User Manual for FaVeST

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1 INSTALLATION

Users do not need to compile FaVeST package by yourself. You only need to uncompress FaVeST archive and change to the newly created directory in your Matlab working environment.

2 FUNCTIONS

FaVeST archive contains three folders: src , drivers , nfft-3.5.2-matlab-openmp , detailed as follows:

- (1) src folder contains the two main functions for FaVeST:
 - FaVeST_fwd.m: forward FFTs computing Fourier coefficients associated with a quadrature rule (e.g., **Algorithm**1 in the paper) with inputs:
 - *T*: tangent field samples;
 - L: degree for vector spherical harmonic;
 - *X*, *w*: quadrature rule used for evaluating FFT.
 - FaVeST_adj.m : adjoint FFTs for vector spherical harmonic expansion with given inputs (e.g., **Algorithm 2** in the paper):
 - alm: Fourier coefficients for divergent-free part;
 - blm: Fourier coefficients of curl-free part;
 - *X*: evaluation points on the sphere.
- (2) drivers folder contains example programs illustrating the use of your code plus those used to generate the results provided in our paper:
 - utils : the folder contains some basic tools/resources/auxiliary functions used for implementing our main functions, including:
 - SD: it contains six examples of [symmetric spherical design points] (https://web.maths.unsw.edu.au/~rsw/Sphere/EffSphDes/ss.html) used in our numerical experiments;
 - tangent_field: the folder that contains several functions for generating three vector fields and their visualization used in our paper. These functions come from Ref. [1];
 - m_map : [mapping package](https://www.eoas.ubc.ca/~rich/map.html#ack) for Matlab. We have used some functions of this package for visualization of tangent fields;
 - QpS2.m : the function is used for computing the weights and quadrature nodes (for a given degree and a specific type of quadrature rule) in either Cartesian coordinates or spherical coordinates;

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- Fig2a, 2b, 2c.m, Fig3a, 3b, 3c.m, Table1.m, Table2_Fig4.m: these routines are used to reproduce the
 numerical results of the corresponding figures and tables of the paper;
- Demo.m: A quick demonstration for using FaVeST_fwd.m and FaVeST_adj.m on a simple tangent field.
- (3) nfft-3.5.2-matlab-openmp folder contains the pre-compiled Matlab interfaces of NFFT 3.5.2 with AVX2 and OpenMP support, downloaded from NFFT library (Download page: https://www-user.tu-chemnitz.de/~potts/nfft/), see Ref. [2]. As noted in the official documentation of NFFT, compiled binaries of the NFFT library and Matlab and Julia interfaces are already offered on the Download page, which means that users can use nfft-3.5.2-matlab-openmp archive directly without any specific installation requirements or further instructions for configuration. Users can refer to NFFT official page for more details.

3 DEMO

Users can run Demo.m to reproduce the figures in Fig.2 (a) in Matlab environment, e.g.,

>> Demo

Then, you can obtain three figures and the following records in Matlab command window:

```
Tangent Field A, Quadrature: GL, L: 30 == Absolute Error: 3.3226e-11, Relative Error: 4.3282e-12
```

All the simulation results can be reproduced by running the corresponding M-scripts in drivers folder as described above.

REFERENCES

- E. J. Fuselier and G. B. Wright. Stability and error estimates for vector field interpolation and decomposition on the sphere with RBFs. SIAM Journal on Numerical Analysis, 47(5):3213–3239, 2009.
- [2] J. Keiner, S. Kunis, and D. Potts. Using NFFT 3 a software library for various nonequispaced fast Fourier transforms. ACM Transactions on Mathematical Software, 36(4):19, 2009.