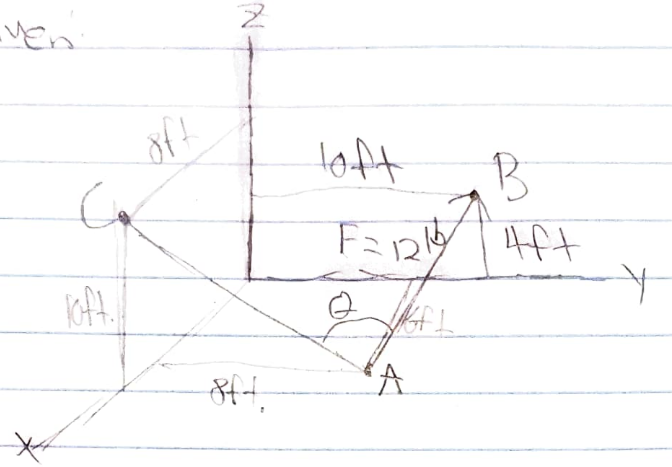
HW 2

IEA 8/30/22 Nicholas Cheng 1

1. Given:



a) Find: angle θ between AC & AB

$r_{AB} = \vec{B} - \vec{A}$ $\quad B = \{6i + 10j + 4k\}$ ft

$A = \{6i + 8j + 0k\}$ ft

$\vec{r}_{AB} = \{-6i + 2j + 4k\}$ ft

$r_{AC} = \vec{C} - \vec{A}$ $\quad C = \{8i + 0j + 10k\}$ ft

$\vec{r}_{AC} = \{2i - 8j + 10k\}$ ft

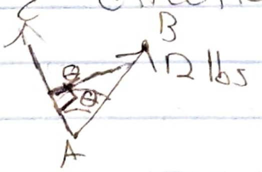
$|r_{AB}| = \sqrt{6^2 + 2^2 + 4^2} = 7.483$ ft

$|r_{AC}| = \sqrt{2^2 + 8^2 + 10^2} = 12.96$ ft

Dot Product: $\vec{r}_{AB} \cdot r_{AC} = -12 - 16 + 40 = 12$

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{AB} \cdot \vec{r}_{AC}}{|r_{AB}| \cdot |r_{AC}|}\right) = 82.89° = \boxed{82.9°}$$

b) Find: Projected component of F = 12 lb in the direction of



F_{AB} component along AC

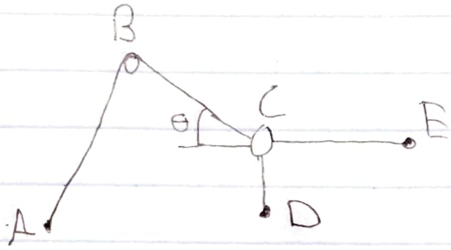
$= \cos(\theta) \cdot 12 = 12 \cdot \cos(82.89°)$

$\boxed{= 1.485 \text{ lb} = 1.48 \text{ lb}}$

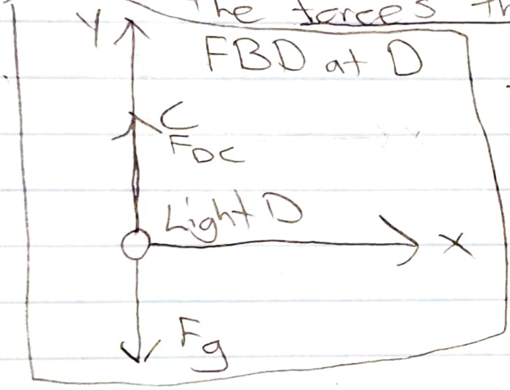
c) Find: $|F_{\text{perpendicular to } AC}| = \sin(82.89°) \cdot 12 =$

$= \boxed{11.9 \text{ lb}}$

2. Given: the light hangs from chain CD

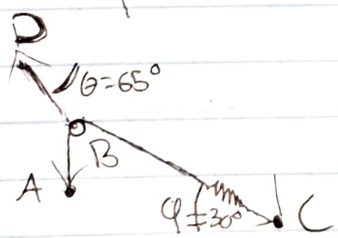


Find: FBD at "the light" by drawing the forces that act on it"

