User manual for software from article

**Algorithm xxx: The OutlierLib – a MATLAB library for outliers’ detection**

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This user manual contains the syntax, descriptions, examples of usage, expected results and references to origin of example data for each function of MATLAB library for outliers’ detection.

Contents

[Tukey test to screen data for outliers 2](#_Toc463516931)

[Modified Z-score to screen data for outliers 4](#_Toc463516932)

[One-sided Dixon test for single outlier 6](#_Toc463516933)

[Two-sided Grubbs test for single outlier 8](#_Toc463516934)

[Two-sided Generalized (extreme Studentized deviate) ESD test for one or more outliers 10](#_Toc463516935)

[Two-sided Tietjen-Moore test for multiple outliers 13](#_Toc463516936)

[Test for outliers in multivariate data using Mahalanobis distance and F-test 15](#_Toc463516937)

# Tukey test to screen data for outliers

**Syntax**

[outliers, idx] = tukey(x, Name, Value)

Input arguments

x – Input array

Name-Value pair arguments

Specify comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify up to 2 name and value pair arguments. Any order of the Name-Value pairs is allowed.

'whisker' – Maximum whisker length (optional)

1.5 (default) | positive numeric value

'plotBoxplot' – Boxplot is plotted (optional)

true | false (default)

Output arguments

outliers – Outliers

idx – Indices of outliers in input array

**Description**

tukey(x, Name, Value) screens the input array x for multiple outliers presence.

**Example of usage**

Screen the input array inputArray for multiple outliers presence and show the boxplot.

[outliers, idx] = tukey(inputArray, 'plotBoxplot', true);

Input:

load carsmall

idx = strmatch('Germany', Origin);

inputArray = MPG(idx);

Expected results:

outliers = 44

idx = 9

Output to screen:

no output

Reference to origin of example data:

MATLAB sample data 'carsmall.mat'. Measurements of cars, 1970, 1976, 1982.

Miles per gallon (MPG) measurements for Germany from sample data is used.

<http://www.mathworks.com/help/stats/boxplot.html>

# Modified Z-score to screen data for outliers

**Syntax**

[outliers, idx] = mzscore(x)

Input arguments

x – Input array

Output arguments

outliers – Potential outliers

idx – Indices of potential outliers in input array

**Description**

mzscore(x) screens the input array x for multiple outliers presence. Test supposes an approximately normal distribution of input data.

**Example of usage**

Screens the input array inputArray for multiple outliers presence.

[outliers, idx] = mzscore(inputArray);

Input:

inputArray = [2.0 0.1 0.8 0.2 3.0 1.9 1.0 14.6 4.8 0.4 0.9 0.1 0.3 0.3];

Expected results:

outliers = 14.6000 4.8000

idx = 8 9

Output to screen:

no output

Reference to origin of example data:

United Nations Office on Drugs and Crime (UNODC). 2011.

Z-Score Report per Substance. Page 2.

Round: 2011/1. Substance: BS-2/6-Monoacetylmorphine (6-MAM).

<https://www.unodc.org/documents/scientific/ZScorePerSubstanceBS.pdf>

# One-sided Dixon test for single outlier

**Syntax**

[outliers, idx] = dixon(x, Name, Value)

Input arguments

x – Input array (size should be between 3 and 30)

Name-Value pair arguments

Specify comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify up to 2 name and value pair arguments. Any order of the Name-Value pairs is allowed.

'alpha' – Significance level (optional)

value between 0 and 1 (if not provided, default is 0.05 for 5% significance)

'verboseOutput' – Verbose output (optional)

'on' | 'off' (default)

Output arguments

outlier – Outlier

idx – Index of outlier in input array

**Description**

dixon(x, Name, Value) tests the input array x for single outlier presence. Test supposes an approximately normal distribution of input data.

**Example of usage**

Test the input array inputArray for outlier presence with significance level 0.10 and verbose output.

