Mathematical Modelling of Race Lines

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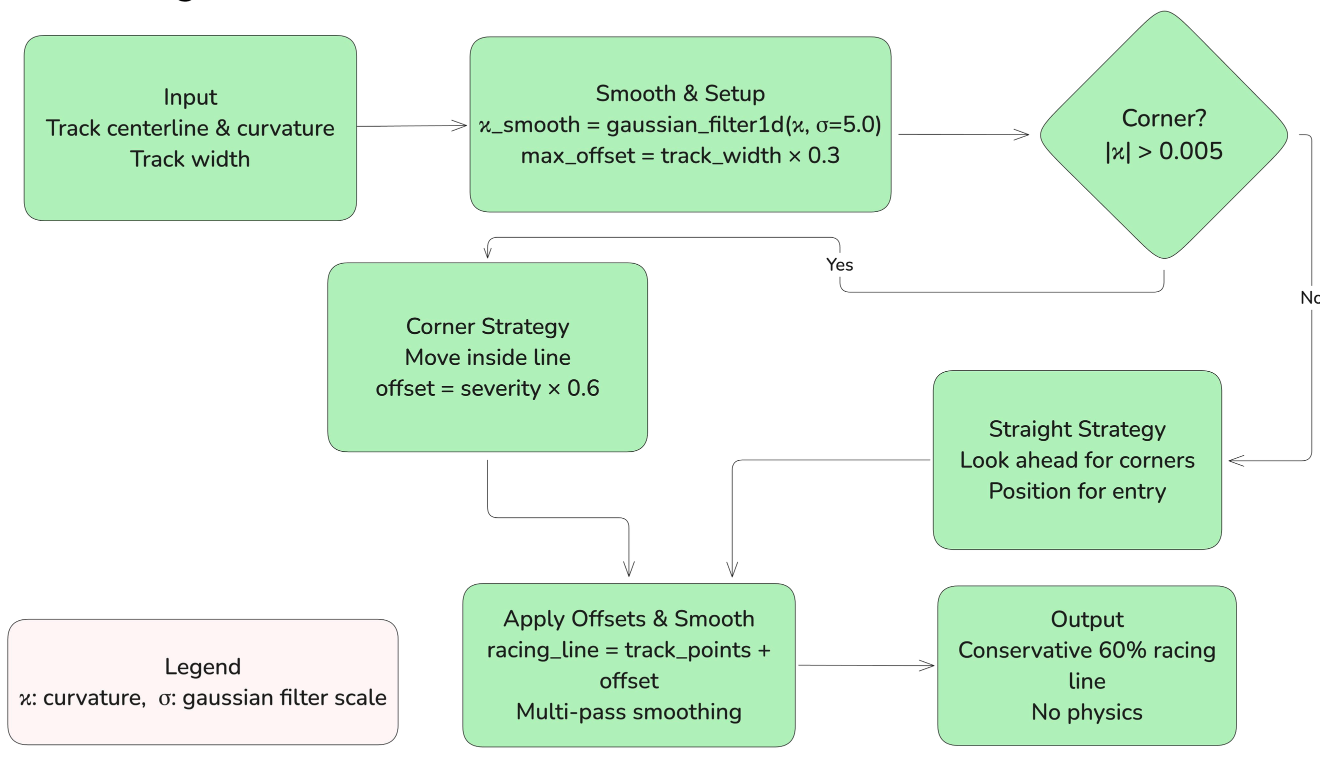
Abstract

Racing performance heavily depends on both track conditions (temperature, friction, slope) and vehicle parameters (torque, acceleration, aerodynamics). Traditional optimal race line models require computationally expensive nonlinear programming, creating barriers to real-time analysis. There is a critical need for fast, flexible, and interactive solutions to model optimal racing lines under varied configurations. This work bridges the gap between theory and application by enabling real-time, user-defined simulations without traditional computational overhead.

