

Recent Research in Psychology

I.I. Bejar R. Chaffin S. Embretson

Cognitive and Psychometric Analysis of Analogical Problem Solving

With 42 Illustrations



Springer-Verlag
New York Berlin Heidelberg London
Paris Tokyo Hong Kong Barcelona

Isaac I. Bejar
Division of Education Policy
Research
Educational Testing Service
Princeton, New Jersey 08541, USA

Roger Chaffin
Trenton State College
Trenton, New Jersey 08625, USA

Susan Embretson
University of Kansas
Lawrence, Kansas 66045, USA

Library of Congress Cataloging-in-Publication Data

Bejar, Isaac I.

Cognitive and psychometric analysis of analogical problem solving

Isaac I. Bejar, Roger Chaffin, Susan Embretson.

p. cm.—(Recent research in psychology)

Includes bibliographical references.

1. Analogy—Psychological aspects—Testing. 2. Problem solving—
Testing. 3. Psychological tests—Design and construction.

4. Graduate Record Examination. I. Chaffin, Roger. II. Embretson,
Susan. III. Title. IV. Series.

BF446.B44 1990

153.4'3—dc20

90-35748

CIP

Printed on acid-free paper

© 1991 Springer-Verlag New York Inc.

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Camera ready copy provided by the authors.

9 8 7 6 5 4 3 2 1

ISBN-13:978-0-387-97321-0

e-ISBN-13:978-1-4613-9690-1

DOI: 10.1007/978-1-4613-9690-1

To John B. Carroll whose contribution to psychometrics and cognition precipitated their merger in understanding aptitude.

Preface

If one were to conduct an analysis of any profession the "ability to think analogically" is more than likely to be one of the requirements for success, be it an architectural studio, a research laboratory, a legal office, or a nuclear plant. Cognitive scientists are aware of the prominence of analogical reasoning in all forms of reasoning and learning, and have devoted substantial effort to ascertaining its nature. Test builders, like cognitive scientists, are aware of the centrality of analogical reasoning and figure, correctly, that a test that samples a student's ability to think analogically may well be a good predictor of success in a variety of fields. This book is the result of a project to investigate analogical reasoning from both an individual differences and a cognitive perspective.

The book is directed to both researchers and practitioners concerned with the nature and measurement of analogical reasoning. Cognitive scientists, linguists, psycholinguists, and natural language researchers will find the semantic taxonomy and accompanying empirical results food for thought. Test developers will find it reassuring that performance on verbal analogy items is not just a reflection of the size of a person's vocabulary, and that tests can be *designed* according to principles, rather than *assembled* to satisfy a set of statistical specifications. Psychometricians will find that content and response modelling can go together and that there are distinct benefits in approaching psychometric response modelling from that integrative perspective.

Although the project was carried out in a "testing context" we believe it complements the related basic research literature and hope that it will be a small contribution to the psychological literature in its own right. The book also illustrates the benefits of combining psychometric and experimental approaches to understanding the nature of individual differences. Lee Cronbach is credited with alerting the rest of us to the need for that integration. However, the effort got started in earnest, with Jack Carroll's *New Structure of Intellect*, and for that reason we dedicate the book to him, with thanks.

The book came about from the confluence of several events. As I.B. was preparing a proposal to study determinants of analogical problem solving he became aware of R.C.'s work on semantic relations. A one-year scholarly visit, which was extended to a second year, was arranged. It was during that visit that a taxonomy of semantic relations was developed, in collaboration with Leslie Peirce of the Test Development staff. The taxonomy can be found in Chapter 3, and played a central role in the development of the project. A second fortunate event was that S.E. was due for a sabbatical leave and decided to spend half a year at ETS. She, of course, had had a long standing interest in analogical reasoning, and had been one of the few researchers to attempt studying the role of semantic class on test performance. Finally, and the most

crucial event, the Graduate Records Examination Board decided to fund the proposal.

We owe thanks to more colleagues and friends than we have room to acknowledge but some individuals deserve special mention. Lawrence Stricker, John De Jong, reviewed early versions of the manuscript, Min hwei Wang carried out many statistical analyses and file transfers to and from the dreaded mainframe; Mary Enright designed and carried out the analyses reported in Chapter 7. Research assistants Lora Moses, Becky Walzer, Som Lok Leung, and Sean Whalen admirably performed a variety of tasks. We are also grateful to subjects from Princeton University and to Jessie Cryer who, in addition to her regular duties, coordinated the recruitment of subjects. The project, of course, could have not been carried out without the help of Test Development staff at ETS, most notably Cheryl Wild, Richard Adams, Charlotte Solomon, and Ned Walthall. Finally, we are especially indebted to Debra Friedman who in addition to acting as a project coordinator also took the responsibility for maintaining the database, carried out many of the statistical analyses, and, as a sign of the times, took responsibility for the production of this manuscript. It is more than likely that this book would have never been completed without her penchant for detail, organizational skills, and good humor.

