# Restricted Multi-Body Monte Carlo Analysis of Earth–Impact Risk

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## Abstract

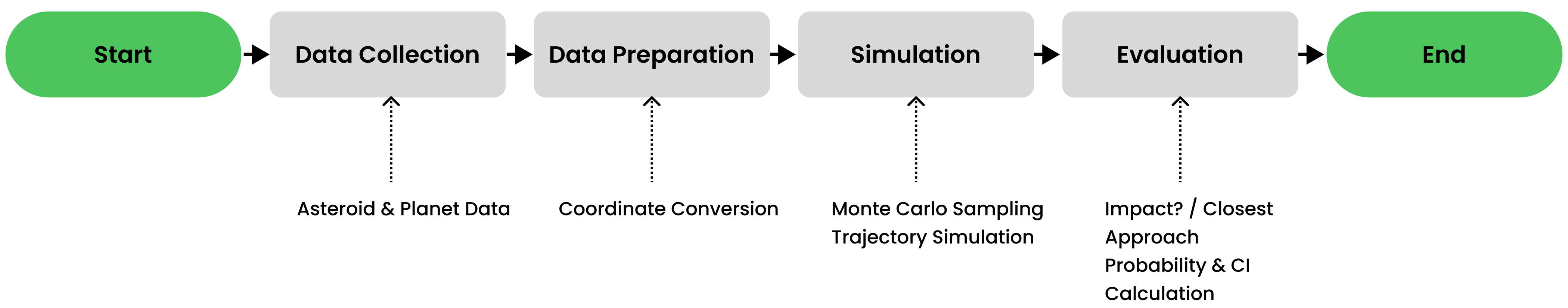
We investigate the orbital dynamics of a selected asteroid using Monte Carlo simulations. Particle trajectories are computed to assess close approaches to Earth, Jupiter, and the Sun, and statistical analyses are performed on minimum distances and collision probabilities. Sensitivity studies explore how small variations in initial position and velocity influence the asteroid's path over time.

## Motivation

Asteroid impacts pose a low-probability but high-consequence hazard to Earth. Predicting them requires accounting for uncertainty in orbital dynamics, since small variations in initial position and velocity can lead to very different outcomes. This project uses Monte Carlo simulations to estimate impact probabilities and analyze trajectory sensitivity. Earth is the primary focus, with Jupiter's gravitational effect included as a test case to evaluate its role in shaping the impact risk.

