Augmenting Dual Decomposition for MAP Inference

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- Still parallelizable, much faster to reach consensus
- Handles *global* structural constraints efficiently
- Experiments: Ising models and natural language parsing

Outline

1. Problem Formulation
2. Dual Decomposition
3. Augmented Lagrangian Method
4. Experiments
5. Conclusions

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Factor Graphs

Let $\mathbf{X} \triangleq (X_1, \ldots, X_n) \in \mathcal{X}$ be a vector of *discrete* random variables

$$P_{\boldsymbol{\theta},\boldsymbol{\phi}}(\mathbf{x}) \propto \exp\left(\sum_{i=1}^{n} \theta_i(x_i) + \sum_{a \in \mathcal{A}} \phi_a(\mathbf{x}_a)\right)$$

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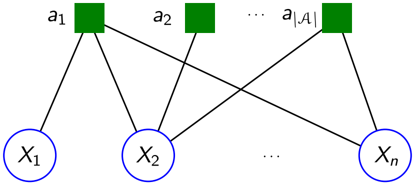

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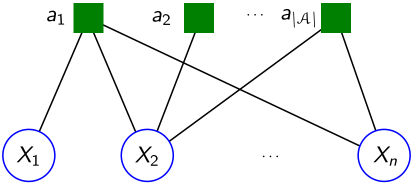

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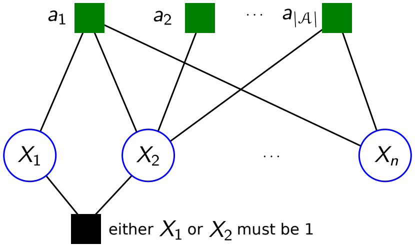

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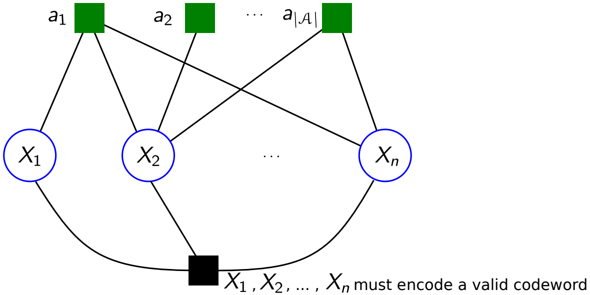

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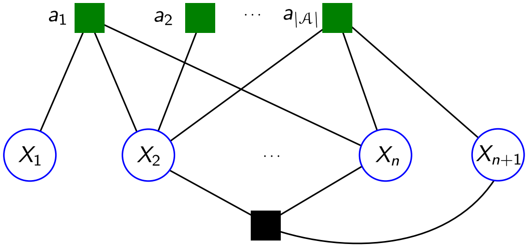

$X_{n+1} \triangleq X_2 \wedge X_n$ (conjunction)

MAP and LP-MAP Inference

Goal: compute $\hat{\mathbf{x}}$ which maximizes $P_{\boldsymbol{\theta},\boldsymbol{\phi}}(\mathbf{x})$

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$$\mathcal{L}(\mathcal{G}) = \left\{(\boldsymbol{\mu},\boldsymbol{\nu}) \mid (\boldsymbol{\mu}_{N(a)},\boldsymbol{\nu}_a) \in \mathcal{M}(\mathcal{G}_a), \quad \forall a \in \mathcal{A}\right\}$$

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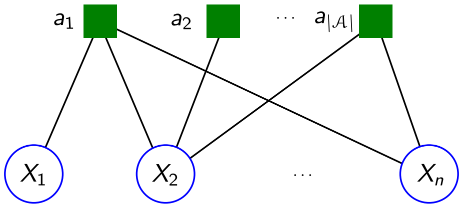

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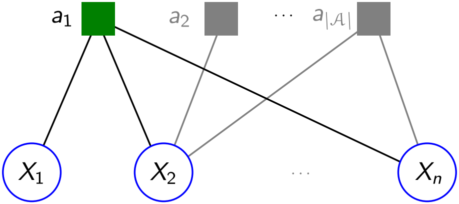

