RENSSELAER POLYTECHNIC INSTITUTE

TROY, NY

EXAM NO. 3 INTRODUCTION TO ENGINEERING ANALYSIS
(ENGR-1100) – Summer 2022

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Problem	Points	Score
1	30	29
2	25	25
3	25	25
4	20	20
Total	100	99

Attention: for every problem, showing all steps is required!

Express your final answers clearly highlight them in <u>boxes</u>.

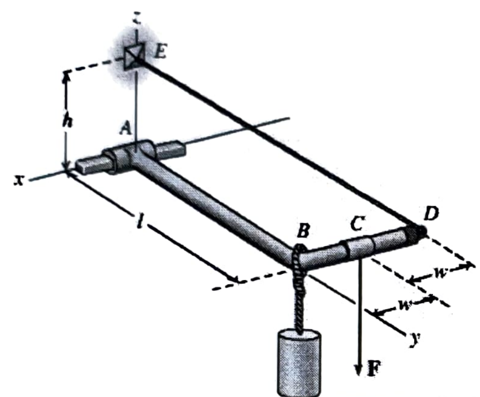
Problem #1 (30 pts)

An L-shaped member ABD is supported by a cable ED and a smooth square rod which fits loosely through the square hole of the collar (which means the member cannot rotate about x-axis). The cylinder has a weight $W = 10$lb. $F = 6$lb is a vertical force applied at C. The dimensions of the member are $w = 2.00$ft, $l = 5.00$ft, and $h = 3.00$ft .

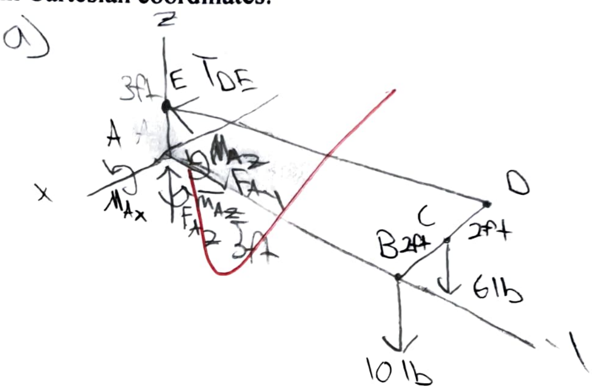
(a) (5pts) Draw a separate and complete FBD of the member ABD.

(b) (10pts) Express all the forces in the FBD in Cartesian vector form.

(c) (15pts) Using the equations of equilibrium to determine tension in cable ED and support reactions (both forces and moments) at A as vectors in Cartesian coordinates.



a)



B) $\boxed{W = \{0i + 0j - 10k\}\,lb \quad F = \{0i + 0j - 6k\}\,lb}$

$r_{DE} = \{4i - 5j + 3k\} \quad |r_{DE}| = 7.071\,lb$

$F_{DE} = \{F_{DE}\frac{4}{7.071}i - F_{DE}\frac{5}{7.071}j + F_{DE}\frac{3}{7.071}k\}\,lb$

$F_{Ay} = \{F_{Ay}j\} \quad F_{Az} = \{F_{Az}k\}$

c) $z: -16 + F_{DE}\frac{3}{7.071} + F_{Az} = 0 \quad F_{Az} = 16$

$Y: F_{Ay} - F_{DE}\frac{5}{7.071} = 0 \quad F_{Ay} = 0$

$X: F_{DE}\frac{4}{7.071} = 0 \quad F_{DE} = 0$

$M_{Ax}: M_{Ax} + \begin{bmatrix} i & j & k \\ 0 & 5 & 0 \\ 0 & 0 & -10 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 2 & 5 & 0 \\ 0 & 0 & -6 \end{bmatrix} = 0 \quad \boxed{M_{Ax} = 80\,lb\cdot ft}$