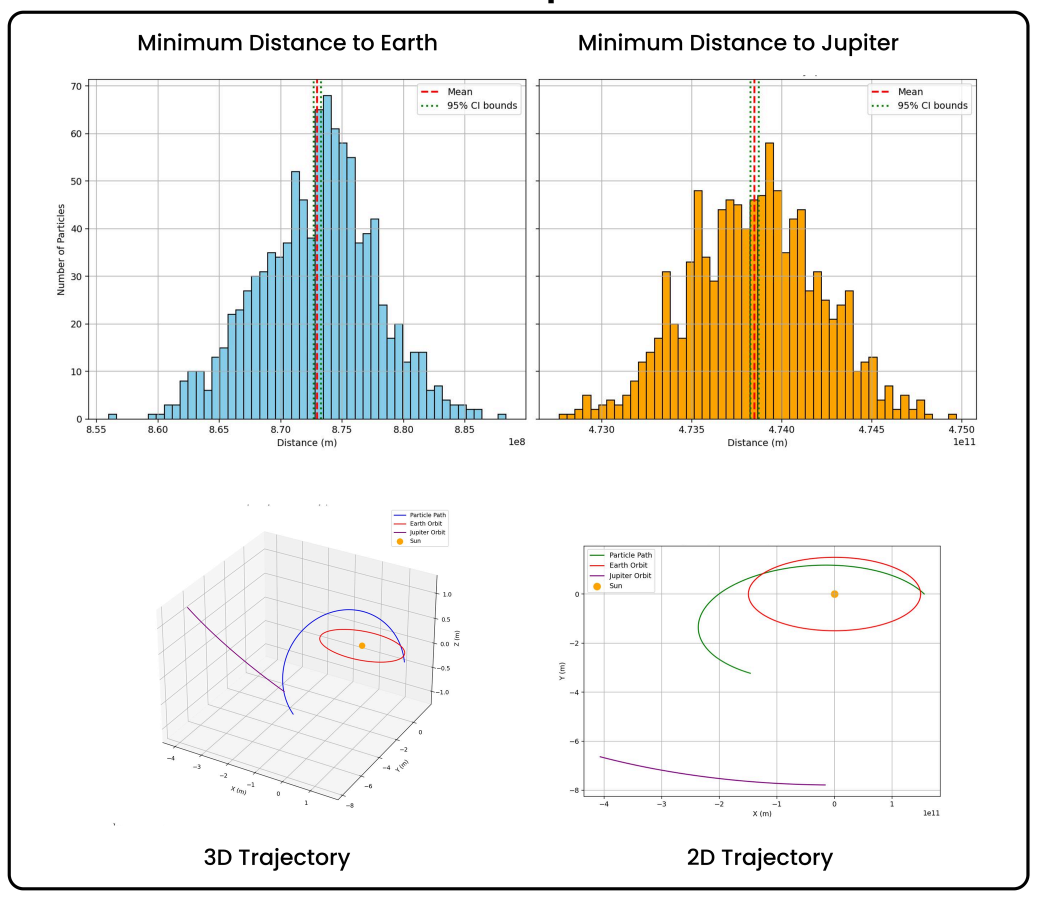
## Methodology

**Asteroid & Planet Data – Gather orbital and physical parameters.**
**Coordinate Transformation – Convert to the reference frame used in the model.**
**Monte Carlo Sampling – Assume normal distributions for position & velocity; generate ~1000 samples of initial conditions.**
**Trajectory Propagation – Simulate asteroid motion under gravitational forces; record whether it impacts Earth or passes safely.**
**Outcome Analysis – Compute impact probability (impacts ÷ total simulations), minimum distance distribution for non-impacts, and confidence intervals for close approaches.**