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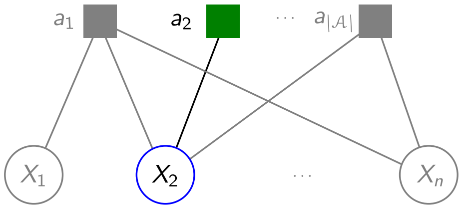

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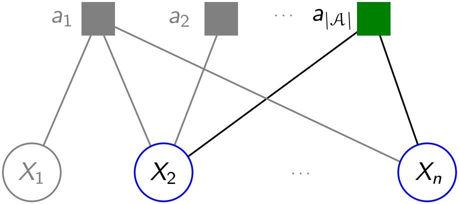

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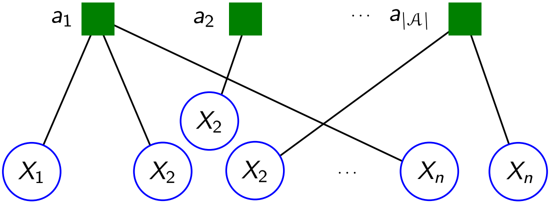

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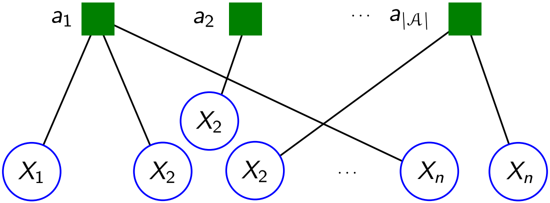

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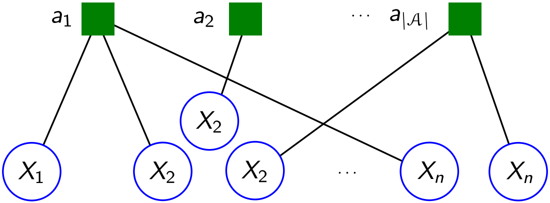

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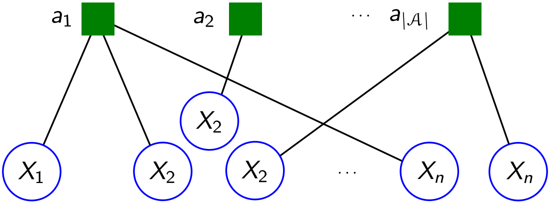

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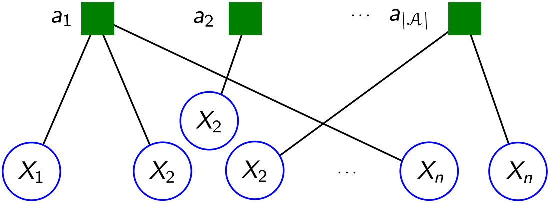

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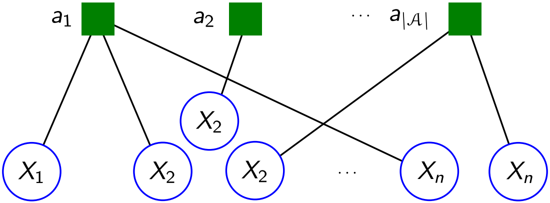

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- Problem would be separable, if not for the *coupling constraints*
- Dualize them out by adding Lagrange multipliers $\boldsymbol{\lambda}_i^a$

Dual Decomposition

Dual formulation:

$$\begin{aligned} \min_{\boldsymbol{\lambda}} \quad & L(\boldsymbol{\lambda}) \triangleq \sum_{a \in \mathcal{A}} \overbrace{\max_{(\boldsymbol{\nu}^a_{N(a)}, \boldsymbol{\nu}_a) \in \mathcal{M}(\mathcal{G}_a)} \sum_{i \in N(a)} \left(d_i^{-1} \boldsymbol{\theta}_i + \boldsymbol{\lambda}_i^a\right)^\top \boldsymbol{\nu}_i^a + \boldsymbol{\phi}_a^\top \boldsymbol{\nu}_a}^{\text{Slave subproblem at factor } a} \\ \text{s.t.} \quad & \boldsymbol{\lambda} \in \Lambda \triangleq \left\{ \boldsymbol{\lambda} \;\middle|\; \sum_{a \in N(i)} \boldsymbol{\lambda}_i^a = 0, \ \forall i \right\} \end{aligned}$$