$M_{Ay}: M_{Ay} - 24 = 0 \quad M_{Ay} = 24$

$M_{Az}: M_{Az} = 0$

$F_{Az} = \{16k\}\,lb$

$F_{Ay} = \{0j\}\,lb$

$F_{DE} = \{0i + 0j + 0k\}\,lb$

$M_{Ax} = \{80i\}\,lb\cdot ft$

$M_{Ay} = \{24j\}\,lb\cdot ft$

$M_{Az} = \{0k\}\,lb\cdot ft$

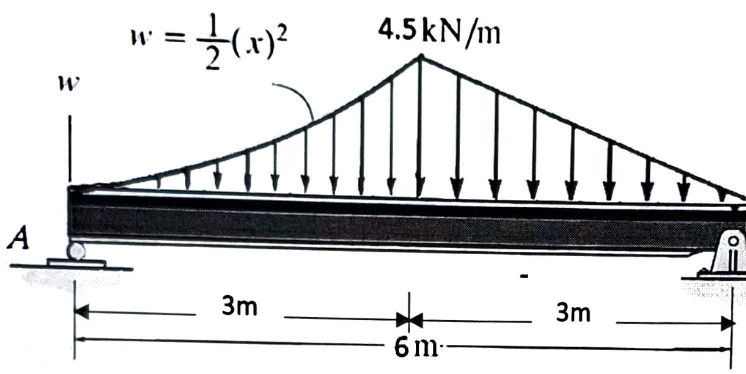
Problem #2 (25 pts)

The beam shown in the figure is supported by a roller at A and a pin at *B*.

(a) (10 pts) Find the magnitude of the equivalent concentrated force of the distributed force acting on the beam.

(b) (10 pts) Find the location of the resultant force from support A.

(c) (5 pts) Find the reaction forces at the supports *A* and *B*.



a) $|F_R| = \int_0^3 \frac{1}{2}x^2 dx + \frac{1}{2}3\cdot 4.5$

$\boxed{= 11.25\ kN}$

b) $\tilde{x}_L = x_w \quad \dfrac{\int_0^3 \frac{1}{2}x^3 dx}{\int_0^3 \frac{1}{2}x^2 dx} = \bar{x}_w$

$\bar{x}_w = 2.25\ m$

$\bar{x}_B = 3 + \frac{1}{3}\cdot 3 = 4$ $\quad M_A: -2.25\cdot \int_0^3 \frac{1}{2}x^2 dx - 4\cdot 6.75 = -37.125$

$37.125 / F_R = \bar{x}_{F_R}$ from A $\quad \boxed{\bar{x}_{F_R} \text{ from } A = 3.3m}$

c) Find reaction forces F_A F_{Bx} F_{By}

$M_A: -10.125 - 27 + F_{By}\cdot 6 = 0$

$\boxed{F_{By} = 6.19\ kN}$

$M_B: 11.25\cdot 2.7 - F_A\cdot 6 = 0$ $\quad \boxed{F_A = 5.06\ kN}$ $\quad \boxed{F_{Bx} = 0}$

Problem #3 (25 pts)

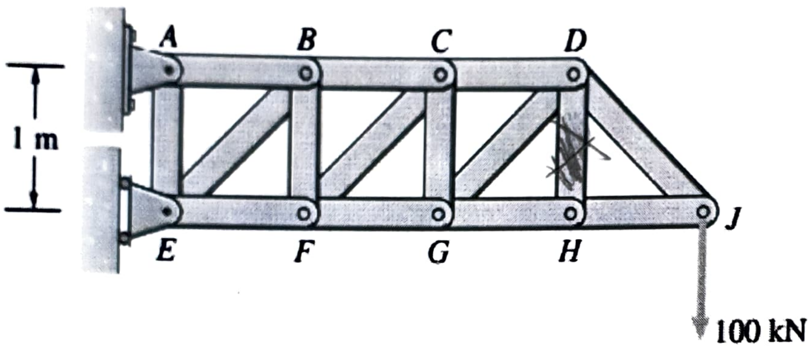
A 100-kN force is supported by a truss that is held in place by a pin A and roller E as shown in the figure. All the vertical and horizontal members are each 1m in length.

(a) (2 pts) Identify all zero-force members.

(b) (8 pts) Using method of joints, determine the force in member DG and specify whether it is in tension or compression.

(c) (12 pts) Using method of sections, determine the force in members BC, CF and FG and specify whether they are in tension or compression.

