relations (CSR). CSR aims to reveal more text semantics than existing semantic parsers by composing new relations out of previously extracted relations. Semantic relations are defined using vectors of semantic primitives, and an algebra is suggested to manipulate these vectors according to a CSR algorithm. Inference axioms that combine two relations and yield another relation are generated automatically. CSR is a language-agnostic, inventory-independent method to extract semantic relations. The formalism has been applied to a set of 26 well-known relations and results are reported.

Categories and Subject Descriptors: I.2.7 [**Artificial Intelligence**]: Natural Language Processing—*Language parsing and understanding*

General Terms: Algorithms, Design, Theory, Experimentation

Additional Key Words and Phrases: Semantic relations, relation extraction, relation inference

ACM Reference Format:
Blanco, E. and Moldovan, D. 2013. Composition of semantic relations: Theoretical framework and case study. ACM Trans. Speech Lang. Process. 10, 4, Article 17 (December 2013), 36 pages.
DOI: http://dx.doi.org/10.1145/2513146

17

1. INTRODUCTION

Semantic representation of text is key to text understanding and reasoning. Improvements to these tasks dramatically impact the performance of natural language processing applications like question answering [Shen and Lapata 2007], text summarization [Gong and Liu 2001], information extraction [Surdeanu et al. 2003], and machine translation [Wu and Fung 2009] to name a few.

Semantic relations are a concise and formal way of representing text semantics. Broadly speaking, they are unidirectional links underlying connections between concepts. For example, the noun phrase *car engine* encodes a PART-WHOLE relation: the *engine* is a part of the *car*. The sentence *The construction slowed down the traffic* encodes a CAUSE relation: *the construction* is the cause of *slowing down* the traffic and detecting it would help answer questions like *Why is traffic slower?*

Semantic parsers attempt to extract meaning representations from free text. The methods of representing the meaning of text vary: some are based on extracting semantic relations, while others consider extensions to first-order logic [Poon and Domingos 2009], logical forms [Allen et al. 2008], and other formal representations [Bos 2008]. A

Authors' addresses: E. Blanco (corresponding author), D. Moldovan, Human Language Technology Research Institute, The University of Texas at Dallas, 800 W. Campbell Rd., Richardson, TX 75080; email: eduardo@hlt.utdallas.edu.

DOI: http://dx.doi.org/10.1145/2513146

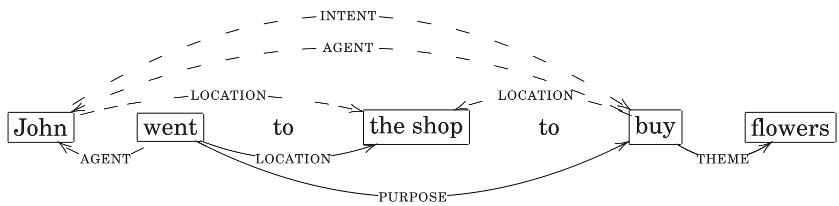

Fig. 1. Semantic representation of the sentence *John went to the shop to buy flowers*. Solid arrows indicate relations extracted by a semantic parser; discontinuous arrows indicate composed relations, that is, relations inferred out of two already known relations.

broad survey of computational semantics with a focus on classical logic can be found in Bos [2011]. Semantic relation extraction has received considerable attention, including numerous competitions with dozens of participants [Carreras and Màrquez 2004; 2005; Diab et al. 2007; Màrquez et al. 2007; Girju et al. 2007; Ruppenhofer et al. 2009; Hendrickx et al. 2009].

We believe that using a fixed set of dyadic semantic relations is better suited to represent the meaning of text than predicate calculus. Dyadic relations have been used by Chklovski and Pantel [2004], Girju et al. [2005], Chang and Choi [2006], Srikumar et al. [2008], Bethard et al. [2008], Tratz and Hovy [2010], and many others.

Consider the following sentence.

(1) John gives Mary flowers.

In first-order logic, (1) may be represented as *Gives*(*John*, *flowers*, *Mary*), where, using their jargon, *Gives* is a predicate that takes three terms. Using dyadic relations, (1) may be represented as AGENT(*gives*, *John*) & THEME(*gives*, *flowers*) & RECIPIENT(*gives*, *Mary*). Both options capture the same meaning, but the latter uses a fixed set of predefined relations. On the other hand, traditional logic uses a large number of predicates which typically grows as more text is processed, for example, *Gives* is added to the list of predicates after processing sentence (1). Using a fixed set of relations known a priori facilitates reasoning.

In this article, we venture beyond semantic relation extraction from text and investigate techniques to infer semantic relations ignored by existing semantic parsers. We explore the idea of inferring a new relation linking the two ends of a chain of relations already known. This scheme, informally used before for combining HYPERNYM with other relations (e.g., [Clark and Harrison 2008]), has not been studied for pairs of relations not involving hypernymy. For example, it seems adequate to state the following: since "*convertible*" has HYPERNYM "*car*" and "*car*" has PART "*engine*", one can infer "*convertible*" has PART "*engine*". Going a step further, we consider nonobvious inferences involving AGENT, PURPOSE, and other relations.

Consider the sentence *John went to the shop to buy flowers*. Figure 1 shows the semantic relations a semantic parser extracts with solid arrows:[1] "*went*" has AGENT "*John*", LOCATION "*the shop*", and PURPOSE "*buy*"; "*buy*" has THEME "*flowers*". By using these relations in composition with automatically obtained inference axioms, one can infer the relations shown with discontinuous arrows: "*buy*" has AGENT "*John*", "*John*" had the INTENT to "*buy*", the "*buying*" event has LOCATION "*the shop*" and "*John*" had LOCATION "*the shop*". This observation that new relations can be composed out of two relations previously extracted was the starting point and the motivation for this work.

This article presents a theoretical framework for composing new semantic relations and a detailed case study with 26 well-known relations. We do not advocate any

[1]Solid arrows correspond to typical semantic roles, that is, relations between a verb and its arguments.

particular relation inventory and have previously applied Composition of Semantic Relations (CSR) to PropBank [Blanco and Moldovan 2011a] and a set of 8 relations [Blanco and Moldovan 2011b]. The work presented here builds on our previous work, while the main novelties are: (1) a generic model to compose any set of semantic relations; (2) an extended set of 14 semantic primitives and a detailed explanation of them, their interactions, and dependencies; (3) an extended algebra to compose the 14 primitives; (4) a detailed case study applying CSR to a set of 26 well-known relations; and (5) the list of 216 inference axioms automatically obtained.

CSR aims to infer unrevealed implicit semantic relations out of previously detected relations. It offers a formalism for Semantic Calculus (SC), a calculus of semantic relations. CSR is based on an extended definition of semantic relations consisting of domain and range restrictions coupled with semantic primitives. This extended definition allows us to reason and manipulate relations based on their semantic properties: a relations R_3 can be composed out of R_1 and R_2 if their domains, ranges, and primitives are congruent. Unlike most approaches to relation extraction, CSR does not take into account lexical or syntactic information. Instead, it works on top of previously extracted semantic relations.

The CSR algorithm presented here automatically obtains inference axioms which allow us to compose with high accuracy relations ignored by existing semantic parsers. The extended definition, primitives, algebra, and the CSR algorithm are independent of any particular set of relations. In other words, the framework presented here can be applied to any relation inventory.

1.1. Terminology

Within this article, we use the term *semantic relation* (*relation* for short) to refer to a labeled dyadic semantic connection between two concepts and *Semantic Parser* (SP) to refer to an automatic tool that extracts semantic relations from text. For example, AGENT(*gives*, *John*) encodes the semantic relation "*gives*" has AGENT "*John*". Examples of semantic parsers are semantic role labelers (e.g., [Koomen et al. 2005]) and the systems presented in SemEval-2010 Task 8 [Hendrickx et al. 2009].

We say that a relation is *used in composition* if it is used to infer another relation. A relation is *composed* if it is inferred by combining two existing relations, or alternatively, a relation is *composed* out of two relations used in composition. Finally, the process of deriving a composed relation is called *composition*. For example, in Figure 1, INTENT(*John*, *buy*) is composed out of AGENT(*went*, *John*) and PURPOSE(*went*, *buy*); AGENT(*went*, *John*) and PURPOSE(*went*, *buy*) are used in composition to infer (or compose) INTENT(*John*, *buy*).

2. COMPARISON WITH PREVIOUS WORK

2.1. Extracting One or More Relations without Composition

There have been many proposals and implementations to detect semantic relations without taking into account composition of relations. All these approaches take as their input text and output semantic relations found in it. On the other hand, CSR extracts relations by combining previously extracted relations.

Generally, efforts to extract semantic relations concentrate either on a set of relations or a particular relation. The cardinalities of sets vary, for example, Szpakowicz et al. [1995] use a set of 9 relations, Turney [2006] a set of 5, and Rosario and Hearst [2004] a set of 38. Proposals to detect a particular relation include CAUSE [Bethard and Martin 2008; Girju and Moldovan 2002; Chang and Choi 2006], INTENT [Tatu 2005], PART-WHOLE [Artale et al. 1996; Girju et al. 2006], and IS-A [Hearst 1992]. They usually focus on specific lexico-syntactic patterns or kind of arguments. There is work on detecting relations within noun phrases [Nulty 2007; Moldovan et al. 2004], named entities [Hirano et al. 2007], clauses [Szpakowicz et al. 1995], and syntax-based comma resolution [Srikumar et al. 2008]. Unsupervised approaches include Davidov et al. [2007], Davidov and Rappoport [2008], and Turney [2006].

Automatic detection of semantic roles has received considerable attention lately [Màrquez et al. 2008; Carreras and Màrquez 2005] and it has been proven to be useful for coreference resolution [Ponzetto and Strube 2006], word sense disambiguation [Dang and Palmer 2005], machine translation [Wu and Fung 2009], and other applications. The SemEval-2007 Task 04 [Girju et al. 2007] and SemEval-2010 Task 08 [Hendrickx et al. 2009] aimed at relations between nominals. The former tackled the problem of deciding whether or not a given relation holds (binary classification); the latter tackled the more complicated problem of deciding which relation (out of 9) holds.

Similar in their goal, but different in their approaches, two recent works are somewhat related to CSR [Ruppenhofer et al. 2009; Gerber and Chai 2010]. The common goal is to detect implicit relations missed by state-of-the-art semantic parsers. Their means, though, are completely different; these proposals are supervised. They are novel because they point out, annotate, and provide supervised models to detect relations ignored by existing SP. Ruppenhofer et al. [2009] focus on null instantiations for events in accordance to PropBank framesets [Palmer et al. 2005] and FrameNet frames [Baker et al. 1998]; Gerber and Chai [2010] focus on implicit arguments for nominals following NomBank [Meyers et al. 2004]. Unlike CSR, both of them depend on the relation inventory, require annotation, and do not reason with relations. Their merit relies on considering relations between concepts not targeted in the original resources (e.g., concepts far apart from each other, concepts from two different sentences).

In contrast to all the preceding references, the CSR proposed here obtains axioms that take previously extracted relations as input and infer new relations. Moreover, CSR is not coupled to any relation inventory.

2.2. Describing Semantic Relations Using Semantic Primitives

Previous research has exploited the idea of using semantic primitives to define and classify semantic relations under different names. Among others, the literature uses *relation elements*, *deep structure*, *aspects*, and *primitives*. In theoretical linguistics, Wierzbicka [1996] introduced semantic primes (often referred to as semantic primitives), but they are unrelated to our semantic primitives.

Chaffin and Herrmann [1987] introduce relation element theory and differentiate a set of 31 relations clustered in five groups (contrast, similars, class inclusion, case-relations, part-whole) with 30 *relation elements* also clustered into five groups (elements of intensional force, dimension elements, elements of agreement, propositional elements, elements of part-whole inclusion). They only use *elements* to define relations, not to compose them. Winston et al. [1987] use 3 *relation elements* (functional, homeomerous, and separable) to distinguish between six subtypes of PART-WHOLE. They only discuss the transitivity of the different subtypes of PART-WHOLE, while they do not consider other relations.

Cohen and Losielle [1988] introduce the notion of *deep structure* of a relation in contrast to the *surface relation*. They only use two *aspects*: *hierarchical* (similar to the primitives *structural* and *intangible* we use in this article) and *temporal*. Huhns and Stephens [1989] extend that previous work by considering a set of 10 primitives.

2.3. Composing Relations

As far as we know, there has not been extensive research on composing relations in the field of computational linguistics. The term *compositional semantics* is used in conjunction with the principle of compositionality, that is, the meaning of a complex expression is determined from the meanings of its parts, and the way in which those parts are combined. These approaches are usually formal and complex, and use a potentially infinite set of predicates to represent meaning. Ge and Mooney [2009] extract semantic representations using syntactic structures while Copestake et al. [2001] develop algebras for semantic construction within grammars. Logic approaches include Lakoff [1970] and Sánchez Valencia [1991], and extensions to natural logic [MacCartney and Manning 2008, 2009]. Also, within compositional semantics, there are ongoing efforts to capture the meaning of text by composing vector representations [Mitchell and Lapata 2008; Erk and Padó 2008; Coecke et al. 2011]. Composition of semantic relations is complimentary to compositional semantics.

Harabagiu and Moldovan [1998] study the composition of WordNet relations and manually extract plausible inference axioms. Their only restriction for combining relations is part-of-speech compatibility. Helbig [2005] transforms chains of relations into theoretical axioms. Unlike these two previous efforts, the CSR framework presented here automatically obtains inference axioms.

Composing relations has been proposed before in the more general field of artificial intelligence, in particular in the context of knowledge bases. Cohen and Losielle [1988] work with a set of nine fairly specific relations (CAUSES, COMPONENT-OF, FOCUS-OF, MECHANISM-OF, PRODUCT-OF, PURPOSE-OF, SETTING-OF, SUBJECT-OF, and SUBFIELD-OF) and two *aspects*. They note the key to determine plausibility is the *transitivity characteristic* of the aspect: two relations shall be used in composition if and only if they do not have contradictory values for any aspect. Huhns and Stephens [1989] were the first to propose an algebra for composing semantic primitives. They use 10 primitives and 21 relations. Their relations are not linguistically motivated and ten of them map to some subtype of PART-WHOLE (e.g., PIECE-OF, SUBREGION-OF). Unlike Cohen and Losielle [1988] and Huhns and Stephens [1989], we use semantic relations widely used to encode the meaning of natural language, propose a method to automatically obtain the inverse, and empirically evaluate the validity of inferred relations.

3. SEMANTIC RELATIONS AND SEMANTIC PRIMITIVES

Semantic relations are the underlying connections between concepts expressed by words or phrases. They are implicit associations between concepts. For example, in the sentence *John drives his truck to work*, "*drives*" has AGENT "*John*", THEME "*his truck*" and LOCATION "*to work*"; additionally, "*John*" has POSSESSION "*truck*".

In general, researchers define semantic relations by stating the kind of connection they encode and giving a few examples. This only provides a broad understanding of the actual semantic connection between the concepts. For example, the PropBank annotation guidelines[2] define MANNER with the following text.

> Manner adverbs specify how an action is performed. For example, "works well with others" is a manner. Manner tags should be used when an adverb be an answer to a question starting with 'how?'.

The PropBank guidelines also provide three annotation examples, for example, in *Among 33 men who worked closely with the substance [. . .]*, "*worked*" has MANNER "*closely*". We note that despite the definition of MANNER, PropBank annotates manners that are not encoded by adverbs, for example, in *The next morning, with a police escort, busloads of executives and their wives raced to the Indianapolis Motor Speedway [. . .]*, "*raced*" has MANNER "*with a police escort*" is annotated.

Similarly, Srikumar et al. [2008] loosely define ATTRIBUTE as "*a relation where one argument describes an attribute of the other*" and give a couple of examples: from *John,*

[2]http://verbs.colorado.edu/∼mpalmer/projects/ace/PBguidelines.pdf, page 26.

who loved chocolate, ate with gusto, they annotate "*John*" has ATTRIBUTE "*loved chocolate*" and "*John*" has ATTRIBUTE "*ate with gusto*".

A third example is VerbOcean [Chklovski and Pantel 2004], which defines ENABLEMENT as a relation that holds "*between two verbs V_1 and V_2 when the pair can be glossed as V_1 is accomplished by V_2*", and provide two examples: "*assess*" ENABLES "*review*" and "*accomplish*" ENABLES "*complete*". We cannot provide a more thorough definition for the relations used by these three previous works because their authors did not provide more details.

We find these widespread kinds of definitions weak and prone to confusion. For example, following the aforesaid MANNER definition in the sentence *The legislation itself noted that it was* [*introduced*]$_{predicate}$ [*"by request"*]$_{PP}$,[3] one could label the link between the predicate and the PP as MANNER whereas we believe it is a CAUSE relation. Indeed, PropBank annotates this as MANNER. The rationale behind our thought is that hadn't the request took place, the law would not have been introduced. Moreover, *request* and *was introduced* both encode events and the former temporally precedes the latter. Intuitively, the manner of an event occurs at the same time than the event (e.g., *walking* [*holding hands*]$_{\text{MANNER}}$): if "*y*" occurred before "*x*", "*y*" cannot be the manner in which "*x*" took place. As we shall see, our extended definition for semantic relations formally and succinctly enforces this kind of properties with semantic primitives.

3.1. Extended Definition of Semantic Relations

We propose an extended definition for semantic relations, including: (1) semantic restrictions on the first and second argument; and (2) semantic primitives. Semantic restrictions are defined using an ontology. Semantic primitives indicate if a certain property holds between the arguments of a relation. For example, the primitive *temporal* indicates if the first argument must happen before the second in order for the relation to hold.

Besides providing a better understanding of each relation, this extended definition allows us to: (1) identify pairs of relations that can be used in composition; (2) mark identified pairs as prohibited based on semantic grounds; (3) determine which relations can be composed out of a nonprohibited identified pair, if any. The CSR algorithm performs the aforementioned tasks and automatically generates inference axioms to compose relations.