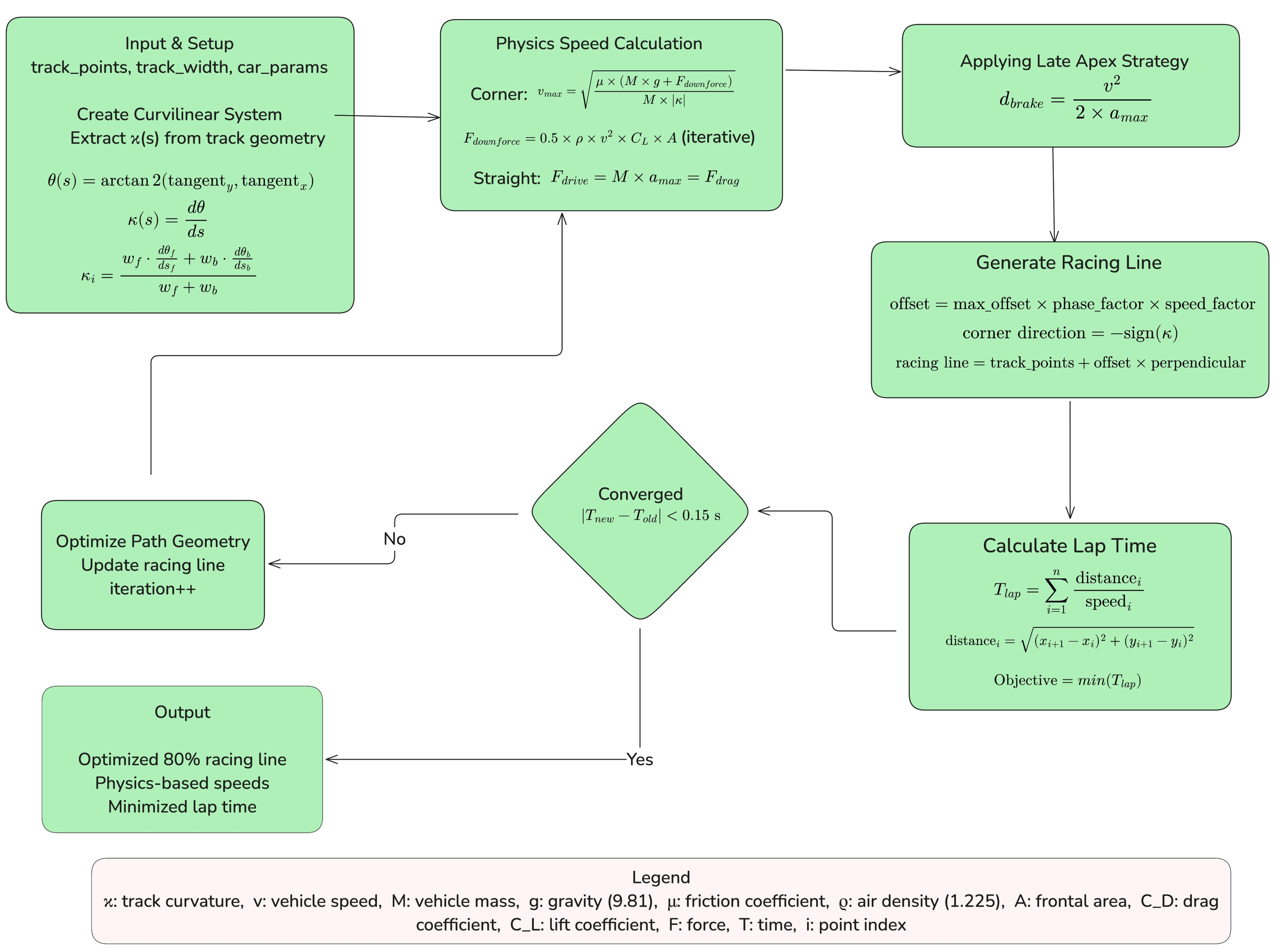
Methodology

This project implements three racing line optimization models with increasing complexity: basic geometric algorithms for smooth track line calculation using corner detection and conservative offsets in Cartesian coordinates, physics-based calculations with iterative lap time optimization using curvilinear coordinate systems and dynamics vehicle parameters, and the advanced sequential two-step algorithm implementing forward-backward integration and convex optimization for research-grade trajectory planning.