Isaac I. Bejar
Roger Chaffin
Susan E. Embretson
Princeton, NJ

Contents

Preface	vii
List of Figures	xiii
Chapter 1: Introduction	1
Objectives of the Project and Overview of the Report	3
Chapter 2: Theories of Memory Representation and Analogical Reasoning	7
A Normative Model	8
Words and Concepts	9
Representation of Word Meaning	11
Prototype Theory	12
Feature Models	14
Semantic Networks	16
Schemata, Frames, and Scripts	18
Summary	19
Relations Between Concepts	20
Relation Expressions	20
Novel Relations	22
Relation Similarity	25
Summary	26
Comparison of Relations	26
Relation Element Theory	26
Reasoning with Analogies	29
Psychometric Models	29
Spearman's Model	29
Sternberg's Models	30
Evans' Model	35
Pellegrino and Collaborators	38
Embretson (Whitely) Models	43
Relation Element Theory	47
Reasoning Models	50
Rumelhart and Abrahamson's Model	51
Gentner's Structural Mapping Theory	51
Holyoak and Thagard's ACME Model	53
Summary	53
Chapter 3: A Taxonomy of Semantic Relations	55
The Taxonomy	57

Completeness of the Taxonomy	64
A Higher-Order Classification of Semantic Relations	67
Intensional and Pragmatic Relations	67
The 10 Relation Families	69
Precedents for the Dichotomy	70
Empirical Evidence for the Dichotomy	74
Previous Studies	74
A Reanalysis of Whitely's 1977 Data	75
A New Sorting Study	78
A Comparison of GRE Candidates With Different Majors	79
Factor Analytic Evidence	83
Earlier Work on Cognitive Dictionaries	85
Uses of the Taxonomy	87
Item Development	87
Computer Generation of Items	88
Theoretical Primitives	89
Research	89
Chapter 4: Description of the Item Pool	93
Distribution of Deltas and r-Biserial	94
Distribution According to Test Development Taxonomies	99
Distribution According to Relational Taxonomy	109
Relationship of the Taxonomy of Semantic Relations to Other Test Development Taxonomies	114
Summary	116
Chapter 5: The Effect of Vocabulary Level and Rationale Complexity on Item Difficulty	119
Data and Procedures	120
Regression of Delta on Stem and Key Frequency for All Items	121
The Role of Complexity in Difficulty	129
Summary	133
Chapter 6: The Relationship Between Delta and r-Biserial	137
The Negative Relationship As an Artifact	138
The Relationship of Delta and r-Biserial for Other GRE Items	139
Other Data Sets	146
Recomputing r-Biserial	147
Polyserial Analysis	148
Summary	150
Chapter 7: Expert Analyses of Analogy Items	153
Method	155
Results	158

Data Analysis	158
Prediction of Item Difficulty	158
Raters' Model of Item Difficulty	160
Prediction of Delta	161
Raters' Performance and Type of Analogy	165
Summary	166
Chapter 8: Cognitive Processing and Item Difficulty	169
Relationship Between Processing Demands and Psychometric Difficulty	171
Subjects and Method for Experiments 1 and 2	171
Experiment 1	172
Experiment 2	176
Lexical Overlap	179
Practical Value of Process Variables in Modeling Difficulty	184
Summary	186
Chapter 9: Cross Validation: Analysis of Pretested Items	187
Establishing Criteria	188
Regression Results	192
A Practical Assessment of Utility	199
Predicting Low r -Biserials	199
Summary	201
Chapter 10: Summary and Conclusions	203
The Study in Perspective	204
Systematic Item Writing	206
Psychological Response Modeling	207
Processing Models	208
Practical Considerations	209
Further Research	210
References	211
Author Index	229
Subject Index	233

