Extracting and Composing Robust Features with Denoising Autoencoders

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The problem

- Building good predictors on complex domains means learning complicated functions.
- These are best represented by multiple levels of non-linear operations i.e. deep architectures.
- Deep architectures are an old idea: multi-layer perceptrons.
- Learning the parameters of deep architectures proved to be challenging!

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 - \rightarrow disappointing performance. Stuck in poor solutions.
- Solution 2: Deep Belief Nets (Hinton, Osindero and Teh, 2006): initialize by stacking Restricted Boltzmann Machines, fine-tune with Up-Down.
 - → impressive performance.

Key seems to be good unsupervised layer-by-layer initialization...

- Solution 3: initialize by stacking autoencoders, fine-tune with gradient descent. (Bengio et al., 2007; Ranzato et al., 2007)
 → Simple generic procedure, no sampling required.
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- .but not quite. Can we do better?

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Open question: what would make a good unsupervised criterion for finding good initial intermediate representations?

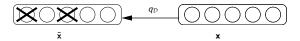
- Inspiration: our ability to "fill-in-the-blanks" in sensory input.
 missing pixels, small occlusions, image from sound, ...
- Good fill-in-the-blanks performance ← distribution is well captured.
- → old notion of associative memory (motivated Hopfield models (Hopfield, 1982))

What we propose:

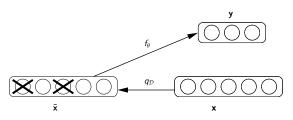
unsupervised initialization by explicit fill-in-the-blanks training.



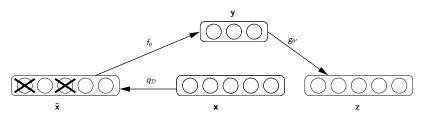
- Clean input $\mathbf{x} \in [0,1]^d$ is partially destroyed, yielding corrupted input: $\tilde{\mathbf{x}} \sim q_D(\tilde{\mathbf{x}}|\mathbf{x})$.
- $\tilde{\mathbf{x}}$ is mapped to hidden representation $\mathbf{y} = f_{\theta}(\tilde{\mathbf{x}})$.
- From **y** we reconstruct a $\mathbf{z} = g_{\theta'}(\mathbf{y})$
- Train parameters to minimize the cross-entropy "reconstruction error" $L_H(x,z) = H(\mathcal{B}_x || \mathcal{B}_z)$, where \mathcal{B}_x denotes multivariate Bernoulli distribution with parameter x.



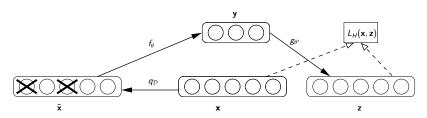
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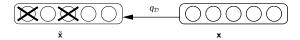


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The input corruption process $q_{\mathcal{D}}(\tilde{\mathbf{x}}|\mathbf{x})$



- Choose a fixed proportion ν of components of **x** at random.
- Reset their values to 0.
- Can be viewed as replacing a component considered missing by a default value.

Other corruption processes are possible.

Form of parameterized mappings

We use standard sigmoid network layers:

•
$$\mathbf{y} = f_{\theta}(\tilde{\mathbf{x}}) = \operatorname{sigmoid}(\underbrace{\mathbf{W}}_{d' \times d} \tilde{\mathbf{x}} + \underbrace{\mathbf{b}}_{d' \times 1})$$

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$$g_{\theta'}(\mathbf{y}) = \operatorname{sigmoid}(\underbrace{\mathbf{W}'}_{d \times d'} \mathbf{y} + \underbrace{\mathbf{b}'}_{d \times 1}).$$

Denoising using classical autoencoders was actually introduced much earlier (LeCun, 1987; Gallinari et al., 1987), as an alternative to Hopfield networks (Hopfield, 1982).

