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Ambiguity and Idiosyncratic Interpretation

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Abstract

This paper discusses logics whose premisses and/or conclusions can contain ambiguous material. Two different kinds of applications are sketched for these logics. First, the paper discusses how logics with ambiguous expressions can shed light on the way in which human hearers or readers understand certain 'paradoxical' logical arguments, in which crucial use is made of ambiguous material. Second, the paper uses practical applications to show how a logic ambiguous expressions can be used to avoid interpretational deadlock in such systems. Here a key role is played by the Principle of Idiosyncratic Interpretation, which states that, in a given context of occurrence, different human interpreters may be unaware of each other's interpretations of an utterance. This principle is shown to have important consequences for the choice between different possible logics. To illustrate how a logic of ambiguous formulas can be used in Natural Language Processing, the case of a Question-Answering system is discussed in some detail.

1 INTRODUCTION

Natural language is ambiguous at various levels of interpretation. At a low (e.g. speech recognition) level, a signal can be ambiguous between various utterances; at a higher (semantic) level, a fully recognized utterance can be used to express various different propositions; and at an even higher (pragmatic) level, a proposition may be used for various different purposes. Ambiguity, understood in this broad way, is the main problem that confronts humans or machines when they try to interpret natural language. This paper will focus on ambiguities of the second kind, which are sometimes called *semantic ambiguities* or also just *ambiguities*, when there is no likelihood of confusion. Semantic ambiguity can arise from various different sources. For example, the source of an ambiguity may be lexical (as in cases of lexical homonymy and polysemy), syntactic (when an expression has more than one possible syntactic derivation), or contextual (as in anaphoric ambiguities). Combinations of such different sources occur as well, for example when a word is ambiguous between different parts of speech.

It has often been observed that semantic ambiguity is one of the most daunting problems for automatic interpretation of natural language. (See

Bar-Hillel 1960 for an early and very outspoken statement of this claim). It is sometimes suggested that the main problem here is one of combinatorial explosion and it is true that the number of calculations to be performed for some types of ambiguities can 'explode' (see e.g. Ristad & Berwick 1989). But the situation is not the one in which considerations of computational complexity usually come to the forefront, namely where one has to perform a great number of calculations each of which is unproblematic in itself. The situation in disambiguation is typically much more complex: the number of interpretations may be small or great, but it is often completely unclear how to decide which ones can be refuted. As a result, disambiguation is *deadlocked*. For example, consider the following sentences.

Three boys carried a piano up the stairs.

None of the standard disambiguation algorithms can refute interpretations according to which each boy carries a separate piano. To discard this interpretation one would have to represent some highly nontrivial bits of common sense knowledge about

- *boys* (How much can they carry?)
- *stairs* (What is their size, shape, and structure?)
- *pianos* (What is their shape and weight?)

Let us assume that all this information may once be formally represented in some common-sense knowledge base, perhaps of the kind advocated in Guha & Lenat 1990. This knowledge would have to be used in inference, along the following lines (see e.g. Buvac 1996), where *cs* represents the knowledge presumably shared by all competent speakers of the language. A 'grammatically possible' interpretation is an interpretation that is logically well formed[1] as well as in accordance with all the linguistic rules governing the sentence as uttered in its linguistic context.

Speaker has uttered sentence S;
$p_1, \ldots, p_n$ are the only grammatically possible interpretations of S;
$cs \models \neg p_i$; therefore,
$p_1, \ldots p_{i-1}, p_{i+1}, \ldots p_n$ are the only remaining interpretations of S.

More intricate versions of the same pattern of reasoning aimed at 'charitable interpretation' would include knowledge of the speaker that is not a part of *cs* but, for example, asserted earlier by the same speaker. Note that the inference presented is only valid if several assumptions are made whose truth can be extremely difficult to assess in practice. For example, it must be assumed that *cs* contains, in principle, sufficient information to refute p_i (e.g. Does common-sense knowledge rule out the possibility of a boy carrying a piano? Even if he is aided by advanced equipment?); it must be assumed that the speaker is not only a competent speaker of the language, but she must also be able to perform the inference $cs \models \neg p_i$ 'on line', and she may not be speaking in irony, etc. Clearly, all of this makes it extremely difficult to exploit common-sense knowledge for the disambiguation of natural language. So difficult, that it is unlikely that this strategy will allow a natural language interpreting system to get rid of all ambiguity.

2 UNDERSPECIFIED REPRESENTATIONS

Mainly to avoid the need for separate storage of all possible interpretations, computational semanticists now routinely use levels of semantic interpretation that contain ambiguous representations. An early example is the PHLIQA system (see e.g. Bronnenberg *et al.* 1979), in which this strategy was applied mainly to cope with lexical ambiguities. An example[2] is the ambiguity of the word *American*, which can mean

1. λx: *Country*(*DeparturePlace*(x)) = *USA* (for a flight),
2. λx: *Country*(*ArrivalPlace*(x)) = *USA* (for a flight),
3. λx: *Country*(*Manufacturer*(x)) = *USA* (for an airplane),
4. λx: *Country*(*PlaceOfBirth*(x)) = *USA* (for a person),
5. λx: *Carrier*(x) = *American Airlines* (for a flight),

and so on. Let us assume that these five options exhaust the possible interpretations of *American*. In Phliqa, the word AMERICAN would be mapped on to an ambiguous constant AMERICAN, which is later mapped to one of the unambiguous paraphrases AMERICAN_1, AMERICAN_2, etc., each of which had one of the interpretations listed. For example, the query 'How many American flights are operated by Quantas?' would be translated into a formula describing the cardinality of the set

$$\{x \in \text{flights: AMERICAN}(x)\ \&\ \mathit{Operator}(x) = \mathit{Quantas}\},$$

where *Operator*(x) stands for 'the carrier operating the Departure of the flight x'. The interpretations corresponding with (3) and (4) above are refuted as a result of type conflicts. (The Predication *Operator*(x) = *Quantas* shows that x can neither be a person nor an airplane.) But (1), (2), and (5) lead to distinct and respectable interpretations:

$\{x \epsilon$ flights: Country(DeparturePlace$(x)) = USA$ &
$Operator(x) = Quantas\}$,

$\{x \epsilon$ flights: Country(ArrivalPlace$(x)) = USA$ &
$Operator(x) = Quantas\}$.

$\{x \epsilon$ flights: Operator$(x) = AA$ & $Operator(x) = Quantas\}$.

Similar ideas were applied to quantifier scope ambiguity in Schubert & Pelletier 1982. This track of research gained popularity in the late 1980s, when linguists at SRI proposed their underspecified 'quasi-logical forms' (see e.g. Alshawi 1990). The advantage of using underspecified logical formulas for the representation of linguistic meaning can be considerable: the syntax is not burdened by purely semantic considerations (Fenstad *et al.* 1987), and the computational load on the system, for storing and processing the meaning of a formula at a stage at which it has not yet been resolved, is reduced considerably. On the other hand, the ambiguous representations are typically not put to real use until a separate module has disambiguated them. In particular, the information in them is not evaluated semantically or used in logical inference.[3] Thus it was impossible for the systems described to exploit the fact that, since Quantas operates only flights whose departure and arrival are in the same country, AMERICAN_1 and AMERICAN_2 lead to basically the same query. We will see in section 5.3 how such facts may be exploited to improve the system.

Since the systems described cannot do much before they have resolved all ambiguities, it is often impossible for them to find an intelligent way of dealing with ambiguities at all, because the information that could, in principle, disambiguate them is unavailable. Underspecified representations by themselves do little to solve the problem of interpretational deadlock, but their use has prompted some important questions:

1. How do *people* process ambiguous information?
2. What kinds of processing are (logically) possible on the basis of underspecified representations? In particular, how can ambiguous information be represented and how can it enter logical inference?

Processing of ambiguous expressions by humans (1) has always been a major concern of psycholinguists. Psycholinguists, however, have usually focused on 'apparent' ambiguities, which can be resolved by linguistic aspects of the context of occurrence (e.g. MacDonald *et al.* 1994). The widespread use of underspecified semantic representations has triggered questions such as, for example, Under what circumstances is disambiguation necessary (Poesio 1994, 1996)? What kinds of inferences do people draw from an utterance before it is fully disambiguated? (For example, if you shout *Watch out! He's trying to shoot you*, the addressee might decide to seek hiding even if he is unable to resolve the referent of the pronoun *he*.) It is intuitively clear that language understanding is not an all-or-nothing affair and one would ultimately want to model the understanding process in a way that reflects this. Many of the relevant issues must be left alone here, but one of them will be discussed in some detail: In section 4, we will illustrate how the logic of underspecified representations can shed light on the difficulty people have in assessing the validity of a certain type of *paradoxical* logical arguments, namely those whose paradoxicality hinges on the ambiguity of some of the expressions making up the argument.