List of Figures

Figure 2.1: Normative Model of Analogy Solution	10
Figure 2.2: Spearman's Model of Analogical Reasoning	31
Figure 2.3: Sternberg's Model III with Alternating Scanning and Justification	32
Figure 2.4: Evan's (1968) Model of Analogical Reasoning	36
Figure 2.5: Figural Analogies	37
Figure 2.6: Pellegrino and Glaser's (1982) Conceptually-driven Model	40
Figure 2.7: Pellegrino and Glaser's Interactive Model of Analogical Reasoning	42
Figure 2.8: Embretson's (1985) Simplified Model for Analogical Reasoning	45
Figure 2.9: Embretson's Model with Alternative Processing Strategies	46
Figure 4.1: Frequency Distribution of Delta for All Items (N=179)	96
Figure 4.2: Frequency Distribution of r-Biserial for All Items (N=179)	96
Figure 4.3: Scatter Plot of Delta and r-Biserial for All Items (N=179)	98
Figure 4.4a: Scatter Plot of Delta and r-Biserial for Items in the Aesthetic Set	102
Figure 4.4b: Scatter Plot of Delta and r-Biserial for Items in the Practical Affairs Set	102
Figure 4.4c: Scatter Plot of Delta and r-Biserial for Items in the Science Set	103
Figure 4.4d: Scatter Plot of Delta and r-Biserial for Items in the Human Relations Set	103
Figure 4.5a: Scatter Plot of Delta and r-Biserial for Items in the Abstract Set	106
Figure 4.5b: Scatter Plot of Delta and r-Biserial for Items in the Concrete Set	106
Figure 4.5c: Scatter Plot of Delta and r-Biserial for Items in the Mixed Set	107
Figure 4.6a: Scatter Plot of Delta and r-Biserial for Items in the Overlapping Set	108
Figure 4.6b: Scatter Plot of Delta and r-Biserial for Items in the Independent Set	108
Figure 4.7: Scatter Plot for the Mean Delta and r-Biserial for the 10 Classes of Analogy Items	111
Figure 4.8: Scatter Plot of Mean SMinl (the log of the frequency of the least frequent word in the stem pair) and Delta for the 10 Classes of Analogy Items	113

Figure 4.9: Scatter Plot of Mean KMinl (the log of the frequency of the least frequent word in the key pair) and Delta for the 10 Classes of Analogy Items	114
Figure 5.1: Frequency Distribution of SMinl	121
Figure 5.2: Frequency Distribution of KMinl	122
Figure 6.1: Scatter Plot of Delta and r-Biserial for Verbal Items	140
Figure 6.2: Scatter Plot of Delta and r-Biserial for Analytical Items	140
Figure 6.3: Scatter Plot of Delta and r-Biserial for Quantitative Items	141
Figure 6.4: Scatter Plot of Delta and r-Biserial for Sentence Completion Items	141
Figure 6.5: Scatter Plot of Delta and r-Biserial for Reading Comprehension Items	142
Figure 6.6: Scatter Plot of Delta and r-Biserial for Antonym Items	142
Figure 6.7: Scatter Plot of Delta and r-Biserial for Analogy Items	143
Figure 6.8: Scatter Plot of Delta and r-Biserial for Analytical Reasoning Items	143
Figure 6.9: Scatter Plot of Delta and r-Biserial for Logical Reasoning Items	144
Figure 6.10: Scatter Plot of Delta and r-Biserial for Data Interpretation Quantitative Items	144
Figure 6.11: Scatter Plot of Delta and r-Biserial for Discrete Quantitative Items	145
Figure 6.12: Scatter Plot of Delta and r-Biserial for Quantitative Comparison Items	145
Figure 6.13: Plot of Delta and r-Biserial for 300 SAT Analogy Items	146
Figure 6.14: Plot of Rasch Difficulty and Discrimination for 216 Items From Embretson and Curtright (1980)	147
Figure 7.1: Input Screen for Rater's Responses	156
Figure 7.2: Improvement in the R2 for Rated Difficulty and Delta as a Function of Practice for Raters A and B	159
Figure 7.3: Standardized Mean Squared Difference as a Function of Practice for Raters A and B	160
Figure 8.1: Plot of Mean Delta and Mean Recognition Time	174
Figure 8.2: Plot of Mean Delta and Mean Time to Instantiate Rationale	177
Figure 8.3: Plot of Log of Recognition Time and Lexical Overlap for Intensional Word Pairs	183
Figure 8.4: Plot of Relatedness Rating and Lexical Overlap for Intensional Word Pairs	183
Figure 8.5: Plot Delta and Lexical Overlap for Intensional Word Pairs	184