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Master problem: minimize $L(\boldsymbol{\lambda})$ via the *projected subgradient algorithm*

- Subgradient given by the slaves:

$$\nabla_{\boldsymbol{\lambda}^a_i} L(\boldsymbol{\lambda}) = \hat{\boldsymbol{\nu}}^a_i \quad \text{(solution of a } \textit{local} \text{ MAP subproblem at } \mathcal{G}_a\text{)}$$

Dual Decomposition

Dual formulation:

$$\begin{array}{ll} \min_{\boldsymbol{\lambda}} & L(\boldsymbol{\lambda}) \triangleq \sum_{a \in \mathcal{A}} \overbrace{\max_{(\boldsymbol{\nu}^a_{N(a)}, \boldsymbol{\nu}_a) \in \mathcal{M}(\mathcal{G}_a)} \sum_{i \in N(a)} \left(d_i^{-1}\boldsymbol{\theta}_i + \boldsymbol{\lambda}_i^a\right)^\top \boldsymbol{\nu}_i^a + \boldsymbol{\phi}_a^\top \boldsymbol{\nu}_a}^{\text{Slave subproblem at factor } a} \\ \text{s.t.} & \boldsymbol{\lambda} \in \Lambda \triangleq \left\{ \boldsymbol{\lambda} \;\middle|\; \sum_{a \in N(i)} \boldsymbol{\lambda}_i^a = 0, \; \forall i \right\} \end{array}$$

Slave subproblems: one per each factor $a \in \mathcal{A}$—parallelizable!

Master problem: minimize $L(\boldsymbol{\lambda})$ via the *projected subgradient algorithm*

- Subgradient given by the slaves:

$$\nabla_{\boldsymbol{\lambda}_i^a} L(\boldsymbol{\lambda}) = \hat{\boldsymbol{\nu}}_i^a \quad \text{(solution of a } \textit{local} \text{ MAP subproblem at } \mathcal{G}_a\text{)}$$

- Projection onto Λ: a simple *centering* operation

DD-Subgradient Algorithm

input: $\mathcal{G}$, $\boldsymbol{\theta}, \boldsymbol{\phi}$, number of iterations T, sequence $(\eta_t)_{t=1}^T$
Initialize $\boldsymbol{\lambda} = \mathbf{0}$
for $t = 1$ **to** T **do**
  for each factor $a \in \mathcal{A}$ **do**
    Set unary potentials $\boldsymbol{\omega}_i^a = d_i^{-1}\boldsymbol{\theta}_i + \boldsymbol{\lambda}_i^a$, for $i \in N(a)$
    Compute $(\hat{\boldsymbol{\nu}}_{N(a)}^a, \hat{\boldsymbol{\nu}}_a) = \mathrm{MAP}\left(\boldsymbol{\omega}_{N(a)}^a, \boldsymbol{\phi}_a\right)$
  end for
  Compute average $\bar{\boldsymbol{\nu}}_i = d_i^{-1} \sum_{a:i\in N(a)} \hat{\boldsymbol{\nu}}_i^a$
  Update $\boldsymbol{\lambda}_i^a \leftarrow \boldsymbol{\lambda}_i^a - \eta_t \left(\hat{\boldsymbol{\nu}}_i^a - \bar{\boldsymbol{\nu}}_i\right)$
end for
output: $\boldsymbol{\lambda}$

- Converges for a suitable stepsize sequence $(\eta_t)_{t\in T}$
- *Slow* when the number of slaves is large