Formally, a semantic relation is represented as R(x, y), where R is the relation type and "x" and "y" the first and second argument. R(x, y) could be read as *"x" has* R *"y"*, for example, AGENT(*came*, *John*) could be read *"came"* has AGENT "*John*". As we shall see, it is sometimes convenient to reverse the arguments of a relation. We introduce the inverse relation for this purpose. Given R(x, y), R^{-1}(y, x) always hold. For example, instead of AGENT(*came*, *John*), we will sometimes use AGENT^{-1}(*John*, *came*). The easiest way to read R^{-1}(y, x) is *"y" is* R *of "x"*, for example, AGENT^{-1}(*John*, *came*) could be read "*John*" is AGENT of "*came*".

3.1.1. Domain and Range Restrictions. Given a semantic relation R, DOMAIN(R) and RANGE(R) are defined as the sorts of concepts that can be the first and second argument respectively. In order to define domains and ranges one may use any existing ontology or define his own. Intuitively, when considering if R(x, y) holds, one should first check for domain and range compatibility. Formally, "x" and "y" are compatible as arguments of R iff

$$x \in \text{DOMAIN}(\text{R}) \text{ and } y \in \text{RANGE}(\text{R}).$$

[3]PropBank, file wsj_0041, sentence 47.

Adding domain and range restrictions has several advantages.

—It helps to distinguish between different relations. Since different relations have different domains and ranges, given a pair of arguments only a few relations can hold. For example, "*John*", which encodes an animate concrete object, can be linked to "*tall*", which is a quality through VALUE, but not POSSESSION.

—It helps to discard potential relations that do not hold. A potential relation R(x, y) can be discarded if its arguments are not compatible with DOMAIN(R) and RANGE(R). For example, "*wind*" (abstract object), cannot be part of DOMAIN(INTENT). Therefore INTENT(*wind*, x) can be discarded even before "x" is instantiated.

—It helps to compose semantic relations. By checking domains and ranges, one can retrieve the pairs of relations that can be used in compostion (Section 4).

3.1.2. Semantic Primitives. Semantic primitives capture deep characteristics of relations. Huhns and Stephens [1989] define them as follows.

> They [primitives] are independently determinable for each relation and relatively self-explanatory. They specify a relationship between an element of the domain and an element of the range of the semantic relation being described.

Semantic primitives are fundamental properties that cannot be explained using other primitives; they are elemental. They specify basic attributes of a relation by stating whether a particular property must hold by definition between the arguments of the relation. Primitives help understand inter-relation differences and clustering relations based on their semantic properties. They can be used as conditions to be fulfilled in order to determine whether a potential relation holds. Primitives indicate properties that hold for all instances of a relation.

Each relation takes a value for each primitive from the set $V = \{-, 0, +\}$, where "$-$" indicates that the primitive does not hold, "0" that it does not apply, and "$+$" that it holds. For example, the primitive *volitional* indicates whether or not a relation requires volition between the arguments; AGENT takes "$+$" and PART-WHOLE takes "0".

3.1.3. The List of Semantic Primitives. Our set of primitives (Table I) is grounded on previous work in knowledge bases [Winston et al. 1987; Cohen and Losielle 1988; Huhns and Stephens 1989], but we consider more primitives. The addition of new primitives is justified by the fact that we aim at composing relations capturing the meaning of natural language. Whatever the set of relations, the primitives describe the characteristics of events (who/what/where/when/why/how something happened), connections between events (e.g., CAUSE, CORRELATION) and associations between concepts (e.g., PART-WHOLE, IS-A). The fourth column in Table I indicates the value of each primitive for the inverse relation. *Id* means that the inverse takes the same value and *op* that it takes the opposite value. The opposite of "$-$" is "$+$", the opposite of "$+$" is "$-$", and the opposite of "0" is "0". For example, if a relation R takes "$+$" for *homeomerous*, then R^{-1} also takes "$+$". However, if R takes "$+$" for *temporal*, then R^{-1} takes "$-$".

AGENT is defined with the following values for the semantic primitives $P_{\text{AGENT}} = \{+, +, -, +, 0, 0, 0, 0, -, -, +, -, 0, 0\}$, indicating that $P^3_{\text{AGENT}} = -$ and $P^{11}_{\text{AGENT}} = +$. Thus, AGENT($x$, y) does not require "x" and "y" to be of the same kind (homeomerous) and it requires volition between the arguments (volitional).

This set of primitives is not guaranteed to be the best for any relation inventory. There is no implicit assumption that primitives are independent of each other (i.e., certain values for a primitive restrict the possible values for other primitives). However, we show that using these primitives is useful (Section 6). The remainder of this section

Table I. Primitives for Characterizing Semantic Relations and Values for the Inverse Relation

No.	Primitive	Description	Inv.	Ref.
1	Composable	Relation between "x" and "y" can be meaningfully used in composition due to its fundamental characteristics	id.	[3]
2	Functional	"x" is in a specific spatial or temporal position with respect to "y" in order for the connection to exist	id.	[1]
3	Homeomerous	"x" must be the same kind of thing as "y"	id.	[1]
4	Separable	"x" can be temporally or spatially separated from "y", "x" can thus exist independently of "y"	id.	[1]
5	Structural	There is a hierarchical relationship in terms of a physical structure; "x" is subsumed by "y"	op.	[2]
6	Temporal	"x" temporally precedes "y"	op.	[2]
7	Intangible	"x" is owned or mentally included in "y"	op.	[3]
8	Near	"x" is physically or temporally close to "y"	id.	[3]
9	Connected	"x" is physically or temporally connected to "y"; connection might be indirect.	id.	[3]
10	Intrinsic	Relation is an attribute of the essence/stufflike nature of "x" or "y"	id.	[3]
11	Volitional	Relation requires volition between the arguments	id.	-
12	Universal	Relation is always true between the arguments	id.	-
13	Fully Implicational	The existence of "x" implies the existence of "y"	op.	-
14	Weakly Implicational	The existence of "x" sometimes, but not necessarily always, implies the existence of "y"	op.	-

Primitives 11–14 are proposed by us, "x" is used to refer to the first argument of a relation, and "y" to the second. In the fifth column, [1] stands for Winston et al. [1987], [2] for Cohen and Losielle [1988], and [3] for Huhns and Stephens [1989].

gives insights on each primitive and exemplifies values for different relations. We use "x" to refer to the first argument of a relation, and "y" to the second argument.

(1) *Composable* indicates if a relation can be used in composition by definition. Simply put, if a relation R has $P_{\text{R}}^{composable} = -$, then R cannot be used to compose another relation. Values are assigned based on the nature of relations. For example, since KINSHIP encodes a very broad connection between two concepts, namely, that two concepts are related by blood or marriage, we have $P_{\text{KINSHIP}}^{composable} = -$.

(2) *Functional* indicates if a relation R requires the arguments to be in specific spatial or temporal positions with respect to each other. We have $P_{\text{AGENT}}^{functional} = +$ because the agent of an action must be in a specific space and time frame with respect to the action it performs. On the other hand, $P_{\text{HYPERNYM}}^{functional} = -$ since a concept and its hypernyms are connected via HYPERNYM regardless of their spatial and temporal positions.

(3) *Homeomerous* specifies if arguments must be of the same kind of thing. We have $P_{\text{AGENT}}^{homeomerous} = -$ (i.e., the concept that performs an action and the action itself are not of the same kind) and $P_{\text{IS-A}}^{homeomerous} = +$ (i.e., a concept and its subsumers are different, but they share significant meaning).

(4) *Separable* holds for most relations, as it is often the case that an argument could exist independently of the other. This primitive was first proposed to differentiate subtypes of PART-WHOLE, for example, PLACE-AREA does not hold this primitive [Winston et al. 1987, PACE-AREA(*Everglades*, *Florida*)].

(5) *Structural* indicates if the first argument subsumes the second one in terms of a hierarchical structure. Note that if $P_{\text{R}}^{structural} = +$, then $P_{\text{R}^{-1}}^{structural} = -$. A typical relation holding this primitive is HYPONYMY since HYPONYM(x, y) indicates that "x" is higher in some taxonomy than "y".

(6) *Temporal* holds if the first argument must temporally precede the second one. If a relation does not consider time, then this primitive does not apply. For example, $P^{temporal}_{\text{PURPOSE}} = +$ because the purpose "y" of an action "x", if it occurs, is guaranteed to happen after action "x". On the other hand, $P^{temporal}_{\text{IS-A}} = 0$ and $P^{temporal}_{\text{PART-WHOLE}} = 0$ since these relations do not require any temporal ordering of the arguments.

(7) *Intangible* is similar to *structural*. It specifies if the first argument is mentally or physically included in the second one. For example, this primitive is relevant to PART-WHOLE since parts are included in their wholes.

(8) *Near* indicates if arguments are physically or temporally close to each other. An instrument has to be close to the action it participates at, so we have $P^{near}_{\text{INSTRUMENT}} = +$. Note that *near* holds if the arguments must be close to each other, and *functional* holds if they must be in a specific temporal or spacial position (possibly far away).

(9) *Connected* is related to both *functional* and *near*. It indicates if arguments are physically or temporally connected, either directly or indirectly. For any relation R, if $P^{near}_{\text{R}} \in \{-, +\}$, then $P^{connected}_{\text{R}} = +$. The location "$y$" of an action "$x$" is physically connected to action "x", so $P^{connected}_{\text{LOCATION}} = +$.

(10) *Intrinsic* applies to relations that state a basic connection between the arguments, something that belongs to the nature of the arguments. For example, actions can be done for a variety of purposes ($P^{intrinsic}_{\text{PURPOSE}} = -$), but their causes are the reason why they exist, and therefore $P^{intrinsic}_{\text{CAUSE}} = +$.

(11) *Volitional* indicates if a relation requires volition between the arguments. Volition holds if there is will, if one argument is aware and conscious of the connection with the other. For example, $P^{volitional}_{\text{AGENT}} = +$ and $P^{volitional}_{\text{INTENT}} = +$. Note that this primitive does not distinguish between "x" or "y" requiring volition, $P^{volitional}_{\text{R}} = P^{volitional}_{\text{R}^{-1}}$.

(12) *Universal* indicates if the relation is always true between the arguments. This primitive holds for taxonomical relations and in general for relations that are true by definition. Examples that hold this primitive are IS-A, PART-WHOLE, and SYNONYMY.

(13) *Fully Implicational* holds if the first argument guarantees the existence of the second. In other words, "x" would not exist if "y" did not exist. This primitive only applies to relations involving two situations when the existence of one is due to the existence of the other, for example, CAUSE. If one argument does not guarantee the existence of the other one this primitive does not apply, for example, $P^{fully\ impl.}_{\text{IS-A}} = 0$.

(14) *Weakly Implicational* is a weaker version of *fully implicational*. If the first argument influences the existence of the other, this primitive holds. Note that if $P^{fully\ impl.}_{\text{R}} = +$, then $P^{weakly\ impl.}_{\text{R}} = +$. Performing an action "$x$" whose purpose is "$y$" makes "$y$" more likely to occur, therefore $P^{weakly\ impl.}_{\text{PURPOSE}} = +$. If one argument does not affect the existence of the other one, this primitive does not apply, for example, $P^{weakly\ impl.}_{\text{SYNONYMY}} = 0$.

These primitives are not orthogonal to each other. Some dependencies have already been outlined, for example, it is impossible to find a relation R such that $P^{weakly\ impl.}_{\text{R}} = 0$ and $P^{fully\ impl.}_{\text{R}} = +$. Other dependencies are more subtle.

—Only relations that hold *intrinsic* might be *universal*. Namely, if a relation R is always true between domain and range ($P^{universal}_{\text{R}} = +$), then it must describe an attribute of the essence of the arguments ($P^{intrinsic}_{\text{R}} = +$). Note that the opposite is not true: $P^{intrinsic}_{\text{CAUSE}} = +$ (the cause of the existence of any action is at the essence of the action), but $P^{universal}_{\text{CAUSE}} = -$ (concepts do not always have the same causes).

—For any relation R, if $P^{fully\ impl.}_{\text{R}} = +$, then $P^{temporal}_{\text{R}} = +$. Namely, if the first argument guarantees the existence of the second, then the first argument must happen before the second.

Table II. Algebra for Composing Semantic Primitives

1: Composable

	R_2		
R_1	$-$	0	$+$
$-$	$\times$	0	$\times$
0	0	0	0
$+$	$\times$	0	$+$

2: Functional

	R_2		
R_1	$-$	0	$+$
$-$	$-$	0	$+$
0	0	0	0
$+$	$+$	0	$+$

3: Homeomerous

	R_2		
R_1	$-$	0	$+$
$-$	$-$	$-$	$-$
0	$-$	0	0
$+$	$-$	0	$+$

4: Separable

	R_2		
R_1	$-$	0	$+$
$-$	$-$	$-$	$-$
0	$-$	0	$+$
$+$	$-$	$+$	$+$

5: Structural

	R_2		
R_1	$-$	0	$+$
$-$	$-$	0	$\times$
0	0	0	0
$+$	$\times$	0	$+$

6: Temporal

	R_2		
R_1	$-$	0	$+$
$-$	$-$	$-$	$\times$
0	$-$	0	$+$
$+$	$\times$	$+$	$+$

7: Intangible

	R_2		
R_1	$-$	0	$+$
$-$	$-$	0	$\times$
0	0	0	$+$
$+$	$\times$	$+$	$+$

8: Near

	R_2		
R_1	$-$	0	$+$
$-$	$-$	$-$	$+$
0	$-$	0	$+$
$+$	$+$	$+$	$+$

9: Connected

	R_2		
R_1	$-$	0	$+$
$-$	$-$	$-$	$+$
0	$-$	0	$+$
$+$	$+$	$+$	$+$

10: Intrinsic

	R_2		
R_1	$-$	0	$+$
$-$	$-$	0	$-$
0	0	0	0
$+$	$-$	0	$+$

11: Volitional

	R_2		
R_1	$-$	0	$+$
$-$	$-$	0	$+$
0	0	0	$+$
$+$	$+$	$+$	$+$

12: Universal

	R_2		
R_1	$-$	0	$+$
$-$	$-$	0	$-$
0	0	0	0
$+$	$-$	0	$+$

13: Fully impl.

	R_2		
R_1	$-$	0	$+$
$-$	$-$	0	$\times$
0	0	0	0
$+$	$\times$	0	$+$

14: Weakly impl.

	R_2		
R_1	$-$	0	$+$
$-$	$-$	$-$	$\times$
0	$-$	0	$+$
$+$	$\times$	$+$	$+$

"$\times$" indicates that R_1 and R_2 cannot be used in composition, "$+$" that the primitive holds for any relation composed out of R_1 and R_2, "$-$" that it does not hold, and "0" that it does not apply.

3.1.4. An Algebra for Composing Semantic Primitives. The key to automatically obtain inference axioms is the ability to analytically determine the resulting primitives of using two relations in composition. This way, one can identify the relations, if any, that can be composed out of any two relations.

We define an algebra that computes the value any relation composed out of R_1 and R_2 takes for the ith primitive based on the values R_1 and R_2 take for the ith primitive. After the algebra is used for all primitives, we obtain the values of all primitives for any relation that can be composed out of R_1 and R_2.

Table II depicts the whole algebra. Regarding the *intrinsic* primitive, we have:

—if R_1 and R_2 are *intrinsic* ($+$), any relation composed out of them is *intrinsic* ($+$);
—else if *intrinsic* does not apply (0) to either R_1 or R_2, *intrinsic* does not apply (0) to any relation composed out of them;
—else any relation composed out of them is not *intrinsic* ($-$).

Regarding other primitives, the algebra states that:

—R_1 and R_2 shall not be used in composition if $P^{temporal}_{R_1} = +$ and $P^{temporal}_{R_2} = -$, or $P^{temporal}_{R_1} = -$ and $P^{temporal}_{R_2} = +$.
—Any relation composed out of R_1 and R_2 is *universal* if and only if both R_1 and R_2 are *universal*.
—Any relation composed out of R_1 and R_2 is *connected* if either R_1 or R_2 are *connected*.

3.2. An Example

In this section, we offer an example using two relations common in the literature: AGENT and PURPOSE. The primitives for these relations are as follows.

$$P_{\text{AGENT}} = \{+, +, -, +, 0,\ 0, 0, 0, -, -, +, -, 0,\ 0\}$$
$$P_{\text{PURPOSE}} = \{+, -, -, +, 0, +, 0, 0, -, -, -, -, 0, +\}$$

Using the algebra, we obtain the following values for any relation composed out of AGENT and PURPOSE ($P_{\text{AGENT}\circ\text{PURPOSE}}$).

$$P_{\text{AGENT}\circ\text{PURPOSE}} = \{+, +, -, +, 0, +, 0, 0, -, -, +, -, 0, +\}$$

One can identify which relations can be composed out of AGENT and PURPOSE by retrieving the relations whose primitives are consistent with $P_{\text{AGENT}\circ\text{PURPOSE}}$. In this case, AGENT is consistent with the result, that is, P_{AGENT} and $P_{\text{AGENT}\circ\text{PURPOSE}}$ do not have contradictory values for any primitive. The only contradictory values are $(+, -)$ and $(-, +)$; 0 is not contradictory to any value.

4. SEMANTIC CALCULUS

4.1. Definitions, Operators and Properties

Following the theory of binary relations in mathematics, semantic calculus distinguishes three classes of semantic relations.

R is reflexive iff $\forall x : \text{R}(x, x)$, for example, SYNONYMY (1)

R is symmetric iff $\forall x, y : \text{R}(x, y)$ implies $\text{R}(y, x)$, for example, KINSHIP (2)

R is transitive iff $\forall x, y, z : \text{R}(x, y) \circ \text{R}(y, z) \rightarrow \text{R}(x, z)$, for example, CAUSE. (3)

That is, a relation R is transitive if and only if whenever $\text{R}(x, y)$ and $\text{R}(y, z)$ hold, $\text{R}(x, z)$ can be composed.

The composition operator is represented by the symbol "$\circ$". It combines two relations and yields a third one. It is the operator used to define an axiom. Formally, we denote inference axioms $\text{R}_1(x, y) \circ \text{R}_2(y, z) \rightarrow \text{R}_3(x, z)$, where R_1 and R_2 are the premises (i.e., relations used in composition) and R_3 is the conclusion (i.e., the relation composed out of the premises).

