HW1

IEA 5/25/22 Nicholas Cheng 1

1.

	SI	U.S. Customary
Length	meters (m)	foot (ft.)
Time	seconds (s)	second (s)
Mass	kilogram (kg)	slug $\frac{lb \cdot s^2}{ft.}$
Force	Newtons (N), $\frac{kg \cdot m}{s^2}$	pound (lb)
Area	m^2 square meters (A)	ft^2 square feet (A)
Velocity	$\frac{m}{s}$, velocity (v)	$\frac{ft}{s}$ velocity (v)
Acceleration	$\frac{m}{s^2}$, acceleration (a)	ft/s^2 acceleration (a)

2. a)

Given: $a = \frac{x}{t^2} - vt + \frac{F}{m}$

Find: the terms that are dimensionally homogeneus

a = is acceleration = $\frac{\text{distance}}{\text{time}^2}$

$x/t^2 = \frac{\text{distance}}{\text{time}^2}$

vt = velocity · time = $\frac{\text{distance}}{\text{time}} \cdot$ time = distance

$F/m = \frac{\text{Force}}{\text{mass}}$; using $F = ma$; $F/m = a$ = acceleration = $\frac{\text{distance}}{\text{time}^2}$

a, $\frac{x}{t^2}$, and F/m are all dimensionally homogeneus However vt does not share dimensions with any other term

b)

Given: $F/A = g/A + pv^2 + \frac{ma}{L^2}$

A = area, L = length, g = acceleration, p = density = $\frac{\text{mass}}{\text{volume}}$

$F/A = \frac{\text{mass} \cdot \text{distance}}{\text{time}^2 \cdot \text{distance}^2} = \frac{\text{mass}}{\text{time}^2 \cdot \text{distance}}$

$g/A = \frac{\text{distance}}{\text{time}^2 \cdot \text{distance}^2} = \frac{1}{\text{time}^2 \cdot \text{distance}}$

$pv^2 = \frac{\text{mass}}{\text{distance}^3} \cdot \frac{\text{distance}^2}{\text{time}^2} = \frac{\text{mass}}{\text{time}^2 \cdot \text{distance}}$

$\frac{ma}{L^2} = \frac{\text{mass} \cdot \text{distance}}{\text{time}^2 \cdot \text{distance}^2} = \frac{\text{mass}}{\text{time}^2 \cdot \text{distance}}$

F/A, pv^2, $\frac{ma}{L^2}$ are all dimensionally homogeneus and g/A isn't with any other term

3.

Given: $\dfrac{0.631\,Mm}{(8.60\,kg)^2}$

Find: the above in SI units with 3 significant figures

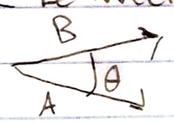
$Mm = 1{,}000{,}000\,m;\ kg^2$

$\Rightarrow \dfrac{631{,}000\,m}{(8.6)^2\,kg^2} = 8{,}532\,\dfrac{m}{kg^2} = \boxed{8{,}530\,\dfrac{m}{kg^2}}$

4.

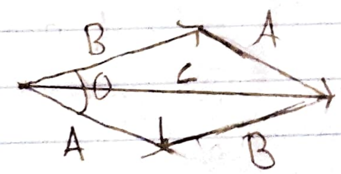
Given: $\vec{A}$ with $|A| = A$ $\quad\vec{B}$ with $|B| = B$

$\angle$ between $\vec{A}$ & $\vec{B} = \theta$ $\quad C = A + B$



a)

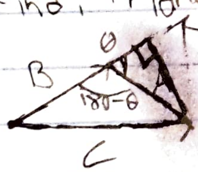
Find: Parallelogram Law to find C



$\vec{C} = (A_x + B_x\,i + A_y + B_y\,j)$

b)

Find: Triangular Law to find $\vec{C}$ and $|C|$



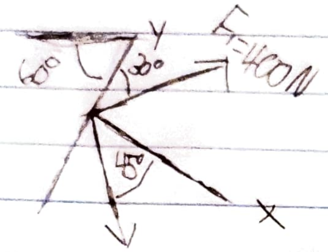
$\vec{C} = ((A_x + B_x)i + (A_y + B_y)j)$

