

---

# Momentum Library

Minified

Competitive Programming Library  
of

<https://github.com/OmarBazaraa/Competitive-Programming>

---

September 13<sup>th</sup>, 2019

# Contents

Data Structures.....	1
Sparse Table.....	1
Monotonic Queue .....	1
Disjoint-Set Union (DSU) .....	2
Fenwick Tree (Binary Indexed Tree).....	3
Segment Tree as Multiset.....	4
Strings .....	8
KMP .....	8
Z-Algorithm.....	8
Trie .....	9
Suffix Array .....	10
Graphs.....	12
Shortest Path (Floyd Warshal's Algorithm) .....	12
Shortest Path (Bellman Ford's Algorithm) .....	12
Shortest Path (Dijkstra's Algorithm).....	13
Minimum Spanning Tree (Kruskal's Algorithm) .....	14
SCC (Kosaraju's Algorithm).....	15
Topological Sort (Khan's Algorithm).....	15
Tree Diameter.....	16
Bipartite Graph Check.....	17
Bridge Tree .....	17
LCA (Euler Walk + RMQ).....	18
LCA (Parent Sparse Table) .....	20
Max Flow (Edmonds Karp's Algorithm) .....	21
Math.....	24

GCD .....	24
LCM.....	24
Extended Euclid .....	24
Fast Power.....	24
Modular Inverse.....	25
Combinations ( $nCr$ ) .....	25
Pascal's Triangle .....	25
Prime Check .....	25
Prime Check (Miller Rabin's Algorithm).....	26
Generate Primes .....	27
Generate Prime Divisors.....	27
Generate Divisors.....	27
Matrix Power.....	28
Others.....	30
Longest Increasing Sub-sequence.....	30

# Data Structures

## Sparse Table

```
// The array to compute its sparse table and its size.
int n, a[N];

// Sparse table related variables. Don't access them directly.
int ST[LOG_N][N], LOG[N];

// Builds the sparse table for computing min/max/gcd/lcm/...etc
// for any contiguous sub-segment of the array in O(n.log(n)).
//
// This is an example of computing the index of the minimum value.
void buildST() {
    LOG[0] = -1;

    for (int i = 0; i < n; ++i) {
        ST[0][i] = i;
        LOG[i + 1] = LOG[i] + !(i & (i + 1));
    }

    for (int j = 1; (1 << j) <= n; ++j) {
        for (int i = 0; (i + (1 << j)) <= n; ++i) {
            int x = ST[j - 1][i];
            int y = ST[j - 1][i + (1 << (j - 1))];

            ST[j][i] = (a[x] <= a[y] ? x : y);
        }
    }
}

// Queries the sparse table for the computed value of the interval [l, r] in O(1).
int query(int l, int r) {
    int g = LOG[r - l + 1];
    int x = ST[g][l];
    int y = ST[g][r - (1 << g) + 1];
    return (a[x] <= a[y] ? x : y);
}
```

## Monotonic Queue

```
/**
 * Monotonic queue to keep track of the minimum and the maximum
 * elements so far in the queue in amortized time of O(1).
 */
template<class T>
class monotonic_queue {
    queue<T> qu;
    deque<T> mx, mn;

public:
```

```

void push(T v) {
    qu.push(v);
    while (mx.size() && mx.back() < v) mx.pop_back();
    mx.push_back(v);
    while (mn.size() && mn.back() > v) mn.pop_back();
    mn.push_back(v);
}

void pop() {
    if (mx.front() == qu.front()) mx.pop_front();
    if (mn.front() == qu.front()) mn.pop_front();
    qu.pop();
}

T front() const {
    return qu.front();
}

T max() const {
    return mx.front();
}

T min() const {
    return mn.front();
}

size_t size() const {
    return qu.size();
}
};

```

## Disjoint-Set Union (DSU)

```

/**
 * Disjoint-set data structure to tracks a set of elements partitioned
 * into a number of disjoint subsets.
 */
class DSU {
    int setsCount;
    vector<int> siz;
    mutable vector<int> par;

public:
    DSU(int n) {
        setsCount = n;
        siz.resize(n, 1);
        par.resize(n);
        iota(par.begin(), par.end(), 0);
    }

    int findSetId(int u) const {
        return u == par[u] ? u : par[u] = findSetId(par[u]);
    }

    bool areInSameSet(int u, int v) const {
        return findSetId(u) == findSetId(v);
    }
}

```

```

bool unionSets(int u, int v) {
    u = findSetId(u);
    v = findSetId(v);

    if (u == v) {
        return false;
    }

    setsCount--;
    siz[v] += siz[u];
    par[u] = v;
    return true;
}

int getSetSize(int u) const {
    return siz[findSetId(u)];
}

int getSetsCount() const {
    return setsCount;
}
};