Outline

Augmented Lagrangian and ADMM

$$
\begin{aligned}
A_\eta(\boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{\lambda}) \quad \triangleq \quad & \overbrace{\sum_{a \in \mathcal{A}} \left(\sum_{i \in N(a)} \left(d_i^{-1} \boldsymbol{\theta}_i + \boldsymbol{\lambda}_i^a \right)^\top \boldsymbol{\nu}_i^a + \boldsymbol{\phi}_a^\top \boldsymbol{\nu}_a \right) - \sum_{a \in \mathcal{A}} \sum_{i \in N(a)} \boldsymbol{\lambda}_i^{a\top} \boldsymbol{\mu}_i}^{\text{Lagrangian function}} \\
& \underbrace{- \frac{\eta}{2} \sum_{a \in \mathcal{A}} \sum_{i \in N(a)} \| \boldsymbol{\nu}_i^a - \boldsymbol{\mu}_i \|^2}_{\text{Residual Term}}
\end{aligned}
$$

Augmented Lagrangian and ADMM

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$$-\frac{\eta}{2} \sum_{a \in \mathcal{A}} \sum_{i \in N(a)} \|\boldsymbol{\nu}_i^a - \boldsymbol{\mu}_i\|^2$$

Residual Term

We want to maximize A_η wrt $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$

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1. Maximize w.r.t. $\boldsymbol{\mu}$ (closed form)
2. Maximize w.r.t. $\boldsymbol{\nu}$ (can be carried out in parallel at each factor)
3. Adjust the Lagrange multipliers $\boldsymbol{\lambda}_i^a \leftarrow \boldsymbol{\lambda}_i^a - \tau\eta \left(\boldsymbol{\nu}_i^a - \boldsymbol{\mu}_i \right)$

DD-ADMM Algorithm

input: $\mathcal{G}$, $\boldsymbol{\theta}, \boldsymbol{\phi}$, number of iterations T, sequence $(\eta_t)_{t=1}^{T}$, parameter τ
Initialize $\boldsymbol{\mu}$ uniformly, $\boldsymbol{\lambda} = \mathbf{0}$
for $t = 1$ **to** T **do**
  for each factor $a \in \mathcal{A}$ **do**
    Set unary potentials $\boldsymbol{\omega}_i^a = d_i^{-1}\boldsymbol{\theta}_i + \boldsymbol{\lambda}_i^a + \eta_t \boldsymbol{\mu}_i$, for $i \in N(a)$
    Update $(\boldsymbol{\nu}_{N(a)}^a, \boldsymbol{\nu}_a) \leftarrow \mathrm{QUAD}_{\eta_t}(\boldsymbol{\omega}_{N(a)}^a, \boldsymbol{\phi}_a)$
  end for
  Update $\boldsymbol{\mu}_i \leftarrow d_i^{-1} \sum_{a:i\in N(a)} (\boldsymbol{\nu}_i^a - \eta_t^{-1}\boldsymbol{\lambda}_i^a)$
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- Instead of MAP, we solve a quadratic problem QUAD at each factor:

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- Under certain conditions, converges even when QUAD is *approximately* solved (Eckstein and Bertsekas, 1992).

Solving the Quadratic Problem

Binary pairwise factors: closed form solution

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Hard constraint factors (Tarlow et al., 2010; Martins et al., 2010):

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- For many hard factors imposing logical constraints, can be done efficiently with *sort* operations

Larger slaves (sequences, trees): solved approximately with primal-dual cyclic projection algorithms (work in progress)

Outline

Binary Pairwise MRFs

Ising model on a random 30×30 toroidal grid; DD-ADMM against:

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Ising model on a random 30 $\times$ 30 toroidal grid; DD-ADMM against:

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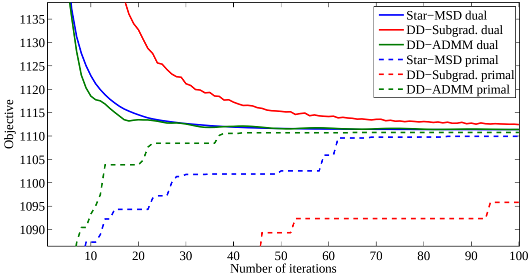

