

# Learning from Measurements in Exponential Families

ICML – Montreal

June 16, 2009

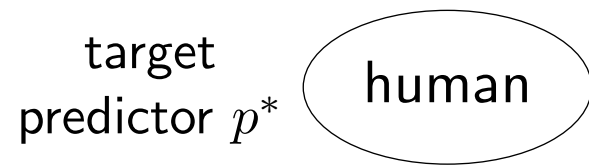
Percy Liang

Michael Jordan

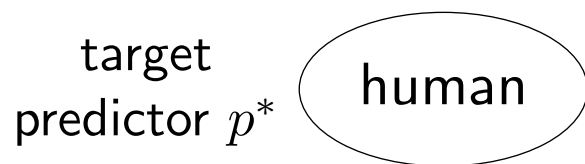
Dan Klein



# The big picture



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Example:

$y$ : FEAT FEAT FEAT FEAT FEAT ...

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Outline:

1. Coherently learn from diverse measurements



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Outline:

1. Coherently learn from diverse measurements
2. Actively select the best measurements

# Measurements

$X_1$  ,  $Y_1$

$X_2$  ,  $Y_2$

$X_3$  ,  $Y_3$

... ..

$X_i$  ,  $Y_i$

... ..

$X_n$  ,  $Y_n$

# Measurements

Measurement features:  $\sigma(x, y) \in \mathbb{R}^k$

$$\sigma( X_1 , Y_1 )$$

$$\sigma( X_2 , Y_2 )$$

$$\sigma( X_3 , Y_3 )$$

$$\begin{matrix} \dots & \dots \\ \sigma( X_i , Y_i ) \end{matrix}$$

$$\begin{matrix} \dots & \dots \\ \sigma( X_n , Y_n ) \end{matrix}$$

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Measurement features:  $\sigma(x, y) \in \mathbb{R}^k$

Measurement values:  $\tau \in \mathbb{R}^k$

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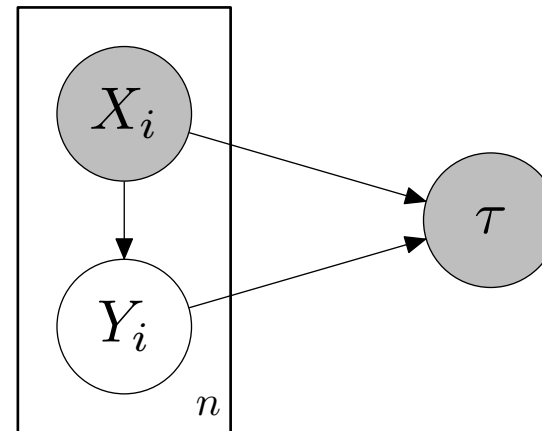
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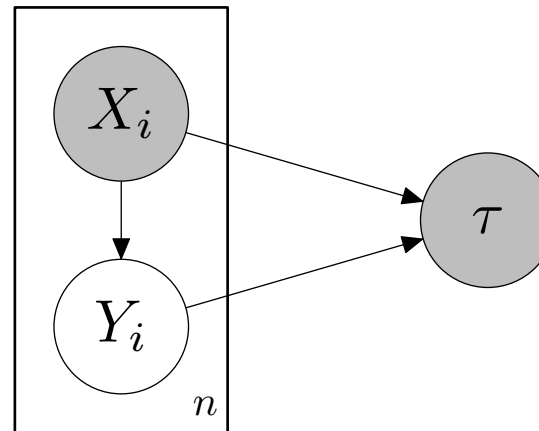
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Set  $\sigma$  to reveal various types of information about  $Y$  through  $\tau$

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Fully-labeled example:

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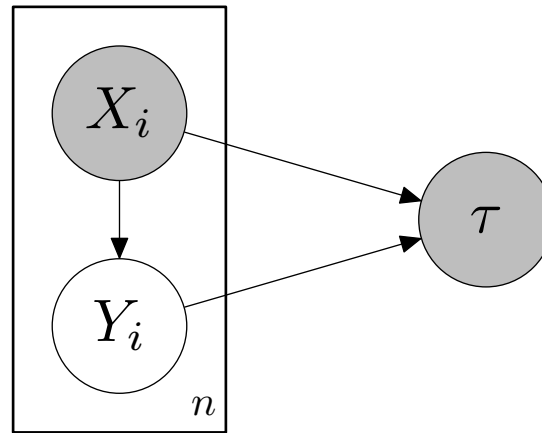
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Next: How to combine these diverse measurements coherently?

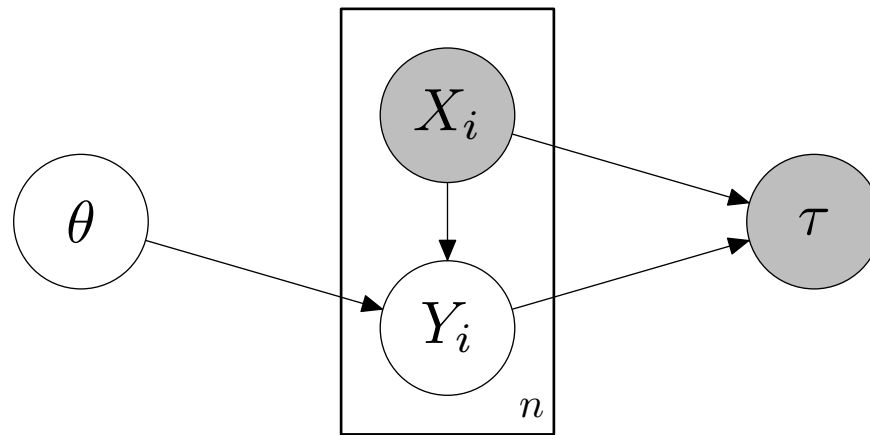
# Prediction model

Bayesian framework:



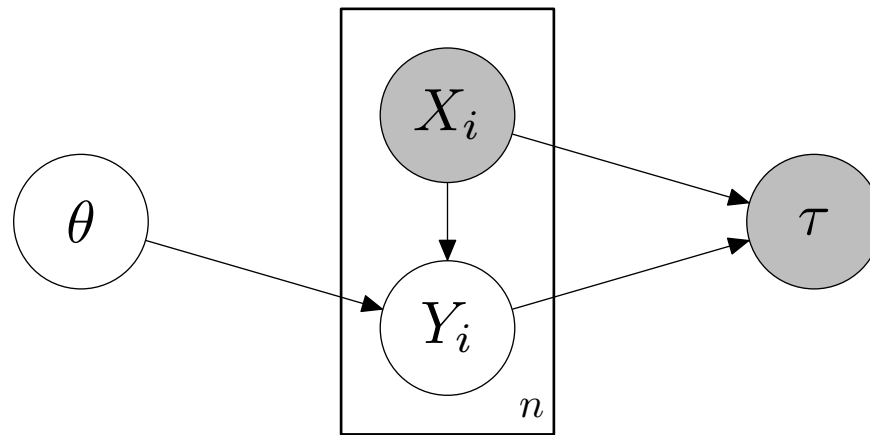
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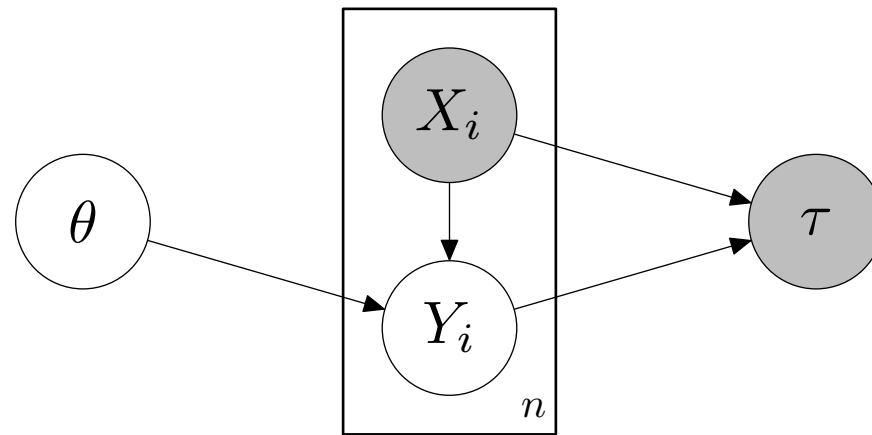
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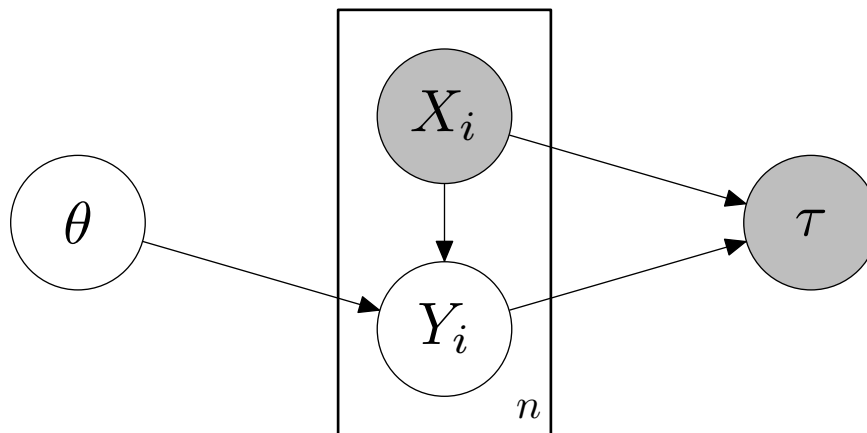
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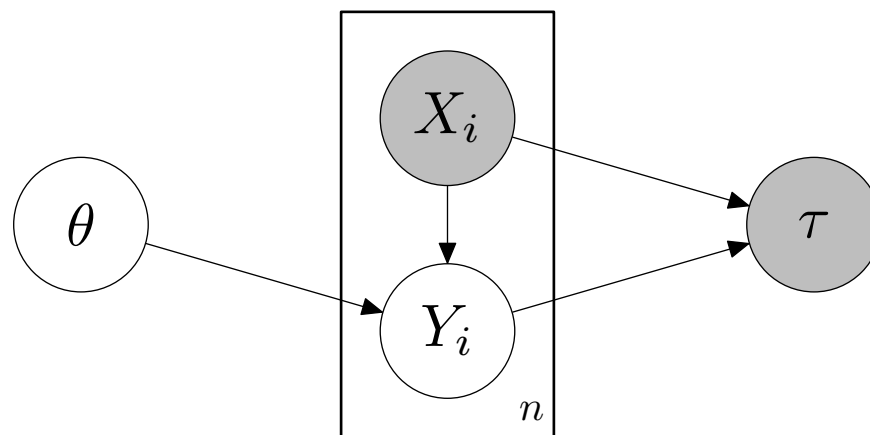
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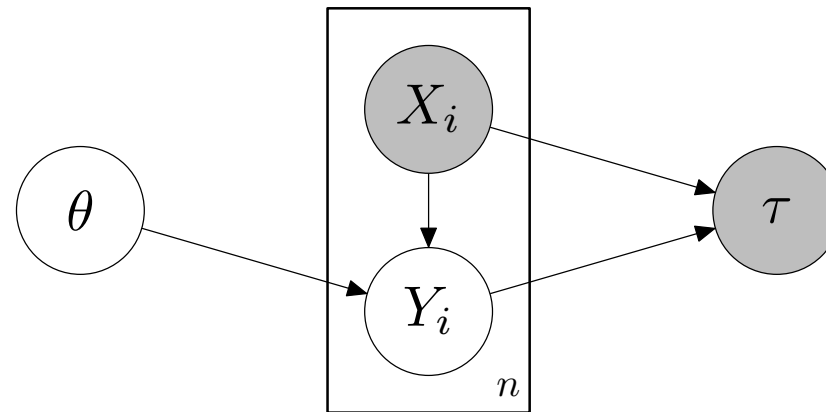
$\phi(x, y) \in \mathbb{R}^d$ : model features

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$A(\theta; x) = \log \int \exp\{\langle \phi(x, y), \theta \rangle\} dy$ : log-partition function