```

## Fenwick Tree (Binary Indexed Tree)

```

/**
 * Regular Fenwick tree class to compute and update prefix sum in  $O(\log(N))$ .
 *
 * Note that the tree is 1-indexed.
 */
template<class T>
class fenwick_tree {
    T BIT[N];

public:
    fenwick_tree() {
        memset(BIT, 0, sizeof(BIT));
    }

    void update(int idx, T val) {
        while (idx < N) {
            BIT[idx] += val;
            idx += idx & -idx;
        }
    }

    T operator[](int idx) {
        T res = 0;
        while (idx > 0) {
            res += BIT[idx];
            idx -= idx & -idx;
        }
        return res;
    }
};

```

```

/**
 * Fenwick tree class to compute and update range sum in  $O(\log(N))$ .
 *
 * Note that the tree is 1-indexed.
 */
template<class T>
class range_fenwick_tree {
    fenwick_tree<T> M, C;

public:
    void update(int l, int r, T val) {
        M.update(l, val);
        M.update(r + 1, -val);
        C.update(l, -val * (l - 1));
        C.update(r + 1, val * r);
    }

    T operator[](int idx) {
        return idx * M[idx] + C[idx];
    }
};

```

## Segment Tree as Multiset

```

/**
 * Segment tree node struct.
 */
struct node {
    int size;
    node* childL, * childR;

    node() {
        size = 0;
        childL = childR = this;
    }

    node(int s, node* l, node* r) {
        size = s;
        childL = l, childR = r;
    }
};

/**
 * Multiset that store integers in the range of  $[-N, N]$ .
 * The multiset is implemented using segment tree.
 *
 * Note that the multiset is 0-indexed.
 * The most complex function in this class is done in time complexity of  $O(\log(N))$ .
 */
class segment_multiset {
    const int N;
    node* nil, * root;

public:
    segment_multiset(int N) : N(N) {
        root = nil = new node();
    }

```

```

~segment_multiset() {
    destroy(root);
    delete nil;
}

void clear() {
    destroy(root);
    root = nil;
}

int size() {
    return root->size;
}

void insert(int val, int cnt = 1) {
    insert(root, val, cnt, -N, N);
}

int erase(int val, int cnt = 1) {
    return erase(root, val, cnt, -N, N);
}

int count(int val) {
    node* cur = root;
    int l = -N, r = N;

    while (l < r) {
        int mid = l + (r - l) / 2;

        if (val <= mid) {
            cur = cur->childL;
            r = mid;
        } else {
            cur = cur->childR;
            l = mid + 1;
        }
    }

    return cur->size;
}

int operator[](int idx) {
    node* cur = root;
    int l = -N, r = N;

    while (l < r) {
        int mid = l + (r - l) / 2;

        if (idx < cur->childL->size) {
            cur = cur->childL;
            r = mid;
        } else {
            idx -= cur->childL->size;
            cur = cur->childR;
            l = mid + 1;
        }
    }
}

```



```

    return r;
}

int lower_bound(int val) {
    node* cur = root;
    int l = -N, r = N, ret = 0;

    while (l < val) {
        int mid = l + (r - l) / 2;

        if (val <= mid) {
            cur = cur->childL;
            r = mid;
        } else {
            ret += cur->childL->size;
            cur = cur->childR;
            l = mid + 1;
        }
    }

    return ret;
}

int upper_bound(int val) {
    return lower_bound(val + 1);
}

private:

void insert(node*& root, int val, int cnt, int l, int r) {
    if (val < l || val > r) {
        return;
    }

    if (root == nil) {
        root = new node(0, nil, nil);
    }

    root->size += cnt;

    if (l == r) {
        return;
    }

    int mid = l + (r - l) / 2;

    insert(root->childL, val, cnt, l, mid);
    insert(root->childR, val, cnt, mid + 1, r);
}

int erase(node*& root, int val, int cnt, int l, int r) {
    if (val < l || val > r) {
        return 0;
    }

    if (root == nil) {
        return 0;
    }
}

```

```

    if (l == r) {
        return remove(root, cnt);
    }

    int mid = l + (r - l) / 2;

    int ret = 0;

    ret += erase(root->childL, val, cnt, l, mid);
    ret += erase(root->childR, val, cnt, mid + 1, r);

    return remove(root, ret);
}

int remove(node*& root, int cnt) {
    int ret = min(cnt, root->size);

    root->size -= cnt;

    if (root->size <= 0) {
        destroy(root);
        root = nil;
    }

    return ret;
}

void destroy(node* root) {
    if (root == nil) return;
    destroy(root->childL);
    destroy(root->childR);
    delete root;
}
};