[outlier, idx] = dixon(inputArray, 'verboseOutput', 'on', 'alpha', 0.1);

Input:

inputArray = [568 570 570 570 572 578 584 596];

Expected results:

no outliers

Output to screen:

Significance level: 0.10

Test kind: right-side

Test statistic: 0.461538

Critical value: 0.478911 (based on 25000 simulations)

Reference to origin of example data:

<http://www.itl.nist.gov/div898/software/dataplot/refman1/auxillar/dixon.htm>

# Two-sided Grubbs test for single outlier

**Syntax**

[outliers, idx] = grubbs(x, Name, Value)

Input arguments

x – Input array

Name-Value pair arguments

Specify comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify up to 2 name and value pair arguments. Any order of the Name-Value pairs is allowed.

'alpha' – Significance level (optional)

value between 0 and 1 (if not provided, default is 0.05 for 5% significance)

'verboseOutput' – Verbose output (optional)

'on' | 'off' (default)

Output arguments

outlier – Outlier

idx – Index of outlier in input array

**Description**

grubbs(x, Name, Value) tests the input array x for single outlier presence. Test supposes an approximately normal distribution of input data.

**Example of usage**

Test the input array inputArray for outlier presence with significance level 0.10 and verbose output.

[outlier, idx] = grubbs(inputArray, 'verboseOutput', 'on', 'alpha', 0.1);

Input:

inputArray = [199.31, 199.53, 200.19, 200.82, 201.92, 201.95, 202.18, 245.57];

Expected results:

outlier = 245.5700

idx = 8

Output to screen:

Significance level: 0.10

Test statistic: 2.468765

Critical value: 2.031652

Reference to origin of example data:

<http://www.itl.nist.gov/div898/handbook/eda/section3/eda35h1.htm>

# Two-sided Generalized (extreme Studentized deviate) ESD test for one or more outliers

**Syntax**

[outliers, idx] = gesd(x, Name, Value)

Input arguments

x – Input array

Name-Value pair arguments

Specify comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify up to 3 name and value pair arguments. Any order of the Name-Value pairs is allowed.

'outliersNumber' – Upper bound for suspected number of outliers (mandatory)

'alpha' – Significance level (optional)

value between 0 and 1 (if not provided, default is 0.05 for 5% significance)

'verboseOutput' – Verbose output (optional)

'on' | 'off' (default)

Output arguments

outliers – Outliers

idx – Indices of outliers in input array

**Description**

gesd(x, Name, Value) tests the input array x for one or more outliers presence. Test supposes an approximately normal distribution of input data.

**Example of usage**

Test the input array inputArray for presence of 10 outliers with verbose output.

[outliers, idx] = gesd(inputArray, 'outliersNumber', 10, 'verboseOutput', 'on');

Input:

inputArray = [-0.25 0.68 0.94 1.15 1.20 1.26 1.26 1.34 1.38 1.43 1.49 1.49 1.55 1.56 1.58 1.65 1.69 1.70 1.76 1.77 1.81 1.91 1.94 1.96 1.99 2.06 2.09 2.10 2.14 2.15 2.23 2.24 2.26 2.35 2.37 2.40 2.47 2.54 2.62 2.64 2.90 2.92 2.92 2.93 3.21 3.26 3.30 3.59 3.68 4.30 4.64 5.34 5.42 6.01];

Expected results:

outliers = 6.0100 5.4200 5.3400

idx = 54 53 52

Output to screen:

Significance level: 0.05

Outliers: 1, Test statistic: 3.119, critical value: 3.159

Outliers: 2, Test statistic: 2.943, critical value: 3.151

Outliers: 3, Test statistic: 3.179, critical value: 3.144

Outliers: 4, Test statistic: 2.810, critical value: 3.136

Outliers: 5, Test statistic: 2.816, critical value: 3.128

Outliers: 6, Test statistic: 2.848, critical value: 3.120

Outliers: 7, Test statistic: 2.279, critical value: 3.112

Outliers: 8, Test statistic: 2.310, critical value: 3.103

Outliers: 9, Test statistic: 2.102, critical value: 3.094

Outliers: 10, Test statistic: 2.067, critical value: 3.085

Number of outliers: 3

Reference to origin of example data:

<http://www.itl.nist.gov/div898/handbook/eda/section3/eda35h3.htm>

# Two-sided Tietjen-Moore test for multiple outliers

**Syntax**

[outliers, idx] = tietjen(x, Name, Value)

Input arguments

x – Input array

Name-Value pair arguments

Specify comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify up to 3 name and value pair arguments. Any order of the Name-Value pairs is allowed.