4.1.1. Inverse Relation. Given $\text{R}(x, y)$, $\text{R}^{-1}(y, x)$ always holds. The extended definition of R^{-1} can be automatically obtained given the definition for R.

—Relation type: the link is the same, but now is defined between the swapped arguments
—$\text{DOMAIN}(\text{R}^{-1}) = \text{RANGE}(\text{R})$
—$\text{RANGE}(\text{R}^{-1}) = \text{DOMAIN}(\text{R})$
—$P_{\text{R}^{-1}}$ is defined according to Table I. For each primitive, R^{-1} takes the same value than R if the fourth column reads "id", or the opposite if it reads "op".

The easiest way to read $\text{R}^{-1}(x, y)$ is *"x" is* R *of "y"*. For example, AGENT^{-1}(*John*, *came*) could be read *"John" is* AGENT *of "came"*.

4.1.2. No Relation and Dominance. We denote that there is no relation from a given set holding between two concepts with the symbol $\perp$. For example, when using most relation inventories, in *John is tall and his wife is blonde*, we have $\perp$(*John*, *blonde*).

If the conclusion of an axiom is one of the premises, one premise dominates the other. There are three kinds of dominance. R_1 left dominates R_2, $\text{R}_1 \triangleright \text{R}_2$, iff the composition of R_1 and R_2 yields R_1. R_1 right dominates R_2, $\text{R}_1 \triangleleft \text{R}_2$, iff the composition of R_2 and R_1 yields

Table III. The Four Unique Possible Axioms Taking as Premises R_1 and R_2

$\mathrm{R}_1 \circ \mathrm{R}_2$	${\mathrm{R}_1}^{-1} \circ \mathrm{R}_2$	$\mathrm{R}_2 \circ \mathrm{R}_1$	$\mathrm{R}_2 \circ {\mathrm{R}_1}^{-1}$
$\begin{tikzcd} x \arrow[r, "\mathrm{R}_1"] \arrow[dr, "\mathrm{R}_3"'] & y \arrow[d, "\mathrm{R}_2"] \\ & z \end{tikzcd}$	$\begin{tikzcd} x \arrow[dr, "\mathrm{R}_3"'] & y \arrow[l, "\mathrm{R}_1"'] \arrow[d, "\mathrm{R}_2"] \\ & z \end{tikzcd}$	$\begin{tikzcd} x \arrow[d, "\mathrm{R}_2"'] \arrow[dr, "\mathrm{R}_3"] & \\ y \arrow[r, "\mathrm{R}_1"'] & z \end{tikzcd}$	$\begin{tikzcd} x \arrow[d, "\mathrm{R}_2"'] \arrow[dr, "\mathrm{R}_3"] & \\ y & z \arrow[l, "\mathrm{R}_1"] \end{tikzcd}$

Conclusions are indicated by R_3 and are not guaranteed to be the same for the four axioms.

R_1. R_1 completely dominates R_2, $\mathrm{R}_1 \bowtie \mathrm{R}_2$, iff $\mathrm{R}_1 \rhd \mathrm{R}_2$ and $\mathrm{R}_1 \lhd \mathrm{R}_2$. In general, if one states R_1 *dominates* R_2, it could be interpreted as R_1 *completely dominates* R_2.

More formally, we have

$$\perp(x, y) \text{ iff } \neg\exists\mathrm{R} \text{ such that } \mathrm{R}_i(x, y) \tag{4}$$

$$\mathrm{R}_1 \rhd \mathrm{R}_2 \text{ iff } \mathrm{R}_1(x, y) \circ \mathrm{R}_2(y, z) \rightarrow \mathrm{R}_1(x, z) \tag{5}$$

$$\mathrm{R}_1 \lhd \mathrm{R}_2 \text{ iff } \mathrm{R}_2(x, y) \circ \mathrm{R}_1(y, z) \rightarrow \mathrm{R}_1(x, z) \tag{6}$$

$$\mathrm{R}_1 \bowtie \mathrm{R}_2 \text{ iff } \mathrm{R}_1(x, y) \circ \mathrm{R}_2(y, z) \rightarrow \mathrm{R}_1(x, z) \text{ and } \mathrm{R}_2(x, y) \circ \mathrm{R}_1(y, z) \rightarrow \mathrm{R}_1(x, z). \tag{7}$$

4.1.3. Derived Properties. Applying the steps to obtain the inverse of a relation, the following properties immediately follow.

$$(\mathrm{R}^{-1})^{-1} = \mathrm{R} \tag{8}$$

$$\perp \text{ is symmetric} \tag{9}$$

Furthermore, other properties can be derived and are useful to consider.

$$\mathrm{R}_i \circ \mathrm{R}_j = ({\mathrm{R}_j}^{-1} \circ {\mathrm{R}_i}^{-1})^{-1} \tag{10}$$

$$\mathrm{R}_i \rhd \mathrm{R}_j \text{ iff } {\mathrm{R}_i}^{-1} \lhd {\mathrm{R}_j}^{-1} \tag{11}$$

$$\mathrm{R}_i \bowtie \mathrm{R}_j \text{ iff } {\mathrm{R}_i}^{-1} \bowtie {\mathrm{R}_j}^{-1} \tag{12}$$

$$\text{If } \mathrm{R}_i \text{ is symmetric and } \mathrm{R}_i \bowtie \mathrm{R}_j,\ {\mathrm{R}_i}^{-1} \bowtie \mathrm{R}_j \tag{13}$$

$$\text{If } \mathrm{R}_j \text{ is symmetric and } \mathrm{R}_i \bowtie \mathrm{R}_j,\ \mathrm{R}_i \bowtie {\mathrm{R}_j}^{-1} \tag{14}$$

For the detailed derivation of properties (10)–(14), refer to Appendix A.

5. INFERENCE AXIOMS

Semantic relations are composed using inference axioms. Axioms are defined using the composition operator "$\circ$". We denote an axiom $\mathrm{R}_1(x, y) \circ \mathrm{R}_2(y, z) \rightarrow \mathrm{R}_3(x, z)$, where $\mathrm{R}_1(x, y)$ and $\mathrm{R}_2(y, z)$ are the premises and $\mathrm{R}_3(x, z)$ the conclusion. In order for an axiom to exist, the premises must have the "y" argument in common.

In general, for n relations there are $\binom{n}{2} = \frac{n(n-1)}{2}$ different pairs. For each pair, taking into account the two relations and their inverses, there are $4 \times 4 = 16$ different possible combinations. Applying property (10) $\mathrm{R}_i \circ \mathrm{R}_j = ({\mathrm{R}_j}^{-1} \circ {\mathrm{R}_i}^{-1})^{-1}$, only 10 combinations are unique: (1) 4 combine R_1, R_2 and their inverses (Table III); (2) 3 combine R_1 and its inverse (namely, $\mathrm{R}_1 \circ \mathrm{R}_1$, ${\mathrm{R}_1}^{-1} \circ \mathrm{R}_1$ and $\mathrm{R}_1 \circ {\mathrm{R}_1}^{-1}$); and (3) 3 combine R_2 and its inverse (namely, $\mathrm{R}_2 \circ \mathrm{R}_2$, ${\mathrm{R}_2}^{-1} \circ \mathrm{R}_2$ and $\mathrm{R}_2 \circ {\mathrm{R}_2}^{-1}$).

The most interesting axioms fall into category (5), since the other two can be resolved by the transitivity property of a relation and its inverse. Therefore, for n relations there are $2n^2 + n$ potential axioms.

$$\binom{n}{2} \times 4 + 3n = \frac{n(n-1)}{2} \times 4 + 3n = 2 \times n(n-1) + 3n = 2n^2 - 2n + 3n = 2n^2 + n$$

Depending on n, the number of axioms to consider can be considerably large. For $n = 20$, there are 820 axioms to explore and for $n = 30$, 1,830. Manual examination of those potential axioms would be time consuming and prone to errors. We avoid these issues using the extended definition and the algebra for composing semantic primitives.

5.1. Necessary Conditions for Defining Inference Axioms

In principle, one could attempt to compose relations out of every single pair of relations. However, there are two necessary conditions in order to compose a third relation R_3 out of two relations R_1 and R_2.

First, they have to be compatible (step 1, CSR algorithm that follows). Two relations are compatible if it is possible, from a theoretical point of view, to apply the composition operator to them. Formally, R_1 and R_2 are compatible iff $\text{RANGE}(\text{R}_1) \cap \text{DOMAIN}(\text{R}_2) \neq \emptyset$. If R_1 and R_2 are compatible but $\text{RANGE}(\text{R}_1) \neq \text{DOMAIN}(\text{R}_2)$, a restriction occurs. Let us denote $\text{RANGE}(\text{R}_1) \cap \text{DOMAIN}(\text{R}_2) = I$. A backward restriction takes place if $\text{RANGE}(\text{R}_1) \neq I$ and a forward restriction takes place if $\text{DOMAIN}(\text{R}_2) \neq I$. In the former case, $\text{RANGE}(\text{R}_1)$ is reduced; in the latter, $\text{DOMAIN}(\text{R}_2)$ is reduced. Note that one can find a forward and backward restriction with the same pair of relations.

Second, a third relation R_3 must fit as conclusion, that is, $\exists \text{R}_3$ such that $\text{DOMAIN}(\text{R}_3) \cap \text{DOMAIN}(\text{R}_1) \neq \emptyset$ and $\text{RANGE}(\text{R}_3) \cap \text{RANGE}(\text{R}_2) \neq \emptyset$ (step 2.1). Furthermore, P_{R_3} must be consistent with $P_{\text{R}_1} \circ P_{\text{R}_2}$ (step 2.2). Two values for a primitive are consistent unless either one is "×" or they are contradictory. The pairs $(-, +)$ and $(+, -)$ are the only contradictory pairs.

5.2. An Algorithm for Obtaining Inference Axioms

Given a set of relations R defined using the extended definition, one can obtain inference axioms applying the CSR algorithm.

CSR ALGORITHM

Input: a set of semantic relations R defined using the extended definition
Output: list of axioms using $(\text{R}_i, \text{R}_j) \in (R \cup R^{-1}) \times (R^{-1} \cup R)$ as their premises

```
inferenceAxioms ← [];
for (R_i, R_j) ∈ R × R do
    for (R_1, R_2) ∈ [(R_i, R_j), (R_i^-1, R_j), (R_j, R_i), (R_j, R_i^-1)] do
        if RANGE(R_1) ∩ DOMAIN(R_2) = ∅ then
            continue;                                                   // Step 1
        end
        P_{R_1∘R_2} ← ComposePrimitives(P_{R_1}, P_{R_2});
        for R_3 ∈ PossibleConclusions(R, R_1, R_2) do
            if DOMAIN(R_3) ∩ DOMAIN(R_1) = ∅ or RANGE(R_3) ∩ RANGE(R_2) = ∅ then
                continue;                                               // Step 2.1
            end
            if CheckConsistency(P_{R_1∘R_2}, P_{R_3}) then
                append R_1(x, y) ∘ R_2(y, z) → R_3(x, z) to inferenceAxioms;   // Step 2.2
            end
        end
    end
end
return inferenceAxioms
```

The inverse of a relation can be obtained as described in Section 4.1. *ComposePrimitives* is implemented using the algebra (Section 3.1.4). The method $PossibleConclusions(R, \text{R}_i, \text{R}_j)$ returns the relations from R that are semantically close to R_i and R_j. If either relation is *universal*, it returns the other relation. If relations are semantically clustered, all the relations belonging to the clusters containing R_i and R_j are returned. If they are not clustered, it returns R_i and R_j. The method *CheckConsistency*(P_{R_i}, P_{R_j}) is a simple procedure that loops through all primitives and checks that there are not contradictory values.

5.3. An Example: PART-WHOLE and LOCATION

In this section, we present an example of applying the CSR algorithm to two well-known relations: PART-WHOLE and LOCATION. PART-WHOLE encodes a meronymy relation between two objects (DOMAIN and RANGE are objects). LOCATION encodes a relation between an object or situation (DOMAIN) and a location (RANGE), where "y" specifies the spatial information of "x".

The primitives are defined as follows.

$$P_{\text{PART-WHOLE}} = \{+, +, -, +, 0, 0, -, -, +, +, 0, +, 0, 0\}$$
$$P_{\text{LOCATION}} = \{+, +, -, 0, 0, 0, 0, 0, +, -, 0, -, 0, 0\}$$

We can automatically obtain the primitives for the inverse relations.

$$P_{\text{PART-WHOLE}^{-1}} = \{+, +, -, +, 0, 0, +, -, +, +, 0, +, 0, 0\}$$
$$P_{\text{LOCATION}^{-1}} = \{+, +, -, 0, 0, 0, 0, 0, +, -, 0, -, 0, 0\}$$

Algorithm CSR loops over the pairs (1) PART-WHOLE, LOCATION, (2) PART-WHOLE^{-1}, LOCATION, (3) LOCATION, PART-WHOLE, and (4) LOCATION, PART-WHOLE^{-1}; and detects that only (1) and (2) fulfill domain and range compatibility (step 1).

Then, the CSR algorithm proceeds to find a conclusion R_3 for the potential axioms PART-WHOLE$(x, y) \circ$ LOCATION$(y, z) \rightarrow R_3(x, z)$ and PART-WHOLE$^{-1}(x, y) \circ$ LOCATION$(y, z) \rightarrow R_3(x, z)$. Composing the primitives using the algebra yields what follows.

$$P_{\text{PART-WHOLE}} \circ P_{\text{LOCATION}} = \{+, +, -, +, 0, 0, 0, -, +, -, 0, -, 0, 0\}$$
$$P_{\text{PART-WHOLE}^{-1}} \circ P_{\text{LOCATION}} = \{+, +, -, +, 0, 0, +, -, +, -, 0, -, 0, 0\}$$

The conclusion match step identifies LOCATION as conclusion for both potential axioms (steps 2.1, 2.2). Therefore, PART-WHOLE$(x, y) \circ$ LOCATION$(y, z) \rightarrow$ LOCATION(x, z) and PART-WHOLE$^{-1}(x, y) \circ$ LOCATION$(y, z) \rightarrow$ LOCATION(x, z) are added to the list.

The CSR algorithm correctly suggests the inference *parts and wholes are at the same location*. Note that PART-WHOLE was defined as meronymy between objects. Other kinds of meronymy, such as HAS-MEMBER or IS-SUBSTANCE, do not follow this rule because they have different meanings and primitives [Winston et al. 1987].

6. CASE STUDY

In this section, we apply CSR over a set of 26 well-known relations that we have experimented with [Badulescu and Moldovan 2009; Moldovan et al. 2010; Blanco et al. 2010]. First, we define the ontology used to define domains and ranges. Then, we define the relations using the extended definition. Finally, we present the inference axioms obtained using the CSR algorithm.

6.1. Ontology

We have adapted the basic structure of the ontology proposed by Helbig [2005], with a few simplifications and some additions. Our goal is to precisely define domains and ranges exclusively for the 26 relations used in this case study.

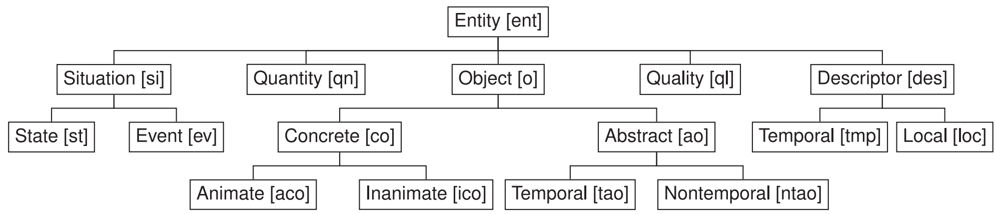

Fig. 2. Ontology used to define domains and ranges.

Figure 2 shows the ontology. The root corresponds to entities, which refer to *all things about which something can be said*. At the highest level of the hierarchy there are objects, situations, descriptors, quantities, and qualities.

Objects can be either concrete or abstract. The former occupy space, are touchable, and tangible. The latter are intangible, somehow a product of human reasoning. If an object can be palpable, we consider it concrete. This distinction is necessary because when we state properties or talk about concrete objects, one could claim that we are stating properties for the abstract representations of the concrete objects in our mind. Actually, most of concrete objects were once an idea in someone's mind and thus an abstract object. For example, a *shirt* is a concrete object even though at some point it was an abstraction in the designer's mind.

Concrete objects are further divided into animate or inanimate. The former have life, vigor, or spirit; the latter are dull, not enlivened. For example, *John* and *Mary's greyhound* both describe animate concrete objects; *paper* and *keyboard* describe inanimate concrete objects. This distinction is useful because only animate objects can perform actions and have volition.

Abstract objects are divided into temporal or nontemporal. The first correspond to abstractions regarding points or periods of time (*July*, *yesterday*, *last month*); the second correspond to other abstractions (*disease*, *weight*, *justice*). Abstract objects can also be sensually perceived, for example, *pain*, *odor*, and *fear* encode abstract objects and can be sensed.

Situations are anything that happens at a time and place. Simply put, if one can think of the time and location of an entity then it is a situation. If they imply a change in the status of other entities they are called events (*run*, *grow*), otherwise states (*be standing next to the door*, *account for 10% of the sales*). Situations can be expressed by verbs (*move*, *print*) or nouns (*party*, *hurricane*, *conference*).

Descriptors complement entities by stating properties about their spatial or temporal context. They are divided into local or temporal descriptors. Local descriptors are formed with an optional noncontent word signaling the local context (typically a preposition) followed by a concrete object or a situation. Typical prepositions denoting spatial context are *at*, *to*, *on*, *onto*, *above*, *over*, *across*, *close to*, and *around* [Quirk et al. 1985]. For example, *above the roof* is divided into $[\textit{above}]_{prep}$ $[\textit{the roof}]_{co}$ and *at the party* is divided into $[\textit{at}]_{prep}$ $[\textit{the party}]_{ev}$. Temporal descriptors follow a similar pattern. They are formed with an optional noncontent word signaling the temporal context followed by a temporal abstract object or a situation. Typical prepositions denoting temporal context are *at*, *on*, *in*, *by*, *for*, *during*, *since*, and *until* [Quirk et al. 1985]. For example, *by the end of the year* is divided into $[\textit{by}]_{prep}$ $[\textit{the end of the year}]_{tao}$ and *during the party* is divided into $[\textit{during}]_{prep}$ $[\textit{the party}]_{ev}$.