```

# Strings

## KMP

```
// KMP Longest match array.
int F[N];

// KMP failure function.
int failure(const char* pat, char cur, int len) {
    while (len > 0 && cur != pat[len]) {
        len = F[len - 1];
    }
    return len + (cur == pat[len]);
}

// Computes the length of the longest suffix ending at the i-th character
// that match a prefix of the string, and fills the results in the global "F" array.
void KMP(const char* pat) {
    for (int i = 1; pat[i]; ++i) {
        F[i] = failure(pat, pat[i], F[i - 1]);
    }
}
```

## Z-Algorithm

```
// Z-Algorithm Longest match array.
int Z[N];

// Computes the length of the longest prefix starting at the i-th character
// that match a prefix of the string, and fills the results in the global "Z" array.
void z_function(const char* str) {
    for (int i = 1, l = 0, r = 0; str[i]; ++i) {
        if (i <= r)
            Z[i] = min(r - i + 1, Z[i - l]);

        while (str[i + Z[i]] && str[Z[i]] == str[i + Z[i]])
            Z[i]++;

        if (i + Z[i] - 1 > r)
            l = i, r = i + Z[i] - 1;
    }
}
```

# Trie

```
// The total length of all the string, and the size of the alphabet.
const int N = 100100, ALPA_SIZE = 255;

int trie[N][ALPA_SIZE];    // The trie.
int nodesCount;            // The number of nodes in the trie.
int distinctWordsCount;    // The number of distinct word in the trie.
int wordsCount[N];         // Number of words sharing node "i".
int wordsEndCount[N];      // Number of words ending at node "i".

// Initializes the trie. This must be called before each test case.
void init() {
    nodesCount = 0;
    memset(trie, -1, sizeof(trie));
}

// Outs a new edge with character "c" from the given node if not exists .
int addEdge(int id, char c) {
    int& nxt = trie[id][c];
    if (nxt == -1) {
        nxt = ++nodesCount;
    }
    return nxt;
}

// Inserts a new word in the trie.
void insert(const char* str) {
    int cur = 0;

    for (int i = 0; str[i]; ++i) {
        wordsCount[cur]++;
        cur = addEdge(cur, str[i]);
    }

    wordsCount[cur]++;
    distinctWordsCount += (++wordsEndCount[cur] == 1);
}

// Removes a word from the trie assuming that it was added before.
void erase(const char* str) {
    int cur = 0;

    for (int i = 0; str[i]; ++i) {
        wordsCount[cur]--;

        int nxt = trie[cur][str[i]];

        if (wordsCount[nxt] == 1) {
            trie[cur][str[i]] = -1;
        }

        cur = nxt;
    }

    wordsCount[cur]--;
    distinctWordsCount -= (--wordsEndCount[cur] == 0);
}
```

```

// Searches for a word in the trie and returns its number of occurrences.
int search(const char* str) {
    int cur = 0;

    for (int i = 0; str[i]; ++i) {
        int nxt = trie[cur][str[i]];

        if (nxt == -1) {
            return 0;
        }

        cur = nxt;
    }

    return wordsEndCount[cur];
}

```

## Suffix Array

```

// n          : the length of the string not the number of suffixes.
// str        : the string itself.
//             Note that "str[n+1]" must be smaller than any value of "str"
// SA         : the suffix array, holding all the suffixes in lexicographical order.
// suffixRank : array holding the order of the i-th suffix after sorting.
// LCP        : array holding the length of the longest common prefix between "SA[i]"
//             and "SA[i - 1]".
int n, SA[N], suffixRank[N], LCP[N];
char str[N];

// Temporary arrays needed while computing the suffix array.
int sortedSA[N], sortedRanks[N], rankStart[N];

// Comparator struct to be used internally from "buildSuffixArray" function.
struct comparator {
    int h;

    comparator(int h) : h(h) {}

    bool operator()(int i, int j) const {
        if (suffixRank[i] != suffixRank[j]) {
            return suffixRank[i] < suffixRank[j];
        }
        return suffixRank[i + h] < suffixRank[j + h];
    }
};

// To be called internally from "buildSuffixArray" function.
void computeSuffixRanks(int h) {
    comparator comp(h);

    for (int i = 1; i <= n; ++i) {
        int& r = sortedRanks[i] = sortedRanks[i - 1];

        if (comp(sortedSA[i - 1], sortedSA[i])) {
            rankStart[++r] = i;
        }
    }
}