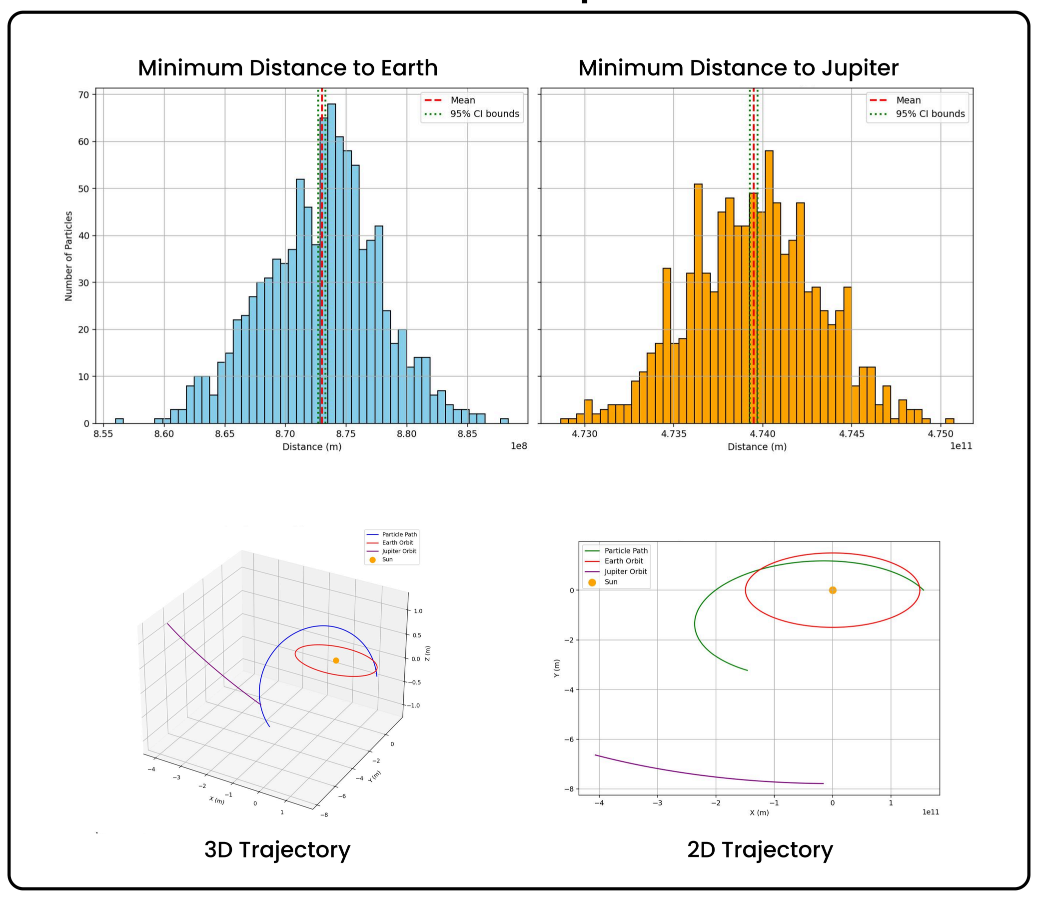



## Plots

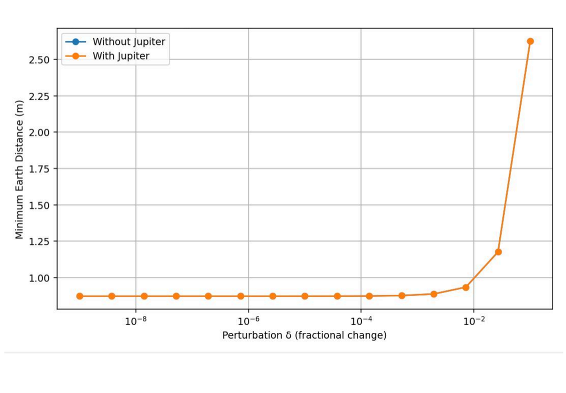
### With Jupiter



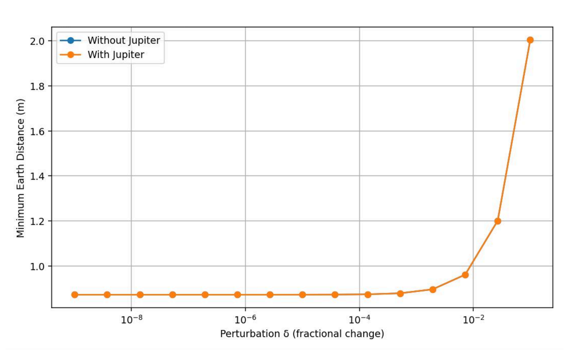

### Without Jupiter




Sensitivity of Minimum Earth Distance to POS Perturbations




Sensitivity of Minimum Earth Distance to VEL Perturbations




### Discussion

- **Impact Probability Monte Carlo simulations give a quantitative estimate of the impact risk for asteroid 2025 PM**
- **For small perturbations (≤$10^{-4}$), Earth distance is nearly constant. Beyond ~$10^{-2}$, minimum Earth distance increases sharply,**
- **Predicted 8.7297x 10^8m vs NASA's prediction 1.052219x10^9m Faulty model or Assumptions?**
  a. Circular orbits have oversimplified the path
  b. Earth's angular velocity varies depending on its distance with the sun. In August its is slow
  c. Just by reducing the angular velocity we got 1.0415x10^9m
- **Where is Jupiter's influence?**
  a. 2025 PM is a NEO- Earth field is dominant here
- **Are 1000 samples enough?**
  a. Sufficient for coarse estimates and saving computational resources—but for determining the fate of the world? Nope
- **Is retarded time actually contributing anything?**
  a. Not really. Planetary motion is slow, and over short timescales the gravitational field barely changes. For close interactions, the delay is negligible. It's about what might work best and it sounded cool when we started the project
- **Why 2025 PM? It flew past Earth on August 17th. We are presenting from August 19th. If it had hit, we wouldn't be here. Since it didn't—let's talk about why.**