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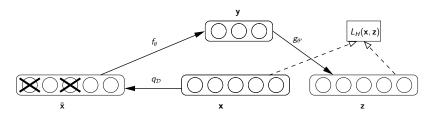
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Learning deep networks Layer-wise initialization



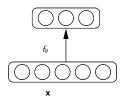
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- ② Remove scaffolding. Use f_{θ} directly on input yielding higher level representation.
- **①** Learn next level mapping $f_{\theta}^{(2)}$ by training denoising autoencoder on current level representation.
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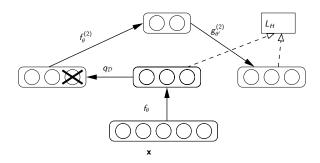
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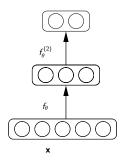
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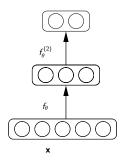
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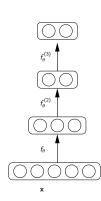


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Learning deep networks Supervised fine-tuning

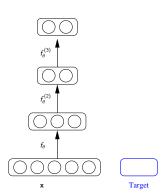
 Initial deep mapping was learnt in an unsupervised way.

- initialization for a supervised task.
- Output layer gets added
- Global fine tuning by gradient descent on supervised criterion.



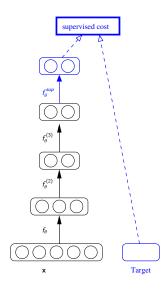
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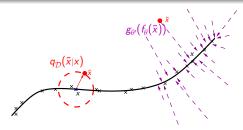


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Perspectives on denoising autoencoders Manifold learning perspective



Denoising autoencoder can be seen as a way to learn a manifold:

- Suppose training data (x) concentrate near a low-dimensional manifold.
- Corrupted examples (•) are obtained by applying corruption process $q_{\mathcal{D}}(\widetilde{X}|X)$ and will lie farther from the manifold.
- The model learns with $p(X|\widetilde{X})$ to "project them back" onto the manifold.
- Intermediate representation Y can be interpreted as a coordinate system for points on the manifold.

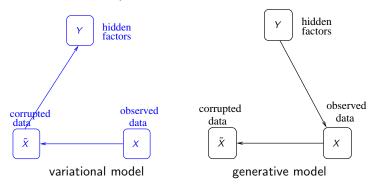
Perspectives on denoising autoencoders

Information theoretic perspective

- Consider $X \sim q(X)$, q unknown. $\widetilde{X} \sim q_{\mathcal{D}}(\widetilde{X}|X)$. $Y = f_{\theta}(\widetilde{X})$.
- It can be shown that minimizing the expected reconstruction error amounts to maximizing a lower bound on mutual information I(X; Y).
- Denoising autoencoder training can thus be justified by the objective that hidden representation Y captures as much information as possible about X even as Y is a function of corrupted input.

Perspectives on denoising autoencoders Generative model perspective

 Denoising autoencoder training can be shown to be equivalent to maximizing a variational bound on the likelihood of a generative model for the corrupted data.



Benchmark problems Variations on MNIST digit classification

basic: subset of original MNIST digits: 10 000 training samples, 2 000 validation samples, 50 000 test samples.



rot: applied random rotation (angle between 0 and 2π radians)



bg-img: background is random patch from one of 20 images









bg-rand: background made of random pixels (value in 0...255)





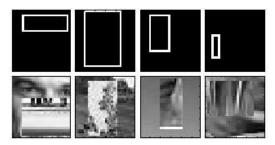




rot-bg-img: combination of rotation and background image

Benchmark problems Shape discrimination

• rect: discriminate between tall and wide rectangles on black background.



- rect-img: borderless rectangle filled with random image patch. Background is a different image patch.
- convex: discriminate between convex and non-convex shapes.



Experiments

We compared the following algorithms on the benchmark problems:

- **SVM**_{rbf}: suport Vector Machines with Gaussian Kernel.
- DBN-3: Deep Belief Nets with 3 hidden layers (stacked Restricted Boltzmann Machines trained with contrastive divergence).
- SAA-3: Stacked Autoassociators with 3 hidden layers (no denoising).
- SdA-3: Stacked Denoising Autoassociators with 3 hidden layers.

Hyper-parameters for all algorithms were tuned based on classification performance on validation set. (In particular hidden-layer sizes, and ν for SdA-3).