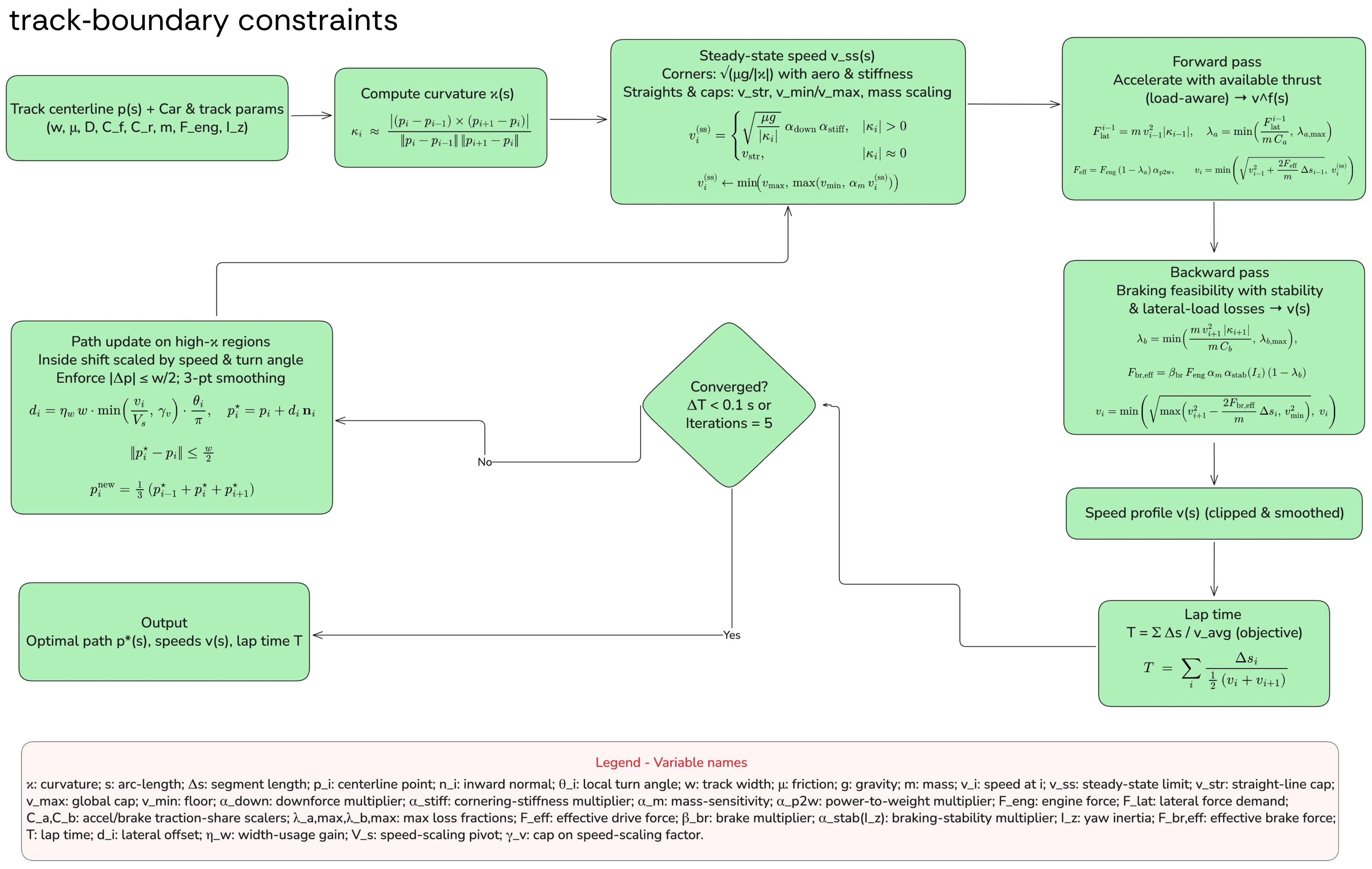
Basic Model: Uses geometric curvature analysis with Gaussian filtering for smooth, conservative racing lines.