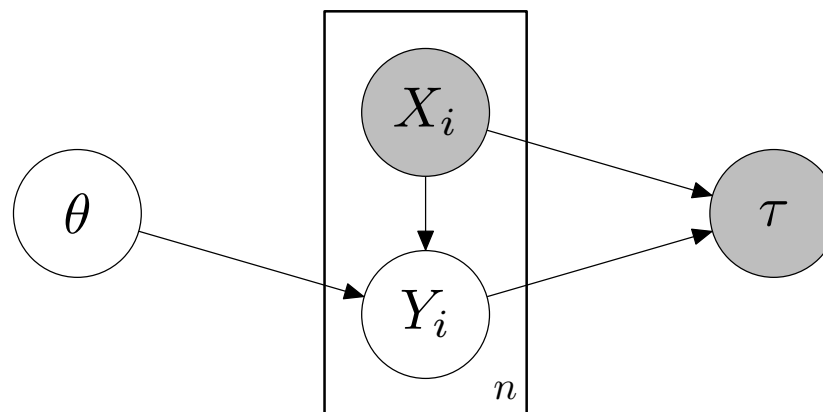
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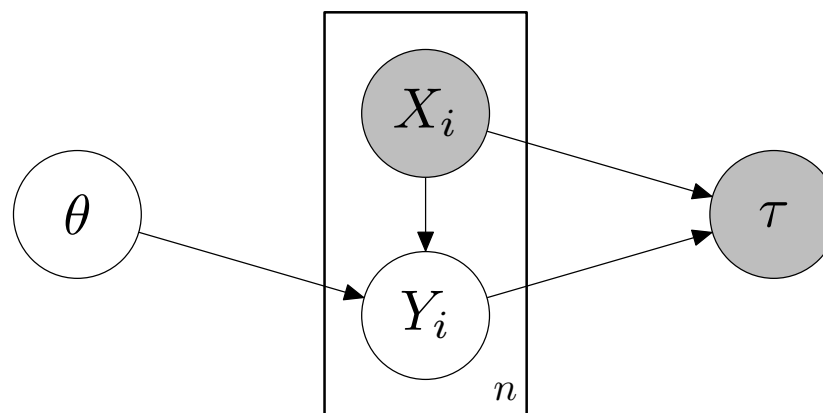


Variational formulation:

$$\min_{q \in \mathcal{Q}_{\theta, Y}} \text{KL} (q(\theta, Y) \parallel p(\theta, Y \mid \tau, X))$$

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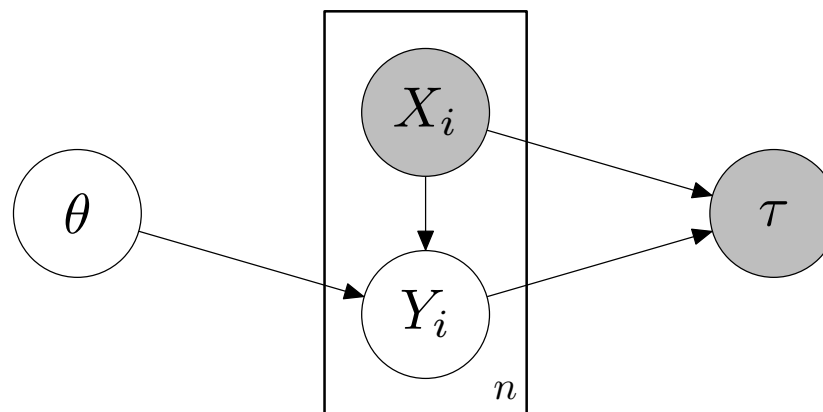
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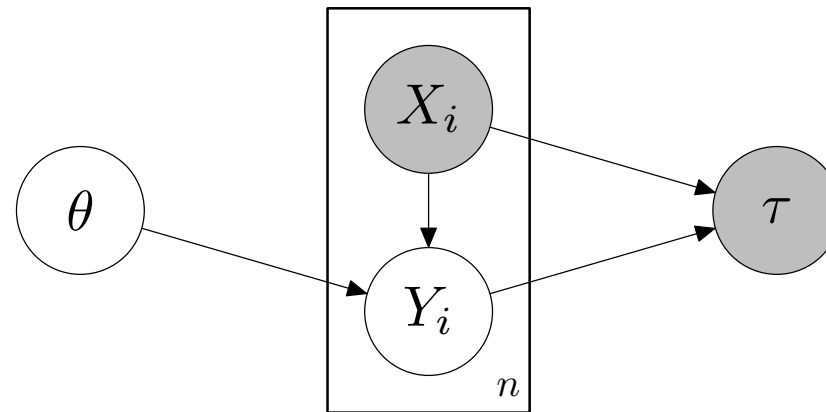
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Algorithm:

Apply Fenchel duality  $\rightarrow$  saddlepoint problem

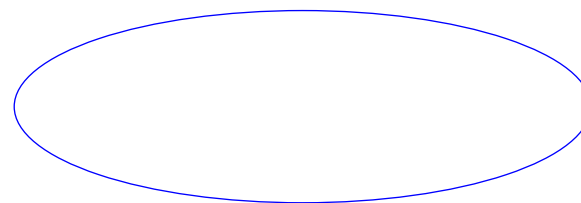
Take alternating stochastic gradient steps



# Information geometry viewpoint

(assume zero measurement noise)

$$\mathcal{P} \stackrel{\text{def}}{=} \{p_{\theta}(y \mid x) : \theta \in \mathbb{R}^d\}$$



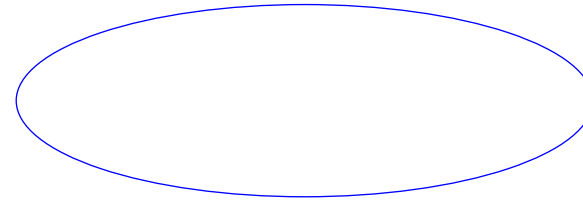
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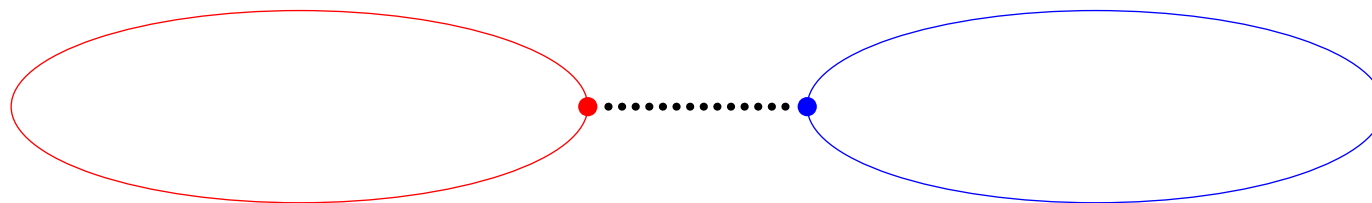