The noncontent word signaling the local or temporal context is usually present, but not always. For example, *Ankara* is a local descriptor describing the event *birthplace* in the following sentence: *The* $[\textit{birthplace}]_{ev}$ *of his mom is* $[\textit{Ankara}]_{loc}$. It is important to note

Table IV. The Set of 26 Relations Clustered, Classified and Their Properties (reflexive, symmetric and transitive)

Cluster	Relation	Abbr.	Class.	Properties *r*	*s*	*t*	DOMAIN × RANGE
Reason	CAUSE	CAU	iv	-	-	✓	[si] × [si]
	JUSTIFICATION	JST	iv	-	-	✓	[si] × [si ∪ ntao]
	INFLUENCE	IFL	iv	-	-	✓	[si] × [si]
Goal	INTENT	INT	i	-	-	-	[aco] × [si]
	PURPOSE	PRP	v	-	-	✓	[si ∪ co ∪ ntao] × [si ∪ ntao]
Object modifiers	VALUE	VAL	v	-	-	-	[o ∪ si] × [ql]
	SOURCE	SRC	ii	-	-	✓	[o] × [loc ∪ ql ∪ ntao ∪ ico]
Typical syntactic subjects	AGENT	AGT	iii	-	-	-	[si] × [aco]
	EXPERIENCER	EXP	iii	-	-	-	[si] × [o]
	INSTRUMENT	INS	iii	-	-	-	[si] × [co ∪ ntao]
Typical direct objects	THEME	THM	iii	-	-	-	[ev] × [o]
	TOPIC	TPC	iii	-	-	-	[ev] × [o ∪ si]
	STIMULUS	STI	iii	-	-	-	[ev] × [o]
Association	ASSOCIATION	ASO	v	✓	✓	✓	[ent] × [ent]
	KINSHIP	KIN	ii	✓	✓	✓	[aco] × [aco]
None	IS-A	ISA	ii	-	-	✓	[o] × [o]
	PART-WHOLE	PW	ii	-	-	*	[o] × [o] ∪ [l] × [l] ∪ [t] × [t]
	MAKE	MAK	ii	-	-	-	[co ∪ ntao] × [co ∪ ntao]
	POSSESSION	POS	ii	-	-	✓	[co] × [co]
	MANNER	MNR	iii	-	-	-	[si] × [ql ∪ st ∪ ntao]
	RECIPIENT	RCP	iii	-	-	-	[ev] × [co]
	SYNONYMY	SYN	v	✓	✓	✓	[ent] × [ent]
	LOCATION	LOC	v	✓	-	*	[o ∪ si] × [loc]
	TIME	TMP	v	✓	-	*	[o ∪ si] × [tmp]
	PROPERTY	PRO	v	-	-	-	[o ∪ si] × [ntao]
	QUANTIFICATION	QNT	v	-	-	-	[o ∪ si] × [qn]

"*l*" stands for *local descriptor, (loc)* and "*t*" for *temporal descriptor, (tmp)*. An asterisk indicates that the property holds under certain assumptions.

the difference between tao and tmp. For example, $[February]_{tao}$ *has 28 days* and *His birthday is* $[in\ February]_{tmp}$.

Qualities represent characteristics than can be assigned to entities. They can be quantifiable, like *tall* and *heavy*, or unquantifiable like *difficult* and *sleepy*.

Quantities represent quantitative characteristics of concepts. For example, *a few pounds*, *22 yards*, *two dozens*.

For more details about the ontology and possible extensions, refer to Helbig [2005].

6.2. Set of Relations

This case study focuses on the set of 26 semantic relations depicted shortly. We found this set specific enough to capture the most frequent semantics of text without bringing unnecessary overspecialization. Table IV depicts domains, ranges, a clustering, and a classification; Table V provides values for the primitives.

Some relations are not composable due to their narrow or broad meanings. For example, QUANTIFICATION cannot be used in composition because it encodes a narrow semantic connection between a concept and a quantification of it (e.g., QUANTIFICATION(*two*, *men*)). Even though this relation is useful to encode the meaning of text, it cannot be combined with another relation to compose a third relation. On the other hand, KINSHIP is not composable because it encodes a relation between two concepts when there is a blood

Table V. The Set of 26 Relations and Values for the Primitives

	Composable	Functional	Homeomerous	Separable	Structural	Temporal	Intangible	Near	Connected	Intrinsic	Volitional	Universal	Fully Impl.	Weakly Impl.
a: CAU	+	+	−	+	0	−	0	0	−	+	0	−	−	−
b: JST	+	+	−	+	0	−	0	0	−	+	0	−	0	−
c: IFL	+	+	−	+	0	−	0	0	−	+	0	−	0	−
d: INT	+	+	−	+	0	+	0	0	−	−	+	−	0	+
e: PRP	+	−	−	+	0	+	0	0	−	−	−	−	0	+
f: VAL	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
g: SRC	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
h: AGT	+	+	−	+	0	0	0	0	−	−	+	−	0	0
i: EXP	+	+	−	+	0	0	0	0	−	−	−	−	0	0
j: INS	+	+	−	+	0	0	0	+	+	−	−	−	0	0
k: THM	+	−	−	+	0	0	0	0	−	−	−	−	0	0
l: TPC	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
m: STI	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
n: ASO	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
o: KIN	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
p: ISA	+	−	+	0	−	0	0	0	0	+	0	+	0	0
q: PW	+	+	−	+	0	0	−	−	+	+	0	+	0	0
r: MAK	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
s: POS	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
t: MNR	+	−	−	+	0	0	0	0	−	−	+	−	0	0
u: RCP	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
v: SYN	+	−	+	0	0	0	0	0	0	+	0	+	0	0
w: LOC	+	+	−	0	0	0	0	0	+	−	0	−	0	0
x: TMP	+	+	−	0	0	0	0	0	+	−	0	−	0	0
y: PRO	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆
z: QNT	−	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆	⋆

For noncomposable relations, we do not specify all values, indicated with "⋆".

or marriage connection between them. This is too broad to be used in composition with our current framework: if we have KINSHIP(*John*, *Mary*) and R(*Mary*, z), we cannot infer any relation between "*John*" and "z".[4]

We decided to include noncomposable relations in the case study for the sake of completeness, coherence with our previous work [Blanco et al. 2010], and to make a fair estimation of how many pairs of relations are composable, how many can be discarded enforcing domain and range restrictions, etc. (Section 6.4). Applying CSR exclusively to composable relations would be misleading since relation inventories currently used do include noncomposable relations.

Relations are clustered such that relations belonging to the same cluster are close in meaning. Working with clusters is useful because it allows us to: (1) map to other proposed relations, justifying the chosen set of relations; (2) work with different levels of specificity; and (3) reason with the relations in a per-cluster basis.

[4]We note that KINSHIP is transitive, thus two KINSHIP relations can be used in composition to compose a third KINSHIP.

The reason cluster includes relations between a concept having a direct impact on the other.

—CAUSE(x, y) holds if "x" would not hold if "y" did not happen. This relation can only hold between situations, since objects can be *generated or created* (encoded by MAK), but *not caused*. For example, *He [got a bad grade]$^x_{ev}$ because he [didn't submit the project]$^y_{ev}$*; CAU(*got a bad grade*, *didn't submit*).
—JUSTIFICATION(x, y) is very close to CAU, while it encodes a moral cause or motive. If JST(x, y), "x" would not hold if "y" did not happen and "y" is a moral reason or socially convened norm. The distinction between CAU and JST depends on the nature of "y". For example, *They [don't smoke]$^x_{ev}$ in the hall because it [is forbidden]$^y_{st}$*; JST(*don't smoke*, *is forbidden*).
—INFLUENCE(x, y) encodes a weaker relation than CAU(x, y). If IFL(x, y), "y" affects the intensity of "x", but it does not affect the occurrence. An event may have several influencers. For example, *[Missing classes]$^y_{st}$ can lead to [a poor grade]$^x_{st}$*; IFL(*poor grade*, *missing classes*).

The goal cluster includes the relations INTENT and PURPOSE. Both relations are very close to each other and sometimes it is difficult to distinguish between them. One *intends* to do something for a *purpose*. For example, *Mary intends to buy a dress to look pretty at the party*. Both relations bring uncertainty, since having an intention or purpose does not guarantee that it will hold.

—INTENT(x, y) encodes intended consequences, which are volitional. Therefore, the domain is restricted to animate concrete objects and it does not make sense to consider intentions for abstract objects or other sorts of concepts. Situations do not have intentions either; their agents or experiencers might. For example, *The [professor]$^x_{aco}$'s goal is to [teach]$^y_{ev}$ students all the material*; INT(*professor*, *teach*).
—PURPOSE(x, y) can be defined for situations, concrete objects, and nontemporal abstract objects; it is somehow a broader relation than INTENT. For example, *Half of the [garage]$^x_{ico}$ is used for [storage]$^y_{ntao}$*; PRP(*garage*, *storage*).

The object modifiers cluster includes relations that describe attributes of objects and situations.

—SOURCE(x, y) holds if "y" expresses the origin of "x". "y" could be either a physical location or a mental, information, or material origin. For example, *We had a great time with the [Mexican]$^y_{ql}$ [students]$^x_{aco}$*; SRC(*students*, *Mexican*).
—VALUE(x, y) holds otherwise. For example, *Not all [smart]$^y_{ql}$ [kids]$^x_{aco}$ get good grades*; VAL(*kids*, *smart*).

The typical syntactic subjects cluster includes relations that encode links between a typical syntactic subject and a situation. The differences rely more on the characteristics of the subject and the connection per se.

—AGENT(x, y). "y" must be volitional and therefore the range is restricted to animate concrete objects. Excluded are the subjects which do not encode an intentional doer; typically encoded by EXP. For example, *[John]$^y_{aco}$ [got married]$^x_{ev}$ last Spring*; AGT(*got married*, *John*).
—EXPERIENCER(x, y). "y" does not change the situation, only experiences "x". "y" does not participate intentionally in "x" either. The difference between AGT and EXP can sometimes be done by the nature of the event. For example, verbs like *drown*, *find*, and *occur* need an EXP, and verbs like *dive*, *search*, and *think about* require an AGT.

The key is whether or not volitionality is involved, for example, *His [cell phone]$^{y}_{ico}$ [suffered]$^{x}_{ev}$ some water damage at the pool party*; EXP(*suffered*, *cell phone*);

—INSTRUMENT(x, y). "y" is used to perform "x", "y" is a tool or device that facilitates "x". For example, *[The hammer]$^{y}_{ico}$ [broke]$^{x}_{ev}$ the window*; INS(*broke*, *thehammer*).

The typical direct objects cluster includes relations between typical direct objects and events.

—THEME(x, y) holds if "y" is affected by or directly involved in "x"; "x" affects "y". For example, *John [read]$^{x}_{ev}$ [the book]$^{y}_{ico}$ twice*; THM(*read*, *the book*).
—TOPIC(x, y) holds if "x" is a communication verb, like *talk* and *argue*. For example, *John [discussed]$^{x}_{ev}$ [the issue]$^{y}_{ntao}$ too late*; TPC(*discussed*, *the issue*).
—STIMULUS(x, y) holds if "x" is a perception verb and "y" is a stimulus that makes "x" happen. For example, *John [perceived]$^{x}_{ev}$ [the ship]$^{y}_{ico}$ coming over the horizon*; STI(*perceived*, *the ship*).

The association cluster includes ASSOCIATION and KINSHIP. ASO is a very broad relation between any pair of entities and KIN encodes a relation between two concepts related by blood or marriage. If KIN(x, y), then ASO(x, y) holds. For example, *[John]$^{x}_{aco}$ and [Mary]$^{y}_{aco}$ work at the same company*; ASO(*John*, *Mary*), and *[John]$^{x}_{aco}$ visited [his parents]$^{y}_{aco}$ for Christmas*; KIN(*John*, *his parents*).

The rest of the relations do not fall into any particular cluster. These relations are specific and have their own unique characteristics. IS-A, PART-WHOLE, SYNONYMY, LOCATION, and TIME have been widely studied in the literature. MAKE(x, y) holds if "y" makes or produces "x" (i.e., "x" has maker "y"; e.g., MAKE(*cars*, *BMW*)). POSSESSION(x, y) holds if "x" owns "y". For example, *[John]$^{x}_{aco}$'s [truck]$^{y}_{ico}$* encodes POS(*John*, *truck*). MANNER encodes the way in which a situation occurs (e.g., MNR(*delivery*, *quick*)). RECIPIENT captures the connection between an event and an object which is the receiver of the event. For example, *John [gave]$^{x}_{ev}$ [Mary]$^{y}_{aco}$ roses* and *John [stole]$^{x}_{ev}$ [Mary]$^{y}_{aco}$'s car*. PROPERTY describes links between a situation or object and its characteristics (e.g., PRO(*John*, *height*)). Values for the characteristics are given through VALUE. QUANTIFICATION(x, y) holds if "x" is quantitatively determined by "y", (e.g., QNT(*eggs*, *a dozen*)).

Relations can also be classified depending on the kind of concepts they describe. The fourth column of Table IV indicates this classification. There are five possible classifications: (i) Intra-Object; (ii) Inter-Objects; (iii) Intra-Situation; (iv) Inter-Situations; and (v) for Object and Situation description.

6.3. Inference Axioms

Given the extended definitions provided in Tables IV and V, obtaining inference axioms using the CSR algorithm is straightforward. The results are shown in Tables VI–VIII. Table VI summarizes axioms $R_1(x, y) \circ R_2(y, z) \rightarrow R_3(x, z)$ and $R_2(x, y) \circ R_1(y, z) \rightarrow R_3(x, z)$; Table VII axioms $R_1^{-1}(x, y) \circ R_2(y, z) \rightarrow R_3(x, z)$; and Table VIII axioms $R_1(x, y) \circ R_2^{-1}(y, z) \rightarrow R_3(x, z)$.

All tables should be indexed with the first premise as the row and the second premise as the column. The meaning of each cell is as follows. A letter indicates an axiom $R_1(x, y) \circ R_2(y, z) \rightarrow R_3(x, z)$ by stating the conclusion R_3. An empty cell signals that the pair cannot be used in composition because R_1 and R_2 are incompatible (step 1, CSR algorithm). A colon ":" indicates that the pair cannot be used in composition because the algebra marks the combination as prohibited ("×"); and a dash "-" that the pair can be used in composition, but a relation that fits as conclusion of $R_1 \circ R_2$ could not be found (steps 2.1 and 2.2, CSR algorithm).

The list of 216 unique inference axioms from Tables VI–VIII can be found in Appendix C. Here are some of the axioms identified by the CSR algorithm.

Table VI. Inference Axioms Obtained with the Set of 26 Relations, Part 1

R$_1$ \ R$_2$	a: CAU	b: JST	c: IFL	d: INT	e: PRP	f: VAL	g: SRC	h: AGT	i: EXP	j: INS	k: THM	l: TPC	m: STI	n: ASO	o: KIN	p: ISA	q: PW	r: MAK	s: POS	t: MNR	y: RCP	v: SYN	w: LOC	x: TMP	y: PRO	z: QNT
a: CAU	a	c,b	c,b		:	:		h	i,h	j	-	:	:	:						-	:	a	w	x	:	:
b: JST	c,b	c,b	c,b		:	:	:	h	i,h	j	-	:	:	:		b	-	:		-	:	b	w	x	:	:
c: IFL	c,b	c,b	c,b		:	:		h	i,h	j	-	:	:	:						-	:	c	w	x	:	:
d: INT	:	:	:		d	:		-	-	-	-	:	:	:						d	:	d	w	x	:	:
e: PRP	:	:	:		e	:	:	h	i	j	k,e	:	:	:		e	-	:		t	:	e	w	x	:	:
f: VAL														:								:				
g: SRC					:	:	:							:		:	:	:	:			:	:	:	:	:
h: AGT				-	-	:	:							:	:	h	-	:	:			h	w	x	:	:
i: EXP				-	i	:	:							:	:	i	-	:	:			i	w	x	:	:
j: INS				-	j	:	:							:	:	j	j	:	:			j	w	x	:	:
k: THM				-	k,e	:	:							:	:	k	-	:	:			k	w	x	:	:
l: TPC	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
m: STI				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
n: ASO	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
o: KIN				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
p: ISA				d	e	:	:							:	:	p	-	:	:			p	w	x	:	:
q: PW				-	-	:	:							:	:	q	q	:	:			q	w	x	:	:
r: MAK				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
s: POS				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
t: MNR	-	-	-		t	:	:	h	h	-				:		t	-	:		t		t	w	x	:	:
u: RCP				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
v: SYN	a	b	c	d	e	:	:	h	i	j	k	:	:	:	:	v	-	:	:	t	:	v	w	x	:	:
w: LOC														:			w					w				
x: TMP														:			x					x				
y: PRO					:	:	:							:		:	:	:				:	:	:	:	:
z: QNT														:								:				

This table summarizes axioms R$_1$ $\circ$ R$_2$ $\rightarrow$ R$_3$ and R$_2$ $\circ$ R$_1$ $\rightarrow$ R$_3$

The upper left corner of Table VI captures the transitivity of CAUSE, JUSTIFICATION, and INFLUENCE. Axiom CAUSE $\circ$ LOCATION $\rightarrow$ LOCATION (Table VI) suggests that the location of an effect is the same as the location of its cause. Table VII identifies the axiom PURPOSE^{-1} $\circ$ MANNER $\rightarrow$ MANNER, whose accuracy is evaluated in Section 7. Table VII proposes the axiom AGENT^{-1} $\circ$ PURPOSE $\rightarrow$ INTENT, capturing the idea that *agents intend the purpose of their actions*.

As expected, composing SYNONYMY with any relation R yields R. This is shown in Tables VI, VII, and VIII in most cells involving either SYNONYMY or SYNONYMY^{-1}. The only exceptions are the composition of SYNONYMY with a noncomposable relation, in which case the inference is blocked by the algebra.

