Simulation of Multi-deck BlackJack

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Abstract:

We develop a reproducible Monte Carlo pipeline to quantify player expected value (EV) in blackjack across deck sizes and house rules. The single-file Python implementation covers dataset generation (decision-point EVs), analysis (EV by dealer upcard, hard/soft action heatmaps, hit-threshold curves), and full-round simulation with two policies: a table-driven Basic Strategy (pairs/soft/hard with splits and doubles) and a Naive baseline. We evaluate rule sensitivities including S17 vs H17, 3:2 vs 6:5 payouts, Double-After-Split, hit-after-split-Aces, and doubling constraints, reporting EV per initial hand with 95% confidence intervals.

Introduction:

We model blackjack as a finite-horizon stochastic control problem.

A state s = (t, soft, u, k_split, d_ok) encodes the player total t, softness flag, dealer upcard u, splits used, and doubling availability.

Actions a ∈ {H, S, D, P} cause a transition s′ ~ T_θ(s′ | s, a), where θ are the house rules (decks, S17/H17, blackjack payout p ∈ {1.5, 1.2}, DAS, etc.).

The round reward R ∈ {–1, 0, +1, +p} is measured per initial hand (splits/doubles change stake and are netted at the round level).

Given a policy π (Basic or Naive), the target is the expected value.

$$V^{\pi}(\theta) = \mathbb{E}_{\pi}[R \mid \theta].$$

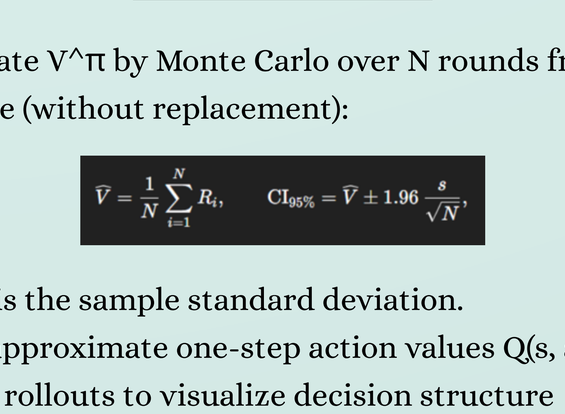
We estimate V^π by Monte Carlo over N rounds from a finite shoe (without replacement):

$$\widehat{V} = \frac{1}{N}\sum_{i=1}^{N} R_i, \qquad \mathrm{CI}_{95\%} = \widehat{V} \pm 1.96\,\frac{s}{\sqrt{N}},$$

Where s is the sample standard deviation.

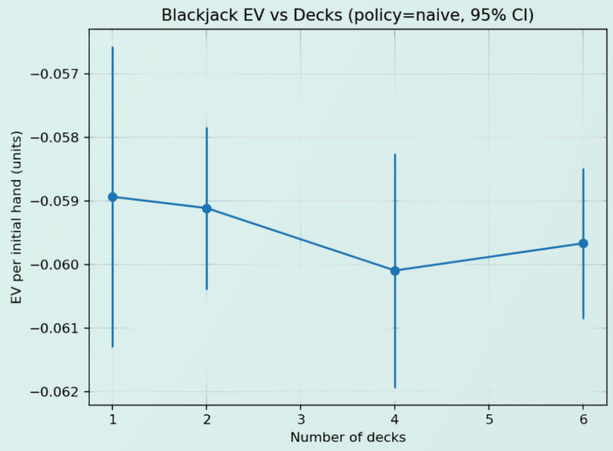
We also approximate one-step action values Q(s, a) via stand/hit rollouts to visualize decision structure (hard/soft heatmaps) and report EV vs. decks with 95% CIs to quantify rule impacts.

Results:



- Deck effect: EV drops as decks increase. Under Basic Strategy, 1–2 decks ≈ break-even (CIs overlap 0), by 4–6 decks EV is negative.
- Statistical note: 95% CI bars show uncertainty; the 4-deck point is below zero even with CI, confirming a real deck penalty.
- Interpretation: EV of –0.005 ≈ –0.5% house edge per hand (per 1-unit bet).

Context: Results are for the listed rules (3:2 payout, dealer soft-17 rule as configured, DAS on, double any two)

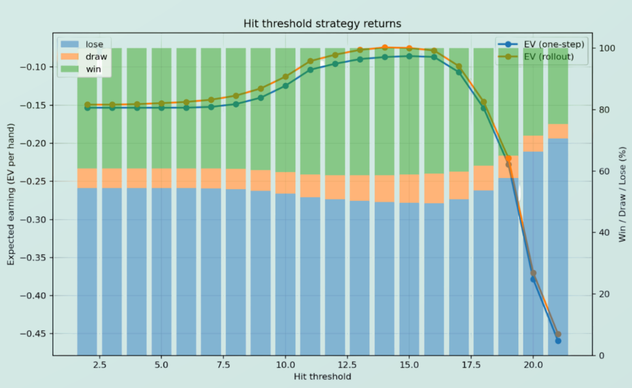


- Large, stable loss: EV ≈ –0.059 across all decks → about a –5.9% house edge per hand (1-unit bet).
- Deck count barely matters here: the 95% CIs overlap, differences between 1–6 decks are not statistically significant under this policy.
- Why it's so bad: the naive rule ignores the dealer upcard and soft/hard context, causing too many bad hits/stands (and missed doubles).
- Context: Switching to Basic Strategy is near break-even under fair rules, rule changes (e.g., S17 ≈ +0.2 pp, 6:5 ≈ –1.3–1.5 pp) affect EV far more than deck count with a naive policy.