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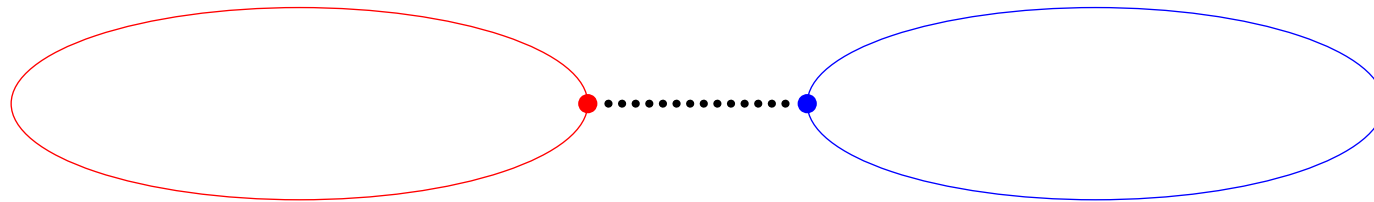
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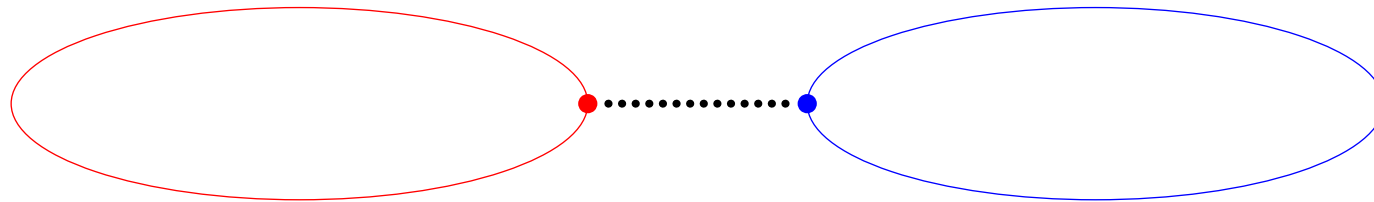
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Two ways to recover supervised learning:

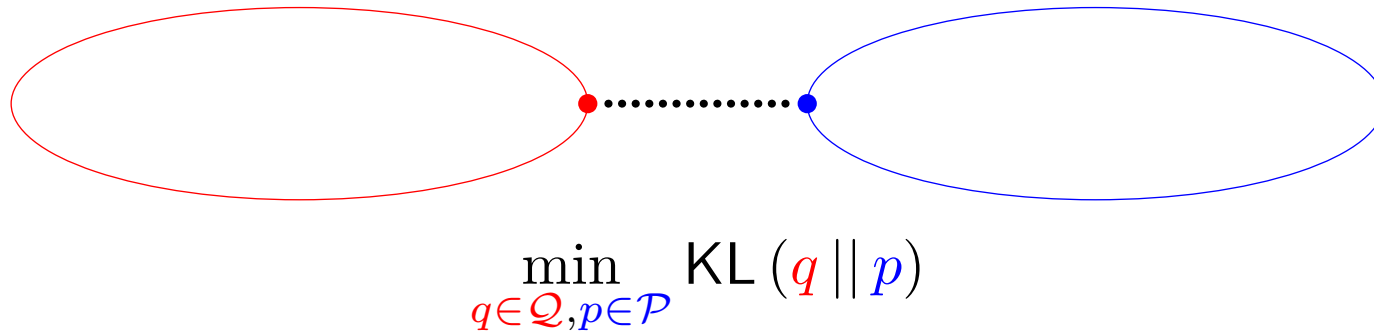
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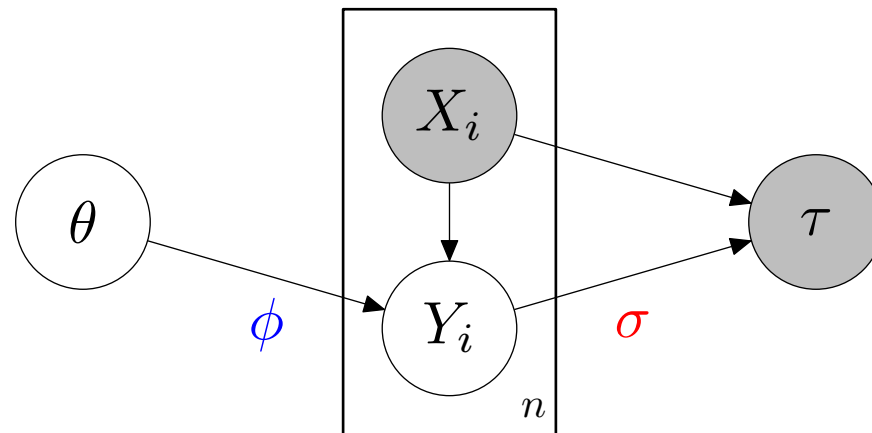
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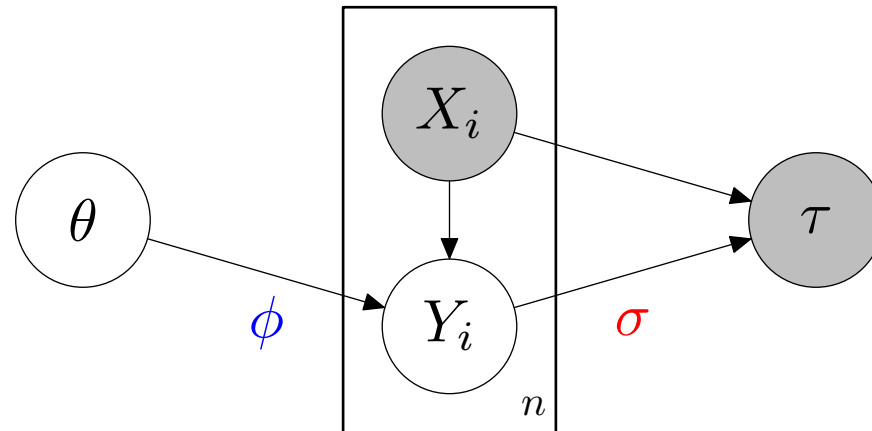
Two ways to recover supervised learning:

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 $\mathcal{Q} = \{\text{empirical distribution}\}$ , project onto  $\mathcal{P}$

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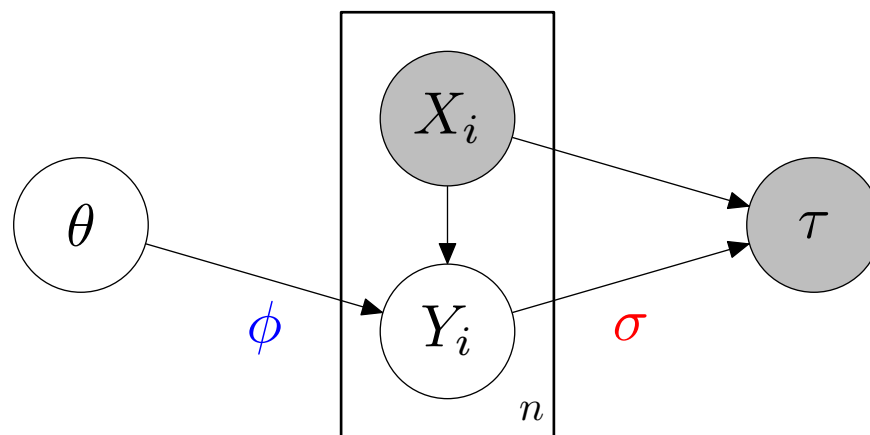