## Theory And Equations

Gravitational Potential at any given point is determined by the cumulative effect of all planetary masses. The instantaneous gravitational field vector at observer's location is given by:

$$\vec{g}(\vec{r}_{\text{observer}}, t) = -\sum_{i \in \mathcal{P}} \frac{GM_i}{|\vec{r}_{\text{observer}}(t) - \vec{r}_i(t)|^2} \cdot \frac{\vec{r}_{\text{observer}}(t) - \vec{r}_i(t)}{|\vec{r}_{\text{observer}}(t) - \vec{r}_i(t)|}$$

Where:

$\vec{g}(\vec{r}_{\text{observer}}, t)$ is the gravitational field vector at the observer's position at time $t$
$G$ is the gravitational constant
$M_i$ is the mass of the $i$-th planet
$\vec{r}_{\text{observer}}(t)$ is the position vector of the observer at time $t$
$\vec{r}_i(t)$ is the position vector of the $i$-th planet at time $t$
$\mathcal{P}$ is the set of all planets contributing to the field

However, gravitational influence does not propagate instantaneously. Instead it travels at the finite speed of light(c) to account for this, we introduce retarded time which reflects the time at which the original gravitational influence originated

$$\vec{g}(\vec{r}_{\text{observer}}, t) = -\sum_{i \in \mathcal{P}} \frac{GM_i}{|\vec{r}_{\text{observer}}(t) - \vec{r}_i(t_{\text{ret}})|^2} \cdot \frac{\vec{r}_{\text{observer}}(t) - \vec{r}_i(t_{\text{ret}})}{|\vec{r}_{\text{observer}}(t) - \vec{r}_i(t_{\text{ret}})|}$$

Where:

$t_{\text{ret}}$ is the retarded time given implicitly by:

$$t_{\text{ret}} = t - \frac{|\vec{r}_{\text{observer}}(t) - \vec{r}_i(t_{\text{ret}})|}{c}$$

This formulation ensures causality by incorporating the finite time it takes for the gravitational effects to propogate

**Model Assumptions:**
**Planets move in circular orbits with constant angular velocity (heliocentric frame).**
**Test particles are massless, affected by gravity but not influencing bodies.**
**Gravity propagates finitely, with positions computed using retarded time.**
**A restricted N-body Newtonian model is applied without external forces.**
**For short intervals (~100 s), velocity and acceleration remain constant.**

## Results

- **Minimum distance from earth 8.7297x$10^8$, CI-[8.7269x$10^8$, 8.7327x$10^8$] and Probability of 0(not going to hit) from Monte Carlo Simulation**
- **With our assumptions we got closest approach as 8.7297x $10^8$m vs NASA's 1.052219x109m**
- **Role of Jupiter-For this NEO, Jupiter didn't play a major role because of its closeness to earth**
- **Updating just the angular velocity yielded a closer result with NASA**
- **A simple model can explain the behaviour of the asteroid for studies but for correct estimations proper modeling is required**

## Future Scope

- **Extend the model to include additional planets (Saturn, Venus, Mars) for more realistic long-term predictions.**
- **Replace circular orbits with elliptical orbits and allow non-constant orbital velocities for greater realism.**
- **Improve accuracy with adaptive time-stepping and higher-order numerical integrators.**
- **Test robustness by validating against a larger set of NASA asteroid trajectory datasets.**
- **Use machine learning or surrogate modeling to speed up large-scale Monte Carlo experiments.**

**References**
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[3] D. E. Vavilov and Y. D. Medvedev, "A fast method for estimation of the impact probability of near-Earth objects," in Monthly Notices of the Royal Astronomical Society, vol. 446, no. 1, pp. 705-709, Nov. 2014, doi: 10.1093/mnras/stu2097. keywords: {methods: numerical;celestial mechanics;minor planets, asteroids: general},