MODERN CONTROL SYSTEM THEORY AND DESIGN

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place transform, $F(s)$	Time function, $f(t)$, $t \geqslant 0$
$\frac{1}{+ Ts)^n}$	$\frac{t^{n-1}e^{-t/T}}{T^n(n-1)!}$
$\frac{1}{1 + Ts)}$	$1 - e^{-t/T}$
$\frac{1}{1 + Ts)^2}$	$1 - \frac{t+T}{T}e^{-t/T}$
$\frac{\omega}{+ a)^2 + \omega^2}$	$e^{-at}\sin \omega t$
$\frac{(s+a)}{+ a)^2 + \omega^2}$	$e^{-at}\cos \omega t$
$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2}\,t + \alpha)$, where $\cos\alpha = \zeta$
$\frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t - \theta)$, where $\theta = 2\tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta}$
$\frac{s}{+ Ts)(s^2 + \omega_n^2)}$	$\frac{-1}{(1+T^2\omega_n^2)}e^{-t/T} + \frac{1}{\sqrt{1+T^2\omega_n^2}}\cos(\omega_n t - \theta)$, where $\theta = \tan^{-1}\omega_n T$
$\frac{s}{^2 + \omega_n^2)^2}$	$\frac{1}{2\omega_n}t\sin\omega_n t$
$\frac{1}{+ b)[(s+a)^2 + \omega^2]}$	$\frac{e^{-bt}}{(b-a)^2+\omega^2} + \frac{e^{-at}\sin(\omega t - \theta)}{\omega[(b-a)^2+\omega^2]^{1/2}}$, where $\theta = \tan^{-1}\frac{\omega}{b-a}$
$\frac{2abs}{^2 + (a+b)^2][s^2 + (a-b)^2]}$	$\sin at \sin bt$
$\frac{1 + as + bs^2}{(1+T_1 s)(1+T_2 s)}$	$t + (a - T_1 - T_2) + \frac{b - aT_1 + T_1^2}{T_1 - T_2}e^{-t/T_1} - \frac{b - aT_2 + T_2^2}{T_1 - T_2}e^{-t/T_2}$

APPENDIX B

PROOF OF THE NYQUIST STABILITY CRITERION

The Nyquist stability criterion can be derived from Cauchy's residue theorem, which states that

$$\frac{1}{2\pi j}\int_C g(s)\,ds = \sum \text{residues of } g(s) \text{ at the poles enclosed by the closed contour } C. \tag{B1}$$

Let us replace $g(s)$ by $f'(s)/f(s)$, where $f(s)$ is a function of s which is single valued on and within the closed contour C and analytic on C. Observe that the singularities of $f'(s)/f(s)$ occur only at the zeros and poles of $f(s)$. The residue may be found at each singularity with multiplicity of the order of zeros and poles taken into account. The residues in the zeros of $f(s)$ are positive and the residues in the poles of $f(s)$ are negative. Therefore, if $f(s)$ is not equal to zero along C, and if there are not at most a finite number of singular points that are all poles within the contour C, then

$$\frac{1}{2\pi j}\int_C \frac{f'(s)}{f(s)}ds = Z - P, \tag{B2}$$

where Z = number of zeros of $f(s)$ within C, with due regard for their multiplicity of order, and P = number of poles of $f(s)$ within C, with due regard for their multiplicity of order. The left-hand side of Eq. (B2) may be written as

$$\frac{1}{2\pi j}\int_C \frac{f'(s)}{f(s)}ds = \frac{1}{2\pi j}\int_C d[\ln f(s)]. \tag{B3}$$

In general, $f(s)$ will have both real and imaginary parts along the contour C. Therefore, its logarithm can be written as

$$\ln f(s) = \ln|f(s)| + j\underline{/f(s)}\,. \tag{B4}$$

If we assume that $f(s)$ is not zero anywhere on the contour C, the integration of Eq. (B3) results in the expression

$$\frac{1}{2\pi j}\int_C d[\ln f(s)] = \frac{1}{2\pi j}\left[\ln|f(s)| + j\underline{/f(s)}\right]_{s_1}^{s_2}, \tag{B5}$$

where s_1 and s_2 denote the arbitrary beginning and end of the closed contour C as it is followed. Because $|f(s)|$ returns to its initial value in completing the closed curve,

$$\frac{1}{2\pi j}\int_C d[\ln f(s)] = \frac{1}{2\pi}\left[\underline{/f(s_2)} - \underline{/f(s_1)}\right]. \tag{B6}$$

Therefore, Eq. (B6) can be rewritten as

$$\frac{1}{2\pi j}\int_C \frac{f'(s)}{f(s)}ds = \frac{1}{2\pi} \times \begin{array}{l}\text{net change in angle of } f(s) \text{ as } s \text{ is}\\ \text{varied over the contour } C\end{array}. \tag{B7}$$

By equating Eqs. (B2) and (B7), we obtain

$$Z - P = \frac{1}{2\pi} \times \begin{array}{l}\text{net change in angle of } f(s) \text{ as } s \text{ is varied}\\ \text{over the contour } C\end{array}. \tag{B8}$$

Equation (B8) states that the excess of zeros over poles of $f(s)$ within the contour C equals $1/2\pi$ times the net change in angle (i.e., equals the number of net encirclements of the origin) of $f(s)$ as s is varied over the contour C. Let N be this number. Then Eq. (B8) can be written as

$$Z - P = N, \tag{B9}$$

where the contour C is traversed in a clockwise direction, and where an encirclement is defined as being positive if it also is in a clockwise direction. The number Z must be zero for the system to be stable.