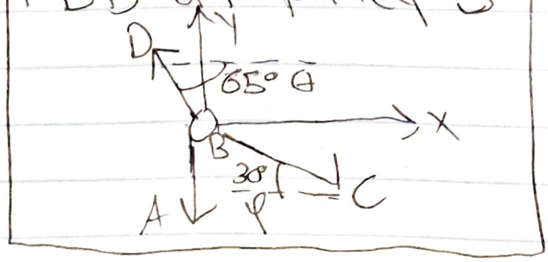


These are the only forces acting on it, the chain CD pulling up and its own weight pulling down

3. Given: B's frictionless & massless pulley
$\varphi = 30^\circ$ $k = 17$ kN/m $\theta = 65^\circ$



a) Draw FBD at pulley B



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b) Find: (Weight of A (Newtons) if spring is stretched 12.3 cm. tension in BD is 3500 N

$k = 17\ kN/m = 17000\ N/m$ $\quad$ $12.3\,cm = 0.123\,m$

Force required to stretch spring $= 17{,}000 \cdot .123\ N$

$= 2091\ N = F_{BC}$

F along AB $= F$ along BC

$\boxed{\therefore F_{AB} = 2091\,N \quad \therefore \text{Weight of } A = 2090\,N}$

c) Find relationship between θ & φ and solve for θ.

$x: -F_{BD} \cdot \cos(\theta) + F_{BC} \cdot \cos(\varphi) = 0$

$y: F_{BD} \cdot \sin(\theta) - F_{BC} \cdot \sin(\varphi) - F_{BA} = 0$

$F_{BC} = F_{BA}$

$x: F_{BD} = \dfrac{F_{BC} \cdot \cos(\varphi)}{\cos(\theta)}$

$y: F_{BD} = \dfrac{F_{BC}\sin(\varphi) - F_{BC}}{\sin(\theta)}$

$$\frac{F_{BC} \cdot \cos(\varphi)}{\cos(\theta)} = \frac{F_{BC} \cdot \sin(\varphi) - F_{BC}}{\sin(\theta)}$$

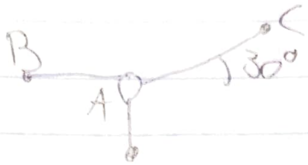
$$\frac{\cos(\varphi)}{\cos(\theta)} = \frac{\sin(\varphi) - 1}{\sin(\theta)}$$

$$\frac{\sin(\theta)}{\cos(\theta)} = \frac{\sin(\varphi) - 1}{\cos(\varphi)}$$

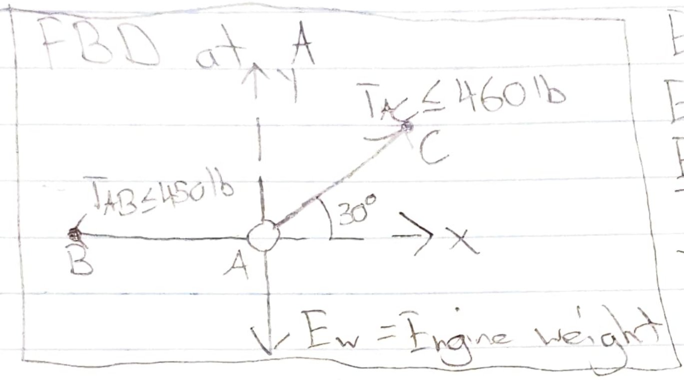
$$\tan(\theta) = \tan(\varphi) - \sec(\varphi)$$