```

```

    for (int i = 0; i <= n; ++i) {
        SA[i] = sortedSA[i];
        suffixRank[SA[i]] = sortedRanks[i];
    }
}

// Builds the suffix array of the given string in time complexity of  $O(n \log(n))$ .
void buildSuffixArray() {
    for (int i = 0; i <= n; ++i) {
        sortedSA[i] = i;
        suffixRank[i] = str[i];
    }

    sort(sortedSA, sortedSA + n + 1, comparator(0));
    computeSuffixRanks(0);

    for (int h = 1; sortedRanks[n] != n; h <= 1) {
        for (int i = 0; i <= n; ++i) {
            int k = SA[i] - h;

            if (k >= 0) {
                sortedSA[rankStart[suffixRank[k]]++] = k;
            }
        }

        computeSuffixRanks(h);
    }
}

// Computes the Longest common prefix (LCP) for every two consecutive suffixes in the
// suffix array in time complexity of  $O(n)$ .
void buildLCP() {
    int cnt = 0;
    for (int i = 0; i < n; ++i) {
        int j = SA[suffixRank[i] - 1];
        while (str[i + cnt] == str[j + cnt]) ++cnt;
        LCP[suffixRank[i]] = cnt;
        if (cnt > 0) --cnt;
    }
}

```

# Graphs

## Shortest Path (Floyd Warshal's Algorithm)

```
int n; // The number of nodes.
int adj[N][N]; // The graph adjacency matrix.
int par[N][N]; // par[u][v] : holds the parent node of "v" in the shortest
               // path from "u" to "v".

// Initializes the graph. Must be called before each test case.
void init() {
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            adj[i][j] = (i == j ? 0 : oo), par[i][j] = i;
}

// Computes all-pair shortest paths using Floyd Warshall's algorithm in  $O(n^3)$ .
void floyd() {
    for (int k = 0; k < n; ++k)
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                if (adj[i][j] > adj[i][k] + adj[k][j])
                    adj[i][j] = adj[i][k] + adj[k][j], par[i][j] = par[k][j];
}

// Checks whether the graph has negative cycles or not.
bool checkNegativeCycle() {
    bool ret = false;
    for (int i = 0; i < n; ++i) {
        ret = ret || (adj[i][i] < 0);
    }
    return ret;
}

// Prints the shortest path from node "u" to node "v".
void printPath(int u, int v) {
    if (u != v) {
        printPath(u, par[u][v]);
    }
    printf("%d ", v + 1);
}
```

## Shortest Path (Bellman Ford's Algorithm)

```
int n; // The number of nodes.
int dis[N]; // dis[v] : holds the shortest distance between
            // the source and node "v".
vector<pair<int, int>> edges[N]; // The graph adjacency list.

// Computes single-source shortest paths using Bellman Ford's algorithm in  $O(n^2)$ .
// And returns whether the graph contains negative cycles or not.
bool bellmanFord(int src) {
    memset(dis, 0x3F, sizeof(dis));
```

```

dis[src] = 0;

bool updated = 1;

for (int k = 0; k < n && updated; ++k) {
    updated = 0;

    for (int u = 1; u <= n; ++u) {
        for (auto& e : edges[u]) {
            int v = e.first;
            int w = e.second;

            if (dis[v] > dis[u] + w) {
                dis[v] = dis[u] + w;
                updated = 1;
            }
        }
    }
}

return updated;
}

```

## Shortest Path (Dijkstra's Algorithm)

```

/**
 * Edge structs to holds the needed information about an edge.
 */
struct edge {
    int to, weight;

    edge() {}
    edge(int t, int w) : to(t), weight(w) {}

    bool operator<(const edge& rhs) const {
        return weight > rhs.weight;
    }
};

int n; // The number of nodes.
int dis[N]; // dis[v] : holds the shortest distance between the source
// and node "v".
vector<edge> edges[N]; // The graph adjacency list.

// Computes single-source shortest paths using Dijkstra's algorithm in O(n.Log(n)).
void dijkstra(int src) {
    priority_queue<edge> q;
    q.push(edge(src, 0));

    memset(dis, 0x3F, sizeof(dis));

    while (!q.empty()) {
        int u = q.top().to;
        int w = q.top().weight;
        q.pop();

        if (dis[u] <= w) {

```



```

        continue;
    }

    dis[u] = w;

    for (edge& e : edges[u]) {
        if (w + e.weight < dis[e.to]) {
            q.push(edge(e.to, w + e.weight));
        }
    }
}
}

```

## Minimum Spanning Tree (Kruskal's Algorithm)

```

/**
 * Edge structs to holds the needed information about an edge.
 */
struct edge {
    int from, to, weight;

    edge() {}
    edge(int f, int t, int w) : from(f), to(t), weight(w) {}

    bool operator<(const edge& rhs) const {
        return (weight < rhs.weight);
    }
};

int n; // The number of nodes.
int par[N]; // The DSU parent array.
vector<edge> edges; // The edges of the graph.

// Finds and returns the set id of an element using the DSU data structure.
int findSetId(int u) {
    return u == par[u] ? u : par[u] = findSetId(par[u]);
}

// Computes and returns the minimum spanning tree of a weighted graph.
int kruskalMST() {
    int MST = 0;

    sort(edges.begin(), edges.end());

    for (int i = 1; i <= n; ++i) {
        par[i] = i;
    }

    for (auto& e : edges) {
        int x = findSetId(e.from);
        int y = findSetId(e.to);

        if (x != y) {
            par[x] = y;
            MST += e.weight;
        }
    }
}