Rule-by-Rule EV Comparison (6 Decks):

Rule	Policy	EV	Replicates	Win%	Draw%	Lose%	EV-Base(pp)
Base	Basic	-0.00581	5	43.2708	8.6733	48.0559	
6:5 Payout	Basic	-0.01944	5	43.2708	8.6733	48.0559	-1.36312
Double on hard	Basic	-0.00739	5	43.2674	8.6833	48.0493	-0.15767
Hit_split_aces	Basic	-0.00474	5	43.3047	8.6469	48.0484	0.10703
2 Maxsplit	Basic	-0.00576	5	43.2728	8.6736	48.0536	0.00550
No double after split	Basic	-0.00750	5	43.2033	8.7481	48.0486	-0.16890
Soft 17	Basic	-0.00334	5	43.3655	8.7295	47.9050	0.24740

Hit threshold

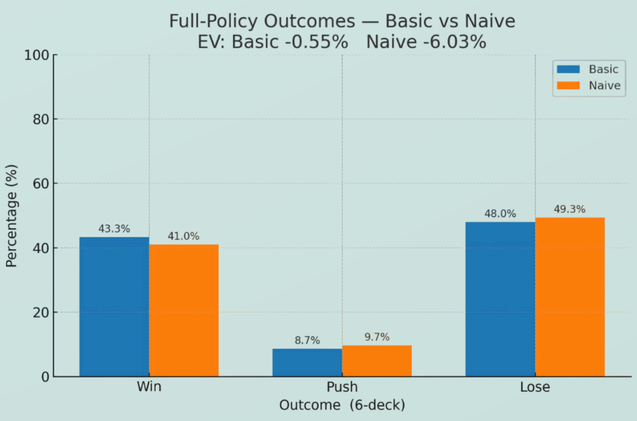


Hit ⩽ T, else Stand: EV (line) peaks around T ≈ 15–16 (soft-only ≈ 17–18). Early increases in T reduce premature stands,beyond ~16, busts rise and EV falls.

Bars = outcome mix: as T rises to the peak, Win% Increases and Loss% decreases; after the peak, Loss climbs.

Even at the best T, EV stays negative because this rule ignores dealer upcard and softness, showing why full Basic Strategy performs better.

Outcome:

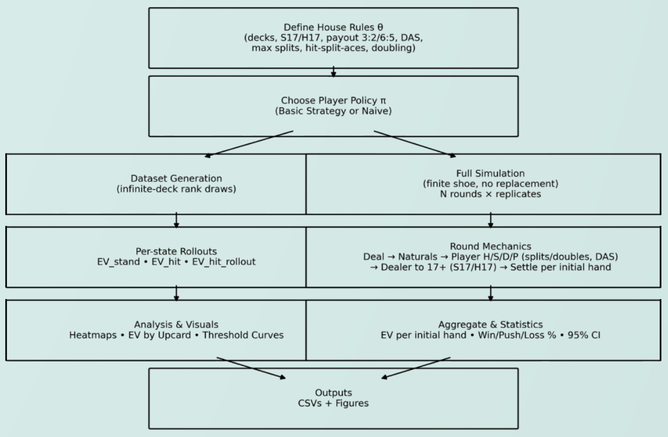


Basic: Win/Push/Lose ≈ 43.28% / 8.68% / 48.04% (EV –0.55%).

Naive: 41.03% / 9.66% / 49.32% (EV –6.03%).

Basic wins more and loses less; Naive misses profitable doubles and makes sub-optimal hit/stand choices, raising busts (slightly higher Push% from more middling totals).

Methodology:



Inputs: Choose house rules (decks, S17/H17, payout, DAS, splits, doubling) and a player policy (Basic or Naive).

Branch A – Dataset/Analysis: Infinite-deck state sampling → compute, rollout EV → visualize decision structure (hard/soft heatmaps, threshold curves).

Branch B – Full Simulation: Finite shoe, no replacement → per round: deal, check naturals, play H/S/D/P (incl. splits & doubles), dealer to 17+ (S17/H17), settle per initial hand.

Outputs: Aggregate over rounds × replicates → EV per initial hand, Win/Push/Loss %, 95% CI. Compare rule variants by changing only.

Discussion:

1. Policy impact: Basic vs Naive improves the edge by ~+5.5pp (–0.55% → –6.03%), with Increase in Win% and Decrease in Lose% from better doubles/splits and fewer marginal hits/stands.
2. Threshold insight: Blind "Hit ⩽ T" peaks near T≈15–16 yet stays negative—dealer upcard and softness are essential.
3. Practical takeaway: Prefer 3:2, S17, DAS tables; avoid 6:5; play Basic to cut losses by ~5–6 units per 100 hands at a 1-unit stake.
4. Next steps: Add surrender/insurance, configurable penetration, and card counting + bet spread.

References:

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[2] Griffin, P. A. (1999). The Theory of Blackjack: The Compleat Card Counter's Guide to the Casino Game of 21 (6th ed.). Huntington Press. Huntington Press

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Software: Python (numpy, pandas, matplotlib, scipy).