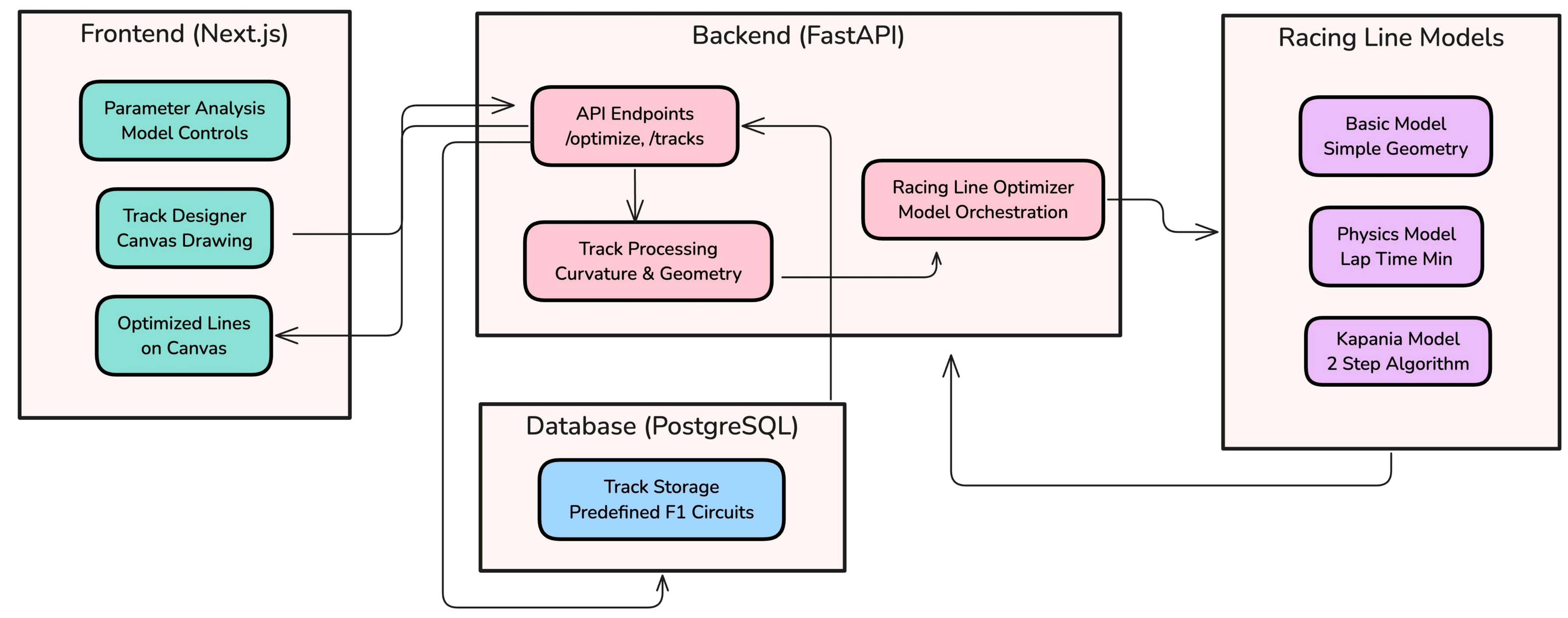
Physics Based Model: Uses curvilinear coordinates with physics equations and 8+ car parameters for iterative lap time optimization.



Sequential two-step algorithm: alternates forward–backward speed-profile integration with convex path optimization to produce minimum-lap-time racing trajectories under tire and track-boundary constraints