```

```

    return MST;
}

```

## SCC (Kosaraju's Algorithm)

```

int n; // Number of nodes.
bool vis[N]; // Nodes visited array.
vector<int> edges[N]; // Graph adjacency list.
vector<int> edgesT[N]; // Transposed graph adjacency list (i.e. reversed edges).
vector<vector<int>> scc; // Strongly connected components.

// Sorts the nodes in a topological order.
void topoSortDFS(int u, vector<int>* edges, vector<int>& nodes) {
    vis[u] = 1;

    for (int v : edges[u]) {
        if (!vis[v]) {
            topoSortDFS(v, edges, nodes);
        }
    }

    nodes.push_back(u);
}

// Extracts the strongly connected components (SCC) of the given directed graph
// using Kosaraju's algorithm, and fills them in the global "scc" vector".
void kosaraju() {
    vector<int> sortedNodes;

    memset(vis, 0, sizeof(vis));

    for (int i = 1; i <= n; ++i) {
        if (!vis[i]) {
            topoSortDFS(i, edges, sortedNodes);
        }
    }

    memset(vis, 0, sizeof(vis));

    for (int i = sortedNodes.size() - 1; i >= 0; --i) {
        int u = sortedNodes[i];

        if (!vis[u]) {
            vector<int> tmp;
            topoSortDFS(u, edgesT, tmp);
            scc.push_back(tmp);
        }
    }
}

```

## Topological Sort (Khan's Algorithm)

```

int n; // Number of nodes.
bool vis[N]; // Nodes visited array.
vector<int> edges[N]; // Graph adjacency list.
vector<int> sortedNodes; // List of topologically sorted nodes.

```

```

// Sorts the graph in a topological order using Khan BFS algorithm,
// and fills the result in the global "sortedNodes" vector
void topoSortBFS() {
    queue<int> q;
    vector<int> inDeg(n + 1, 0);

    for (int i = 1; i <= n; ++i) {
        for (int v : edges[i]) {
            ++inDeg[v];
        }
    }

    for (int i = 1; i <= n; ++i) {
        if (inDeg[i] == 0) {
            q.push(i);
        }
    }

    while (!q.empty()) {
        int u = q.front();
        q.pop();

        sortedNodes.push_back(u);

        for (int v : edges[u]) {
            if (--inDeg[v] == 0) {
                q.push(v);
            }
        }
    }
}

```

## Tree Diameter

```

int dis[N]; // dis[v] : holds the shortest distance between the source
// and node "v".
vector<int> edges[N]; // The graph adjacency list.

// Returns the farthest node from the source node.
int bfs(int u) {
    queue<int> q;
    q.push(u);

    memset(dis, -1, sizeof(dis));
    dis[u] = 0;

    while (!q.empty()) {
        u = q.front();
        q.pop();

        for (auto v : edges[u]) {
            if (dis[v] == -1) {
                dis[v] = dis[u] + 1;
                q.push(v);
            }
        }
    }
}

```

```

    return u;
}

// Computes and returns the length of the diameter of the tree.
int calcTreeDiameter(int root) {
    int u = bfs(root);
    int v = bfs(u);
    return dis[v];
}

```

## Bipartite Graph Check

```

int color[N];           // The set each node belongs to.
vector<int> edges[N];    // The graph adjacency list.

// Do not call this directly.
bool dfs(int u = 1) {
    for (int v : edges[u]) {
        if (color[v] == color[u]) {
            return false;
        }

        if (color[v] == -1) {
            color[v] = color[u] ^ 1;

            if (!dfs(v)) {
                return false;
            }
        }
    }

    return true;
}

// Checks whether the given graph is bipartite or not.
bool isBipartiteGraph() {
    memset(color, -1, sizeof(color));
    color[1] = 0;
    return dfs();
}

```

## Bridge Tree

```

int n;                  // The number of nodes.
vector<int> edges[N];    // The graph adjacency list.

//
// Bridge tree related variables
//
int T
int root
int par[N];
int tin[N];
int low[N];
vector<int> tree[N];
vector<pair<int, int>> bridges;

```

```

// Do not call this directly.
int findSetId(int u) {
    return (par[u] == u ? u : par[u] = findSetId(par[u]));
}

// Do not call this directly.
void findBridges(int u = 1, int p = -1) {
    tin[u] = low[u] = ++T;

    for (int v : edges[u]) {
        if (v == p) {
            continue;
        }

        if (tin[v] == 0) {
            findBridges(v, u);

            if (low[v] > tin[u]) {
                bridges.push_back({u, v});
            } else {
                par[findSetId(u)] = findSetId(v);
            }
        }

        low[u] = min(low[u], low[v]);
    }
}

// Builds the bridge tree of a graph in O(n+m).
void buildBridgeTree() {
    for (int i = 1; i <= n; ++i) {
        par[i] = i;
    }

    findBridges();

    for (auto& b : bridges) {
        int u = findSetId(b.first);
        int v = findSetId(b.second);

        tree[u].push_back(v);
        tree[v].push_back(u);

        root = u;
    }
}