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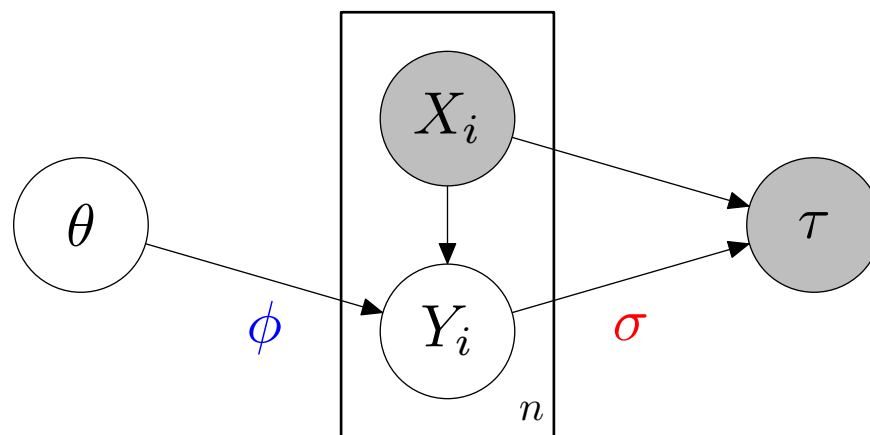


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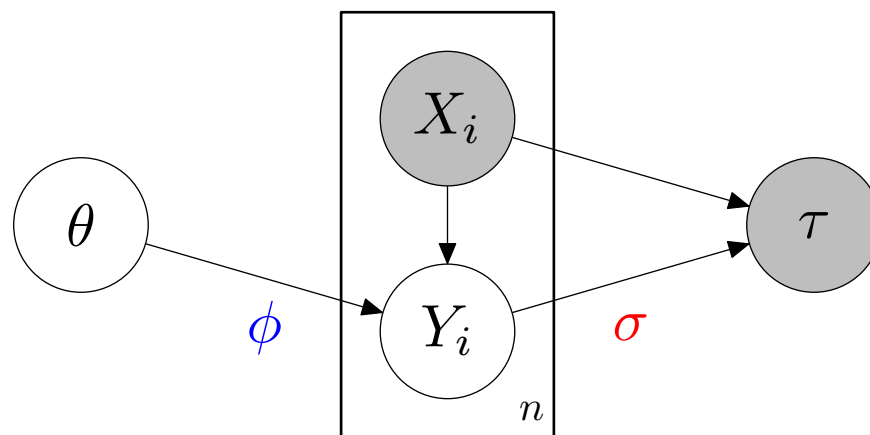
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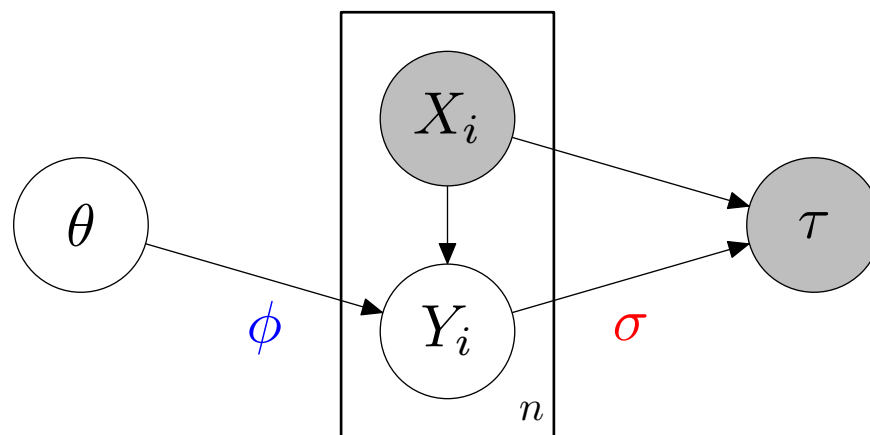
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If  $f$  is a model feature (**indirect**):

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$n = 1000$  total examples (ads), 11 possible labels

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## Per-position test accuracy (on 100 examples):

# labeled examples	10	25	100
General Expectation Criteria	74.6	77.2	80.5
Constraint-Driven Learning	<b>74.7</b>	<b>78.5</b>	81.7
Measurements	71.4	76.5	<b>82.5</b>

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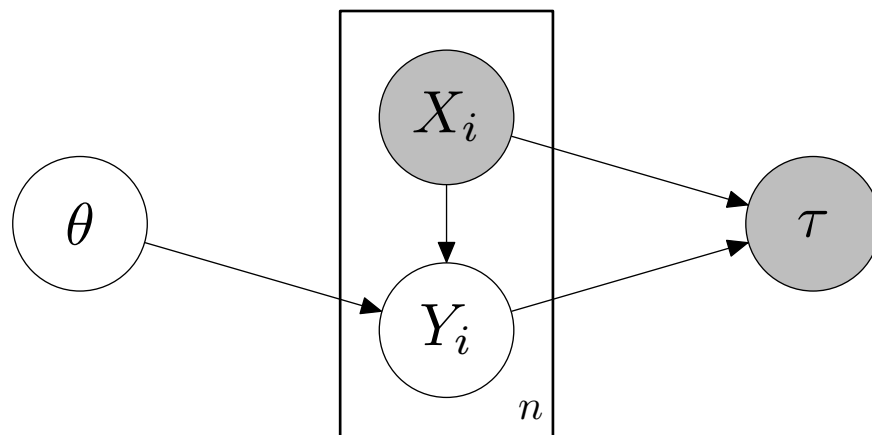
Able to integrate labeled examples and predicates gracefully