$$\boxed{\theta = \tan^{-1}(\tan(\varphi) - \sec(\varphi))}$$

4. Given: $T_{AB} \leq 450\,lb$ $T_{AC} \leq 460\,lb$



Find: FBD & maximum weight of engine



Equilibrium: $E_w = T_{AC} \cdot \sin(30°)$

$E_w = 460 \cdot \sin(30°)$ $T_{AC} = 460$

$E_w = 230\,lb.$

$T_{AB} = T_{AC} \cdot \cos(30°)$

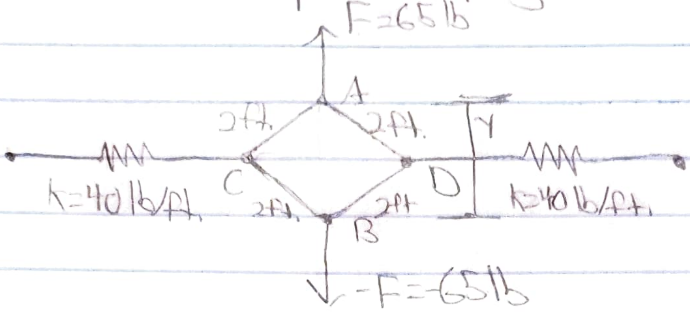
$T_{AB} = 460 \cdot \cos(30°)$

$T_{AB} = 398.4 \leq 450$ ✓

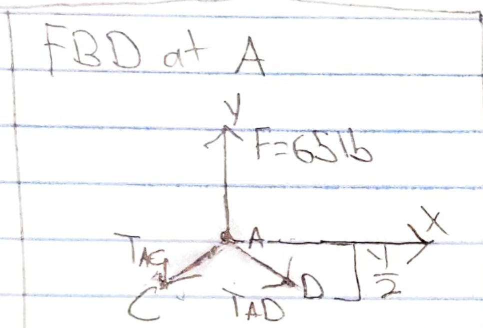
Setting T_{AC} to max $= 460\,lb$ we have $E_w = 230\,lb$. If E_w is any higher then according to equilibrium equation $E_w = T_{AC} \cdot \sin(30°)$ T_{AC} would increase to above 460lb i.e. $T_{AC} > 460\,lb$, violating the maximum T_{AC}. $\therefore$ E_w cannot be higher than 230lb. Since $E_w = 230$ makes $T_{AB} = 398.4$ & $T_{AB} = 398.4 \leq 450$, the weight satisfies all requirements.

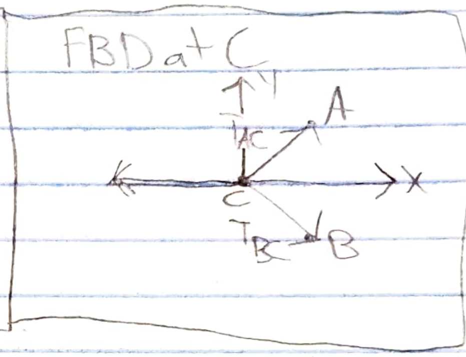
$\boxed{\therefore \text{ Engine weight} = 230\,\text{lbs}}$

5. Given: when $y=0$ springs are 1.5ft Δx $F=65lb$



a) Find FBD at A & C





b) Find: y

FBD A: y: $65 - T_{AC}\cdot\frac{y}{4} - T_{AD}\cdot\frac{y}{4} = 0$ $\quad T_{AC}=T_{AD}=T$

$65 - 2T\frac{y}{4}=0 \quad 65 = T\frac{y}{2} \quad 130 = Ty$

x: $T_{AD}\cdot\sqrt{4-\frac{y^2}{4}} - T_{AC}\cdot\sqrt{4-\frac{y^2}{4}}=0$

FBD C: x: $-40\cdot\left(1.5+\left(2-\sqrt{4-\frac{y^2}{4}}\right)\right) + 2T\cdot\frac{\sqrt{4-\frac{y^2}{4}}}{2}=0$

