Mathematical Appendix: Vehicular Emissions Monte Carlo Model

A. Problem Definition

We estimate the total CO_2 emissions from conflict-related vehicular activity across scenarios, with explicit treatment of uncertainty in activity levels, fuel use, emission factors, and campaign phasing. Let vehicle classes be indexed by $c \in \{1, \ldots, C\}$ (e.g., aircraft, tanks, trucks), days by $t \in \{1, \ldots, T\}$, and scenarios by $s \in \{A, B, C\}$.

B. Emissions Identity

For class c on day t in Monte Carlo iteration i:

$$E_{c,t}^{(i)} = A_{c,t}^{(i)} \, F_c^{(i)} \, \gamma_c^{(i)}. \tag{1}$$

Here $A_{c,t}^{(i)}$ is activity (km driven or hours flown), $F_c^{(i)}$ is fuel consumption per unit activity (L/km or L/h), and $\gamma_c^{(i)}$ is the emission factor (kg CO_2/L). Daily and cumulative emissions are

$$E_t^{(i)} = \sum_{c=1}^{C} E_{c,t}^{(i)}, \tag{2}$$

$$E^{(i)} = \sum_{t=1}^{T} E_t^{(i)}. \tag{3}$$

C. Uncertainty Sources

We treat $A_{c,t}$, F_c, and γ_c as random variables:

$$A_{c,t}^{(i)} \sim \mathrm{Dist}_{s,c,t}(\theta), \tag{4}$$

$$F_c^{(i)} \sim \mathrm{Dist}_F(\mu_c, \sigma_c^2), \tag{5}$$

$$\gamma_c^{(i)} \sim \mathrm{Dist}_\gamma(\mu_c, \sigma_c^2), \tag{6}$$

where θ are scenario-specific parameters (means, variances, autocorrelation or clustering).

D. Phase Length Sampling via Dirichlet Distribution

To encode conflict tempo, the horizon T is partitioned into K phases with lengths $(L_1, \ldots, L_K)$ satisfying $\sum_{k=1}^{K} L_k = T$. We first draw phase proportions

$$(p_1, \ldots, p_K) \sim \mathrm{Dir}(\alpha_1, \ldots, \alpha_K), \tag{7}$$

and set $L_k = \lfloor p_k T \rfloor$ with a final adjustment so that $\sum_k L_k = T$. Within phase k, activity is drawn from class-specific distributions,

$$A_{c,t} \,\big|\, t \in \text{phase } k \;\sim\; \text{Dist}(\mu_{c,k}, \sigma^2_{c,k}). \tag{8}$$

Dirichlet concentration controls structure: $\alpha_k = 1$ yields a uniform prior over partitions; $\alpha_k > 1$ favors equal-length phases; $\alpha_k < 1$ induces unequal partitions with bursts and lulls.

E. Monte Carlo Estimator

For $i = 1, \ldots, N$ iterations:

1. Sample $(A^{(i)}_{c,t}, F^{(i)}_c, \gamma^{(i)}_c)$ and a phase partition $(L_1, \ldots, L_K)$ via the Dirichlet scheme.
2. Compute $E^{(i)}$ and $\{E^{(i)}_t\}_{t=1}^T$.

Point and interval estimates are

$$\hat{E} = \frac{1}{N} \sum_{i=1}^{N} E^{(i)}, \tag{9}$$

$$CI_p = \text{Quantile}_p\Big(\{E^{(i)}\}_{i=1}^N\Big), \tag{10}$$

$$\hat{E}_t = \frac{1}{N} \sum_{i=1}^{N} E^{(i)}_t. \tag{11}$$

F. Variance Decomposition

A pragmatic variance decomposition attributes variability to activity, fuel rates, emission factors, and phase lengths:

$$\text{Var}(E) \approx \text{Var}_A(E) + \text{Var}_F(E) + \text{Var}_\gamma(E) + \text{Var}_L(E), \tag{12}$$

where $\text{Var}_L(E)$ arises from Dirichlet-sampled phase lengths. Empirically, $\text{Var}_A(E)$ and $\text{Var}_L(E)$ dominate the total.

G. Scenario Specification

Each scenario s specifies

- number of phases K_s and Dirichlet parameters $\boldsymbol{\alpha}_s$,
- per-phase activity parameters $\{(\mu_{c,k}, \sigma^2_{c,k})\}_{c,k}$,
- (optionally) correlation or autoregression within phases.

Outputs include distributions of $E^{(i)}$, daily trajectories $E^{(i)}_t$, and credible bands.

H. Equivalence Metrics

Totals can be normalized for comparability:

$$E_{\text{cars}} = \frac{E}{E_{\text{car}}}, \qquad E_{\text{trees}} = \frac{E}{E_{\text{tree}}}, \qquad E_{\text{phones}} = \frac{E}{E_{\text{phone}}}, \tag{13}$$

where denominators are benchmark annual or lifecycle quantities.

Reproducibility. This appendix can be compiled with LaTeX; Quarto or R Markdown variants will render the same mathematics when targeting PDF.