So far: given measurements, how to learn

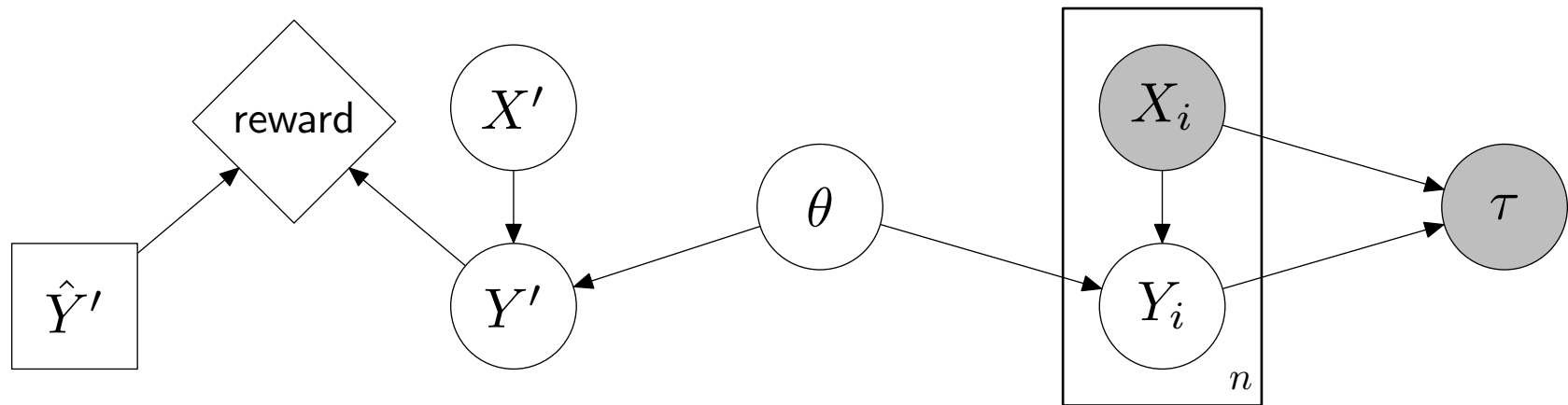
Next: how to choose measurements?

# Bayesian decision theory



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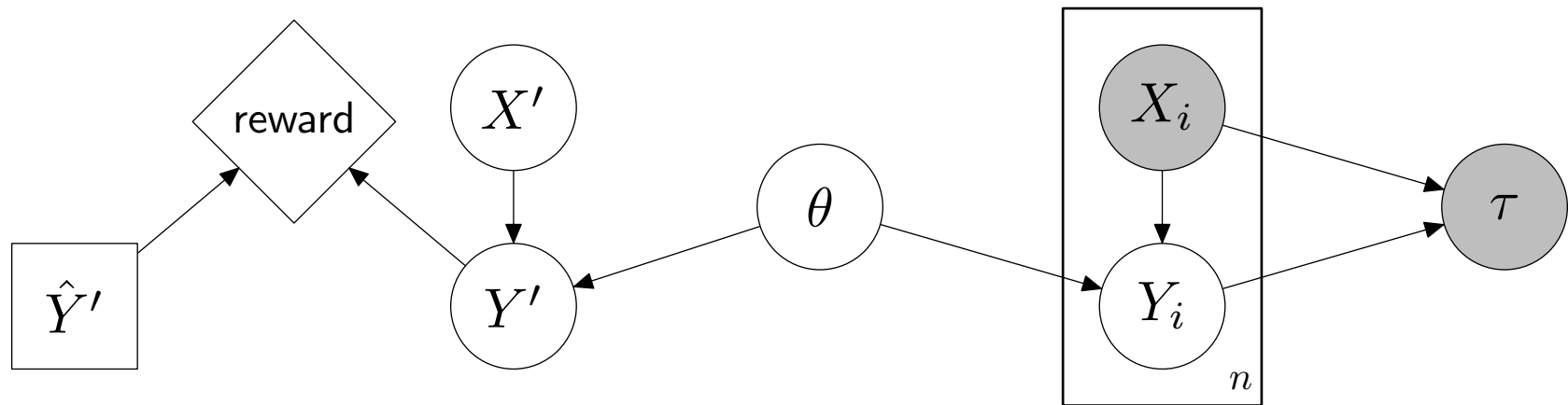


What do we do with an (approximate) posterior  $p(Y, \theta \mid X, \tau)$ ?

Bayes-optimal predictor:

average over  $X'$ , max over  $\hat{Y}'$ , average over  $Y'$  of reward

# Bayesian decision theory



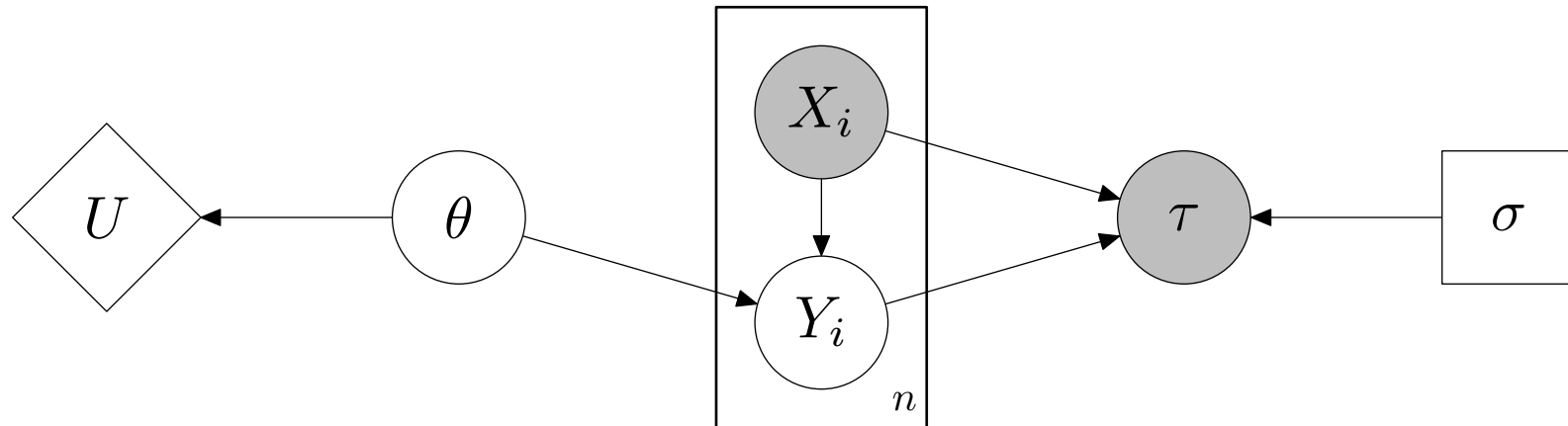
What do we do with an (approximate) posterior  $p(Y, \theta \mid X, \tau)$ ?