The second issue (2), which concerns the kinds of processing for which underspecified representations can be used, will be the main topic of this paper, which will also underlie what we have to say about paradoxical arguments. We will focus on one kind of processing, namely logical inference. Many computational linguists (for references, see section 3) are now trying to move one step beyond merely representing ambiguous information in a natural-language processing system and to wait it out until disambiguating information comes along. This next step, which, to the best of my information, has not been applied in any real system yet, is to use ambiguous sentences as premisses or conclusions of logical reasoning. An important motivation for this work is the idea that if and when a reasoning component is in place that can deal with ambiguous premisses, then complete disambiguation is no longer always a necessity. To give the flavour of the idea, suppose a user of a question-answering system addresses the system by asking an ambiguous yes-no question, and suppose the system is designed to answer questions by trying to prove their truth or falsity from premisses stored in a database. Now if, for example, the negation of *each* interpretation of the question can be proven, then the reasoning component could infer that the answer to the question has to be negative. Consequently, disambiguation of the question is not necessary, and a negative answer can be given. The case of an affirmative answer is analogous, of course.

In what follows, we will discuss the various ways in which a logic for underspecified representations (an 'ambiguous logic', for short) can be useful. Usefulness will be taken in a rather broad sense, since we will not only show how ambiguous logics can help us cope with the problems of natural language understanding, but also how they can be used to shed light on some issues in philosophical logic.

The aims of this paper, as stated here, differ from those of most other work on underspecification. Most of this work focuses on specific linguistic phenomena such as the ambiguities of quantifier scope. The present paper, by contrast, will abstract away from what the different interpretations of a given utterance are, how they may be discovered and represented. Having done this, we will ask what role the utterance, or its formal representation, can play in logical inference. We will try to answer this question by looking at specific problems. We will suggest that a general appeal to intuitions about the logical validity of an inference is sometimes ineffective since, in the case of ambiguous logic, the validity of an inference can depend on the setting and purpose of the inference.[4]

Section 3 will introduce the main concepts that we will need in our discussion of ambiguous logic. Section 4 will show how these concepts can be used as an instrument for the analysis of philosophical questions. Section 5 will discuss the issue of what are appropriate notions of ambiguous consequence. The concluding section will discuss a number of objections that have been advanced against some of the ideas in the preceding sections.

3 LOGICS FOR UNDERSPECIFIED REPRESENTATIONS

The basic fact underlying the notion of ambiguity is that some expressions can be used for more than one purpose and that this can cause indeterminacy of interpretation. More precisely, and focusing on sentences, a sentence S is ambiguous between two nonequivalent propositions φ and ψ if S can be uttered to express φ but also to express ψ. If φ and ψ can be expressed unambiguously by sentences S_1 and S_2 respectively, it is also said that S is ambiguous between S_1 and S_2. When no other information about the speaker's intentions is available, the situation can be summed up conveniently in a formalism often used in natural language generation (e.g. Moore & Paris 1994):

Intend (Speaker, (MB (Speaker, Hearer, φ))) $\vee$
Intend (Speaker, (MB (Speaker, Hearer, ψ))).

Intend (x, ϕ) says that x intends to make it true that ϕ; MB(x, y, ϕ) says that ϕ is mutually believed between x and y. The disjunction reflects that the interpreter of S is uncertain whether φ or ψ is intended.[5]

In accordance with the tradition of logical semantics (e.g. Montague 1973), we will use logic as a mediator between linguistic form and model-theoretic meaning. From this perspective, the natural move if one wants to study the ambiguity of natural language is to set up a new logic and to define a mapping in which each sentence of natural language is mapped on to one potentially ambiguous formula of the logic. Logicians have, until recently, never had much use for ambiguous formulas. Surprisingly, this is also true for logicians who work on the semantics of natural language. An example is Thomason who wrote, explaining Montague's notion of a 'disambiguated language': 'there is no serious point to constructing an artificial language that is not disambiguated' (Thomason 1974). As a result, there still are few formal tools to study logical consequences as a relation between sentences some of which may be ambiguous. But recently, some tentative efforts in this direction were made, including Reyle (1993, 1995, 1996), van Deemter (1991, 1996), van Eijck & Jaspars (1995), Fernando (1995), Alshawi (1990), Poesio (1994, 1996). So far, there is little unanimity about what is or are acceptable notions of logical consequence in an ambiguous setting (ambiguous consequence, for short). In what follows, we will introduce some basic terminology and, drawing on insights from the literature, we will chart the main avenues for defining truth and ambiguous consequence. A discussion of the relative merits of the different definitions will be postponed until section 5.2.

The logic of underspecified representations has been studied using different underlying representational formalisms (predicate logic, DRSs, etc.). In addition, ambiguity has been modeled in different ways. In our discussion of proposals in the literature, we will neglect representational differences between them, pretending instead that one and the same fragment of predicate logic underlies all these proposals. In addition, we will—for reasons explained in the previous section—adopt a rather simple, 'syntactic' modeling of ambiguity.[6] Consider a fragment of predicate logic L and imagine the addition of a class of ambiguous formulas to L. This may be done, for example, by the addition of new constants, or by a loosening of syntax. (For example, one may allow that certain brackets are omitted.) The resulting language, whose sentences form a superset of L, will be called L'. We will assume, for simplicity, that every reading of every sentence of $L' - L$ can be expressed unambiguously by some sentence of L. Let a (syntactic) *disambiguation* d be a function that maps every unambiguous sentence of L' to itself, and that maps every ambiguous sentence φ of L' to some sentence $d(\varphi)\epsilon L$ that expresses one of the readings of φ. In addition, let d also map sets of sentences of L' to sets of unambiguous sentences, in the obvious way: Let $\Sigma \subseteq L'$ then $d(\Sigma) = \{\varphi\epsilon L\colon\ \exists\varphi'\epsilon\Sigma(\varphi = d(\varphi'))\}$. Given this setting, several ways of defining truth and falsity are possible, including[7]

Truth, relative to a disambiguation:
φ is true$_d$ (false$_d$) with respect to a model $M \Leftrightarrow$
$d(\varphi)$ is true (false) w.r.t. M,

Strong truth:
φ is true$_\forall$ (false$_\forall$) with respect to a model $M \Leftrightarrow$
all paraphrases of φ are true (false) w.r.t. M,

Weak truth:
φ is true$_\exists$ (false$_\exists$) with respect to a model $M \Leftrightarrow$
at least one paraphrase of φ is true (false) w.r.t. M.

The first version speaks for itself. As for the other two, true$_\forall$ can be glossed as *irrefutable* and true$_\exists$ as *defensible*. Each option makes sense in its own way. Note that in the case, where every sentence has exactly one meaning, all three notions of truth/falsity coincide with classical truth/falsity: If φ is unambiguous and d is an arbitrary disambiguation, then:

φ is true (false) with respect to $M \Leftrightarrow$
φ is true$_d$ (false$_d$) with respect to $M \Leftrightarrow$
φ is true$_\forall$ (false$_\forall$) with respect to $M \Leftrightarrow$
φ is true$_\exists$ (false$_\exists$) with respect to a model M.

There are several ways of defining logical consequence in the new setting, each of which has to be plausible for ambiguous as well as unambiguous expressions. So suppose, furthermore, that φ and ψ are elements of L, that is, they do not contain ambiguous material. Then if $\models_a$ is the notion of ambiguous consequence, we require

$$\varphi \models_a \psi \Leftrightarrow \varphi \models \psi.$$

for some relation of (unambiguous) logical consequence $\models$ that is itself plausible. This might be called the *Conservativity* requirement. This requirement still leaves open some very different approaches to defining ambiguous consequence. One way of guaranteeing Conservativity is to define ambiguous consequence as the relation that preserves truth, where truth may be defined in any of the three ways mentioned above.

$\varphi \models_a \psi \Leftrightarrow_{Def}$ For all models M, if φ is *true* with respect to M then ψ is *true* with respect to M.

More sophisticated versions of this approach arise when ambiguous consequence exploits the partiality of the situation. After all, if truth and falsity are defined in the strong sense, a sentence may fail to be true *and* fail to be false and if truth and falsity are defined in the weak sense, a sentence may be true *and* false. For example,

Van Eijck & Jaspars (vE & J): $\varphi \models_{vE\,\&\,J} \psi \Leftrightarrow_{Def}$ For all models M, if φ is true$_\forall$ with respect to M then ψ is true$_\forall$ with respect to M *and* if ψ is false$_\forall$ with respect to M then φ is false$_\forall$ with respect to M.