Architecture



Results

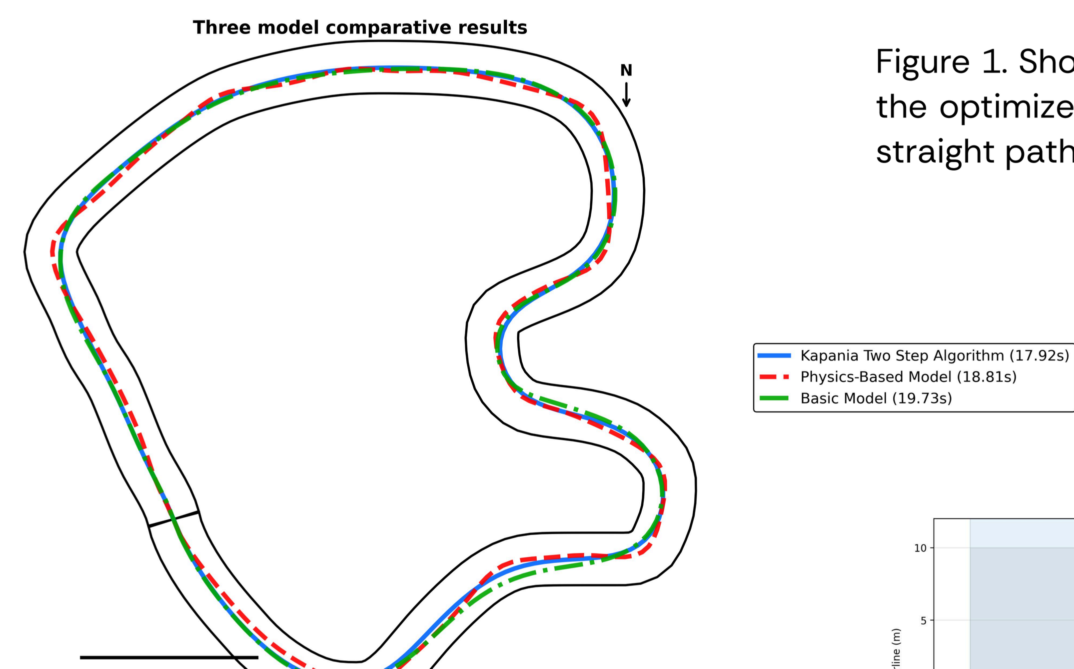


Figure: 1

Figure 1. Shows the three model's comparison on how the optimized racing lines are achieved at curves and straight paths with lap times in legend.

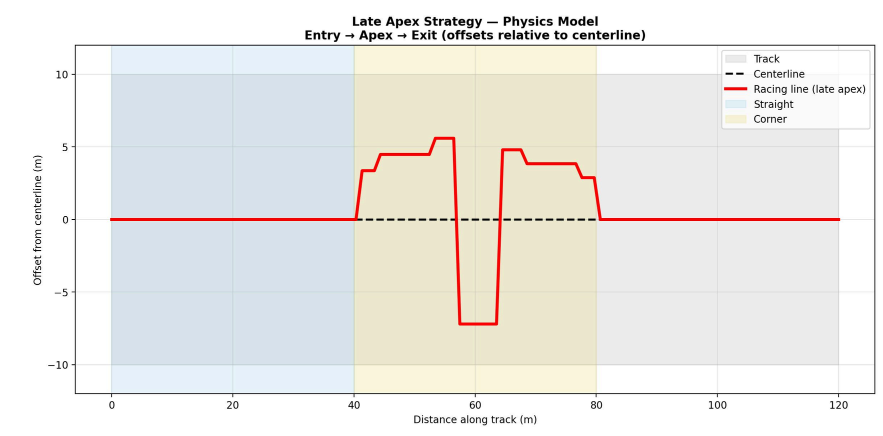


Figure: 2a

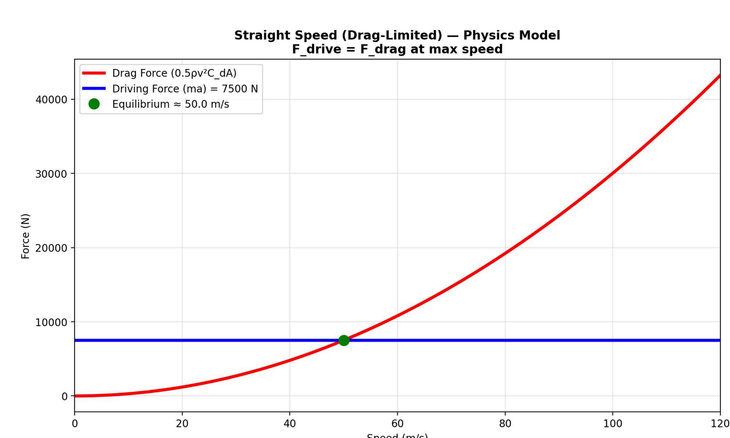


Figure: 2b

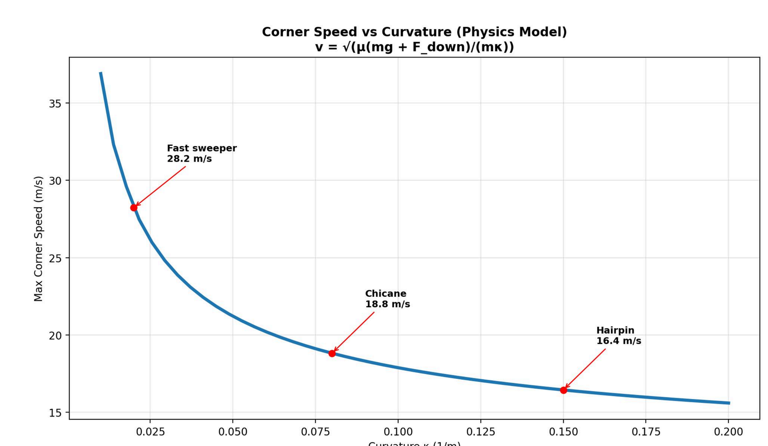


Figure: 2c

Figure 2. (a) shows the late apex racing line strategy using 80% track width for optimal lap time. (b) demonstrates aerodynamic equilibrium where drag force intersects driving force at 50.0 m/s maximum speed. (c) illustrates the inverse relationship between corner speed and curvature, from 28.2 m/s (sweepers) to 16.4 m/s (hairpins).