Dataset	SVM _{rbf}				
	3.03±0.15				
rot	11.11±0.28	10.30±0.27	10.30±0.27	10.29±0.27 (10%)	11.62 (10%)
bg-rand	14.58±0.31	6.73±0.22	11.28±0.28	10.38±0.27 (40%)	15.63 (25%)
bg-img	22.61±0.37	16.31±0.32	23.00±0.37	16.68±0.33 (25%)	23.15 (25%)
rot-bg-img	55.18±0.44	47.39±0.44	51.93±0.44	44.49 _{±0.44} (25%)	54.16 (10%)
rect	2.15±0.13	2.60±0.14	2.41±0.13	$1.99_{\pm0.12} \; (10\%)$	2.45 (25%)
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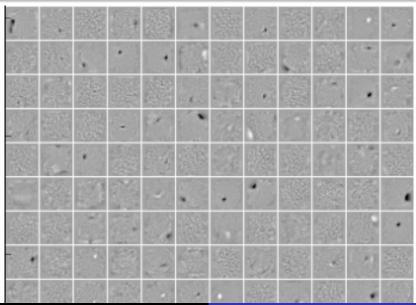
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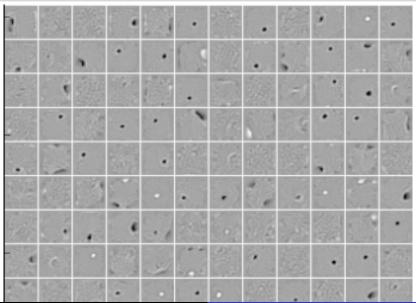
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Learnt filters

0 % destroyed

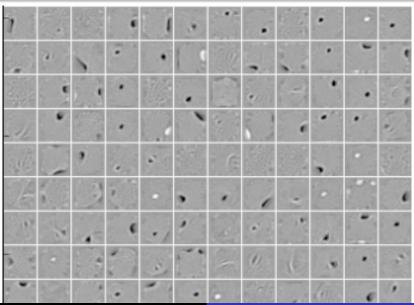


Learnt filters 10 % destroyed



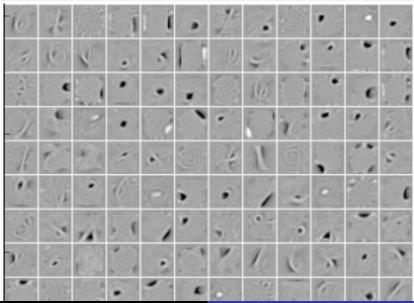
Learnt filters

25 % destroyed



Learnt filters

50 % destroyed



Conclusion and future work

- Unsupervised initialization of layers with an explicit denoising criterion appears to help capture interesting structure in the input distribution.
- This leads to intermediate representations much better suited for subsequent learning tasks such as supervised classification.
- Resulting algorithm for learning deep networks is simple and improves on state-of-the-art on benchmark problems.
- Although our experimental focus was supervised classification, SdA
 is directly usable in a semi-supervised setting.
- Future work will investigate the effect of different types of corruption process.

THANK YOU!

Dataset	SVM _{rbf}	SVM _{poly}	DBN-1	DBN-3	SAA-3	SdA-3 (u)
basic	3.03±0.15	3.69±0.17	3.94±0.17	3.11±0.15	3.46±0.16	2.80±0.14 (10%)
rot	11.11±0.28	15.42±0.32	14.69±0.31	10.30±0.27	10.30±0.27	10.29±0.27 (10%)
bg-rand	14.58±0.31	16.62±0.33	9.80±0.26	6.73±0.22	11.28±0.28	10.38±0.27 (40%)
bg-img	22.61±0.37	24.01±0.37	16.15±0.32	16.31±0.32	23.00±0.37	16.68±0.33 (25%)
rot-bg-img	55.18±0.44	56.41±0.43	52.21±0.44	47.39±0.44	51.93±0.44	44.49±0.44 (25%)
rect	2.15±0.13	2.15±0.13	4.71±0.19	2.60±0.14	2.41±0.13	1.99±0.12 (10%)
rect-img	24.04±0.37	24.05±0.37	23.69±0.37	22.50±0.37	24.05±0.37	21.59±0.36 (25%)
convex	19.13±0.34	19.82±0.35	19.92±0.35	18.63±0.34	18.41±0.34	19.06±0.34 (10%)

red when confidence intervals overlap.

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