```

## LCA (Euler Walk + RMQ)

```

int n; // The number of nodes.
vector<int> edges[N]; // The graph adjacency list.

//
// LCA related variables.
//
int dep[N];
int ST[LOG_N][N << 1];
int LOG[N << 1];

```

```

int F[N];
vector<int> E;

// Do not call this directly.
void dfs(int u = 1, int p = -1, int d = 0) {
    dep[u] = d;
    F[u] = E.size();
    E.push_back(u);

    for (int v : edges[u]) {
        if (v != p) {
            dfs(v, u, d + 1);
            E.push_back(u);
        }
    }
}

// Do not call this directly.
void buildRMQ() {
    LOG[0] = -1;

    for (int i = 0; i < E.size(); ++i) {
        ST[0][i] = i;
        LOG[i + 1] = LOG[i] + !(i & (i + 1));
    }

    for (int j = 1; (1 << j) <= E.size(); ++j) {
        for (int i = 0; (i + (1 << j)) <= E.size(); ++i) {
            int x = ST[j - 1][i];
            int y = ST[j - 1][i + (1 << (j - 1))];
            ST[j][i] = (dep[E[x]] < dep[E[y]]) ? x : y;
        }
    }
}

// Builds the LCA data structure once per test case in O(n.Log(n)).
void buildLCA() {
    dfs();
    buildRMQ();
}

// Do not call this directly.
int query(int l, int r) {
    if (l > r) swap(l, r);
    int g = LOG[r - l + 1];
    int x = ST[g][l];
    int y = ST[g][r - (1 << g) + 1];
    return (dep[E[x]] < dep[E[y]]) ? x : y;
}

// Returns the LCA of node "u" and node "v" in O(1).
int getLCA(int u, int v) {
    return E[query(F[u], F[v])];
}

// Returns the distance between node "u" and node "v" in O(1).
int getDistance(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[getLCA(u, v)];
}

```

## LCA (Parent Sparse Table)

```
int n; // The number of nodes.
vector<int> edges[N]; // The graph adjacency list.

//
// LCA related variables.
//
int dep[N];
int par[LOG_N][N];
int LOG[N];

// Do not call this directly.
void dfs(int u = 1, int p = -1, int d = 0) {
    dep[u] = d;
    par[0][u] = p;

    for (int i = 1; (1 << i) <= d; ++i) {
        par[i][u] = par[i - 1][par[i - 1][u]];
    }

    for (int v : edges[u]) {
        if (v != p) {
            dfs(v, u, d + 1);
        }
    }
}

// Do not call this directly.
void computeLog() {
    LOG[0] = -1;
    for (int i = 1; i <= n; ++i) {
        LOG[i] = LOG[i - 1] + !(i & (i - 1));
    }
}

// Builds the LCA data structure once per test case in O(n.Log(n)).
void buildLCA() {
    dfs();
    computeLog();
}

// Returns the k-th ancestor of a node "u".
int getAncestor(int u, int k) {
    while (k > 0) {
        int x = k & -k;
        k -= x;
        u = par[LOG[x]][u];
    }
    return u;
}

// Returns the LCA of node "u" and node "v" in O(Log(n)).
int getLCA(int u, int v) {
    if (dep[u] > dep[v]) {
        swap(u, v);
    }
}
```

```

v = getAncestor(v, dep[v] - dep[u]);

if (u == v) {
    return u;
}

for (int i = LOG[dep[u]]; i >= 0; --i) {
    if (par[i][u] != par[i][v]) {
        u = par[i][u];
        v = par[i][v];
    }
}

return par[0][u];
}

// Returns the distance between node "u" and node "v" in O(Log(n)).
int getDistance(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[getLCA(u, v)];
}

```

## Max Flow (Edmonds Karp's Algorithm)

```

int n, m;           // The number of nodes and number of edges

int edgeId;         // The next edge id to be inserted.
int head[N];        // head[u] : the id of the last edge added from node "u".
int nxt[M];         // nxt[e] : the next edge id pointed from the same node
                    // as "e".
int to[M];          // to[e] : the id of the node pointed by edge "e".
int capacity[M];    // capacity[e] : the maximum capacity of edge "e".
int flow[M];        // flow[u] : the current flow of edge "e".

int src, snk;       // The id of source and sink nodes.
int dist[N];        // dist[u] : the shortest distance between the source and
                    // node "u".
int from[N];        // from[u] : the id of the edge that leads to node "u" in the
                    // path from source to sink.

// Initializes the graph. Must be called before each test case.
void init() {
    edgeId = 0;
    memset(head, -1, sizeof(head));
}

// Adds a new directed edge in the graph from node "f" to node "t"
// with maximum capacity "c".
void addEdge(int f, int t, int c) {
    int e = edgeId++;

    to[e] = t;
    capacity[e] = c;
    flow[e] = 0;

    nxt[e] = head[f];
    head[f] = e;
}