It is important to note that property (10) (Section 4), R$_i$ $\circ$ R$_j$ = (R$_j$$^{-1}$ $\circ$ R$_i$$^{-1}$)$^{-1}$, does not apply to Table VI since it depicts axioms without inverse relations. If R$_1$ and R$_2$ are compatible, there is no guarantee that R$_2$ and R$_1$ are also compatible. For example, CAUSE and AGENT are compatible and yet AGENT and CAUSE are not (Table VI).

On the other hand, property R$_i$ $\circ$ R$_j$ = (R$_j$$^{-1}$ $\circ$ R$_i$$^{-1}$)$^{-1}$ applies to Tables VII and VIII. However, there are a few cases in which the CSR algorithm is not able to enforce this property. For example, Table VIII identifies axioms SYNONYMY(x, y) $\circ$ IS-A$^{-1}(y, z)$ $\rightarrow$ IS-A$^{-1}(x, z)$ and IS-A(x, y) $\circ$ SYNONYMY$^{-1}(y, z)$ $\rightarrow$ SYNONYMY$^{-1}(x, z)$; intuitively, the latter axiom should conclude IS-A. Possible causes are the manual assignment of values to

Table VII. Inferences Axioms Obtained with the Set of 26 Relations, Part 2

	R_2																									
R_1	a: CAU	b: JST	c: IFL	d: INT	e: PRP	f: VAL	g: SRC	h: AGT	i: EXP	j: INS	k: THM	l: TPC	m: STI	n: ASO	o: KIN	p: ISA	q: PW	r: MAK	s: POS	t: MNR	y: RCP	v: SYN	w: LOC	x: TMP	y: PRO	z: QNT
a^{-1}: CAU^{-1}	:	:	:		-	:		h	h,i	j	-	:	:	:						-	:	a^{-1}	w	x	:	:
b^{-1}: JST^{-1}	:	:	:		-	:		h	h,i	j	-	:	:	:						-	:	b^{-1}	w	x	:	:
c^{-1}: IFL^{-1}	:	:	:		-	:		h	h,i	j	-	:	:	:						-	:	c^{-1}	w	x	:	:
d^{-1}: INT^{-1}				:	:	:	:							:	:	d^{-1}	-	:	:			d^{-1}	w	x	:	:
e^{-1}: PRP^{-1}	-	-	-	:	:	:	:	d^{-1},h	i	j	e^{-1},k	:	:	:	:	e^{-1}	-	:	:	t	:	e^{-1}	w	x	:	:
f^{-1}: VAL^{-1}	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
g^{-1}: SRC^{-1}				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
h^{-1}: AGT^{-1}	h^{-1}	h^{-1}	h^{-1}		h^{-1},d	:		-	-	-	-	:	:	:						h^{-1}	:	h^{-1}	w	x	:	:
i^{-1}: EXP^{-1}	h^{-1},i^{-1}	h^{-1},i^{-1}	h^{-1},i^{-1}		i^{-1}	:		-	-	-	-	:	:	:						h^{-1}	:	i^{-1}	w	x	:	:
j^{-1}: INS^{-1}	j^{-1}	j^{-1}	j^{-1}		j^{-1}	:		-	-	-	-	:	:	:						-	:	j^{-1}	w	x	:	:
k^{-1}: THM^{-1}	-	-	-		k^{-1},e	:		-	-	-	-	:	:	:						-	:	k^{-1}	w	x	:	:
l^{-1}: TPC^{-1}	:	:	:		:	:		:	:	:	:	:	:	:						:	:	:	:	:	:	:
m^{-1}: STI^{-1}	:	:	:		:	:		:	:	:	:	:	:	:						:	:	:	:	:	:	:
n^{-1}: ASO^{-1}	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
o^{-1}: KIN^{-1}				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
p^{-1}: ISA^{-1}				d	e	:	:							:	:	:	-	:	:			p^{-1}	w	x	:	:
q^{-1}: PW^{-1}				-	-	:	:							:	:	q^{-1}	:	:	:			q^{-1}	w	x	:	:
r^{-1}: MAK^{-1}				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
s^{-1}: POS^{-1}				:	:	:	:							:	:	:	:	:	:			:	:	:	:	:
t^{-1}: MNR^{-1}	-	-	-		t^{-1}	:		h	h	-	-	:	:	:						t^{-1},t	:	t^{-1}	w	x	:	:
u^{-1}: RCP^{-1}	:	:	:		:	:		:	:	:	:	:	:	:						:	:	:	:	:	:	:
v^{-1}: SYN^{-1}	a	b	c	d	e	:	:	h	i	j	k	:	:	:	:	v^{-1}	-	:	:	t	:	v^{-1}	w	x	:	:
w^{-1}: LOC^{-1}	w^{-1}	w^{-1}	w^{-1}	w^{-1}	w^{-1}	:	:	w^{-1}	w^{-1}	w^{-1}	w^{-1}	:	:	:	:	w^{-1}	w^{-1}	:	:	w^{-1}	:	w^{-1}	-	-	:	:
x^{-1}: TMP^{-1}	x^{-1}	x^{-1}	x^{-1}	x^{-1}	x^{-1}	:	:	x^{-1}	x^{-1}	x^{-1}	x^{-1}	:	:	:	:	x^{-1}	x^{-1}	:	:	x^{-1}	:	x^{-1}	-	-	:	:
y^{-1}: PRO^{-1}	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
z^{-1}: QNT^{-1}	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:

This table summarizes axioms $R_1{}^{-1} \circ R_2 \rightarrow R_3$.

the primitives, the manual definition of the algebra, and the fact that primitives are composed orthogonally.

6.4. Numeric Analysis

Tables VI–VIII summarize 1,378 unique pairs of premises (recall $R_i \circ R_j = (R_j{}^{-1} \circ R_i{}^{-1})^{-1}$). Domain and range restrictions mark 491 pairs (35.63%) as incompatible and 887 as compatible (64.37%). The algebra labels 608 pairs as prohibited, that is, the composition of at least one primitive yields "×" (44.12%, [68.55% of the compatible pairs]).

The CSR algorithm is unable to find a conclusion for 84 compatible and nonprohibited pairs (6.10%, [9.47%]). Finally, conclusions are found for 195 pairs (14.15%, [21.98%]). Since more than one conclusion might be proposed for the same pair of premises, 216 inference axioms are ultimately identified (Appendix B).

7. EVALUATION

We have evaluated the accuracy and productivity of a partial set of eight axioms using the PropBank annotation [Palmer et al. 2005]. The set of inference axioms evaluated involves CAUSE or PURPOSE and a second relation as premises.

Table VIII. Inferences Axioms after Composing the Set of 26 Relations, Part 3

	R_2																									
R_1	a^{-1}: CAU^{-1}	b^{-1}: JST^{-1}	c^{-1}: IFL^{-1}	d^{-1}: INT^{-1}	e^{-1}: PRP^{-1}	f^{-1}: VAL^{-1}	g^{-1}: SRC^{-1}	h^{-1}: AGT^{-1}	i^{-1}: EXP^{-1}	j^{-1}: INS^{-1}	k^{-1}: THM^{-1}	l^{-1}: TPC^{-1}	m^{-1}: STI^{-1}	n^{-1}: ASO^{-1}	o^{-1}: KIN^{-1}	p^{-1}: ISA^{-1}	q^{-1}: PW^{-1}	r^{-1}: MAK^{-1}	s^{-1}: POS^{-1}	t^{-1}: MNR^{-1}	u^{-1}: RCP^{-1}	v^{-1}: SYN^{-1}	w^{-1}: LOC^{-1}	x^{-1}: TMP^{-1}	y^{-1}: PRO^{-1}	z^{-1}: QNT^{-1}
a: CAU	:	:	:	d^{-1}	d^{-1}							:		:						-		a				
b: JST	:	:	:	d^{-1}	d^{-1}		:		-	-	-	:	:	:		b	-	:		-		b			:	
c: IFL	:	:	:	d^{-1}	d^{-1}							:		:						-		c				
d: INT	d	d	d	:	:							:		:						d		d				
e: PRP	d	d	d	:	:		:		i^{-1}	j^{-1}	k^{-1},e	:	:	:		e	-	:		t^{-1}		e			:	
f: VAL						:	:							:						:		:				
g: SRC		:			:	:	:		:	:	:	:	:	:		:	:	:	:	:	:	:	:		:	
h: AGT								-	-	-	-	:	:	:	:	h	-	:	:		:	h				
i: EXP		-			i		:	-	-	-	-	:	:	:	:	i	-	:	:	-	:	i			:	
j: INS		-			j		:	-	-	-	-	:	:	:	:	j	j	:	:	-	:	j			:	
k: THM		-			e^{-1},k		:	-	-	-	-	:	:	:	:	k	-	:	:	-	:	k			:	
l: TPC	:	:	:	:	:		:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:			:	
m: STI		:			:		:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:			:	
n: ASO	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
o: KIN								:	:	:	:	:	:	:	:	:	:	:	:		:	:				
p: ISA		b^{-1}			e^{-1}		:	h^{-1}	i^{-1}	j^{-1}	k^{-1}	:	:	:	:	:	q^{-1}	:	:	t^{-1}	:	v^{-1}			:	
q: PW		-			-		:	-	-	j^{-1}	-	:	:	:	:	-	:	:	:	-	:	-	w^{-1}	x^{-1}	:	
r: MAK		:			:		:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:			:	
s: POS							:	:	:	:	:	:	:	:	:	:	:	:	:		:	:				
t: MNR	-	-	-	d^{-1}	t	:	:		-	-	-	:	:	:		t	-	:		t^{-1},t		t			:	
u: RCP							:	:	:	:	:	:	:	:	:	:	:	:	:		:	:				
v: SYN	a^{-1}	b^{-1}	c^{-1}	d^{-1}	e^{-1}	:	:	h^{-1}	i^{-1}	j^{-1}	k^{-1}	:	:	:	:	p^{-1}	q^{-1}	:	:	t^{-1}	:	v^{-1}	w^{-1}	x^{-1}	:	:
w: LOC							:							:			w					w	-			
x: TMP														:			x					x		-		
y: PRO		:			:		:		:	:	:	:	:	:		:	:	:		:		:			:	
z: QNT														:								:				:

This table summarizes axioms $R_1 \circ {R_2}^{-1} \rightarrow R_3$.

(1) $\text{CAUSE}^{-1}(x, y) \circ \text{AGENT}(y, z) \rightarrow \text{AGENT}(x, z)$
The agent of an action is inherited by its cause.

(2) $\text{CAUSE}^{-1}(x, y) \circ \text{LOCATION}(y, z) \rightarrow \text{LOCATION}(x, z)$
The spatial context of an action is inherited by its cause.

(3) $\text{CAUSE}^{-1}(x, y) \circ \text{TIME}(y, z) \rightarrow \text{TIME}(x, z)$
The temporal context of an action is inherited by its cause.

(4) $\text{PURPOSE}^{-1}(x, y) \circ \text{AGENT}(y, z) \rightarrow \text{AGENT}(x, z)$
The agent of an action is inherited by its purpose.

(5) $\text{PURPOSE}^{-1}(x, y) \circ \text{THEME}(y, z) \rightarrow \text{THEME}(x, z)$
The theme of an action is inherited by its purpose.

(6) $\text{PURPOSE}^{-1}(x, y) \circ \text{LOCATION}(y, z) \rightarrow \text{LOCATION}(x, z)$
The spatial context of an action is inherited by its purpose.

(7) $\text{PURPOSE}^{-1}(x, y) \circ \text{TIME}(y, z) \rightarrow \text{TIME}(x, z)$
The temporal context of an action is inherited by its purpose.

(8) $\text{PURPOSE}^{-1}(x, y) \circ \text{MANNER}(y, z) \rightarrow \text{MANNER}(x, z)$
The manner of an action is inherited by its purpose.

Table IX. Subset of Relations in PropBank and Their Counts for the First 1,000 Sentences Instantiating Any of the 8 Axioms Used during Evaluation

Relation	No. Instances
AGENT	828
THEME	1,052
LOCATION	72
TIME	239
MANNER	107

We instantiate the preceding axioms using as premises annotation from PropBank. For each instantiation, the validity of the conclusion is checked by hand. This process is not automatable since the inferred relations have not been annotated before. Accuracy is calculated as the number of correct conclusions over the total number of instantiations. Productivity for any axiom $\text{R}_1(x, y) \circ \text{R}_2(y, z) \rightarrow \text{R}_3(x, z)$ is calculated as number of instantiations over the number of relations R_3 annotated in PropBank.

CAUSE, PURPOSE, and MANNER are labeled in PropBank using ARGM-CAU, ARGM-PRP, and ARGM-MNR. For AGENT and THEME, we use ARG0 and ARG1, which correspond to the prototypical AGENT and THEME [Palmer et al. 2005, page 75].

7.1. All Instances of Axiom 8

First, we evaluated all the instantiations of axiom 8. This axiom can be instantiated 237 times, yielding 189 new valid MANNER not present in PropBank. The overall accuracy is 0.80, superior to state-of-the-art semantic role labelers extracting MANNER between a verb and its syntactic arguments.

Consider the following example (wsj_0118, 48).

> [. . .] the traders $[place]_y$ orders $[via\ computers]_{z,\ \text{MNR}}$ $[to\ buy\ the\ basket\ of\ stocks\ \ldots]_{x,\ \text{PRP}}$.

Basic semantic role annotation states that "*place*" has MANNER "*via computers*" and PURPOSE "*to buy the basket of stocks* [. . .]". After applying axiom $\text{PURPOSE}^{-1}(x, y) \circ \text{MANNER}(y, z) \rightarrow \text{MANNER}(x, z)$, the new relation MANNER(*to buy the basket* [. . .], *via computers*) is inferred. This relation is obvious when reading the sentence, but it was ignored before.

Several sentences support the validity of the axiom, for example, *The classics have* $[zoomed]_y$ $[in\ price]_z$ $[to\ meet\ the\ competition]_x$, *[. . .]* (wsj_0071, 9), *[. . .] the government* $[curtailed]_y$ *production* $[with\ land\text{-}idling\ programs]_z$ $[to\ reduce\ price\text{-}depressing\ surpluses]_x$ (wsj_0113, 12). In both examples, basic annotation encodes $\text{PURPOSE}^{-1}(x, y)$ and $\text{MANNER}(y, z)$. After using PRP^{-1} and MNR in composition, two new valid MANNER arise: MANNER(*to meet the competition*, *in price*) and MANNER (*to reduce price-depressing surpluses*, *with land-idling programs*).

7.2. First 1,000 Sentences Instantiating Any of the Eight Axioms

Since PropBank is a large corpus, the number of instantiations found for all axioms is too large to be checked by hand. We have manually evaluated the validity of the eight axioms given before for the first 1,000 sentences instantiating any axiom. Since a sentence may instantiate several axioms, we have actually evaluated 1,412 instantiations.

The first 1,000 sentences instantiating any of the eight axioms are found within the first 31,450 sentences in PropBank. Table IX shows the number of AGENT, THEME,

Table X. Axioms Used during Evaluation, Number of Instantiations, Accuracy and Productivity

		no heuristic			with heuristic		
No.	Axiom	No. Inst.	Acc.	Produc.	No. Inst.	Acc.	Produc.
1	$\text{CAU}^{-1} \circ \text{AGT} \rightarrow \text{AGT}$	201	0.40	24.28%	75	0.67	9.06%
2	$\text{CAU}^{-1} \circ \text{LOC} \rightarrow \text{LOC}$	17	0.82	23.61%	15	0.93	20.83%
3	$\text{CAU}^{-1} \circ \text{TMP} \rightarrow \text{TMP}$	72	0.85	30.13%	69	0.87	28.87%
1-3	$\text{CAU}^{-1} \circ \text{R}_2 \rightarrow \text{R}_3$	290	0.53	25.46%	159	0.78	13.96%
4	$\text{PRP}^{-1} \circ \text{AGT} \rightarrow \text{AGT}$	375	0.89	45.29%	347	0.94	41.91%
5	$\text{PRP}^{-1} \circ \text{THM} \rightarrow \text{THM}$	489	0.12	46.48%	87	0.65	8.27%
6	$\text{PRP}^{-1} \circ \text{LOC} \rightarrow \text{LOC}$	49	0.90	68.06%	48	0.92	66.67%
7	$\text{PRP}^{-1} \circ \text{TMP} \rightarrow \text{TMP}$	138	0.84	57.74%	129	0.88	53.97%
8	$\text{PRP}^{-1} \circ \text{MNR} \rightarrow \text{MNR}$	71	0.82	66.36%	70	0.83	65.42%
4-8	$\text{PRP}^{-1} \circ \text{R}_2 \rightarrow \text{R}_3$	1,122	0.54	48.83%	681	0.88	29.63%
1-8	**All**	1,412	0.54	61.44%	840	0.86	36.55%

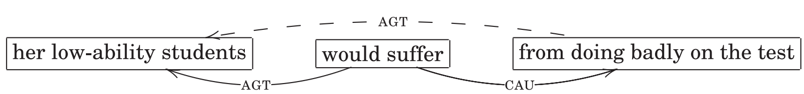

Fig. 3. Example of axiom instantiation 1. *Mostly, she says, she wanted to prevent the damage to self-esteem that her low-ability students would suffer from doing badly on the test* (wsj_0044, 91).

LOCATION, TIME, and MANNER instances already annotated in PropBank in those 1,000 sentences. These counts are used to calculate the productivity of each axiom.

Table X depicts the total number of instantiations for each axiom, their accuracy and productivity (columns 3–5). Accuracies range from 0.12 to 0.90, showing that the validity of an inference axiom depends on the axiom. The average accuracy for axioms involving CAUSE^{-1} is 0.53 and for axioms involving PURPOSE^{-1} is 0.54.

Axiom $\text{CAUSE}^{-1} \circ \text{AGENT} \rightarrow \text{AGENT}$ adds 201 relations, which corresponds to 24.28% in relative terms. Its accuracy is low, 0.40. Other axioms are less productive overall, but have a greater relative impact and accuracy. For example, axiom $\text{PURPOSE}^{-1} \circ \text{MANNER} \rightarrow \text{MANNER}$, only yields 71 new MANNER relations, and yet it is adding 66.36% in relative terms with an accuracy of 0.82.