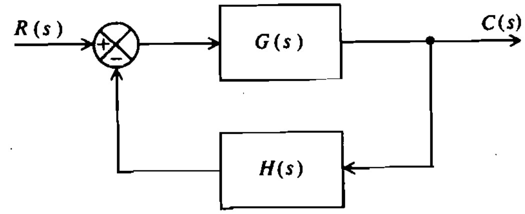

Figure B1

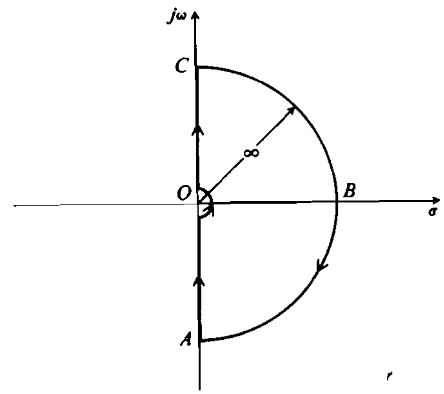

Figure B2

This relationship, which is known as Cauchy's principle of the argument, is the basis of Nyquist's stability criterion. To make use of this principle in applying Nyquist's stability criterion, let us consider the feedback control system of Figure B1. Let

$$f(s) = 1 + G(s)H(s)$$

and examine the number of times $f(s)$ encircles the origin as s traverses the contour of Figure B2. (It is assumed that there are no poles on the imaginary axis, except at the origin.) Observe that the origin of $f(s)$ corresponds to $G(s)H(s) = -1$. Therefore, if $G(s)H(s)$ is plotted for the contour of Figure B2, the number of times that $G(s)H(s)$ encircles the point $-1 + j0$ equals the number of zeros minus the number of poles of $1 + G(s)H(s)$ for s in the right half-plane.

Let us examine the contour of Figure B2 in detail. Because

$$\lim_{s\to\infty} |G(s)H(s)| = 0 \tag{B10}$$

for all physical systems, the points along the infinite arc ABC contract to the origin for $\omega = 0$. Along the small semicircle about the origin, let

$$s = re^{j\theta} = \sigma + j\omega. \tag{B11}$$

Because $G(s)H(s)$ is usually a rational function with a denominator of higher power than that of the numerator, then

$$G(s)H(s) = \lim_{r \to 0} G(re^{j\theta})H(re^{j\theta}) \tag{B12}$$

will map into a segment of an infinite circle. Along the imaginary axis, $s = j\omega$, so that we are concerned with $G(j\omega)H(j\omega)$. Because $G(-j\omega)H(-j\omega)$ is a conjugate of $G(j\omega)H(j\omega)$, the two functions are symmetric about the real axis and $G(-j\omega)H(-j\omega)$ is a reflection of $G(j\omega)H(j\omega)$ about the real axis. Therefore, it is only necessary to plot $G(j\omega)H(j\omega)$ from $\omega = 0$ to ∞.

Therefore, N can be determined by plotting $G(j\omega)H(j\omega)$ and observing the number of encirclements of the $-1 + j0$ point; P can be determined by inspection of the $G(s)H(s)$ expression; and Z can be found by using Eq. (B9).

ANSWERS TO SELECTED PROBLEMS

CHAPTER 1

1.1. An electrical signal proportional to the difference between the desired heading (gyroscopic setting) and original heading is amplified by the power amplifier. The amplified signal drives the motor which turns the rudder until the desired heading and actual heading of the ship are in agreement, and the corresponding electrical signal that is proportional to the difference between these two headings is zero. This is an open-loop control system as there is no feedback signal.

1.2. The rudder's positioning can be made into a closed-loop control system by fastening a resistor to the rudder in a similar manner as the resistor that is fixed to the ship's frame. An electrical feedback signal can then be obtained of the actual position, which can be appropriately compared with that of the desired, or reference, position.

1.3. If the reference temperature of the thermostat is changed, the reference input to the control system changes, and an electrical error signal results. The electric hot-water heater will then change the temperature of the water until the difference between the reference input and actual temperatures is zero.

1.4. A change in the ambient temperature surrounding the tank manifests itself as a disturbance input within the heating control system. The explanation of the system's resulting control action is similar to that discussed in the book for a disturbance occurring in an automatic speed-control system (see Figure 1.10).