Each of the resulting notions of ambiguous consequence respects the Conservativity requirement in the strong sense that it leads to a purely classical logic for nonambiguous expressions.

But, very different avenues for defining ambiguous consequence can be and have been explored. In particular, one can stick to the idea of quantifying over possible paraphrases, while abandoning the idea that premisses and conclusion must be judged independently of each other. This approach makes it possible to define a notion ($\models_a$) of ambiguous consequence *based on* a notion of nonambiguous logical consequence ($\models$). Reyle (1993), for example, defined essentially

Reyle '93: $\varphi \models_a \psi \Leftrightarrow_{Def}$
For *each* paraphrase of φ, *there is* a paraphrase of ψ that makes the argument valid in the sense of $\models$.

This definition is equivalent to what one gets if both the premisses and the conclusion are represented by means of the disjunction of their interpretations.[8] It instantiates the schema

Quantificational schema 1:
$\varphi \models_a \psi \Leftrightarrow_{Def} Q_1 \chi \in A\ Q_2 \eta \in B\colon\ \chi \models \eta,$

where A is the set of paraphrases of φ, B is the set of paraphrases of ψ, and $\models$ is the underlying notion of logical consequence. Note that $\models$ can be any notion of logical consequence defined on nonambiguous expressions. It seems plausible to exclude 'nonlogical' quantifiers such as *most* and focus on the logical quantifiers (i.e. *some*, *all*, *no*, *not all*). But even then, some remaining instantiations of the scheme are clearly implausible. Conservativity, for example,[9] rules out the combinations covered by the following scheme:

$Q_1 =$ *some* or *all*, and
$Q_2 =$ *no* or *not all*.

As a result, the quantificational schema has four remaining logically distinct instantiations. For example, the combination $Q_1 =$ *no*, $Q_2 =$ *no* is equivalent to the second version.

1. $Q_1 = Q_2 =$ *some*
2. $Q_1 = Q_2 =$ *all*
3. $Q_1 =$ *some*, $Q_2 =$ *all*
4. $Q_1 =$ *all*, $Q_2 =$ *some*.

Version (1) says that the argument can be made valid by a properly chosen disambiguation; in other words, the argument is defensible. Version (2) says that the relation *cannot* be *in*validated by such a disambiguation; in other words, the argument is irrefutable. Version (3) says that the premisses can be disambiguated in such a way that the conclusion cannot be disambiguated in such a way that it fails to follow; (4) expresses that no matter how the premisses are disambiguated, a validating disambiguation of the conclusion can be found. Below, (1) will be denoted as $\models_{\exists\exists}$, (2) as $\models_{\forall\forall}$, (3) as $\models_{\exists\forall}$, and (4) as $\models_{\forall\exists}$.

Variants of the quantificational schema are also possible. In particular, the scope of the quantifiers Q_1 and Q_2 may be reversed to produce the following schema:

Quantificational schema 2:
$\varphi \models_a \psi \Leftrightarrow_{Def} Q_2\eta \in B\ Q_1\chi \in A{:}\ \ \chi \models \eta$

where, as before, A is the set of paraphrases of φ while B is the set of paraphrases of ψ. Since quantifier scope will only make a difference if $Q_1 \neq Q_2$, the only cases this new schema adds are the one where Q_1 = *all* and Q_2 = *some*, and the one where Q_1 = *some* and Q_2 = *all*. Informally, the first of the two can be glossed as saying that the conclusion of the argument may be disambiguated in such a way that the argument cannot fail. The second says that, no matter how the conclusion is disambiguated, a validating disambiguation of the premisse can be found. To illustrate the difference between the two quantificational schemas, consider two ambiguous formulas ϕ and ψ. Assume ϕ is ambiguous between p_1, p_2, and p_3 while ψ is ambiguous between p_1 and p_2 only. Assume, furthermore, that p_1, p_2, and p_3 are logically unrelated. Then, on the first quantificational schema, $\models_{\exists\exists}$ is the only of the four relations of ambiguous consequence that validates the inference $\phi \models \psi$. On the second quantificational schema, however (where the quantification over paraphrases of the conclusion takes wide scope), $\phi \models \psi$ is also validated by the relation $\models_{\exists\forall}$.

Finally, one might use Boolean combinations of the clauses featuring in the two schemas.[10] For example, combining a clause from the first with one from the second,

$\varphi \models_a \psi \Leftrightarrow_{Def}$
For *each* paraphrase χ of φ, *there is* a paraphrase η of ψ that makes the argument $\chi \models \eta$ valid, AND
There is a paraphrase χ of ψ such that *each* paraphrase η of φ makes the argument $\chi \models \eta$ valid.

This survey probably does not exhaust all the possibilities (indeed, it is not easy to be sure what the possibilities are), but it seems to include the more obvious ones, including all those proposed in the literature so far. Moreover, the survey includes some notions of ambiguous consequence that are theoretically very useful (section 4), as well as the one notion of ambiguous consequence that will emerge from our discussions in section 5 as most appropriate in an important class of applications. In what follows we will focus on some of the most salient options, leaving it to the reader to work out the implications for the other notions of ambiguous consequence, which are mostly straightforward.

There is one group of consequence relations that is not covered by the survey in this section. These are the relations of ambiguous consequence that take coherence relations (i.e. contextual restrictions on interpretation) into account. For example, different notions of logical consequence result when different occurrences of the same ambiguous constant are required to be interpreted in the same way. The resulting nonmonotonic notions of logical consequence are discussed briefly in section 5.2.

At this point, some of the terminology of ambiguous logic has been introduced and some of the main perspectives on truth and ambiguous consequence have been surveyed. Before turning to the question of which notion of ambiguous consequence is most plausible (or even the 'correct' one), we will show how the terminology of ambiguous logic can be used to analyse logical fallacies.

4 AMBIGUOUS LOGIC AS AN ANALYTICAL TOOL

In this section, we will briefly sketch how the tools developed in the previous section may be used for analytical purposes. Many ancient logical paradoxes hinge on the ambiguity of one or more premisses in the argument. For example, consider the following argument, where the expression 'The king' refers to different persons (say, the present and the previous king of a given country) in the two premisses:

p: *The king* $= Mr.\ X$;
q: *The king* $= Mr.\ Y$; therefore,
r: $Mr.\ X = Mr.\ Y$.

Let us dub this argument the Fallacy of Description. What makes this a fallacy in the sense of a *plausible* piece of *invalid* reasoning[11] is the fact that one might get confused by the situation: p has an interpretation that makes it true, q has an interpretation that makes it true. If p and q were unambiguous the reasoning from p and q to r would have been valid. But it is somewhat unclear what this means for the argument as it stands, that is, with ambiguous premisses.

Aristotle distinguished several situations in which ambiguity can lead to paradox: *amphiboly* if the ambiguity involved grammatical constructions and *ambiguity* if the ambiguity (modern usage) involved the meaning of a word. It is not immediately clear which of the two is more appropriate in the case of an anaphoric ambiguity but the latter does not seem inappropriate given current usage of the word 'ambiguity'.

If we analyse the Fallacy of Description making use of the logics sketched in the previous section, we find that these induce different answers to the question of whether the premisses or the reasoning itself should be blamed for the paradox. First, we have to generalize the logics in such a way that they cover *sets* of premisses. This is trivial in all except one case, namely the version by van Eijck & Jaspars, which now goes

$\Gamma \models_{vE\,\&\,J} \psi \Leftrightarrow_{Def}$ For all models M,
if all $\varphi \epsilon \Gamma$ are strongly true with respect to M then ψ is strongly true with respect to M *and*
if ψ is strongly false with respect to M then at least one $\varphi \epsilon \Gamma$ is strongly false with respect to M.

In particular, $\models_{\forall\exists}$ and $\models_{\forall\forall}$ find fault with the reasoning, but $\models_{\exists\exists}$, $\models_{\exists\forall}$, and $\models_{vE\,\&\,J}$ do not, as may easily be verified:

(a) $p, q \not\models_{\forall\exists} r$
(b) $p, q \not\models_{\forall\forall} r$
(c) $p, q \models_{\exists\exists} r$
(d) $p, q \models_{\exists\forall} r$
(e) $p, q \models_{vE\,\&\,J} r$.

The 'reverse scope' variants of $\models_{\forall\exists}$ and $\models_{\exists\forall}$ (along the lines of quantificational schema 2) behave just like $\models_{\forall\exists}$ and $\models_{\exists\forall}$ themselves.