$(|C|)^2 = (B + A\cos(\theta))^2 + (A\sin(\theta))^2$

$\boxed{|C| = \sqrt{B^2 + A^2 + 2AB\cos(\theta)}}$

5. Given: $F_1 = 400N$ $30°$ from y
$F_2 = 250N$ $-45°$ from x

Find: magnitude of F & direction measured counter clockwise from positive x



$F_{1x} = 400 \sin(30°)\, N = 200N$
$F_{1y} = 400 \cos(30°)\, N = 346.4\, N$
$\vec{F}_1 = \{200i + 346.4j\}\, N$

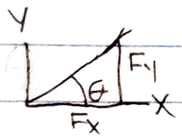
$F_2 = 250N$

$F_{2x} = 250 \cos(45°)\, N = 176.8N$
$F_{2y} = 250 \sin(45°)\, N = -176.8N$
$\vec{F}_2 = \{176.8i - 176.8j\}\, N$

$\vec{F} = \vec{F}_1 + \vec{F}_2$
$\vec{F} = \{376.8i + 169.6j\}\, N$

3 sig fig 2 sig fig

$|F| = \sqrt{(376.8)^2 + (169.6)^2} = 413.2N \approx \boxed{413N = 410N}$

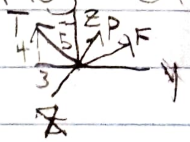


$\tan(\theta) = \frac{F_y}{F_x}$ $\theta = \tan^{-1}\left(\frac{169.6}{376.8}\right) = \boxed{24°}$ 2 sig fig

6.

Given: $F = \{9i + 10j + 8k\}\, N$ $|P| = 6N$ $\alpha = 100°$ $\beta = 30°$ $\gamma = 65°$

y-z plane $|T| = 10N$



Find: R = resultant force find $|R|$ & R in cartesian

$\vec{R} = \vec{F} + \vec{P} + \vec{T}$

$P_x = \cos(100°) \cdot 6N = -1.042$
$P_y = 6 \cdot \cos(30°) = 5.196$
$P_z = 6 \cdot \cos(65°) = 2.536$

$T_x = 0$ $T_y = \frac{-3}{5} \cdot 10N = -6N$
$T_z = \frac{4}{5} \cdot 10N = 8N$

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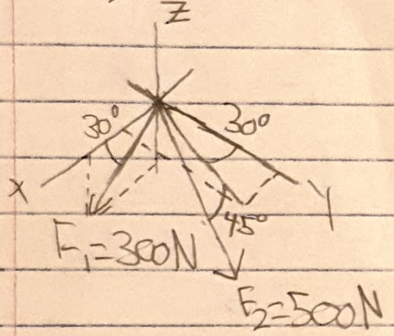
$\vec{F} = \{9i + 10j + 8k\}$ N
$\vec{P} = \{-1.042i + 5.196j + 2.536k\}$ N
$\vec{T} = \{0i - 6j + 8k\}$ N

$\vec{R} = \{7.958i + 9.196j + 18.54k\}$ N

$\boxed{|R| = 22.17\text{ N} = 22.2\text{ N} \quad \vec{R} = \{7.96i + 9.2j + 18.5k\}\text{ N}}$

7. Given: $F_1 = 300$N on xz-plane with 30° from x
$F_2 = 500$ N with 45° from $F^{2'}$ that's 30° from y



Find: magnitude of resultant Force & the coordinate direction angles α, β, γ

$F_{1x} = 300 \cdot \cos(30°) = 259.8$ $\quad F_{1z} = -300 \cdot \sin(30°) = -150$ $\quad F_{1y} = 0$
$F_1 = \{259.8i + 0j - 150k\}$ N

$F_{2z} = -500\sin(45°) = -353.6$ $\quad F_2' = 500\cos(45°) = 353.6$
$F_{2x} = F_2' \cdot \sin(30°) = 176.8$ $\quad F_{2y} = F_2' \cdot \cos(30°) = 306.2$

$F_2 = \{176.8i + 306.2j - 353.6k\}$ N

Resultant Force $(R) = F_1 + F_2 = \{436.6i + 306.2j - 503.6k\}$ N
$|R| = \sqrt{436.6^2 + 306.2^2 + (-503.6)^2} = 733.47 = 733.5$
$\alpha = \cos^{-1}\left(\frac{436.6}{733.5}\right) = 53.5°$ $\quad \beta = \cos^{-1}\left(\frac{306.2}{733.5}\right) = 65.3°$ $\quad \gamma = \cos^{-1}\left(\frac{-503.6}{733.5}\right) = 133°$
$\boxed{|R| = 733 \quad \alpha = 53.5° \quad \beta = 65.3° \quad \gamma = 133°}$

8\.