```



```

// Adds a new augmented edge in the graph between node "f" and node "t"
// with maximum capacity "w".
void addAugEdge(int f, int t, int c) {
    addEdge(f, t, c);
    addEdge(t, f, 0);
}

// Do not call this directly.
bool findPath() {
    queue<int> q;
    q.push(src);

    memset(dist, -1, sizeof(dist));

    dist[src] = 0;

    while (!q.empty()) {
        int u = q.front();
        q.pop();

        for (int e = head[u]; ~e; e = nxt[e]) {
            int v = to[e];
            int c = capacity[e];
            int f = flow[e];

            if (f >= c) {
                continue;
            }

            if (dist[v] == -1) {
                dist[v] = dist[u] + 1;
                from[v] = e;
                q.push(v);
            }

            if (v == snk) {
                return true;
            }
        }
    }

    return false;
}

// Do not call this directly.
int augmentPath() {
    int f = INT_MAX;

    for (int u = snk, e, r; u != src; u = to[r]) {
        e = from[u];    // x ---e--> u
        r = e ^ 1;      // x <--r--- u

        f = min(f, capacity[e] - flow[e]);
    }

    for (int u = snk, e, r; u != src; u = to[r]) {
        e = from[u];    // x ---e--> u
        r = e ^ 1;      // x <--r--- u
    }
}

```

```

        flow[e] += f;
        flow[r] -= f;    // Reversed edge for flow cancelation
    }

    return f;
}

// Returns the the maximum flow/minimum cut of the graph.
int maxFlow() {
    int f = 0;

    while (findPath()) {
        f += augmentPath();
    }

    return f;
}

```

# Math

## GCD

```
// Computes the greatest common divisors GCD(a, b).
template<class T>
T gcd(T a, T b) {
    while (b) {
        int tmp = a % b;
        a = b;
        b = tmp;
    }
    return a;
}
```

## LCM

```
// Computes the Least common multiple LCM(a, b).
template<class T>
T lcm(T a, T b) {
    return a / gcd(a, b) * b;
}
```

## Extended Euclid

```
// Computes the coeffs. of the smallest positive linear combination of "a" and "b".
// (i.e. GCD(a, b) = s.a + t.b).
template<class T>
pair<T, T> extendedEuclid(T a, T b) {
    if (b == 0) {
        return {1, 0};
    }

    pair<T, T> p = extendedEuclid(b, a % b);

    T s = p.first;
    T t = p.second;

    return {t, s - t * (a / b)};
}
```

## Fast Power

```
// Computes ((base^exp) % mod).
template<class T>
T power(T base, T exp, T mod) {
    T ans = 1;
    base %= mod;

    while (exp > 0) {
        if (exp & 1) ans = (ans * base) % mod;
    }
}
```

```

        exp >>= 1;
        base = (base * base) % mod;
    }

    return ans;
}

```

## Modular Inverse

```

// Computes the modular inverse of "a" modulo "m".
template<class T>
T modInverse(T a, T m) {
    return power(a, m - 2, m);
}

```

## Combinations (nCr)

```

// Computes "n" choose "r".
int nCr(int n, int r) {
    if (n < r)
        return 0;

    if (r == 0)
        return 1;

    return n * nCr(n - 1, r - 1) / r;
}

```

## Pascal's Triangle

```

// comb[n][r] : holds the value of "n" choose "r" modulo "mod".
int comb[N][N];

// Builds Pascal's triangle for computing combinations (i.e. "nCr").
void buildPT(int n, int mod) {
    for (int i = comb[0][0] = 1; i <= n; ++i)
        for (int j = comb[i][0] = 1; j <= i; ++j)
            comb[i][j] = (comb[i - 1][j] + comb[i - 1][j - 1]) % mod;
}

```

## Prime Check

```

// Checks whether an integer is prime or not.
template<class T>
bool isPrime(T n) {
    if (n < 2)
        return 0;
    if (n % 2 == 0)
        return (n == 2);
    for (int i = 3; i * i <= n; i += 2)
        if (n % i == 0)
            return 0;
    return 1;
}