So much for the soundness of the reasoning. How about the premisses? Here a similar story can be told. One has to conclude that the question of whether the premisses are true has no unique answer. Instead, they are (1) neither $\text{true}_\forall$ nor $\text{false}_\forall$ but (2) $\text{true}_\exists$. What does this mean? If (ambiguous) truth can be interpreted in such a way that the premisses are true, and if (ambiguous) logical consequence can be interpreted in such a way that the reasoning comes out valid, then does that mean that there is nothing wrong with the Fallacy of Description? Of course not. There are disambiguations that make the reasoning valid and there are disambiguations that make the premisses true, but *there are none that do both.* The problem is that received terminology is not very suitable for making this point. Suppose one dubbed an inference *good* if it is not only logically valid, but if its premisses are true as well:

An inference 'p, q therefore r' is *good* iff it is logically valid and both p and q are true.

In the situation in which premisses may be ambiguous, this definition can take at least four different forms, depending on which notion of ambiguous consequence and which notion of ambiguous truth are selected. Suppose, for instance, one selects $\models_{\exists}$ and true$_{\exists}$, then the definition makes our paradoxical argument *good*, which makes it hard to understand what is wrong with it; on the other hand, if one selects $\models_{\forall}$ and true$_{\forall}$, then the argument comes out *not good* for the double reason that it is not logically valid and its premisses are not true, which makes it hard to see why the argument is at all plausible.

One way to look at this situation is the following: modern logic has never made much fuss about the truth of premisses. The notion of truth can simply be paired with that of logical consequence. The two can be kept separate. On the other hand, as soon as one looks at ambiguous formulas, a simple pairing of truth and logical consequence obfuscates some important issues: saying that the premisses 'can be true' and that the argument 'can be valid' (i.e. they are true/valid on at least one paraphrase) does not tell you whether the argument can be valid, having true premisses *at the same time*. Here is a bit of terminology that helps to fill this gap, by introducing 'context-dependent' versions of truth and logical consequence:

Truth, relative to a disambiguation:
φ is true$_d$ (false$_d$) with respect to a model $M \Leftrightarrow$
$d(\varphi)$ is true (false) w.r.t. M,

Logical consequence, relative to a disambiguation:
$\Gamma \models_d \psi$ iff $d(\Gamma) \models d(\psi)$.

Goodness, relative to a disambiguation:
An argument '$\Gamma \models \psi$' is good$_d$ w.r.t. a model M iff $\Gamma \models_d \psi$ and all the elements of Γ are true$_d$ w.r.t. M.

'Absolute' *goodness* can then be defined as *good*ness with respect to at least one disambiguation:

An argument $\Gamma \models \psi$ is *good* iff $\exists d$ such that $\Gamma \models \psi$ is *good*$_d$.

If the relation $\Gamma \models_d \psi$ holds between premisses and conclusion, we will also say that the argument is valid$_d$. An argument $\Gamma \models \psi$ is valid$_{\exists}$ in case there is a d such that $\Gamma \models_d \psi$. The notion of a sentence being *true*$_{\exists}$ with respect to a model has been defined in the previous section. This terminology allows the following analysis of the paradox: it is valid$_d$ for some d and its premisses are true$_d$ for some d; however, there is no d such that both conditions are met. Consequently, the argument is not *good*. Any argument that is valid$_{\exists}$ and whose premisses are true$_{\exists}$, but that is not *good* is a Fallacy of Ambiguity. The Descriptive Fallacy instantiate this scheme. In van Deemter (1993), it has been shown that some of the most compelling versions of the *sorites* argument can be analysed along exactly analogous lines and that this leads to a good explanation of why they represent a fallacy.[12]

This section has demonstrated how the terminology of ambiguous logic can be useful for the analysis of conceptual problems. The following section will highlight a very different question, namely How to choose between the different notions of logical consequence that are available for ambiguous logic.

5 CHOOSING APPROPRIATE NOTIONS OF AMBIGUOUS CONSEQUENCE

We now turn to the question of which notions of ambiguous logic are most plausible. First we will make a quick comparison of some of the notions of ambiguous consequence advocated in the literature (section 5.1). Then we will discuss the requirements posed by practical NLP applications on the notion of ambiguous logic (section 5.2). Finally, we will focus on one particular type of application, namely Question-Answering, to see how ambiguous logic can be used to deal with ambiguities that threaten to lead to interpretational deadlock (section 5.3).

5.1 *Initial Comparison of Proposals*

We have seen in section 3 that there are different possible ways of defining relations of ambiguous consequence and that different authors have made different choices from among the set of possibilities. Let us briefly contrast three of the main proposals that have been advanced, namely (1) the relation $\models_{\forall\exists}$, proposed by Reyle (1993); (2) the relation $\models_{vE\,\&\,J}$, proposed by van Eijck & Jaspars (1995), and (3), the relation $\models_{\forall\forall}$, which is one of the options discussed by van Deemter (1991, 1996). Some of the most striking properties of each of the three will be listed, but an evaluation will have to wait until the next section.

As was noted above, each of the three logics fulfil the requirement of Conservativity, so if none of the formulas in the premisses and the conclusion are ambiguous then all three are equivalent. Likewise, if only the premisses can be ambiguous, it does not matter how one quantifies over paraphrases of the conclusion. Consequently, $\models_{\forall\forall} = \models_{\forall\exists} = \models_{vE\,\&\,J}$. Conversely, if only the conclusion is ambiguous, it does not matter how one quantifies over paraphrases of the premisses. Consequently, in that situation, $\models_{\forall\forall} = \models_{\exists\forall} = \models_{vE\,\&\,J}$.

Having noted these areas of overlap, let us briefly discuss the logics one by one, stressing some of the differences between them. Firstly, $\models_{\forall\exists}$. This relation is monotonic, reflexive, and transitive and this has been taken as an important argument in favour of it (Reyle 1993). However, it is also *inconsistent* in the sense that contradictory conclusions can sometimes follow from noncontradictory premisses (van Deemter 1991, 1996, Reyle 1995).[13] To see this, we have to allow that an expression can be ambiguous between contradictory paraphrases. Suppose, for example, the predicate F is ambiguous between $\lambda x G(x)$ and $\lambda x \neg G(x)$. Then

$G(a) \models_{\forall\exists} F(a)$ and $G(a) \models_{\forall\exists} \neg F(a)$.

Analogous problems affect the relations $\models_{\exists\exists}$ and $\models_{\exists\forall}$ and the same is true if the scope of the quantifiers $\exists$ and $\forall$ is reversed: inconsistency results whenever paraphrases are quantified over existentially.

Secondly, $\models_{\nu E \,\&\, J}$. This relation is also monotonic, reflexive, and transitive. In addition, it makes all expressions that are ambiguous between contradictory paraphrases equivalent, because in this case the expression can neither be true$_{\forall}$ nor false$_{\forall}$.

Thirdly, $\models_{\forall\forall}$. It is easy to see that this relation is monotonic as well as transitive, but it is not *reflexive*. Stronger even, if a sentence p has (nonequivalent) paraphrases p_1 and p_2, then $p_1 \not\models p_2$ and consequently $p \not\models_{\forall\forall} p$. Thus, we have

Let p be ambiguous between nonequivalent paraphrases p_1 and p_2.
Then $p \models_{\forall\forall} p \Leftrightarrow p \varepsilon L$,

where L, as before, is the nonambiguous subset of L'.

Given what has just been observed about the (non)reflexiveness of the relations proposed in the literature, it is good to look at a simple example. Consider the situation in which dates can be written in either the American (m-d-y) or the European style (d-m-y) and consider the sentences

p: The meeting took place on 10-04-98.
q: The meeting took place on 04-10-98.

In this situation, p is ambiguous between the tenth of April (European style) and the fourth of October (American style). But q is ambiguous between the tenth of April (American style) and the fourth of October (European style) as well! Sentences p and q are ambiguous between the same two paraphrases. None of the notions of ambiguous consequence considered so far distinguishes between different expressions as long as these expressions have the *same sets of interpretations*. As a result, we have

$p \models_{\forall\exists} p$
$p \not\models_{\forall\forall} p$

$p \models_{\nu E \,\&\, J} p.$
$p \models_{\forall\exists} q$
$p \not\models_{\forall\forall} q$
$p \models_{\nu E \,\&\, J} q.$

In the next section we will see that this invalidates the relation $\models_{\forall\exists}$, while it restricts the usefulness of the relation $\models_{\forall\forall}$ to situations in which we cannot assume coherence between the expressions in the premisses and the conclusion of an argument.

Our observations concerning these three relations of ambiguous consequence are fairly symptomatic of the class of all the relations mentioned in section 3. For example, all the relations that quantify existentially over paraphrases of the conclusion lead to inconsistency. Furthermore, all the relations that make preservation of (strong or weak) truth and/or falsity the defining property of ambiguous consequence (as, for example, $\models_{\nu E \,\&\, J}$, or its analogon that replaces true$_\forall$ and false$_\forall$ by true$_\exists$ and false$_\exists$) produce unexpected results for expressions that are ambiguous between paraphrases that are logically related. This is even more true for 'nonpartial' versions of this approach. For example, consider the relation

$\varphi \models_a \psi \Leftrightarrow_{Def}$ For all models M, if φ is strongly true with respect to M then ψ is (at least) weakly true with respect to M.