Bayes-optimal predictor:

average over  $X'$ , max over  $\hat{Y}'$ , average over  $Y'$  of reward

$R(\sigma, \tau)$  = expected reward of Bayes-optimal predictor  
(i.e., how happy we are with the given situation)

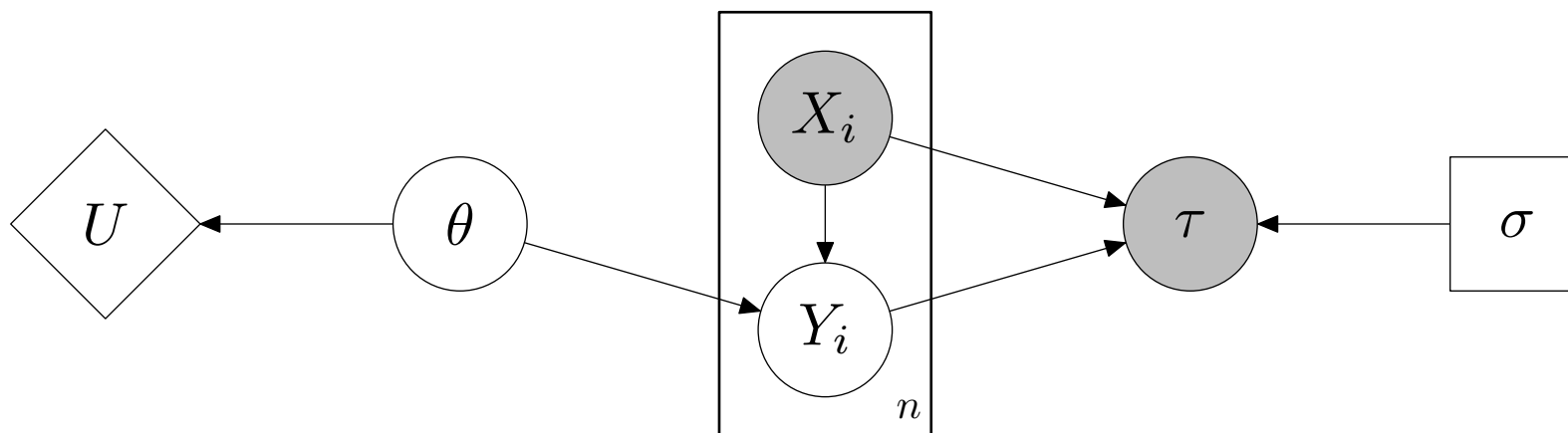
# Experimental design (active learning)



Utility of measurement  $(\sigma, \tau)$ :

$$U(\sigma, \tau) = \underbrace{R(\sigma, \tau)}_{\text{reward}} - \underbrace{C(\sigma)}_{\text{cost}}$$

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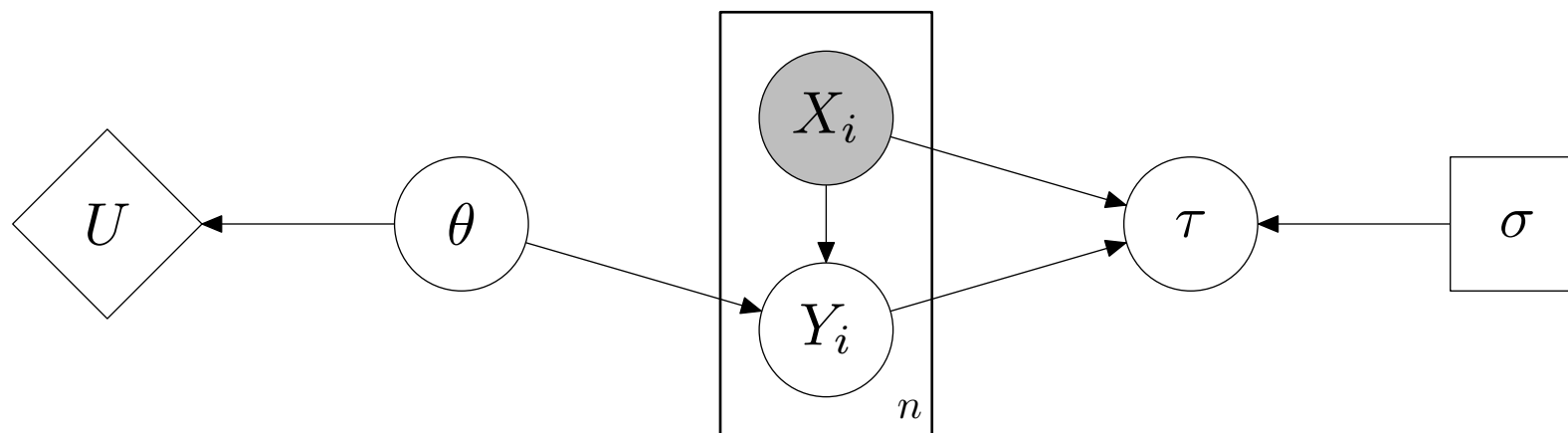
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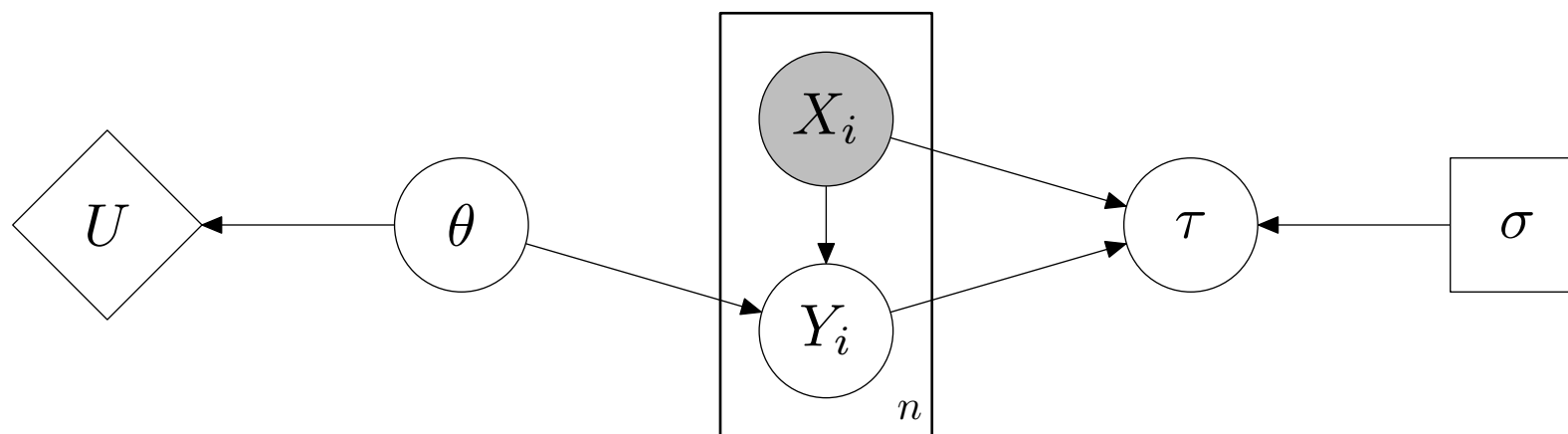
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Choose best measurement feature  $\sigma$ :