```

## Prime Check (Miller Rabin's Algorithm)

*// Do not call this directly.*

```
template<class T>
bool millerRabinTest(T a, T k, T q, T n) {
    T x = power<long long>(a, q, n);

    if (x == 1) {
        return true;
    }

    while (k-- > 0) {
        if (x == n - 1) {
            return true;
        }

        x = (x * 1LL * x) % n;
    }

    return false;
}
```

*// Checks whether an integer is prime or not using a deterministic  
// version of Miller Rabin's algorithm in  $O(\log(n))$ .*

```
template<class T>
bool isPrimeMillerRabin(T n) {
    if (n == 2) {
        return true;
    }

    if (n < 2 || n % 2 == 0) {
        return false;
    }

    T k = 0;
    T q = n - 1;
    while ((q & 1) == 0) {
        k++;
        q >>= 1;
    }

    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23}) {
        if (n == a) {
            return true;
        }

        if (!millerRabinTest(a, k, q, n)) {
            return false;
        }
    }

    return true;
}
```

## Generate Primes

```
// prime[i] : true if integer "i" is prime; false otherwise.
bool prime[N];

// Generates all the prime numbers of the integers from 1 to "n"
void generatePrimes(int n) {
    memset(prime, true, sizeof(prime));
    prime[0] = prime[1] = false;

    for (int i = 2; i * i <= n; ++i) {
        if (!prime[i]) continue;

        for (int j = i * i; j <= n; j += i) {
            prime[j] = false;
        }
    }
}
```

## Generate Prime Divisors

```
// divs[i] : holds a list of all the prime divisors of integer "i".
vector<int> primeDivs[N];

// Generates all the prime divisors of the integers from 1 to "n".
void generatePrimeDivisors(int n) {
    for (int i = 2; i <= n; ++i) {
        if (primeDivs[i].size()) continue;

        for (int j = i; j <= n; j += i) {
            primeDivs[j].push_back(i);
        }
    }
}
```

## Generate Divisors

```
// Computes all the divisors of a positive integer.
template<class T>
vector<T> getDivisors(T n) {
    vector<T> divs;
    for (T i = 1; i * i <= n; ++i) {
        if (n % i != 0) continue;
        divs.push_back(i);
        if (i * i == n) continue;
        divs.push_back(n / i);
    }
    sort(divs.begin(), divs.end());
    return divs;
}

// divs[i] : holds a list of all the divisors of integer "i".
vector<int> divs[N];

// Generates all the divisors of the integers from 1 to "n".
```

```

void generateDivisors(int n) {
    for (int i = 1; i <= n; ++i)
        for (int j = i; j <= n; j += i)
            divs[j].push_back(i);
}

```

## Matrix Power

```

/**
 * Matrix class for fast matrix power operation in  $O((M^3) \cdot \log(\exp))$ .
 */
class matrix {
    int rows, cols;
    int mat[M][M] = {};

public:

    static matrix eye(int n) {
        matrix res(n, n);
        for (int i = 0; i < n; ++i) {
            res.mat[i][i] = 1;
        }
        return res;
    }

    matrix(int n = 1, int m = 1, const vector<int>& data = {}) {
        rows = n;
        cols = m;
        set(data);
    }

    void set(const vector<int>& data) {
        int k = 0;
        for (int i = 0; i < rows; ++i) {
            for (int j = 0; j < cols; ++j) {
                if (k >= data.size()) {
                    return;
                }
                mat[i][j] = data[k++];
            }
        }
    }

    int& operator()(int i, int j) {
        return mat[i][j];
    }

    matrix operator*(const matrix& rhs) const {
        matrix res(rows, rhs.cols);

        for (int i = 0; i < res.rows; ++i)
            for (int j = 0; j < res.cols; ++j)
                for (int k = 0; k < rhs.rows; ++k)
                    res.mat[i][j] = (res.mat[i][j] + mat[i][k] * 1LL * rhs.mat[k][j])

% MOD;

        return res;
    }
}

```

```

matrix operator^(long long exp) const {
    matrix base = *this;
    matrix res = eye(rows);

    while (exp > 0) {
        if (exp & 1) res = (res * base);
        exp >>= 1;
        base = (base * base);
    }

    return res;
}
};

```



# Others

## Longest Increasing Sub-sequence

```
// The array to compute its LIS and its length.
int n, a[N];

// Computes and returns the length of the Longest increasing subsequence (LIS) of
// the global array "a" in time complexity of  $O(n \cdot \log(n))$ .
int getLIS() {
    int len = 0;
    vector<int> LIS(n, INT_MAX);

    for (int i = 0; i < n; ++i) {
        // To get the length of the Longest non decreasing subsequence
        // replace function "lower_bound" with "upper_bound"
        int idx = lower_bound(LIS.begin(), LIS.end(), a[i]) - LIS.begin();
        LIS[idx] = a[i];
        len = max(len, idx);
    }

    return len + 1;
}
```