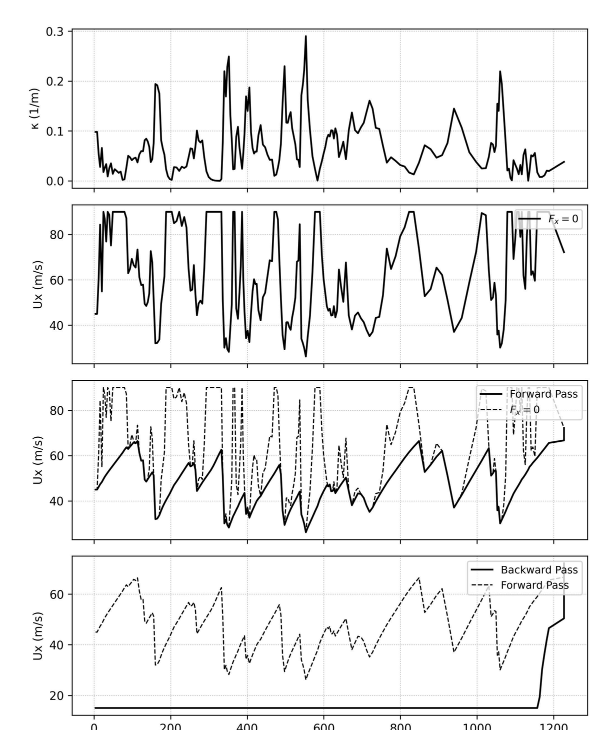


Figure: 3 (a, b, c, d)

(a) Sample curvature profile. (b) Velocity profile given zero longitudinal force. (c) Velocity profile after forward pass (d) Final velocity profile after backward pass.

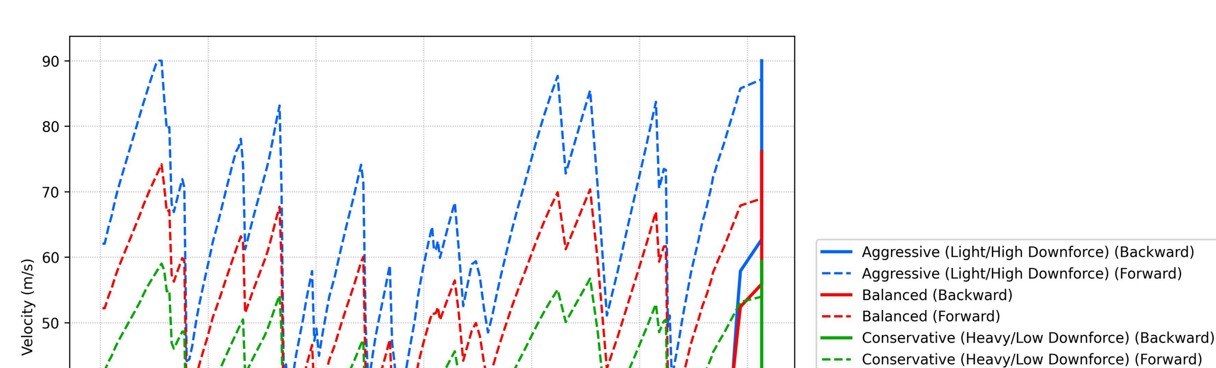


Figure: 4

Figure. 4: Velocity profile Ux(s) vs. distance along the path for the two-step method for three cars with different parameters to gauge behaviour

Conclusion and Future Scope

We implemented research-based mathematical models for race line optimization, achieving real-time, high-speed calculations for accurate lap times, speed profiles, and optimized racing lines. The application accepts user-defined tracks and parameters, making it suitable for autonomous racing, simulation-based driver training, and interactive design tools.

Future enhancements include adding tire degradation and weather effects, with generated data feeding into machine learning models to optimize pit stop strategies, tire selection, and overall race performance.

References

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2. Kapania, N. R., Subosits, J., and Christian Gerdes, J. (June 2, 2016). "A Sequential Two-Step Algorithm for Fast Generation of Vehicle Racing Trajectories."
3. Jain A, Morari M (2020) Computing the racing line using bayesian optimization. 2020 59th IEEE Conference on Decision and Control (CDC).