Let us go back to the example involving dates. Let φ represent the meaning of sentence p. Then clearly, its two paraphrases cannot both be true. Consequently, the argument

$\varphi \models_a \psi$

is valid for arbitrary ψ, which seems undesirable.[14]

5.2 *A Practical Perspective*

We are left with a difficult situation. Different and incompatible proposals have been made for a formalization of ambiguous consequence. Several of these proposals were accompanied by arguments in defence of them involving, for example, the structural properties of the logics concerned. One wonders what this means. Perhaps there does exist an ambiguous logic that combines all the properties featuring in these discussions (i.e. conservativity, monotonicity, reflexivity, transitivity, and consistency) as well as doing justice to what it means for an expression to be ambiguous (section 3). In that case this logic has not been discovered yet. Or else there does not exist such a logic. In the remainder of this paper it will be argued that there *does* exist a logic that is appropriate for all different kinds of situations but this logic does not have all the properties listed above, and that some situations allow a strengthening of this logic.

Apparently, intuitions on ambiguous inference can conflict. We will try to resolve these conflicts by making the inferences more concrete. In particular, we will add some context to them that will make it clear *who is the author* of the premisses and the conclusion of the argument. Imagine a situation in which the premisses and the conclusion of an inference can be ambiguous, while the premisses of the argument stem from a person that is different from the one interpreting the conclusion. For example, consider a person x producing a database filled with potentially ambiguous discourse. One of the expressions he or she uses, at a given point of the discourse, is the expression p discussed in the previous section:

p: The meeting took place on 10-04-98.

Suppose x intends this in the European sense, meaning that the meeting took place on the tenth of April, unaware of the other interpretation that the sentence may have. Now a second person, y, comes along and asks '(Is it the case that) p?', intending p to mean that the meeting took place on the fourth of October, and unaware of the ambiguity. Now consider $\models_{\forall\exists}$. We have seen that reflexivity is a theorem of $\models_{\forall\exists}$ and, more specifically, $p \models_{\forall\exists} p$. Consequently, a Question-Answering system built on $\models_{\forall\exists}$ will answer the query in the affirmative and thereby end up misleading y. The same problem affects $\models_{\exists\exists}$ and $\models_{vE\,\&\,J}$, as the reader may verify.[15] In a picture:

$$\text{p!} \xRightarrow[\texttt{(fills database)}]{\texttt{User x (p)}} \text{DB} \xRightarrow[\texttt{(queries database)}]{\texttt{User y (p)}} \text{p?}$$

A variant of the same example can be constructed in a setting where x has not entered an ambiguous expression into the database but, for example, the unambiguous expression *The meeting took place on the tenth of April 1998*, which we will abbreviate as p_{Eu}. The relevant inference is $p_{Eu} \models_{\forall\exists} p$. Now suppose the system has inferred that, if the meeting is on the tenth of April it cannot also be on the fourth of October. Abbreviating *The meeting took place on the fourth of October 1998* as p_{Am}, this means that $p_{Eu} \models \neg p_{Am}$. Therefore, $p_{Eu} \models_{\forall\exists} \neg p_{Am}$. We also have $\neg p_{Am} \models_{\forall\exists} \neg p$, and then, due to transitivity, $p_{Eu} \models_{\forall\exists} \neg p$. Thus, the premisse p_{Eu} has led to contradictory conclusions. In a case like this it is totally unclear how the system should respond to the query p.

These examples point at an important principle governing the interpretation of ambiguous material. We will call it the Principle of Idiosyncratic Interpretation:

> **Principle of Idiosyncratic Interpretation** (PII). *If an expression is ambiguous then different people may assign different preferred interpretations to it, even in one and the same context of occurrence. In such a situation, each of them may be unaware of the existence of legitimate interpretations that differ from their own preferred interpretation.*[16]

The Principle stresses that the ambiguity of an utterance is not guaranteed to disappear when all the publicly accessible circumstances of the utterance (i.e., its linguistic and nonlinguistic context) are taken into account. An ambiguity may be unresolvable. Moreover, it may be that neither the speaker nor the hearer of an utterance realizes that the ambiguity is there.

This example shows that, in a situation where both the premisses and the conclusion of an argument can be ambiguous, and where unwarranted inferences on the part of the user of the system must be avoided, the PII forces one to use the weakest of the logics discussed, namely $\models_{\forall\forall}$. This can be seen as follows. Ambiguity arises when there is uncertainty about the intention of a speaker (cf. section 1). In situations like this, there is often equal uncertainty about how a hearer will interpret the utterance. Therefore, suppose an agent S enters a premiss φ that is ambiguous between paraphrases $\varphi_1, \ldots, \varphi_m$ and an agent H interprets the conclusion ψ that is ambiguous between paraphrases $\psi_1, \ldots, \psi_n$. In accordance with the PII, let us assume that each of S and H have their own preferred interpretation of the ambiguous expressions involved. Let $\alpha(\phi)$ denote the interpretation of ϕ that is preferred by an agent α. Then the system performing the inference is in the dark about the values of $S(\varphi)$ and $H(\psi)$. Therefore, the only way to guarantee that $S(\varphi) \models H(\psi)$ is by guaranteeing that $\varphi \models_{\forall\forall} \psi$. To sum up the argument: unresolvable ambiguity implies indeterminacy of interpretation and indeterminacy is best modeled by a supervaluational account (cf. van Fraassen 1969).

Natural language understanding applications may be categorized along the following lines: either (1) some of the queries they respond to are ambiguous, while the information sources that contain the answer to the query ('the database', for short) are nonambiguous; or (2) some of the information in the database is ambiguous, while the query itself cannot be ambiguous; or (3) both the query and the information in the database can be ambiguous, or (4) neither can be ambiguous. An example of (1) is the situation of a natural language question-answering system, such as the PHLIQA system discussed in section 2. Note that in such systems, the query may typically contain ambiguous material, whereas the database containing the answers does not. These systems—no matter how they operate internally—may be described as deriving *ambiguous conclusions* from *unambiguous premisses.*

An example of (2) is the situation of a discourse interpreting system, such as one that reads scientific abstracts to summarize them. Such systems may be described as deriving nonambiguous conclusions (i.e. the information derived from the text) from ambiguous data (i.e. the abstracts interpreted). We have just discussed an example of the most difficult kind of application (3), which occurs when a user types an ambiguous query, which is used by the system to search a body of natural language. This situation may be described as one in which an ambiguous conclusion is derived from ambiguous premisses. An example of (4), finally, is the standard one, in which ambiguity plays no role. To summarize:

	Ambiguous premisses:	Nonambiguous premisses:
Ambiguous conclusions:	(3) NL QUESTION-ANSWERING (NL DATABASE)	(1) NL QUESTION-ANSWERING (STANDARD DATABASE)
Nonambiguous conclusions:	(2) DISCOURSE INTERPRETATION	(4) FORMAL DATABASE QUERY (e.g. SQL)

With this classification in mind, it is easy to generalize what has just been said about fitting logics and applications: if the premisses of a logical argument can be ambiguous, then one has to quantify universally over interpretations of the premisses; if the conclusion can be ambiguous, one has to quantify universally over interpretations of the conclusion.

We have so far assumed that the systems under discussion have to be fault-proof and this is, in a sense, the normal situation. But there are applications where 'faulty' output is not that problematic. One example is that of a cooperative dialogue of the kind where misunderstandings are unlikely as a result of an abundance of common sense information or where politeness is more important than accuracy. Thus, for example, if participant A in a dialogue says *The meeting is on the fourth of August 1998* and participant B responds *OK, so the meeting is on 04-10-98*, then we might count this as a legitimate inference on B's part, even if we have no prior information about whether B uses the American or the European notation for dates. After all, it is unlikely that A's initial statement was misunderstood by B and consequently, A has no reason to doubt that the proposition B intends to convey is correct. But in the general case, where no such assumptions can be made (and where the inference can be taken up by another interpreter who has not heard A's original statement), it is unsafe to count *B*'s statement as a valid inference from what *A* said. In the general case, therefore, we have to use a notion of inference such as $\models_{\forall\forall}$, that does not warrant the inference.