$$\sigma^* = \operatorname{argmax}_{\sigma} U(\sigma)$$



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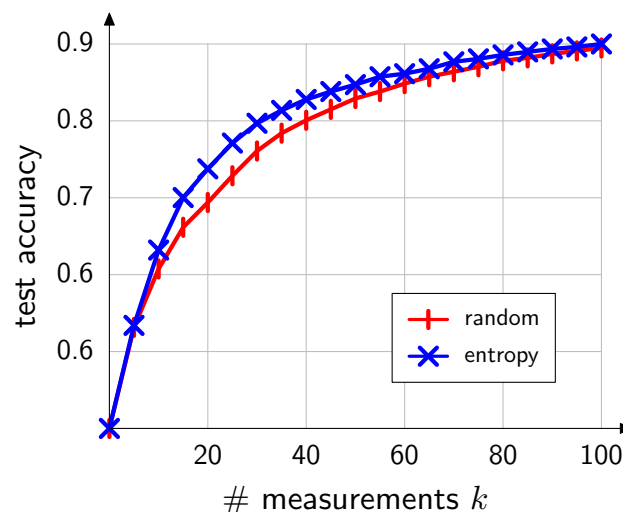
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(a) Labeling examples

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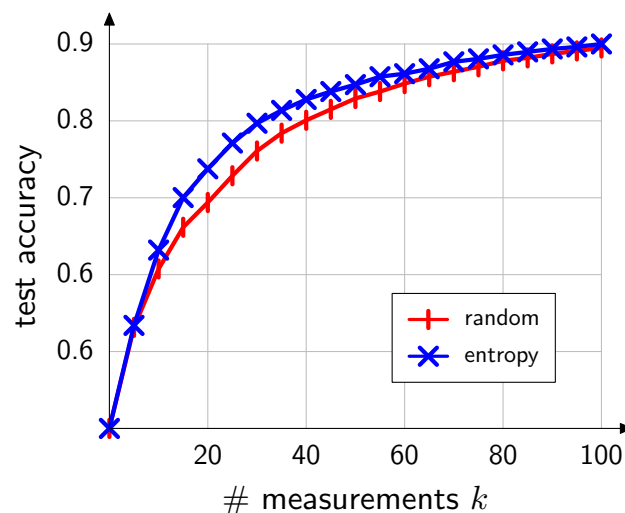
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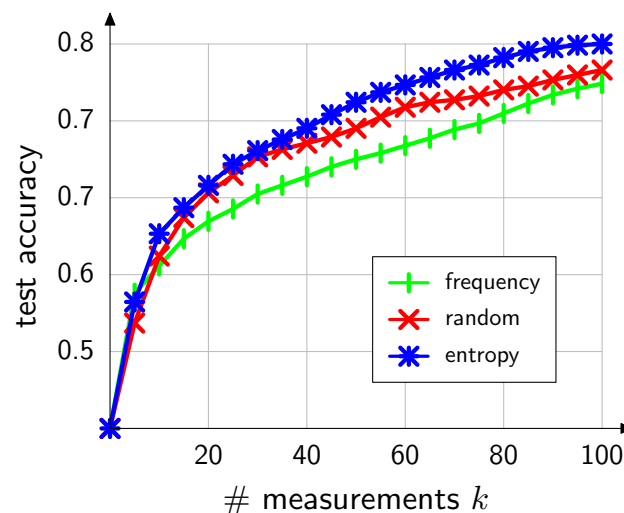
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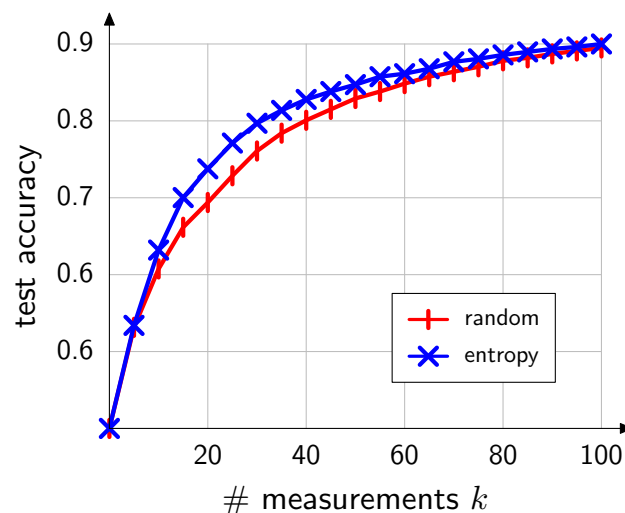
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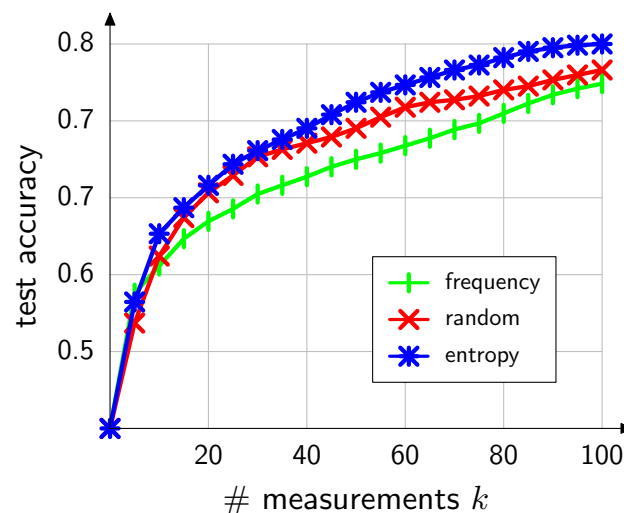
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# Summary



Measurements

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Measurements

Bayesian model

# Summary



Measurements

variational approx. — Bayesian model



# Summary



Measurements

variational approx. — Bayesian model

information  
geometry

# Summary



Measurements

variational approx. — Bayesian model — decision theory

information  
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Measurements

variational approx. — Bayesian model — decision theory

information  
geometry

active  
learning