It is worth noting that overall, applying the eight axioms used during evaluation adds 1,412 relations on top of the ones already present (61.44% in relative terms) with an accuracy of 0.54. Section 7.4 introduces a heuristic that improves the accuracy (columns 6–8).

7.3. Examples

Several examples of inferences are provided in Figures 3–7. Solid arrows indicate basic annotation present in PropBank; discontinuous arrows indicate inferred relations. An asterisk signals erroneous inferences (Figure 7).

Figures 3 and 4 show instantiations of $\text{CAU}^{-1} \circ \text{AGT} \rightarrow \text{AGT}$. Figure 5 illustrates instantiations of the three axioms involving CAUSE^{-1} as premise: $\text{CAU}^{-1} \circ \text{AGT} \rightarrow \text{AGT}$, $\text{CAU}^{-1} \circ \text{LOC} \rightarrow \text{LOC}$, and $\text{CAU}^{-1} \circ \text{TMP} \rightarrow \text{TMP}$.

Figures 6 and 7 show instantiations of the axioms involving PURPOSE^{-1}. The sentence in Figure 6 exemplifies axioms $\text{PRP}^{-1} \circ \text{AGT} \rightarrow \text{AGT}$, $\text{PRP}^{-1} \circ \text{LOC} \rightarrow \text{LOC}$, and $\text{PRP}^{-1} \circ \text{TMP} \rightarrow \text{TMP}$.

Figure 7 instantiates axioms $\text{PRP}^{-1} \circ \text{AGT} \rightarrow \text{AGT}$, $\text{PRP}^{-1} \circ \text{THM} \rightarrow \text{THM}$, and $\text{PRP}^{-1} \circ \text{MNR} \rightarrow \text{MNR}$. Note that the second inference obtained with the second axiom is wrong, indicated with "*". However, this inference is blocked by a heuristic described next.

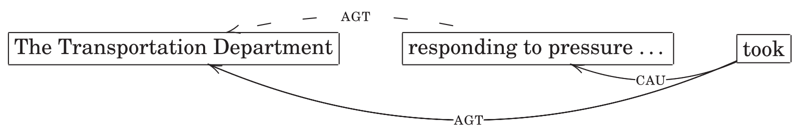

Fig. 4. Example of axiom instantiation 2. *The Transportation Department, responding to pressure from safety advocates, took further steps to impose on light trucks and vans the safety requirements used for automobiles* (wsj_0064 0).

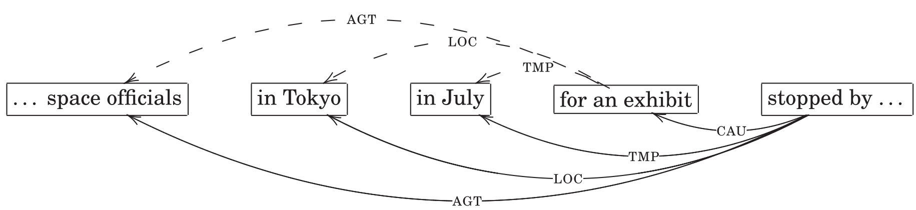

Fig. 5. Example of axiom instantiation 3. *A half-dozen Soviet space officials, in Tokyo in July for an exhibit, stopped by to see their counterparts at the National Space Development Agency of Japan* (wsj_0405, 1).

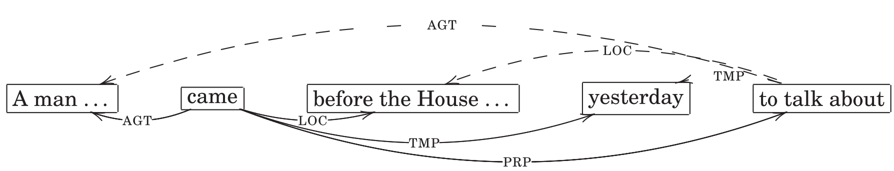

Fig. 6. Example of axiom instantiation 4. *A man from the Bush administration came before the House Agriculture Committee yesterday to talk about the U.S.'s intention to send some $100 million in food aid to Poland, with more to come from the EC* (wsj_0134, 0).

7.4. Error Analysis

Because of the low accuracy of axioms 1 and 5, an error analysis was performed. We found that, unlike other axioms, these often yield a relation type that is already present in the semantic representation with similar arguments. Specifically, axioms 1 and 5 often yield $\textsc{r}(x, z)$ when $\textsc{r}(x, z')$ is already known.

An example can be found in Figure 7, where axiom 5 yields THEME(*tobuy*, *orders*) and the relation THEME(*to buy*, *the basket* [...]) is already present. We use the following heuristic in order to improve the accuracy.

> *Heuristic*. Do not instantiate an axiom $\textsc{r}_1(x, y) \circ \textsc{r}_2(y, z) \rightarrow \textsc{r}_3(x, z)$ if a relation $\textsc{r}_3(x, z')$ is already known.

This simple heuristic increases the accuracy of the inferences at the cost of lowering the productivity. The last three columns in Table X show results when using this heuristic. The eight axioms add 840 relations (36.55% in relative terms) with an accuracy of 0.86.

Even though most relations inferred using the aforesaid heuristic are correct, mistakes are made. Some errors could be further avoided using another heuristic that we did not implement: do not instantiate an axiom $\textsc{r}_1(x, y) \circ \textsc{r}_2(y, z) \rightarrow \textsc{r}_3(x, z)$ if a relation $\textsc{r}_3'(x, z)$ is already known. This heuristic simply avoids inferring $\textsc{r}_3(x, z)$ if the arguments "x" and "z" are already connected by another relation $\textsc{r}_3'$.

Other errors are due to complicated linguistic phenomena difficult to overcome. Coreference resolution is sometimes needed and we do not consider it. For example, consider the sentence *These products require monitoring because they must be valued separately*. Basic annotation includes the relations: AGENT(*require*, *These products*),

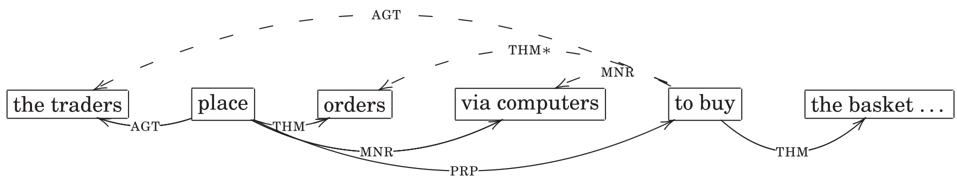

Fig. 7. Example of axiom instantiation 5. *When it occurs, the traders place orders via computers to buy the basket of stocks (such as the 500 stocks that constitute the Standard & Poor's 500 stock index) in whichever market is cheaper and sell them in the more expensive market; they lock in the difference in price as profit* (wsj_0118, 48).

THEME(*require*, *monitoring*), CAUSE(*require*, *because they must be valued separately*), and THEME(*valued*, *they*), resolving the coreference, THEME(*valued*, *These products*). Axiom CAUSE$^{-1}(x, y) \circ$ AGENT$(y, z) \rightarrow$ AGENT(x, z) infers AGENT(*valued*, *These products*), which is wrong and could be blocked using the second proposed heuristic since THEME(*valued*, *These products*) is already known.

Some inferences are wrong because the annotation used as premises is not sound. Let us consider the sentence "[*Our intensive discussions with Jaguar*]$_{z,\ \text{AGENT}}$*," GM said,* "[*have*]$_y$ [*as their objectives*]$_{x,\text{PURPOSE}}$ *to create [. . .]*". PropBank annotates PURPOSE(*have*, *as their objectives*), which we believe does not hold. Because of the presence of this relation, axiom PURPOSE$^{-1}(x, y) \circ$ AGENT$(y, z) \rightarrow$ AGENT(x, z) can be instantiated, yielding AGENT(*as their objective*, *Our intensive discussions with Jaguar*). This inference is counted as wrong in our evaluation, even though the fault relies on the validity of the premises.

Another source of errors could be solved using a finer-grained ontology to define domains and ranges. Consider the sentence [*Due to the earthquake*]$_{x,\text{CAUSE}}$ *in San Francisco,* [*Nissan*]$_{z,\text{AGENT}}$ *is* [*donating*]$_y$ *its commercial air time to broadcast American Red Cross Emergency Relief messages*. Basic annotation includes CAUSE(*donating*, *due to the earthquake*) and AGENT(*donating*, *Nissan*). Instantiating axiom CAUSE$^{-1}(x, y) \circ$ AGENT$(y, z) \rightarrow$ AGENT(x, z), we infer AGENT(*due to the earthquake*, *Nissan*), which is simply wrong even though the premises are correct. "*Earthquake*" encodes an event, but cannot have an AGENT. The ontology used to define domains and ranges does not allow us to make the distinction between events that accept agents and those that do not.

8. CONCLUSION

In this article, we have presented a model for composing semantic relations and a detailed case study using 26 well-known relations. The model is independent of any particular set of relations and automatically extracts inference axioms. Axioms allow us to compose new relations out of previously extracted relations.

The formal framework is based on an extended definition of semantic relations, including restrictions on domains and ranges. Relations are also defined assigning values for a set of semantic primitives. Semantic primitives state elemental properties between the arguments of a relation. For example, if a relation takes "+" for the primitive *temporal*, the first argument is guaranteed to occur before the second.

We have defined a semantic calculus, a framework for composing semantic relations. Semantic calculus uses the extended definition of semantic relations in order to automatically extract inference axioms. An algebra has been manually defined for composing semantic primitives. The CSR algorithm identifies the pairs of relations that can be used in composition, and which relations can be composed out of them, if any.

Our case study shows that CSR applied to a set of 26 semantic relations identifies 216 axioms. We have evaluated a subset of 8 axioms involving CAUSE and PURPOSE and the overall accuracy is 0.86 using a heuristic. We believe these results are worthwhile for a totally unsupervised approach to semantic relation extraction. First, there is no need of annotation for obtaining inference axioms; the only requirement is to define the relation inventory using the extended definition. Second, once inference axioms are obtained, instantiating them is a simple procedure that does not require modifications to any other tool.

The model has limitations and is not always correct. First, relations are defined manually and mistakes could be made while assigning values to primitives. Second, the algebra for composing primitives is manually defined as well.

The first problem is easy to overcome. Whatever the set of relations one might use, thinking in terms of primitives helps to understand the nature of the relations and their differences. It will help annotation efforts and specifying the boundaries between different relations. An issue might be that the proposed set of primitives is not enough for a particular set, but more primitives could be added to solve this situation.

A further issue with the algebra is the fact that primitives are composed orthogonally. This is a simplification, but we have shown that this simplified algebra works. The authors believe that a good analogy is a Naive Bayes classifier. Even though this classifier is assuming something wrong (in most cases, attributes used to predict a class are not independent of each other), its performance is often astonishingly good.

Even though different sets of relations may call for different ontologies to define domains and ranges, and possibly an extended set of primitives, we believe the framework presented in this article is applicable to any set. To the best of our knowledge, this is a novel way to compose semantic relations in the field of NLP.

APPENDIXES

A. SEMANTIC-CALCULUS-DERIVED PROPERTIES

Semantic Calculus has the following definitions and properties (Section 4).

$$\text{R is reflexive iff } \forall x : \text{R}(x, x)\text{, for example, SYNONYMY} \tag{1}$$

$$\text{R is symmetric iff R}(x, y) \text{ implies R}(y, x)\text{, for example, KINSHIP} \tag{2}$$

$$\text{R is transitive iff R}(x, y) \circ \text{R}(y, z) \rightarrow \text{R}(x, z)\text{, e.g., CAUSE} \tag{3}$$

$$\perp(x, y) \text{ iff } \neg\exists \text{R}_i \text{ such that } \text{R}_i(x, y) \tag{4}$$

$$\text{R}_1 \triangleright \text{R}_2 \text{ iff } \text{R}_1(x, y) \circ \text{R}_2(y, z) \rightarrow \text{R}_1(x, z) \tag{5}$$

$$\text{R}_1 \triangleleft \text{R}_2 \text{ iff } \text{R}_2(x, y) \circ \text{R}_1(y, z) \rightarrow \text{R}_1(x, z) \tag{6}$$

$$\text{R}_1 \bowtie \text{R}_2 \text{ iff } \text{R}_1(x, y) \circ \text{R}_2(y, z) \rightarrow \text{R}_1(x, z) \text{ and } \text{R}_2(x, y) \circ \text{R}_1(y, z) \rightarrow \text{R}_1(x, z) \tag{7}$$

$$(\text{R}^{-1}(x, y))^{-1} = \text{R} \tag{8}$$

$$\perp(x, y)^{-1} = \perp(x, y)\text{, that is, } \perp \text{ is symmetric} \tag{9}$$

Section 4 also provides five derived properties.

$$\text{R}_i \circ \text{R}_j = (\text{R}_j{}^{-1} \circ \text{R}_i{}^{-1})^{-1} \tag{10}$$

$$\text{R}_i \triangleright \text{R}_j \text{ iff } \text{R}_i{}^{-1} \triangleleft \text{R}_j{}^{-1} \tag{11}$$

$$\text{R}_i \bowtie \text{R}_j \text{ iff } \text{R}_i{}^{-1} \bowtie \text{R}_j{}^{-1} \tag{12}$$

$$\text{If } \text{R}_i \text{ is symmetric and } \text{R}_i \bowtie \text{R}_j,\ \text{R}_i{}^{-1} \bowtie \text{R}_j \tag{13}$$

$$\text{If } \text{R}_j \text{ is symmetric and } \text{R}_i \bowtie \text{R}_j,\ \text{R}_i \bowtie \text{R}_j{}^{-1} \tag{14}$$

The proofs for the five derived properties are as follows.

—Property (10) states that the composition of two relations is equal to the inverse composition of the swapped inverse relations. Assume (a) $\text{R}_i(x, y) \circ \text{R}_j(y, z) \rightarrow \text{R}_k(x, z)$ and (b) $\text{R}_j{}^{-1}(x, y) \circ \text{R}_i{}^{-1}(y, z) \rightarrow \text{R}_l^{-1}(x, z)$. We will prove that $\text{R}_k = \text{R}_l^{-1}$ by contradiction.

Assumption (a)	$\text{R}_i(x, y) \circ \text{R}_j(y, z) \rightarrow \text{R}_k(x, z)$	(i)
Using inverses [i]	$\text{R}_i{}^{-1}(y, x) \circ \text{R}_j{}^{-1}(z, y) \rightarrow \text{R}_k(x, z)$	(ii)
Reordering [ii]	$\text{R}_j{}^{-1}(z, y) \circ \text{R}_i{}^{-1}(y, x) \rightarrow \text{R}_k(x, z)$	(iii)
Assumption (b)	$\text{R}_j{}^{-1}(x, y) \circ \text{R}_i{}^{-1}(y, z) \rightarrow \text{R}_l^{-1}(x, z)$	(iv)
Assuming $\text{R}_k \neq \text{R}_l^{-1}$	we reach a contradiction with [iii,iv]	(v)
Therefore	$\text{R}_k = \text{R}_l^{-1}$ and $\text{R}_i \circ \text{R}_j = (\text{R}_j{}^{-1} \circ \text{R}_i{}^{-1})^{-1}$	(vi)

—Property (11) asserts that R_i left dominates R_j if and only if $\text{R}_i{}^{-1}$ right dominates $\text{R}_j{}^{-1}$. Assume (a) $\text{R}_i \rhd \text{R}_j$. The property can be derived as follows.

Assumption (a)	$\text{R}_i \rhd \text{R}_j$	(i)
By (5) [i]	$\text{R}_i \rhd \text{R}_j$ iff $\text{R}_i \circ \text{R}_j \rightarrow \text{R}_i$	(ii)
Applying (10) [ii]	$\text{R}_i \rhd \text{R}_j$ iff $\text{R}_j{}^{-1} \circ \text{R}_i{}^{-1} \rightarrow \text{R}_i{}^{-1}$	(iii)
Therefore (6) [iii]	$\text{R}_i \rhd \text{R}_j$ iff $\text{R}_i{}^{-1} \lhd \text{R}_j{}^{-1}$	(iv)

—Property (12) reads R_i completely dominates R_j if and only if $\text{R}_i{}^{-1}$ completely dominates $\text{R}_j{}^{-1}$. Assume (a) $\text{R}_i \bowtie \text{R}_j$.

Assumption (a)	$\text{R}_i \bowtie \text{R}_j$	(i)
By (7) [i]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i \circ \text{R}_j \rightarrow \text{R}_i$ and $\text{R}_j \circ \text{R}_i \rightarrow \text{R}_i$	(ii)
Applying (10) [ii]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_j{}^{-1} \circ \text{R}_i{}^{-1} \rightarrow \text{R}_i{}^{-1}$ and $\text{R}_i{}^{-1} \circ \text{R}_j{}^{-1} \rightarrow \text{R}_i{}^{-1}$	(iii)
Reordering [iii]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i{}^{-1} \circ \text{R}_j{}^{-1} \rightarrow \text{R}_i{}^{-1}$ and $\text{R}_j{}^{-1} \circ \text{R}_i{}^{-1} \rightarrow \text{R}_i{}^{-1}$	(iv)
Therefore (7) [iv]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i{}^{-1} \bowtie \text{R}_j{}^{-1}$	(v)

—Property (13) states that if R_i is symmetric and it completely dominates R_j, the inverse of R_i completely dominates R_j as well. Assume (a) R_i is symmetric and (b) $\text{R}_i \bowtie \text{R}_j$.

Assumption (a)	R_i is symmetric	(i)
By (2) [i]	$\text{R}_i(x, y)$ implies $\text{R}_i(y, x)$	(ii)
Applying inverse [ii],	$\text{R}_i(x, y)$ implies $\text{R}_i{}^{-1}(x, y)$	(iii)
Assumption (b)	$\text{R}_i \bowtie \text{R}_j$	(iv)
By (7) [iv]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i(x, y) \circ \text{R}_j(y, z) \rightarrow \text{R}_i(x, z)$	(v)
and	$\text{R}_j(x, y) \circ \text{R}_i(y, z) \rightarrow \text{R}_i(x, z)$	(vi)
Therefore [iii,v]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i{}^{-1}(x, y) \circ \text{R}_j(y, z) \rightarrow \text{R}_i{}^{-1}(x, z)$	(vii)
and [iii,vi]	$\text{R}_j(x, y) \circ \text{R}_i{}^{-1}(y, z) \rightarrow \text{R}_i{}^{-1}(x, z)$	(viii)
Therefore (7) [vii,viii]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i{}^{-1} \bowtie \text{R}_j$	(ix)

—Property (14) states that if R_j is symmetric and it is completely dominated by a relation R_i, then the inverse of R_j is completely dominated by R_i as well. Assume (a) R_j is symmetric and (b) $\text{R}_i \bowtie \text{R}_j$.