Another example is literary *translation*. Translation may be modeled as a situation in which an expression S in the source language is related to an expression S' in the target language such that $S \models_a S'$ and $S' \models_a S$. Clearly, for this purpose, $\models_{\forall\forall}$ is not a realistic notion of logical consequence to play the role of $\models_a$, since this would cause *all ambiguous sentences* to become untranslatable. Assuming that translation requires mutual consequence (i.e., both $\varphi \models_a \psi$ and $\psi \models_a \varphi$), a notion of logical consequence more appropriate for modeling translation might be:

$$\varphi \models_a \psi \Leftrightarrow \forall p\epsilon d(\varphi)\exists p'\epsilon d(\psi):\ p \models p' \ \&\ p' \models p.$$

It would follow that a source text can be translated into a target text if and only if each interpretation of the source text is equivalent to one of the target text and conversely.

Text retrieval systems represent another area where $\models_{\forall\forall}$ is not strong enough to be useful and where users might be sufficiently forgiving to allow a stronger notion of consequence. The task of a text retrieval system may be modeled as trying to find constructive proofs for existential formulas of the form $\exists x\varphi(x)$, where φ characterizes a set of texts. For example, $\varphi(x)$ might say 'x is about pitchers' (where 'pitcher' is ambiguous between a jug and a type of baseball player), reflecting the user's interest in texts that are about 'pitchers'. In such applications, users are used to getting everything that constitutes an answer to their query in any of its senses, that is, in this case: texts about baseball players as well as documents about jugs. In this setting, false positives do not hurt too much. Consequently, the relation $\models_{\exists\exists}$ seems most appropriate in this case.

We have seen that $\models_{\forall\forall}$ is the only ambiguous logic that is generally safe. We have also seen that, in some situations, we can be certain that only the premisses or only the conclusion of an argument can be ambiguous, in which case $\models_{\forall\forall}$ becomes indistinguishable from some of its contenders. Furthermore, we have seen that there may be exceptional situations in which other logics, such as $\models_{\exists\exists}$, are more useful. Most interestingly, perhaps, there are also situations in which a genuine strengthening of the logic is called for. This issue can be illustrated with our earlier example involving the American/European interpretation of calendar dates. Recall the sentences

p: The meeting took place on 10-04-98,
q: The meeting took place on 04-10-98,

where some of the systems discussed had

$p \models_a p$
$p \models_a q$

while others had

$p \not\models_a p$
$p \not\models_a q$

We are now in a position to see that it is impossible to determine the correct inference pattern unless we take into account the source of the premiss and the conclusion. If the premiss and the conclusion stem from different agents (as when agent 1 enters the premiss while agent 2 interprets the conclusion), then the second pattern is correct, due to the PII. If, however, the premiss and the conclusion stem from one and the same agent, then it is reasonable to assume that premiss and conclusion are interpreted making use of one and the same 'system' of interpretation: the American (m-d-y) or the European system (d-m-y). In this case, the intuitively correct pattern of inferences is

$p \models_a p$
$p \not\models_a q$ (and even $p \models_a \neg q$).

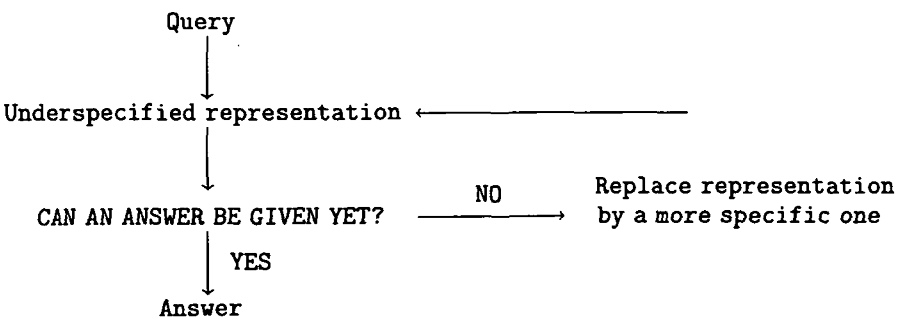
By requiring that different occurrences of the same ambiguous material are interpreted in the same way, one can define 'coherent' versions of our relations of logical consequence. The challenge is to define coherence in such a way that it can be overruled by contextual factors, such as in 'The pitcher drank wine from the pitcher', where the two occurrences of *pitcher* are forced to have different interpretations. To explain how 'imperfect' coherence may be defined is outside the scope of the present paper and the reader is referred to van Deemter (1991, 1996) for an exploration and to Asher & Fernando (1997) for related work.[17] As was shown in van Deemter (1991), even imperfectly 'coherent' relations of logical consequence tend to support reflexivity. For example, consider the example with the calendar dates. The date appears in the same linguistic context in the premisse and the conclusion. Therefore, if both are associated with the same author, coherence ends up rescuing reflexivity, since there are only two possibilities: both occurrences of a date are interpreted in American style or both are interpreted in European style. As a result, we obtain the pattern $p \models_a p$, $p \not\models_a q$, which is appropriate for the situation where no coherence between premisses and conclusion can be taken for granted.

Our discussion of how ambiguous logic can be used in practical applications has been sketchy so far. To show in more detail what role ambiguous logic can play, we will focus on Question-Answering systems in the next section. For simplicity, we will continue to abstract away from any coherence requirements.[18]

5.3 *Ambiguous Logic for Question-Answering*

Question-Answering, as we have seen in section 2, is the type of application that originally motivated the introduction of underspecified representations and it still is the application in which they are most used. It is therefore worthwhile to see how reasoning with underspecified expressions could affect the way in which such a system operates.

Let us assume that $\models_{\forall\forall}$ (that is, the system that quantifies universally over paraphrases of all the ambiguous expressions in a logical argument) has been implemented in a natural language interpreting system and draw some conclusions for the architecture of the system. The **old-fashioned** (i.e. pre-underspecification) architecture for a Question-Answering system is one in which a query is translated into a set of disambiguated expressions in a formal language. The system tries to discard all but one of these, selecting the remaining one as the chosen interpretation. The now **dominating** architecture replaces the set of disambiguated representations by one or more underspecified representations, trying to resolve all sources of ambiguity in it. As before, the remaining interpretation is selected if the disambiguation process is successful. If it is unsuccessful, nothing can be done. As an alternative to the traditional and the dominating architecture we propose the following, more flexible architecture, which makes use of **ambiguous logic**: A query is—as in the dominating approach—translated into an underspecified representation. Also as before, the system tries to disambiguate the representation but it stops as soon as the resulting representation is specific enough to warrant an answer. This approach allows the system to display behaviour that is helpful as well as, in an important sense, 'correct', even in situations in which disambiguation would be extremely difficult. The sense of correctness intended here derives from the Principle of idiosyncratic interpretation: No matter what legitimate interpretation the user selects as the preferred one, the answer provided by the system will be correct with respect to this interpretation.



An underspecified representation can be replaced by a more specific one through the use of any common disambiguation technique, including, for example, querying the user ('Did you mean . . . or . . . ?'). In the approaches discussed in this paper, an 'answer can be given' if the answer (e.g., the truth or falsity of a statement in the case of a yes/no question) follows logically from the information in the database, using the relation of logical consequence denoted by $\models_{\forall\forall}$.

To illustrate, let us return to our earlier example of an ambiguous query: 'How many American flights are operated by Quantas?' (section 2). As we have seen, this question can be translated into an underspecified formula of the form

$$\|\{x \in \mathit{flights}\colon\ \textsc{American}(x)\ \&\ \mathit{Operator}(x) = \mathit{Quantas}\}\|,$$

which denotes the cardinality of a set, the precise identity of which has yet to be established. Using the single stroke ('|') to list alternative paraphrases of an ambiguous constant, this formula may also be written as

$$\|\{x \in \mathit{flights}\colon\ \textsc{American}_1|\textsc{American}_2|\textsc{American}_5(x)\ \&\ \mathit{Operator}(x) = \mathit{Quantas}\}\|$$

since the remaining two interpretations, $\textsc{American}_3$ and $\textsc{American}_4$, do not correspond to contextually viable interpretations of AMERICAN. Let us assume that not all of the three viable interpretations lead to the same answer. This is extremely plausible, since $\{x \in \mathit{flights}\colon \textsc{American}_5(x)\ \&\ \mathit{Operator}(x) = \mathit{Quantas}\}$ must always be the empty set. No flight can be operated by Quantas and American Airlines at the same time, so under this interpretation, the query contains an inconsistency. Let us assume that the interpretation of the query resulting from the interpretation $\textsc{American}_5$ can be refuted on the basis of this inconsistency. (See e.g. Buvač 1996.) The result of this disambiguation step is a second, more specific representation.

$$\|\{x \in \mathit{flights}\colon\ \textsc{American}_1|\textsc{American}_2(x)\ \&\ \mathit{Operator}(x) = \mathit{Quantas}\}\|,$$

