Monte Carlo Simulation of a Football League using a Bivariate Poisson Model with Elo

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Abstract

We simulate full football seasons using a *Bivariate Poisson* goals model augmented with *Elo* as a covariate. Team attack/defense strengths and home advantage are estimated via penalized likelihood; Elo evolves match-to-match with decay. Monte Carlo rollouts of an entire fixture list generate distributions of points and finishing positions. We backtest on recent Premier League seasons and present key outcome probabilities.

Introduction

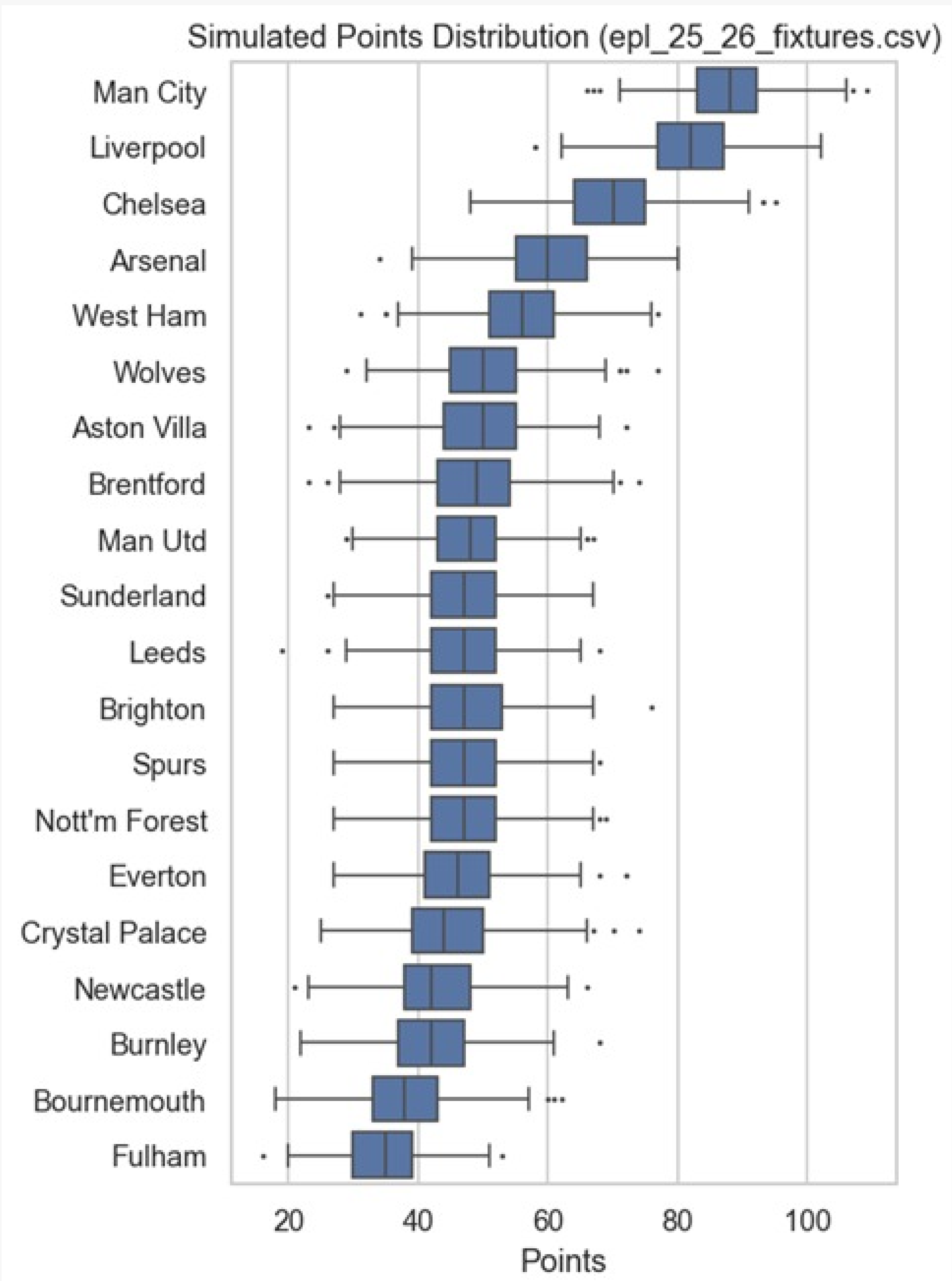
Data. Match results (date, home/away teams, goals) and fixture lists for backtests / projections.