Assumption (a)	R_j is symmetric	(i)
By (2) [i]	$\text{R}_j(x, y)$ implies $\text{R}_j(y, x)$	(ii)
Applying inverse [ii],	$\text{R}_j(x, y)$ implies $\text{R}_j{}^{-1}(x, y)$	(iii)
Assumption (b)	$\text{R}_i \bowtie \text{R}_j$	(iv)
By (7) [iv]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i(x, y) \circ \text{R}_j(y, z) \rightarrow \text{R}_i(x, z)$	(v)
and	$\text{R}_j(x, y) \circ \text{R}_i(y, z) \rightarrow \text{R}_i(x, z)$	(vi)
Therefore [iii,v]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i(x, y) \circ \text{R}_j{}^{-1}(y, z) \rightarrow \text{R}_i(x, z)$	(vii)
and [iii,vi]	$\text{R}_j{}^{-1}(x, y) \circ \text{R}_i(y, z) \rightarrow \text{R}_i(x, z)$	(viii)
Therefore (7) [vii,viii]	$\text{R}_i \bowtie \text{R}_j$ iff $\text{R}_i \bowtie \text{R}_j{}^{-1}$	(ix)

B. INFERENCE AXIOMS OBTAINED USING THE 26 SEMANTIC RELATIONS

This appendix lists the 216 unique inference axioms (Tables VI–VIII) obtained with the CSR algorithm and the set of 26 relations (Tables IV, V).

1:	CAUSE	○ CAUSE	→ CAUSE
2:	CAUSE	○ JUSTIFICATION	→ JUSTIFICATION
3:	CAUSE	○ JUSTIFICATION	→ INFLUENCE
4:	CAUSE	○ INFLUENCE	→ JUSTIFICATION
5:	CAUSE	○ INFLUENCE	→ INFLUENCE
6:	CAUSE	○ AGENT	→ AGENT
7:	CAUSE	○ EXPERIENCER	→ AGENT
8:	CAUSE	○ EXPERIENCER	→ EXPERIENCER
9:	CAUSE	○ INSTRUMENT	→ INSTRUMENT
10:	CAUSE	○ SYNONYMY	→ CAUSE
11:	CAUSE	○ LOCATION	→ LOCATION
12:	CAUSE	○ TIME	→ TIME
13:	JUSTIFICATION	○ CAUSE	→ JUSTIFICATION
14:	JUSTIFICATION	○ CAUSE	→ INFLUENCE
15:	JUSTIFICATION	○ JUSTIFICATION	→ JUSTIFICATION
16:	JUSTIFICATION	○ JUSTIFICATION	→ INFLUENCE
17:	JUSTIFICATION	○ INFLUENCE	→ JUSTIFICATION
18:	JUSTIFICATION	○ INFLUENCE	→ INFLUENCE
19:	JUSTIFICATION	○ AGENT	→ AGENT
20:	JUSTIFICATION	○ EXPERIENCER	→ AGENT
21:	JUSTIFICATION	○ EXPERIENCER	→ EXPERIENCER
22:	JUSTIFICATION	○ INSTRUMENT	→ INSTRUMENT
23:	JUSTIFICATION	○ IS-A	→ JUSTIFICATION
24:	JUSTIFICATION	○ SYNONYMY	→ JUSTIFICATION
25:	JUSTIFICATION	○ LOCATION	→ LOCATION
26:	JUSTIFICATION	○ TIME	→ TIME
27:	INFLUENCE	○ CAUSE	→ JUSTIFICATION
28:	INFLUENCE	○ CAUSE	→ INFLUENCE
29:	INFLUENCE	○ JUSTIFICATION	→ JUSTIFICATION
30:	INFLUENCE	○ JUSTIFICATION	→ INFLUENCE
31:	INFLUENCE	○ INFLUENCE	→ JUSTIFICATION
32:	INFLUENCE	○ INFLUENCE	→ INFLUENCE
33:	INFLUENCE	○ AGENT	→ AGENT
34:	INFLUENCE	○ EXPERIENCER	→ AGENT

35:	INFLUENCE	∘ EXPERIENCER	→ EXPERIENCER
36:	INFLUENCE	∘ INSTRUMENT	→ INSTRUMENT
37:	INFLUENCE	∘ SYNONYMY	→ INFLUENCE
38:	INFLUENCE	∘ LOCATION	→ LOCATION
39:	INFLUENCE	∘ TIME	→ TIME
40:	INTENT	∘ PURPOSE	→ INTENT
41:	INTENT	∘ MANNER	→ INTENT
42:	INTENT	∘ SYNONYMY	→ INTENT
43:	INTENT	∘ LOCATION	→ LOCATION
44:	INTENT	∘ TIME	→ TIME
45:	PURPOSE	∘ PURPOSE	→ PURPOSE
46:	PURPOSE	∘ AGENT	→ AGENT
47:	PURPOSE	∘ EXPERIENCER	→ EXPERIENCER
48:	PURPOSE	∘ INSTRUMENT	→ INSTRUMENT
49:	PURPOSE	∘ THEME	→ PURPOSE
50:	PURPOSE	∘ THEME	→ THEME
51:	PURPOSE	∘ IS-A	→ PURPOSE
52:	PURPOSE	∘ MANNER	→ MANNER
53:	PURPOSE	∘ SYNONYMY	→ PURPOSE
54:	PURPOSE	∘ LOCATION	→ LOCATION
55:	PURPOSE	∘ TIME	→ TIME
56:	AGENT	∘ IS-A	→ AGENT
57:	AGENT	∘ SYNONYMY	→ AGENT
58:	AGENT	∘ LOCATION	→ LOCATION
59:	AGENT	∘ TIME	→ TIME
60:	EXPERIENCER	∘ PURPOSE	→ EXPERIENCER
61:	EXPERIENCER	∘ IS-A	→ EXPERIENCER
62:	EXPERIENCER	∘ SYNONYMY	→ EXPERIENCER
63:	EXPERIENCER	∘ LOCATION	→ LOCATION
64:	EXPERIENCER	∘ TIME	→ TIME
65:	INSTRUMENT	∘ PURPOSE	→ INSTRUMENT
66:	INSTRUMENT	∘ IS-A	→ INSTRUMENT
67:	INSTRUMENT	∘ PART-WHOLE	→ INSTRUMENT
68:	INSTRUMENT	∘ SYNONYMY	→ INSTRUMENT
69:	INSTRUMENT	∘ LOCATION	→ LOCATION
70:	INSTRUMENT	∘ TIME	→ TIME
71:	THEME	∘ PURPOSE	→ THEME
72:	THEME	∘ PURPOSE	→ PURPOSE
73:	THEME	∘ IS-A	→ THEME
74:	THEME	∘ SYNONYMY	→ THEME
75:	THEME	∘ LOCATION	→ LOCATION
76:	THEME	∘ TIME	→ TIME
77:	IS-A	∘ INTENT	→ INTENT
78:	IS-A	∘ PURPOSE	→ PURPOSE
79:	IS-A	∘ IS-A	→ IS-A
80:	IS-A	∘ SYNONYMY	→ IS-A
81:	IS-A	∘ LOCATION	→ LOCATION
82:	IS-A	∘ TIME	→ TIME
83:	PART-WHOLE	∘ IS-A	→ PART-WHOLE
84:	PART-WHOLE	∘ PART-WHOLE	→ PART-WHOLE
85:	PART-WHOLE	∘ SYNONYMY	→ PART-WHOLE
86:	PART-WHOLE	∘ LOCATION	→ LOCATION

87:	PART-WHOLE	$\circ$ TIME	$\rightarrow$ TIME
88:	MANNER	$\circ$ PURPOSE	$\rightarrow$ MANNER
89:	MANNER	$\circ$ AGENT	$\rightarrow$ AGENT
90:	MANNER	$\circ$ EXPERIENCER	$\rightarrow$ AGENT
91:	MANNER	$\circ$ IS-A	$\rightarrow$ MANNER
92:	MANNER	$\circ$ MANNER	$\rightarrow$ MANNER
93:	MANNER	$\circ$ SYNONYMY	$\rightarrow$ MANNER
94:	MANNER	$\circ$ LOCATION	$\rightarrow$ LOCATION
95:	MANNER	$\circ$ TIME	$\rightarrow$ TIME
96:	SYNONYMY	$\circ$ CAUSE	$\rightarrow$ CAUSE
97:	SYNONYMY	$\circ$ JUSTIFICATION	$\rightarrow$ JUSTIFICATION
98:	SYNONYMY	$\circ$ INFLUENCE	$\rightarrow$ INFLUENCE
99:	SYNONYMY	$\circ$ INTENT	$\rightarrow$ INTENT
100:	SYNONYMY	$\circ$ PURPOSE	$\rightarrow$ PURPOSE
101:	SYNONYMY	$\circ$ AGENT	$\rightarrow$ AGENT
102:	SYNONYMY	$\circ$ EXPERIENCER	$\rightarrow$ EXPERIENCER
103:	SYNONYMY	$\circ$ INSTRUMENT	$\rightarrow$ INSTRUMENT
104:	SYNONYMY	$\circ$ THEME	$\rightarrow$ THEME
105:	SYNONYMY	$\circ$ IS-A	$\rightarrow$ SYNONYMY
106:	SYNONYMY	$\circ$ MANNER	$\rightarrow$ MANNER
107:	SYNONYMY	$\circ$ SYNONYMY	$\rightarrow$ SYNONYMY
108:	SYNONYMY	$\circ$ LOCATION	$\rightarrow$ LOCATION
109:	SYNONYMY	$\circ$ TIME	$\rightarrow$ TIME
110:	LOCATION	$\circ$ PART-WHOLE	$\rightarrow$ LOCATION
111:	LOCATION	$\circ$ SYNONYMY	$\rightarrow$ LOCATION
112:	TIME	$\circ$ PART-WHOLE	$\rightarrow$ TIME
113:	TIME	$\circ$ SYNONYMY	$\rightarrow$ TIME
114:	CAUSE^{-1}	$\circ$ AGENT	$\rightarrow$ AGENT
115:	CAUSE^{-1}	$\circ$ EXPERIENCER	$\rightarrow$ AGENT
116:	CAUSE^{-1}	$\circ$ EXPERIENCER	$\rightarrow$ EXPERIENCER
117:	CAUSE^{-1}	$\circ$ INSTRUMENT	$\rightarrow$ INSTRUMENT
118:	CAUSE^{-1}	$\circ$ SYNONYMY	$\rightarrow$ CAUSE^{-1}
119:	CAUSE^{-1}	$\circ$ LOCATION	$\rightarrow$ LOCATION
120:	CAUSE^{-1}	$\circ$ TIME	$\rightarrow$ TIME
121:	JUSTIFICATION^{-1}	$\circ$ AGENT	$\rightarrow$ AGENT
122:	JUSTIFICATION^{-1}	$\circ$ EXPERIENCER	$\rightarrow$ AGENT
123:	JUSTIFICATION^{-1}	$\circ$ EXPERIENCER	$\rightarrow$ EXPERIENCER
124:	JUSTIFICATION^{-1}	$\circ$ INSTRUMENT	$\rightarrow$ INSTRUMENT
125:	JUSTIFICATION^{-1}	$\circ$ SYNONYMY	$\rightarrow$ JUSTIFICATION^{-1}
126:	JUSTIFICATION^{-1}	$\circ$ LOCATION	$\rightarrow$ LOCATION
127:	JUSTIFICATION^{-1}	$\circ$ TIME	$\rightarrow$ TIME
128:	INFLUENCE^{-1}	$\circ$ AGENT	$\rightarrow$ AGENT
129:	INFLUENCE^{-1}	$\circ$ EXPERIENCER	$\rightarrow$ AGENT
130:	INFLUENCE^{-1}	$\circ$ EXPERIENCER	$\rightarrow$ EXPERIENCER
131:	INFLUENCE^{-1}	$\circ$ INSTRUMENT	$\rightarrow$ INSTRUMENT
132:	INFLUENCE^{-1}	$\circ$ SYNONYMY	$\rightarrow$ INFLUENCE^{-1}
133:	INFLUENCE^{-1}	$\circ$ LOCATION	$\rightarrow$ LOCATION
134:	INFLUENCE^{-1}	$\circ$ TIME	$\rightarrow$ TIME
135:	INTENT^{-1}	$\circ$ IS-A	$\rightarrow$ INTENT^{-1}
136:	INTENT^{-1}	$\circ$ SYNONYMY	$\rightarrow$ INTENT^{-1}
137:	INTENT^{-1}	$\circ$ LOCATION	$\rightarrow$ LOCATION
138:	INTENT^{-1}	$\circ$ TIME	$\rightarrow$ TIME

139:	PURPOSE^{-1}	∘ AGENT	→ INTENT^{-1}
140:	PURPOSE^{-1}	∘ AGENT	→ AGENT
141:	PURPOSE^{-1}	∘ EXPERIENCER	→ EXPERIENCER
142:	PURPOSE^{-1}	∘ INSTRUMENT	→ INSTRUMENT
143:	PURPOSE^{-1}	∘ THEME	→ PURPOSE^{-1}
144:	PURPOSE^{-1}	∘ THEME	→ THEME
145:	PURPOSE^{-1}	∘ IS-A	→ PURPOSE^{-1}
146:	PURPOSE^{-1}	∘ MANNER	→ MANNER
147:	PURPOSE^{-1}	∘ SYNONYMY	→ PURPOSE^{-1}
148:	PURPOSE^{-1}	∘ LOCATION	→ LOCATION
149:	PURPOSE^{-1}	∘ TIME	→ TIME
150:	AGENT^{-1}	∘ MANNER	→ AGENT^{-1}
151:	AGENT^{-1}	∘ SYNONYMY	→ AGENT^{-1}
152:	AGENT^{-1}	∘ LOCATION	→ LOCATION
153:	AGENT^{-1}	∘ TIME	→ TIME
154:	EXPERIENCER^{-1}	∘ MANNER	→ AGENT^{-1}
155:	EXPERIENCER^{-1}	∘ SYNONYMY	→ EXPERIENCER^{-1}
156:	EXPERIENCER^{-1}	∘ LOCATION	→ LOCATION
157:	EXPERIENCER^{-1}	∘ TIME	→ TIME
158:	INSTRUMENT^{-1}	∘ SYNONYMY	→ INSTRUMENT^{-1}
159:	INSTRUMENT^{-1}	∘ LOCATION	→ LOCATION
160:	INSTRUMENT^{-1}	∘ TIME	→ TIME
161:	THEME^{-1}	∘ SYNONYMY	→ THEME^{-1}
162:	THEME^{-1}	∘ LOCATION	→ LOCATION
163:	THEME^{-1}	∘ TIME	→ TIME
164:	IS-A^{-1}	∘ SYNONYMY	→ IS-A^{-1}
165:	IS-A^{-1}	∘ LOCATION	→ LOCATION
166:	IS-A^{-1}	∘ TIME	→ TIME
167:	PART-WHOLE^{-1}	∘ SYNONYMY	→ PART-WHOLE^{-1}
168:	PART-WHOLE^{-1}	∘ LOCATION	→ LOCATION
169:	PART-WHOLE^{-1}	∘ TIME	→ TIME
170:	MANNER^{-1}	∘ MANNER	→ MANNER^{-1}
171:	MANNER^{-1}	∘ MANNER	→ MANNER
172:	MANNER^{-1}	∘ SYNONYMY	→ MANNER^{-1}
173:	MANNER^{-1}	∘ LOCATION	→ LOCATION
174:	MANNER^{-1}	∘ TIME	→ TIME
175:	SYNONYMY^{-1}	∘ SYNONYMY	→ SYNONYMY^{-1}
176:	SYNONYMY^{-1}	∘ LOCATION	→ LOCATION
177:	SYNONYMY^{-1}	∘ TIME	→ TIME
178:	CAUSE	∘ INTENT^{-1}	→ INTENT^{-1}
179:	CAUSE	∘ PURPOSE^{-1}	→ INTENT^{-1}
180:	CAUSE	∘ SYNONYMY^{-1}	→ CAUSE
181:	JUSTIFICATION	∘ INTENT^{-1}	→ INTENT^{-1}
182:	JUSTIFICATION	∘ PURPOSE^{-1}	→ INTENT^{-1}
183:	JUSTIFICATION	∘ IS-A^{-1}	→ JUSTIFICATION
184:	JUSTIFICATION	∘ SYNONYMY^{-1}	→ JUSTIFICATION
185:	INFLUENCE	∘ INTENT^{-1}	→ INTENT^{-1}
186:	INFLUENCE	∘ PURPOSE^{-1}	→ INTENT^{-1}
187:	INFLUENCE	∘ SYNONYMY^{-1}	→ INFLUENCE
188:	INTENT	∘ MANNER^{-1}	→ INTENT
189:	INTENT	∘ SYNONYMY^{-1}	→ INTENT
190:	PURPOSE	∘ EXPERIENCER^{-1}	→ EXPERIENCER^{-1}

191:	PURPOSE	$\circ$ INSTRUMENT^{-1}	$\rightarrow$ INSTRUMENT^{-1}
192:	PURPOSE	$\circ$ THEME^{-1}	$\rightarrow$ THEME^{-1}
193:	PURPOSE	$\circ$ THEME^{-1}	$\rightarrow$ PURPOSE
194:	PURPOSE	$\circ$ IS-A^{-1}	$\rightarrow$ PURPOSE
195:	PURPOSE	$\circ$ MANNER^{-1}	$\rightarrow$ MANNER^{-1}
196:	PURPOSE	$\circ$ SYNONYMY^{-1}	$\rightarrow$ PURPOSE
197:	AGENT	$\circ$ IS-A^{-1}	$\rightarrow$ AGENT
198:	AGENT	$\circ$ SYNONYMY^{-1}	$\rightarrow$ AGENT
199:	EXPERIENCER	$\circ$ IS-A^{-1}	$\rightarrow$ EXPERIENCER
200:	EXPERIENCER	$\circ$ SYNONYMY^{-1}	$\rightarrow$ EXPERIENCER
201:	INSTRUMENT	$\circ$ IS-A^{-1}	$\rightarrow$ INSTRUMENT
202:	INSTRUMENT	$\circ$ PART-WHOLE^{-1}	$\rightarrow$ INSTRUMENT
203:	INSTRUMENT	$\circ$ SYNONYMY^{-1}	$\rightarrow$ INSTRUMENT
204:	THEME	$\circ$ IS-A^{-1}	$\rightarrow$ THEME
205:	THEME	$\circ$ SYNONYMY^{-1}	$\rightarrow$ THEME
206:	IS-A	$\circ$ PART-WHOLE^{-1}	$\rightarrow$ PART-WHOLE^{-1}
207:	IS-A	$\circ$ MANNER^{-1}	$\rightarrow$ MANNER^{-1}
208:	IS-A	$\circ$ SYNONYMY^{-1}	$\rightarrow$ SYNONYMY^{-1}
209:	PART-WHOLE	$\circ$ LOCATION^{-1}	$\rightarrow$ LOCATION^{-1}
210:	PART-WHOLE	$\circ$ TIME^{-1}	$\rightarrow$ TIME^{-1}
211:	MANNER	$\circ$ MANNER^{-1}	$\rightarrow$ MANNER^{-1}
212:	MANNER	$\circ$ MANNER^{-1}	$\rightarrow$ MANNER
213:	MANNER	$\circ$ SYNONYMY^{-1}	$\rightarrow$ MANNER
214:	SYNONYMY	$\circ$ SYNONYMY^{-1}	$\rightarrow$ SYNONYMY^{-1}
215:	SYNONYMY	$\circ$ LOCATION^{-1}	$\rightarrow$ LOCATION^{-1}
216:	SYNONYMY	$\circ$ TIME^{-1}	$\rightarrow$ TIME^{-1}

REFERENCES

ALLEN, J. F., SWIFT, M., AND DE BEAUMONT, W. 2008. Deep semantic analysis of text. In *Proceedings of the Conference on Semantics in Text Processing (STEP'08).* J. Bos and R. Delmonte, Eds., Research in Computational Semantics Series, vol. 1, College Publications, 343–354.