This representation happens to be specific enough to warrant an answer. Intuitively, this is because both remaining interpretations of the query lead to the same answer. This intuition is captured by the inference relation $\models_{\forall\forall}$, which allows us to prove that the cardinality of the set described by the underspecified formula does not depend on how AMERICAN is resolved. More specifically, the system may prove, for a certain number n, that $\|\{x \in \mathit{flights}\colon \textsc{American}(x)\ \&\ \mathit{Operator}(x) = \mathit{Quantas}\}\| = n$. The inference goes

$$1, 2 \models_{\forall\forall} 3,$$

where 1, 2, and 3 are as follows:

1. $\|\{x\epsilon \text{ flights: } \textsc{American}_1(x) \ \& \ \textit{Operator}(x) = \textit{Quantas}\}\| = n.$
2. $\|\{x\epsilon \text{ flights: } \textsc{American}_2(x) \ \& \ \textit{Operator}(x) = \textit{Quantas}\}\| = n.$
3. $\|\{x\epsilon \text{ flights: } \textsc{American}_1|\textsc{American}_2(x) \ \& \ \textit{Operator}(x) = \textit{Quantas}\}\| = n.$

The number n is the number that the system can safely return to the user to answer the ambiguous question.[19]

It may be good to compare this strategy with that of existing question-answering systems. Such systems are confronted with basically two options: to 'throw a dice' or to query the user for clarification. In the *first* case, an unintended interpretation may be selected and a misleading answer generated. (This will not happen if all interpretations lead to the same answer, but traditional systems do not check whether this is the case.) In the *second* case, where the system asks something like 'Do you mean flights arriving in the US or flights flying in from the US?', the incorrect implicature will be that it matters which of the two interpretations is selected. For example, if the user selects the second interpretation and the system then answers 'Five', the user will be invited to infer that this answer depends crucially on the result of the clarification dialogue and that, consequently, the number of flights arriving from the US is either unknown to the system or unequal to 5.

6 DISCUSSION

The growing number of recent proposals in the area of reasoning with underspecified formulas notwithstanding, underspecification is often still only used as a way to speed up the disambiguation process or to cut down on computer memory needed for disambiguation. (See Bos 1996 for a recent example.) In this paper, we have sketched some ways in which reasoning with underspecified representations can be put to use. Doing this, a distinction has been made between theoretical and practical applications. The conclusions of our 'practical' discussion may trigger a number of objections, a few of which we will briefly try to pre-empt.

1. '*The present account of ambiguous consequence does nothing to solve the combinatorial explosion problem.*' This is correct. All the notions of logical consequence discussed in this paper hinge on enumeration of interpretations (or paraphrases) of premisses and conclusion. But this is in accordance with the way in which the problem with interpreting ambiguous material was defined in the Introduction: the combinatorial explosion is only a secondary problem; the primary problem (i.e. the

problem of deadlock) is how to make sense of the notion of inference in an ambiguous domain. If and when this primary issue has been resolved, the question of finding equivalent, easily implementable, characterizations of the notion of logical consequence will come to the fore.

2. '*Reasoning with ambiguous expressions hinges on people's ability to perform supervaluational reasoning.*' Incorrect. The idea behind this objection is that, presumably, people are bad at the kind of reasoning in which different interpretations have to be kept apart. *First*, this incorrectly assumes that inspecting the different possible interpretations is the only way to establish that a conclusion follows from certain premisses using the relation $\models_{\forall\forall}$. That this is not necessarily the case may be made plausible most simply by looking at examples which do not hinge on ambiguity, such as the inference $p \models_{\forall\forall} p \vee q$, where only q contains ambiguous material. Even if q is ambiguous between many interpretations, the irrelevance of q is so evident that the reasoning is easy to perform. But *second*, our discussion of the practical applications in section 5 does not assume that the *user* of these systems can perform the reasoning involved. Quite possibly, people have other resolution strategies at their disposal than present-day computers. The only thing our discussion in section 5.3 did assume was an ability on the part of the *computer* to perform supervaluational reasoning. In addition, it was claimed that using this ability is the best way for the computer to similate human behaviour. (See especially the closing remarks of section 5.)

3. '*Isn't reasoning with underspecified expressions limited to polysemy?*' The example worked out in section 5.3 concerned the polysemy of the word *American*. It may be plausible that people leave the ambiguity of polysemous words unresolved if the situation does not require total disambiguation, but does this also apply to lexical homonymy and to other (e.g. non-lexical) kinds of ambiguity? This is a difficult question, especially in so far as it focuses on human interpretive behaviour. It has been argued that it is possible to make a linguistic distinction between types of ambiguity that do and types of ambiguity that do not require disambiguation (Pinkal 1995, Poesio 1996). If we assume that it is possible to make this distinction precise then it is still unclear that this constitutes a problem for the approach outlined in section 5.3. Consider a question by a user U involving a homonymous source of ambiguity, such as

U: *Is this a picture of the pitchers?*

and assume the system S is unable to determine whether *pitchers* refers to jugs or a baseball players. Suppose, furthermore, that both interpretations led to negative answers and that, following the recipe of section 5.3, the system answered in the negative. In terms of direct information content, this would be a perfectly correct answer. On the other hand, it may be argued that this answer licences the inference that the system was unaware of the ambiguity of the sentence. If this is considered problematic, however, the inference is easily blocked by the explicit acknowledgement that the system has found some 'importantly different' interpretations:

> **S:** *I'm not exactly sure what you mean, but the answer to your query must be negative either way.*

This illustrates an important point, namely that the strategy outlined in section 5.3 leaves it open in what way the information is passed on to the user: it determines under what circumstances an affirmative/negative answer may be given, but it does not say how this answer is presented. In systems that allow follow-up questions, it can happen that a follow-up question contains the same ambiguity or refers anaphorically to material in the original question in which case a clarification dialogue may be needed after all. Just as likely, however, the follow-up question will contain sufficient information to decide which of the two interpretations is the intended one. In that case, the fact that a clarification dialogue has been avoided has led to a shorter, less complex dialogue, at no informational 'cost' to the user whatsoever.

4. '$\models_{\forall\forall}$ *is a very weak logic.*' It has been noted that the logic arising from the relation $\models_{\forall\forall}$ is weaker than most other systems discussed. Although this does not refute the logic, it might be seen as reducing its value for practical applications. Let us observe three things, however.

Firstly, the logic based on the relation $\models_{\forall\forall}$ allows more nontrivial and practically useful conclusions than one may at first realize. The reason is that many different kinds of ambiguities tend to lead to interpretations that are logically ordered. This is true, for example, for polysemy. Consider a word like *write*, as in *Has John written any interesting papers recently?* The verb may mean 'being the sole author of' or 'being one of the authors of', which is subsumed by the first interpretation. The same is true for quantifier scope ambiguities. Take any sequence of n logical quantifiers:

$$(*)\quad Q_2 x_1 Q_2 x_2 \ldots Q_n x_n \varphi,$$

where Q_i is either $\forall$ or $\exists$. Then all $n!$ interpretations of $(*)$ are linearly ordered.

$\varphi_1 \models \varphi_2 \models \ldots \models \varphi_{n!}$.

For example, $\exists x \forall y Rxy \models \forall y \exists x Rxy$. Now suppose either the strongest of $\varphi_1, \ldots, \varphi_{n!}$ is true or the weakest of $\varphi_1, \ldots, \varphi_{n!}$ are false. In the first case, it follows that all of $\varphi_1, \ldots, \varphi_{n!}$ are true. In the second case, it follows that all of $\varphi_1, \ldots, \varphi_{n!}$ are false. Either way, all $n!$ interpretations give the same result and no logical 'strength' is lost by quantifying universally over them. More generally,

> If φ_i (where $1 \leq i \leq n!$) is true then all φ_j are true, for $i \leq j \leq n!$; and
> If φ_i (where $1 \leq i \leq n!$) is false then all φ_j are false, for $1 \leq j \leq i$.

Analogous remarks apply to the case of polysemy.[20] So, the relation $\models_{\forall\forall}$ may be weak but it is not too weak to be useful.

Secondly, we have seen that in some cases, it is possible to strengthen the relation of ambiguous consequence to one that is reflexive. This may be realistic, for example, in the case of short, well-written texts (Gale *et al.* 1992). For example, by requiring that different occurrences of the same ambiguous material are interpreted in the same way, one may define 'coherent' versions of $\models_{\forall\forall}$. For a discussion of the consequences of this move, which tends to upset other structural rules such as monotonicity, see van Deemter (1991, 1996).