Model overview.

- **Scoring:** (X, Y) follow a Bivariate Poisson with shared component K.
- **Rates:** λ_1, λ_2 use team attack/defense, home advantage, and Elo ratio $\left(\frac{\text{Elo}_h}{\text{Elo}_a}\right)^{\gamma}$.
- **Elo:** Updated each match with step K and decay δ.
- **Fit:** Penalized (ridge) MLE with sum-to-zero constraints.
- **Simulation:** Draw one outcome per fixture $\Rightarrow$ season table; repeat n times.

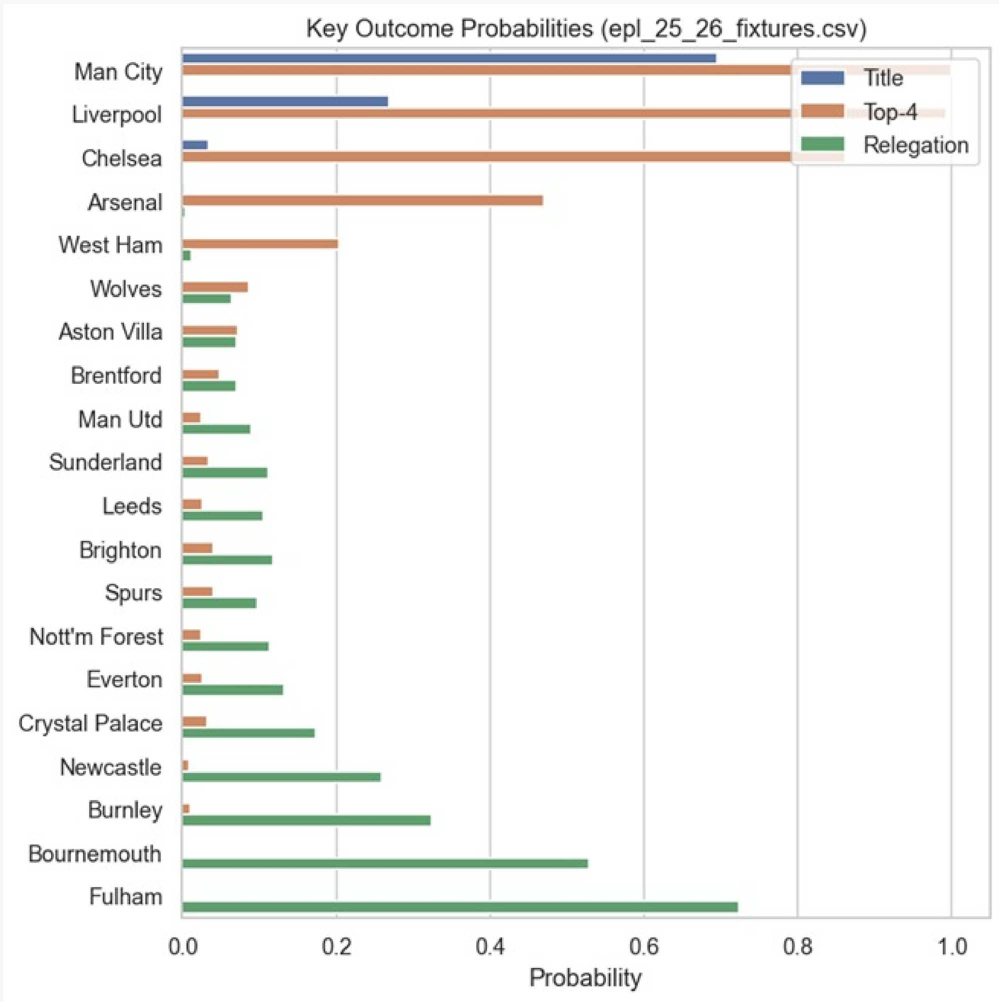
Hyperparameters. K=40, γ=0.06, λ_{ridge}=0.02, δ=0.995 (effective step $\approx$ 0.20).