(d) (3 pts) What is the reaction force at pin A?



a) 1 zero-force member $\boxed{DH}$

b) FBD at J

F_{JD} 45° J

F_{JH}

100kN

$F_{JDy} = 100kN = \frac{1}{\sqrt{2}} F_{JD}$

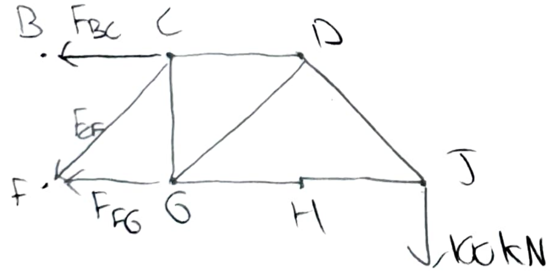
$F_{JD} = 141.4\ kN\ (T)$ FBD at D

F_{CD} F_{DG} F_{JD}

$F_{DGy} = F_{JDy}$ $F_{DG} = F_{JD}$

$\boxed{F_{DG} = 141.4\ kN\ (C)}$

c)



$M_F: -100 \cdot 3 + 1F_{BC} = 0$

$M_C: -100 \cdot 2 - F_{FG} = 0$

$F_{CFy} = -100\ kN$

$F_{CFx} = -100\ kN$

$\boxed{F_{BC} = 300\ kN\ (T)}$

$\boxed{F_{FG} = 200\ kN\ (C)}$

$\boxed{F_{CF} = 141.4\ kN\ (C)}$

d) $M_A: -4 \cdot 100 + F_E = 0$ $F_E = 400$

$\boxed{F_{Ax} = -400\ kN}$

$\boxed{F_{Ay} = 100\ kN}$

Problem #4 (20 pts)

Consider the system of three linear equations:

$$\begin{aligned} 2x + y - z &= 8 \\ 5x + \quad 2z &= 5 \\ 3x + y + z &= 1 \end{aligned}$$

(a) (3 pts) Write the system of linear equations in matrix form, Ax = B
(b) (7 pts) Find the determinant of the matrix A using cofactor expansion along column #1.
(c) (7 pts) Determine A^{-1}, the inverse of A, using only row operations.
)d) (3 pts) Using A^{-1}, solve the system of equations for the variables x, y, and z.

Note: You need to show your work to receive credit.

a)

$$Ax = B \quad \begin{bmatrix} 2 & 1 & -1 \\ 5 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 5 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \quad \det(A)_{\text{column } 1} = 2 \cdot \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$$

$$= 2 \cdot (-2) - 5(1+1) + 3(2) = \boxed{-8 = \det(A)}$$

c)

$$\left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 5 & 0 & 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ \cdot 2 - 5R_1 \\ \cdot 2 - 3R_1 \end{array} \quad \left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 0 & -5 & 9 & -5 & 2 & 0 \\ 0 & -1 & 5 & -3 & 0 & 2 \end{array}\right] \begin{array}{l} \cdot 5 + R_2 \\ \\ \cdot 5 - R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 10 & 0 & 4 & 0 & 2 & 0 \\ 0 & -5 & 9 & -5 & 2 & 0 \\ 0 & 0 & 16 & -10 & -2 & 10 \end{array}\right] \begin{array}{l} -4/16 \cdot R_3 \\ -9/16 \cdot R_3 \\ /16 \end{array} \quad \left[\begin{array}{ccc|ccc} 10 & 0 & 0 & \frac{5}{2} & \frac{5}{2} & -\frac{5}{2} \\ 0 & -5 & 0 & \frac{5}{8} & \frac{25}{8} & -\frac{45}{8} \\ 0 & 0 & 16 & -\frac{5}{8} & -\frac{1}{8} & \frac{5}{8} \end{array}\right] \begin{array}{l} /10 \\ /-5 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{8} & -\frac{5}{8} & \frac{9}{8} \\ 0 & 0 & 1 & -\frac{5}{8} & -\frac{1}{8} & \frac{5}{8} \end{array}\right] \quad \boxed{A^{-1} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{8} & -\frac{5}{8} & \frac{9}{8} \\ -\frac{5}{8} & -\frac{1}{8} & \frac{5}{8} \end{bmatrix}}$$

d)

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{8} & \frac{5}{8} & \frac{9}{8} \\ -\frac{5}{8} & -\frac{1}{8} & \frac{5}{8} \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -5 \end{bmatrix} \quad \boxed{\begin{aligned} x &= 3 \\ y &= -3 \\ z &= -5 \end{aligned}}$$