Thirdly, it is important to realize that, as has been shown in section 5.3, there are situations in which simply no strengthening of the logical consequence relation is warranted. In applications where the two occurrences of F stem from different authors—as when one user fills the database and another asks a query—there simply is no guarantee that any kind of coherence between the two occurrences exists. This is the kind of situation that is set aside for future research in Reyle (1995), but that is crucial in many practical applications, as we have seen.[21] The situation is reminiscent of the so-called *Schoenmakers paradox* of distributed database theory. This paradox exploits the different ways in which the facts in standard databases can be interpreted. When two databases are combined, one of which contains the information p, while the other contains the information that $p \rightarrow q$, then the inference q is only warranted if all the terms in p are defined in the same way in the two databases. Thus one may easily construct a case in which q could be inferred, using Modus Ponens, while the negation of q is also represented. If problems like this lead to a breakdown of logical principles (*Modus Ponens*, *Reflexivity*, etc.), then this may be regrettable. But to *deny* the problem and propose a more convenient logic that fails to reflect the facts that the logic is meant to model would be a bad way of expressing one's regret.

5. *'Is reasoning with underspecified expressions meant to replace disambiguation as a strategy to come to terms with ambiguity?'* No, it is not. Ambiguities that can be resolved, by taking linguistic context, prosody (see e.g. Price *et al.* 1991), or even common sense knowledge into account, are probably resolved. Also, if the number of ambiguities in an utterance is limited and the ambiguity is easily explainable in nontechnical terms, then a clarification dialogue may effectively take care of the problem. Finally, if probabilistic information makes a number of remaining interpretations extremely unlikely and if the application permits occasional errors, then this may also be used to cut down on the number of interpretations. These methods will sometimes allow one to hypothesize the speaker's intention with some degree of confidence. But in other cases, the strategy outlined in this paper (i.e. the strategy of reasoning with ambiguous information) can prevent unnecessary deadlock.

At this point it may be useful to return to what was said in Section 1 about charitable interpretation. Suppose, once more, that $p_1, \ldots, p_n$ are the only grammatically possible paraphrases of S, while $cs \models \neg p_i$. Let φ be an expression that is ambiguous between paraphrases $p_1, \ldots, p_n$ and φ' one that is ambiguous between $p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n$. Then

$$\varphi \mathbin{\&} \neg p_i \models_{\forall\exists} \varphi'$$

is a valid inference that can be used to perform disambiguation. Thus, while one type of ambiguous logic can sometimes replace disambiguation, another can come to its aid.

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NOTES

[1] In particular, well-formedness implies that the 'grammatically possible' interpretations contain no type-conflicts. This gets rid of a considerable number

of logically devious quasi-interpretations without significant effort. See section 2 for an example.

2 This example is taken from the TENDUM flight information system (Bunt 1985), a follow-up to the PHLIQA system which focused on pragmatic aspects of interpretation.

3 Partial exceptions to this rule can be found in the literature (e.g. Fenstad *et al.* 1987, Westerstahl *et al.* 1993, Asher & Fernando 1997) but not in actual systems, it seems.

4 This, of course, is true for other types of logics as well. Witness, for example, the varieties of logical inference that are studied in linear logic (Troelstra 1992).

5 Note that this is very different from Intend (Speaker, MB (Speaker, Hearer, $\varphi \vee \psi$)), where the disjunction has moved from the object level to the meta level. See Poesio (1996) or van Deemter (1996b) for more elaborate comparison.

6 In the semantic approach of van Deemter (1991, 1996b), an ambiguous expression is associated with a model-theoretic device that assigns an unambiguous interpretation to each of the ambiguous constants in a formula. The semantic approach becomes superior to the syntactic approach when coherence phenomena are modeled, but these will not be studied in detail here. (See the end of section 5.2 for some sketchy remarks.)

7 Note that this leaves out some interesting possibilities, such as the case where Q is the quantifier *most*. For a mathematical characterization of the four logical quantifiers, cf. van Benthem 1986.

8 If the set A consists of $A_1, \ldots, A_n$, each of which corresponds to a way of disambiguating the conclusion, then this instantiation can also be written as $\varphi \models_a \psi \Leftrightarrow_{Def} (A_1 \vee \ldots \vee A_n \models B_1 \vee \ldots \vee B_m)$, where $\models$ denotes a version of unambiguous logical consequence.

9 In van Deemter (1996b), another contraint on $\models_a$ was discussed, namely Monotonicity. Monotonicity says, informally, that when the underlying (unambiguous) notion of logical consequence is relaxed (i.e. made extensionally larger) then the same must be true for the notion of ambiguous consequence based on it. If Q_1 and Q_2 are 'logical' quantifiers, as we have assumed, then Monotonicity is equivalent with Conservativity.

10 Boolean combinations of this kind were suggested by one of the anonymous reviewers of this paper.

11 See, for example, Mackie's contribution on 'Fallacies' in the Encyclopedia of Philosophy (J. L. Mackie 1967). Note that the criterion of *plausibility* gives the notion of 'fallacy' its psychological flavour. For an example involving a fallacy resting on lexical ambiguity, see van Deemter (1993).

12 A key assumption in these versions of the sorites argument is that if two objects are *indistinguishable* in terms of, for example, their size, then if one of the two is 'small', the other must also be 'small'. The analysis of the sorites paradox in van Deemter (1993) hinges on the idea that indistinguishability can either be understood as an absolute notion or as a contextual one, in which it matters whether the context provides sufficiently many other objects to tell the two objects apart. (Only) the first of the two is shown to lead to a valid sorites argument, while (only) the second leads to true premisses.

13 Reyle (1995) uses the following example to refute $\models_{\forall\exists}$: *Everybody slept or everybody didn't sleep.* Assuming the second disjunct to be scopally ambiguous, he observes that this sentence is intuitively contingent, while the relation $\models_{\forall\exists}$ would wrongly cause it to be classified as a tautology.

14 Nothing in this argument hinges on the relation between p and q. Consequently, the argument can be duplicated for

simples examples such as the one where $\varphi = Pitcher(j)$, assuming that $Pitcher(j)$ is ambiguous between $Baseball\text{-}player(j)$ and $Jug(j)$, which are incompatible in the sense that no admissible model can verify both formulas, provided some Meaning Postulates forbid that something is both a baseball player and a jug.

15 Nothing hinges on the particulars of this example, since the same point can be made using any ambiguous sentence.

16 A plausible strengthening of the PII would involve one and the same person (instead of different people) at different moments. Note that the principle makes use of the (semantic) notion of an interpretation, instead of the (syntactic) notion of a paraphrase. Reformulation in syntactic terms is, of course, possible.

17 It is unclear to me how the proposals in Reyle (1995, 1996) come to terms with such 'imperfections' in the notion of coherence. See Reyle (1996, p. 240) for a comparison between the treatments of coherence in van Deemter (1991) and Reyle (1995).

18 Usually, the *premisses* of a Question-Answering system (i.e., the facts on the basis of which the system proves or rejects a statement) stem from another source than the *conclusion* (i.e., the statement proved or rejected). Consequently, coherence between premisses and conclusion is not an issue in such systems. Coherence between expressions within the premisses, however, is an issue, and the same is true for coherence between expressions within the conclusion. Taking even this limited type of coherence into account would change the structural properties of the logic (especially monotonicity and transitivity, cf. van Deemter 1991, 1996b), but *not* the way the logic can be used in a Question-Answering system.

19 Note that it was assumed in section 2 that there is no separate interpretation of *American* which corresponds with the disjunction $\text{AMERICAN}_1(x) \vee \text{AMERICAN}_2(x)$. If such an interpretation were found to exists (which seems very unlikely) then this would destroy the unanimity between the different interpretations and consequently, a clarification dialogue would be needed before the query could be answered.

20 Other ambiguities, such as those arising from pronominal anaphora, cannot normally be ordered linearly according to logical strength, but in these cases it is often possible to find a simple weakening that is false or a simple strengthening that is true. For example, to answer the question *Has he co-authored a paper on long-distance dependencies?*, one may observe that a formula corresponding to a weakened question *Has someone co-authored a paper on long-distance dependencies* is false. Hence, the answer must be negative no matter how the pronoun is resolved.

21 Reyle (1995) focuses on the very different situation, the goal of which is to draw (possibly ambiguous) conclusions from someone's (possibly ambiguous) mental state, to make his or her beliefs explicit. In such situations, it would make sense to assume certain principles of coherence, similar to the ones discussed above. On the other hand, why would one be interested in drawing *ambiguous* inferences about someone's mental state? Like in discourse interpretation (cf. section 5.2), it often seems more useful to draw *unambiguous* conclusions from a mental state. As far as I can see, ambiguous conclusions only deserve a place in applications in which these conclusions themselves are somehow 'given', for example when a user has expressed an ambiguous query which the system then seeks to prove or disprove.

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