Results: Points Distribution

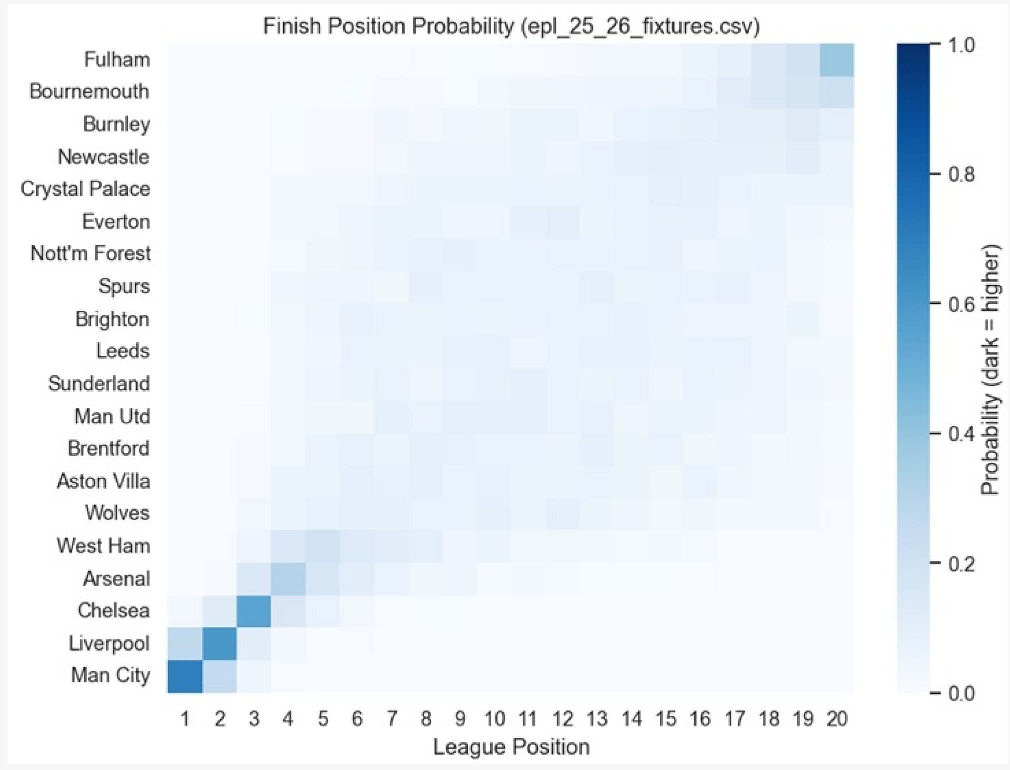


- Elite clubs sit far right with higher medians; spreads are relatively tight.
- Mid-table teams overlap substantially, reflecting competitive balance.
- Lower-table teams show wider variance, indicating higher outcome uncertainty.

Results:Outcome Probabilities & Finish Position



- Title chances concentrated among 1–2 teams.
- Top-4 dominated by small elite group.
- Relegation risk borne by bottom six.



- Strong diagonal: mass near expected positions.
- Top clubs in 1–6; relegation in 18–20.
- Off-diagonal haze = plausible surprises.

Methodology

1. **Data prep:** Parse dates; build team set across train/backtest; compute pre-kickoff Elo per match.
2. **Model:** Bivariate Poisson with $\lambda_1 = \exp(\alpha_h - \beta_a + \eta_h)\left(\frac{\text{Elo}_h}{\text{Elo}_a}\right)^{\gamma}$, $\lambda_2 = \exp(\alpha_a - \beta_h)\left(\frac{\text{Elo}_h}{\text{Elo}_a}\right)^{-\gamma}$, shared $\lambda_3 = \exp(\rho)$.
3. **Elo:** Logistic expectation; step K with decay δ applied to updates.
4. **Estimation:** Penalized MLE (ridge) with sum-to-zero constraints.
5. **Simulation:** Draw a scoreline per fixture; award points; repeat full season n times.
6. **Evaluation:** Compare simulated mean points/finish probs to actuals; MAE + calibration checks.

Discussion

- **Interpretation:** Shared BP component captures tempo/co-movement; Elo stabilizes team effects over time.
- **Drivers:** γ (Elo influence) shifts competitive balance; ρ (shared) adjusts draw frequency.
- **Generalization:** Ridge improves out-of-sample; W/D/L calibration good with mild draw underestimation.
- **Caveats:** Missing injuries/transfers/congestion; home advantage not time-varying.
- **Future:** Add Dixon–Coles decay, richer covariates, Bayesian shrinkage; extend to multi-league settings.

Literature cites / Resources

Dixon, M. J., & Coles, S. G. (1997). *Modelling association football scores...* JRSS C, 46(2), 265–280.
Groll, A., Ley, C., Schauberger, G., & Van Eetvelde, H. (2021). *A hybrid random forest to predict soccer matches.* Royal Society Open Science 8(2):210617.

Software: `R` (`data.table`, `ggplot2`, `lubridate`).

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