Given: Point A: $x_A = 3.2\,ft$ $z_A = 2.3\,ft$
Point B: $x_B = 2.7\,ft$ $y_B = 1.4\,ft$ $z_B = 2.\,ft$

a) Find: position vector of A from origin

$$\boxed{\vec{A} = \{3.2i + 0j + 2.3k\}\,ft}$$

b) Find: ijk components of position vector starting at A and ending at B

$$\vec{B} = \{-2.7i + 1.4j - 2k\}\,ft.$$

$$\vec{r}_{AB} = \vec{B} - \vec{A} \quad \boxed{\vec{r}_{AB} = \{-5.9i + 1.4j - 4.3k\}\,ft.}$$

9\. Given: Support A: $x_A = 3.25\,ft$ $z_A = 3.5\,ft$
Support B: $x_B = 1.08\,ft$ $y_B = 4.15\,ft$

Find: unit vector of line of action from A to B

$$r_{AB} = \vec{B} - \vec{A} \quad \vec{B} = \{1.08i + 4.15j + 0k\}\,ft$$

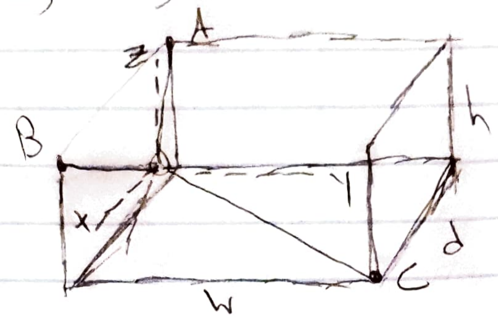
$$\vec{A} = \{3.25i + 0j + 3.5k\}\,ft. \quad \vec{r}_{AB} = \{-2.17i + 4.15j - 3.5k\}\,ft$$

$$|r_{AB}| = \sqrt{(-2.17)^2 + 4.15^2 + (-3.5)^2} = 5.846\,ft.$$

$$\vec{u}_{AB} = \left\{\frac{-2.17}{5.846}i + \frac{4.15}{5.846}j - \frac{3.5}{5.846}k\right\}ft$$

$$\boxed{\vec{u}_{AB} = -0.371i + 0.710j - 0.599k}$$

10. Given: 3.3 lb-ball at D = Origin: (0,0,0)
$A = (-4.4, -3.8, 2.8)$ ft $B = (2.3, -3.8, 1.8)$ ft
$C = (2.3, 5.3, -1.7)$ ft tension $T_{DA} = 10.5$ lb



a) Find: vector $\vec{T}_{DA}$ from D to A

$$\vec{u}_{DA} = \frac{\vec{T}_{DA}}{|T_{DA}|} \qquad \vec{T}_{DA} = |T_{DA}| \times \vec{u}_{DA}$$

$$\vec{u}_{DA} = \frac{A_x}{A} i + \frac{A_y}{A} j + \frac{A_z}{A} k \qquad A = \sqrt{(-4.4)^2 + (-3.8)^2 + (2.8)^2}$$

$A = 6.453$

$$\vec{u}_{DA} = \frac{-4.4}{6.453} i - \frac{3.8}{6.453} j + \frac{2.8}{6.453} k$$

$$\vec{u}_{DA} = -0.6819 i - 0.5889 j + 0.4339 k$$

$$T_{DA} = \{-7.159 i - 6.183 j + 4.556 k\} \text{ lb}$$
$$= \boxed{\{-7.16 i - 6.18 j + 4.56 k\} \text{ lb}}$$

b) Find: tension in cable DC if $T_{DA} = 10.7$ lb and $T_{DB} = 5.55$ lb

The ball is not in motion, therefore net force must be 0, therefore the tension of C must be the opposite of the combined tension of $T_{DA} + T_{DB}$. However, this is only true if gravity is not considered, considering gravity we must accont for the 3.3 lb downward force on D.