ARTALE, A., ARTALE, R., FRANCONI, E., AND GUARINO, N. 1996. Open problems with part-whole relations. In *Proceedings of the Description Logic Workshop (DL'96).* AAAI Press, 70–73.

BADULESCU, A. AND MOLDOVAN, D. 2009. A semantic scattering model for the automatic interpretation of english genitives. *Natural Lang. Engin.15*, 02, 215–239.

BAKER, C. F., FILLMORE, C. J., AND LOWE, J. B. 1998. The berkeley framenet project. In *Proceedings of the 17th International Conference on Computational Linguistics*. Morgan Kaufmann, San Fransisco, CA.

BETHARD, S., CORVEY, W., KLINGENSTEIN, S., AND MARTIN, J. H. 2008. Building a corpus of temporal-causal structure. In *Proceedings of the 6th International Language Resources and Evaluation (LREC'08).* European Language Resources Association.

BETHARD, S. AND MARTIN, J. H. 2008. Learning semantic links from a corpus of parallel temporal and causal relations. In *Proceedings of the 46th Annual Meeting of the Association for Computational Linguistics on Human Language Technologies: Short Papers*. Association for Computational Linguistics, 177–180.

BLANCO, E., CANKAYA, H. C., AND MOLDOVAN, D. 2010. Composition of semantic relations: Model and applications. In *Proceedings of the 23rd International Conference on Computational Linguistics (COLING'10).* Poster volume. Association for Computational Linguistics, 72–80.

BLANCO, E. AND MOLDOVAN, D. 2011a. A model for composing semantic relations. In *Proceedings of the 9th International Conference on Computational Semantics (IWCS'11).* Association for Computational Linguistics, 45–54.

BLANCO, E. AND MOLDOVAN, D. 2011b. Unsupervised learning of semantic relation composition. In *Proceedings of the 49th Annual Meeting of the Association for Computational Linguistics: Human Language Technologies (ACL-HLT'11).* Association for Computational Linguistics, 1456–1465.

BOS, J. 2008. Wide-coverage semantic analysis with boxer. In *Proceedings of the Conference on Semantics in Text Processing (STEP'08).* J. Bos and R. Delmonte, Eds., Research in Computational Semantics Series, vol. 1, College Publications, 277–286.

BOS, J. 2011. A survey of computational semantics: Representation, inference and knowledge in wide-coverage text understanding. *Lang. Linguist. Compass 5*, 6, 336–366.

CARRERAS, X. AND MARQUEZ, L. 2004. Introduction to the conll-2004 shared task: Semantic role labeling. In *Proceedings of the 8th Conference on Computational Natural Language Learning (CoNLL'04).* H. T. Ng and E. Riloff, Eds., Association for Computational Linguistics, 89–97.

CARRERAS, X. AND MARQUEZ, L. 2005. Introduction to the conll-2005 shared task: Semantic role labeling. In *Proceedings of the 9th Conference on Computational Natural Language Learning (CONLL'05).* Association for Computational Linguistics, 152–164.

CHAFFIN, R. AND HERRMANN, D. J. 1987. Relation element theory: A new account of the representation and processing of semantic relations. In *Memory and Learning. The Ebbinghaus Centennial Conference*, D. S. Gorfein and R. R. Hoffman, Eds., Lawrence Erlbaum Associates.

CHANG, D. S. AND CHOI, K. S. 2006. Incremental cue phrase learning and bootstrapping method for causality extraction using cue phrase and word pair probabilities. *Inf. Process. Manag. 42*, 3, 662–678.

CHKLOVSKI, T. AND PANTEL, P. 2004. VerbOcean: Mining the web for fine-grained semantic verb relations. In *Proceedings of the Conference on Empirical Methods in Natural Language Processing (EMNLP'04),* D. Lin and D. Wu, Eds., Association for Computational Linguistics, 33–40.

CLARK, P. AND HARRISON, P. 2008. Recognizing textual entailment with logical inference. In *Proceedings of the Text Analysis Conference (TAC'08).* Association for Computational Linguistics.

COECKE, B., SADRZADEH, M., AND CLARK, S. 2011. Mathematical foundations for a compositional distributed model of meaning. *Linguist. Anal. 36*, 1–4, 345–384.

COHEN, P. R. AND LOSIELLE, C. L. 1988. Beyond isa: Structures for plausible inference in semantic networks. In *Proceedings of the 7th National Conference on Artificial Intelligence*. AAAI Press.

COPESTAKE, A., LASCARIDES, A., AND FLICKINGER, D. 2001. An algebra for semantic construction in constraint-based grammars. In *Proceedings of the 39th Annual Meeting of the Association for Computational Linguistics.* Association for Computational Linguistics, 140–147.

DANG, H. T. AND PALMER, M. 2005. The role of semantic roles in disambiguating verb senses. In *Proceedings of the 43rd Annual Meeting of the Association for Computational Linguistics (ACL'05).* Association for Computational Linguistics, 42–49.

DAVIDOV, D. AND RAPPOPORT, A. 2008. Unsupervised discovery of generic relationships using pattern clusters and its evaluation by automatically generated sat analogy questions. In *Proceedings of the 46th Annual Meeting of the Association of Computational Linguistics.* Association for Computational Linguistics, 692–700.

DAVIDOV, D., RAPPOPORT, A., AND KOPPEL, M. 2007. Fully unsupervised discovery of concept-specific relationships by web mining. In *Proceedings of the 45th Annual Meeting of the Association of Computational Linguistics.* Association for Computational Linguistics, 232–239.

DIAB, M., ALKHALIFA, M., ELKATEB, S., FELLBAUM, C., MANSOURI, A., AND PALMER, M. 2007. SemEval-2007 task 18: Arabic semantic labeling. In *Proceedings of the 4th International Workshop on Semantic Evaluations (SemEval'07).* Association for Computational Linguistics, 93–98.

ERK, K. AND PADO, S. 2008. A structured vector space model forword meaning in context. In *Proceedings of the Conference on Empirical Methods in Natural Language Processing.* Association for Computational Linguistics, 897–906.

GE, R. AND MOONEY, R. 2009. Learning a compositional semantic parser using an existing syntactic parser. In *Proceedings of the Joint Conference of the 47th Annual Meeting of the ACL and the 4th International Joint Conference on Natural Language Processing of the AFNLP*. Association for Computational Linguistics, 611–619.

GERBER, M. AND CHAI, J. 2010. Beyond nombank: A study of implicit arguments for nominal predicates. In *Proceedings of the 48th Annual Meeting of the Association for Computational Linguistics*. Association for Computational Linguistics, 1583–1592.

GIRJU, R., BADULESCU, A., AND MOLDOVAN, D. 2006. Automatic discovery of part-whole relations. *Comput. Linguist. 32*, 1, 83–135.

GIRJU, R. AND MOLDOVAN, D. 2002. Mining answers for causation questions. In *Proceedings of the American Association for Artificial Intelligence: Spring Symposium (AAAI'02).* AAAI Press.

GIRJU, R., MOLDOVAN, D., TATU, M., AND ANTOHE, D. 2005. On the semantics of noun compounds. *Comput. Speech Lang. 19*, 4, 479–496.

GIRJU, R., NAKOV, P., NASTASE, V., SZPAKOWICZ, S., TURNEY, P., AND YURET, D. 2007. SemEval-2007 task 04: Classification of semantic relations between nominals. In *Proceedings of the 4th International Workshop on Semantic Evaluations (SemEval'07).* Association for Computational Linguistics, 13–18.

GONG, Y. AND LIU, X. 2001. Generic text summarization using relevance measure and latent semantic analysis. In *Proceedings of the 24th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR'01).* ACM Press, New York, 19–25.

HARABAGIU, S. AND MOLDOVAN, D. 1998. Knowledge processing on an extended wordnet. In *WordNet: An Electronic Lexical Database and Some of its Applications*, C. Fellbaum, Ed., The MIT Press, Cambridge, MA, 684–714.

HEARST, M. A. 1992. Automatic acquisition of hyponyms from large text corpora. In *Proceedings of the 14th International Conference on Computational Linguistics*. Association for Computational Linguistics, 539–545.

HELBIG, H. 2005. *Knowledge Representation and the Semantics of Natural Language* 1st Ed. Springer.

HENDRICKX, I., KIM, S. N., KOZAREVA, Z., NAKOV, P., DIARMUID, PADO, S., PENNACCHIOTTI, M., ROMANO, L., AND SZPAKOWICZ, S. 2009. SemEval-2010 task 8: Multi-way classification of semantic relations between pairs of nominals. In *Proceedings of the Workshop on Semantic Evaluations: Recent Achievements and Future Directions (SEW'09).* Association for Computational Linguistics, 94–99.

HIRANO, T., MATSUO, Y., AND KIKUI, G. 2007. Detecting semantic relations between named entities in text using contextual features. In *Proceedings of the 45th Annual Meeting of the Association for Computational Linguistics, Demo and Poster Sessions*. Association for Computational Linguistics, 157–160.

HUHNS, M. N. AND STEPHENS, L. M. 1989. Plausible inferencing using extended composition. In *Proceedings of the 11th International Joint Conference on Artificial Intelligence (IJCAI'89).* Morgan Kaufmann, San Fransisco, CA, 1420–1425.

KOOMEN, P., PUNYAKANOK, V., ROTH, D., AND YIH, W. T. 2005. Generalized inference with multiple semantic role labeling systems. In *Proceedings of the 9th Conference on Computational Natural Language Learning (CONLL'05).* Association for Computational Linguistics, 181–184.

LAKOFF, G. 1970. Linguistics and natural logic. *Synthese 22*, 1, 151–271.

MACCARTNEY, B. AND MANNING, C. D. 2008. Modeling semantic containment and exclusion in natural language inference. In *Proceedings of the 22nd International Conference on Computational Linguistics (COLING'08).* Association for Computational Linguistics, 521–528.

MACCARTNEY, B. AND MANNING, C. D. 2009. An extended model of natural logic. In *Proceedings of the 8th International Conference on Computational Semantics.* Association for Computational Linguistics, 140–156.

MARQUEZ, L., CARRERAS, X., LITKOWSKI, K. C., AND STEVENSON, S. 2008. Semantic role labeling: An introduction to the special issue. *Comput. Linguist. 34*, 2, 145–159.

MARQUEZ, L., VILLAREJO, L., MARTI, M. A., AND TAULE, M. 2007. SemEval-2007 task 09: Multilevel semantic annotation of catalan and spanish. In *Proceedings of the 4th International Workshop on Semantic Evaluations (SemEval'07).* Association for Computational Linguistics, 42–47.

MEYERS, A., REEVES, R., MACLEOD, C., SZEKELY, R., ZIELINSKA, V., YOUNG, B., AND GRISHMAN, R. 2004. The nombank project: An interim report. In *Proceedings of the Workshop on Frontiers in Corpus Annotation*. A. Meyers, Ed., Association for Computational Linguistics, 24–31.

MITCHELL, J. AND LAPATA, M. 2008. Vector-based models of semantic composition. In *Proceedings of the Annual Meeting of the Association for Computational Linguistics*. Association for Computational Linguistics, 236–244.

MOLDOVAN, D., BADULESCU, A., TATU, M., ANTOHE, D., AND GIRJU, R. 2004. Models for the semantic classification of noun phrases. In *Proceedings of the Workshop on Computational Lexical Semantics.* Association for Computational Linguistics, 60–67.

MOLDOVAN, D., TATU, M., AND CLARK, C. 2010. Role of semantics in question answering. In *Semantic Computing*, P. Sheu, H. Yu, C. V. Ramamoorthy, A. K. Joshi, and L. A. Zadeh, Eds., Wiley/IEEE Press.

NULTY, P. 2007. Semantic classification of noun phrases using web counts and learning algorithms. In *Proceedings of the Student Research Workshop*. Association for Computational Linguistics, 79–84.

PALMER, M., GILDEA, D., AND KINGSBURY, P. 2005. The proposition bank: An annotated corpus of semantic roles. *Comput. Linguist. 31*, 1, 71–106.

PONZETTO, S. P. AND STRUBE, M. 2006. Semantic role labeling for coreference resolution. In *Companion Volume to the Proceedings of the 11th Conference of the European Chapter of the Association for Computational Linguistics*. Association for Computational Linguistics, 143–146.

POON, H. AND DOMINGOS, P. 2009. Unsupervised semantic parsing. In *Proceedings of the Conference on Empirical Methods in Natural Language Processing*. Association for Computational Linguistics, 1–10.

QUIRK, R., GREENBAUM, S., LEECH, G., AND SVARTVIK, J. 1985. *A Comprehensive Grammar of the English Language.* Longman, London.

ROSARIO, B. AND HEARST, M. 2004. Classifying semantic relations in bioscience texts. In *Proceedings of the 42nd Meeting of the Association for Computational Linguistics (ACL'04)*. Association for Computational Linguistics, 430–437.

RUPPENHOFER, J., SPORLEDER, C., MORANTE, R., BAKER, C., AND PALMER, M. 2009. SemEval-2010 task 10: Linking events and their participants in discourse. In *Proceedings of the Workshop on Semantic Evaluations: Recent Achievements and Future Directions (SEW'09)*. Association for Computational Linguistics, 106–111.

SANCHEZ VALENCIA, V. M. 1991. Studies on natural logic and categorial grammar. Ph.D. thesis, University of Amsterdam.

SHEN, D. AND LAPATA, M. 2007. Using semantic roles to improve question answering. In *Proceedings of the Joint Conference on Empirical Methods in Natural Language Processing and Computational Natural Language Learning (EMNLP-CoNLL'07)*. Association for Computational Linguistics, 12–21.

SRIKUMAR, V., REICHART, R., SAMMONS, M., RAPPOPORT, A., AND ROTH, D. 2008. Extraction of entailed semantic relations through syntax-based comma resolution. In *Proceedings of the 46th Annual Meeting of the Association for Computational Linguistics: Human Language Technologies*. 1030–1038.

SURDEANU, M., HARABAGIU, S., WILLIAMS, J., AND AARSETH, P. 2003. Using predicate-argument structures for information extraction. In *Proceedings of the 41st Annual Meeting on Association for Computational Linguistics (ACL'03)*. Vol. 1, Association for Computational Linguistics, 8–15.

SZPAKOWICZ, B., BARKER, K., AND SZPAKOWICZ, S. 1995. Interactive semantic analysis of clause-level relationships. In *Proceedings of the 2nd Conference of the Pacific Association for Computational Linguistics*. Association for Computational Linguistics, 22–30.

TATU, M. 2005. Automatic discovery of intentions in text and its application to question answering. In *Proceedings of the ACL Student Research Workshop*. Association for Computational Linguistics, 31–36.

TRATZ, S. AND HOVY, E. 2010. A taxonomy, dataset, and classifier for automatic noun compound interpretation. In *Proceedings of the 48th Annual Meeting of the Association for Computational Linguistics*. Association for Computational Linguistics, 678–687.

TURNEY, P. D. 2006. Expressing implicit semantic relations without supervision. In *Proceedings of the 21st International Conference on Computational Linguistics and 44th Annual Meeting of the Association for Computational Linguistics*. Association for Computational Linguistics, 313–320.

WIERZBICKA, A. 1996. *Semantics: Primes and Universals*. Oxford University Press.

WINSTON, M. E., CHAFFIN, R., AND HERRMANN, D. 1987. A taxonomy of part-whole relations. *Cogn. Sci. 11*, 4, 417–444.

WU, D. AND FUNG, P. 2009. Semantic roles for smt: A hybrid two-pass model. In *Proceedings of the Human Language Technologies: The Annual Conference of the North American Chapter of the Association for Computational Linguistics, Companion Volume: Short Papers*. Association for Computational Linguistics, 13–16.

Received November 2011; revised September 2012; accepted May 2013