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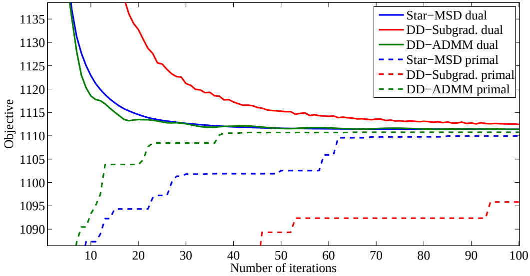
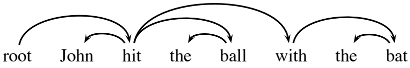


- DD-subgradient is the slowest (many slaves!)
- DD-ADMM outperforms the others: it approaches a near optimal primal-dual solution in a few tens of iterations

Non-projective Dependency Parsing

A problem with heavily constrained outputs, to which DD has recently been applied (Rush et al., 2010; Koo et al., 2010)



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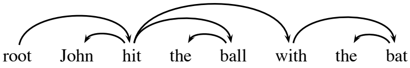
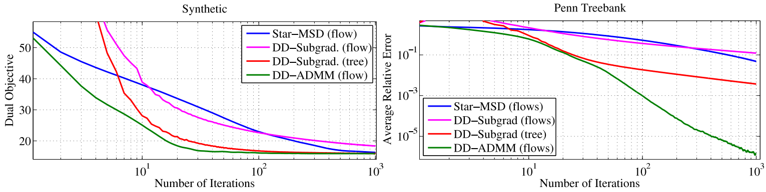


Our model: 2nd-order with $O(n^3)$ slaves (distinct from Koo et al. 2010)



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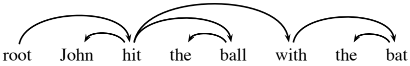

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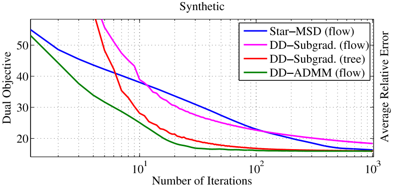

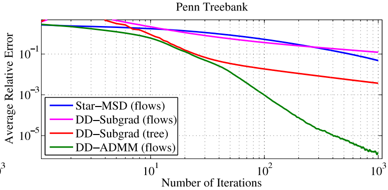

- DD-ADMM wins with both synthetic (10 words) and real sentences

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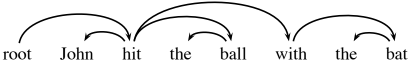

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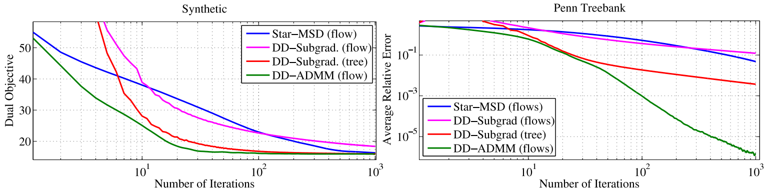

- DD-ADMM wins with both synthetic (10 words) and real sentences
- Also outperforms DD-Subgradient with a TREE model (fewer slaves)

Outline

Conclusions

- DD-ADMM: a new algorithm for LP-MAP inference
- Dual decomposable, hence the slaves can all be solved in parallel
- Allies the simplicity of DD with the effectiveness of AL methods
- Suitable for problems with many slaves, outperforming Komodakis et al. (2007)
- Optimality certificates for LP-MAP (not just MAP)
- A significant amount of computation can be saved by *caching* and *warm-starting* the subproblems
- Related work in accelerating DD and in quadratic projections: Jojic et al. (2010); Ravikumar et al. (2010)
- Future work: larger slaves and *approximate* ADMM steps

Thank you!

- Questions?

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