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First we will find $\vec{T}_{DA}$ + $\vec{T}_{DB}$ in vector form
Then we will consider the force of gravity.
Then reverse final force to obtain DC's tension

$\vec{T}_{DA} = |T_{DA}| \cdot \vec{u}_{DA}$ $\quad \vec{r}_{DA} = (-4.4i - 3.8j + 2.8k)\,ft$

$|r_{DA}| = 6.453$ $\quad \vec{u}_{DA} = -.6819i - .5889j + .4339k$

$\vec{T}_{DA} = \{-7.296i - 6.301j + 4.643k\}\,lb$

$\vec{T}_{DB} = |T_{DB}| \cdot \vec{u}_{DB}$ $\quad \vec{r}_{DB} = (2.3i - 3.8j + 1.8k)\,ft$

$|r_{DB}| = 4.793$ $\quad \vec{u}_{DB} = 0.4799i - 0.7928j + 0.3755k$

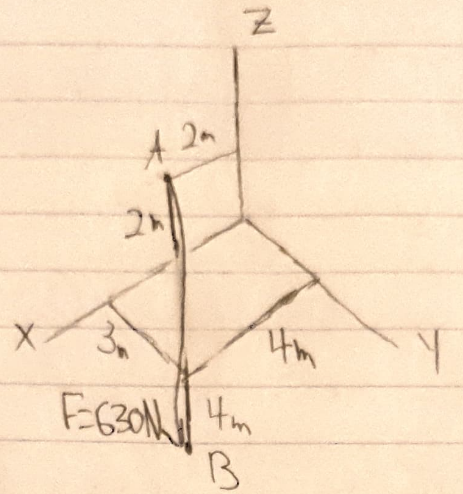
$\vec{T}_{DB} = \{2.663i - 4.4j + 2.084k\}\,lb$

$R_{DB+DA} = \{-4.633i - 10.7j + 6.727k\}\,lb$

$R_{DB+DA+gravity} = \{-4.633i - 10.7j + 3.427k\}\,lb$

$\boxed{\begin{array}{l} \vec{T}_{DC \text{ without gravity}} = \{4.63i + 10.7j - 6.73k\}\,lb \\ |T_{DC \text{ without gravity}}| = 13.46\,lbs = 13.5\,lbs \\ \vec{T}_{DC \text{ with gravity}} = \{4.63i + 10.7j - 3.43k\}\,lb \\ |T_{DC \text{ with gravity}}| = 12.15\,lbs = 12.2\,lbs \end{array}}$

11. Given:



Find: Force in Cartesian vector from

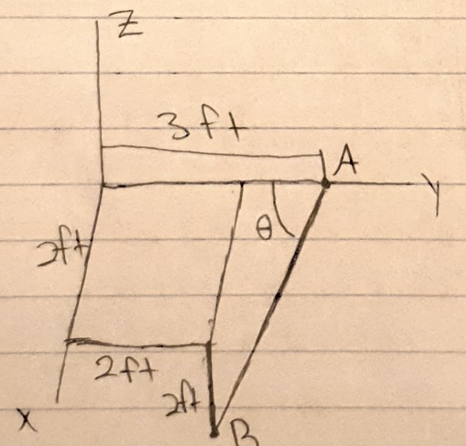
$A=(2,0,2)$ $B=(4,3,-4)$ $B-A=(2,3,-6)$

$|B-A| = \sqrt{2^2+3^2+(-6)^2} = 7$

$\vec{u}_{AB} = \frac{2}{7}i + \frac{3}{7}j - \frac{6}{7}k$

$\boxed{\vec{F} = |F| \cdot \vec{u}_{AB} = \{180i + 270j - 540k\}\ N}$

12. Given:



Find: θ between y-axis and AB wire

Plan: move origin to A and find $\vec{r}_{AB}$ where A is origin then find θ i.e. angle from r_{AB_y} & $|r_{AB}|$

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$A=(0,3,0)$ $B=(2,2,-2)$ $B-A=(2,-1,-2)$
$\vec{r}_{AB}=(2i-1j-2k)ft$ $|r_{AB}|=\sqrt{9}=3$

$\beta=\cos^{-1}\left(\frac{-1}{3}\right)=109.5$ $\theta+\beta=180$ $\theta+109.5=180^\circ$
$\boxed{\theta=70.5^\circ}$