'outliersNumber' – Number of outliers (mandatory)

'alpha' – Significance level (optional)

value between 0 and 1 (if not provided, default is 0.05 for 5% significance)

'verboseOutput' – Verbose output (optional)

'on' | 'off' (default)

Output arguments

outliers – Outliers

idx – Indices of outliers in input array

**Description**

tietjen(x, Name, Value) tests the input array x for multiple outliers presence. Test supposes an approximately normal distribution of input data.

**Example of usage**

Test the input array inputArray for presence of 2 outliers with verbose output.

[outliers, idx] = tietjen(inputArray, 'outliersNumber', 2, 'verboseOutput', 'on');

Input:

inputArray = [-1.40 -0.44 -0.30 -0.24 -0.22 -0.13 -0.05 0.06 0.10 0.18 0.20 0.39 0.48 0.63 1.01];

Expected results:

outliers = -1.4000 1.0100

idx = 1 15

Output to screen:

Significance level: 0.05

Test statistic: 0.291999

Critical value: 0.316691 (based on 10000 simulations)

Reference to origin of example data:

<http://www.itl.nist.gov/div898/handbook/eda/section3/eda35h2.htm>

# Test for outliers in multivariate data using Mahalanobis distance and F-test

**Syntax**

[outliers, idx] = mahdist(x, Name, Value)

Input arguments

x – Array of multivariate data (samples of each variate in separate row)

Name-Value pair arguments

Specify comma-separated pairs of Name, Value arguments. Name is the argument name and Value is the corresponding value. Name must appear inside single quotes (' '). You can specify up to 2 name and value pair arguments. Any order of the Name-Value pairs is allowed.

'alpha' – Significance level (optional)

value between 0 and 1 (if not provided, default is 0.05 for 5% significance)

'verboseOutput' – Verbose output (optional)

'on' | 'off' (default)

Output arguments

outliers – Outliers

idx – Indices of outliers in input array

**Description**

mahdist(x1, x2, Name, Value) tests the input arrays x1 and x2 for outliers presence. Test supposes an approximately normal multivariate distribution of input data.

**Example of usage**

Test the input bivariate array inputArray for outliers presence with verbose output.

[outlier, idx] = mahdist(inputArray, 'verboseOutput', 'on');

Input:

inputArray = [154 136 91 125 133 125 93 80 132 107 142 115 114 120 141; 108 90 54 89 93 77 43 50 125 76 96 74 79 71 90];

Expected results:

outlier =

132 93

125 43

idx = 9 7

Output to screen:

Significance level: 0.05

Variates: 2

Samples: 15

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Sample Test statistic P

1 1.52 0.258058

2 0.36 0.705487

3 1.09 0.367818

4 0.08 0.921651

5 0.17 0.844478

6 0.27 0.767123

7 1.95 0.185275

8 2.64 0.112036

9 16.38 0.000372

10 0.41 0.674741

11 0.61 0.560950

12 0.05 0.952880

13 0.10 0.904111

14 0.36 0.705578

15 0.87 0.441904

min P: 0.000372, Sample: 9 (132.0 125.0)

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Sample Test statistic P

1 1.56 0.253543

2 0.30 0.746988

3 0.96 0.411185

4 0.68 0.526699

5 0.43 0.662707

6 0.39 0.683226

7 5.62 0.020854

8 2.82 0.102589

9 1.36 0.296418

10 0.54 0.595561

11 0.02 0.978085

12 0.50 0.617706

13 0.71 0.511994

14 0.96 0.413978

min P: 0.020854, Sample: 7 (93.0 43.0)

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Sample Test statistic P

1 1.74 0.224137

2 0.26 0.779609

3 1.61 0.246982

4 0.87 0.446666

5 0.50 0.619839

6 0.85 0.454451

7 2.91 0.101114

8 1.41 0.288811

9 0.47 0.638841

10 0.09 0.913226

11 0.48 0.632789

12 1.87 0.203993

13 1.09 0.374218

min P: 0.101114, Sample: 7 (80.0 50.0)

Reference to origin of example data:

Afifi, A.A. and Azen, S.P. 1979. *Statistical Analysis: A Computer Oriented Approach* (2nd ed.). Academic Press. Chapter 5.1.