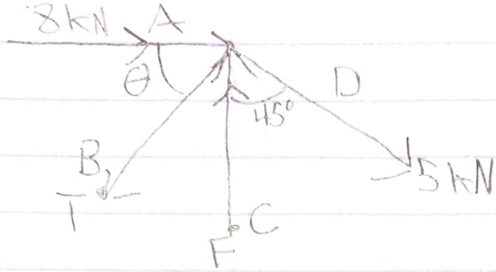
$T\cdot\sqrt{4-\frac{y^2}{4}} = 60+80-40\cdot\sqrt{4-\frac{y^2}{4}}$

$T=\frac{140}{\sqrt{4-\frac{y^2}{4}}}-40 \quad T=\frac{130}{y}$

$\frac{130}{y}=\frac{140}{\sqrt{4-\frac{y^2}{4}}}-40$ Solving for y

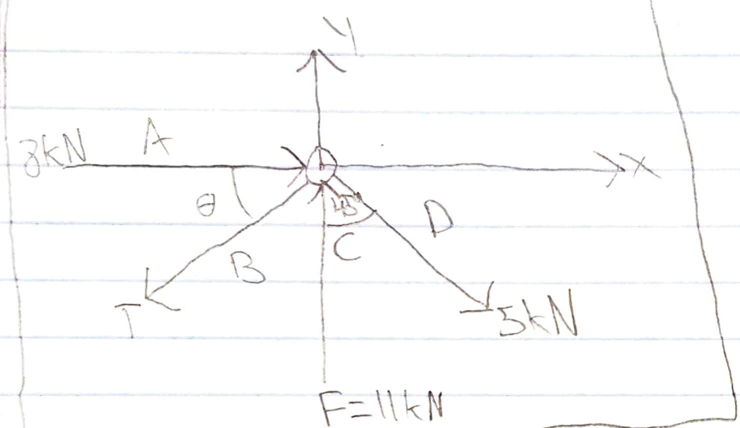
We get $\boxed{y=2.55ft}$

6. Given:



a) Find: FBD at O

FBD at O:



b) Find: Force in O for equilibrium.

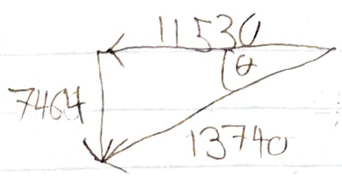
x: 8000 + 5000 · sin(45°) = 11535 N

y: 11,000 − 5000 · cos(45°) = 7464.5 N

∴ for equilibrium $T_x = -11535$ N & $T_y = -7464.5$ N

∴ T = 13740 N = $\boxed{13700 \text{ N or } 13.7 \text{ kN}}$

c) Find: θ



$\cos^{-1}\left(\frac{7464}{13740}\right) = 57°$

$\boxed{\theta = 57°}$

7. a) Given:

$4x_1 + 5x_2 + 3x_3 = -6$

$2x_1 + 6x_2 + 2x_3 = 12$

$2x_1 - x_2 + x_3 = 5$

Find: solution x_1, x_2, x_3

$$\begin{bmatrix} 4 & 5 & 3 & -6 \\ 2 & 6 & 2 & 12 \\ 2 & -1 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 & 2 & 12 \\ 2 & -1 & 1 & 5 \\ 4 & 5 & 3 & -6 \end{bmatrix} \quad \begin{array}{l} R_1 = \frac{R_1}{2} \\ R_2 = R_2 - R_1 \\ R_3 = R_3 - 2R_1 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 1 & 6 \\ 0 & -7 & -1 & -7 \\ 0 & -7 & -1 & -30 \end{bmatrix} \Rightarrow \boxed{\begin{array}{l} -7x_2 - x_3 = -7 \\ -7x_2 - x_3 = -30 \\ \text{therefore No Solution} \end{array}}$$

X contradiction, therefore No Solution

b) Given:

$2x_1 + 5x_2 + 3x_3 = -1$

$10x_1 + 30x_2 + 10x_3 = 1$

$30x_1 + 10x_2 + 30x_3 = -8$

Find: Solution x_1, x_2, x_3

$$\begin{bmatrix} 2 & 5 & 3 & -1 \\ 10 & 30 & 10 & 1 \\ 30 & 10 & 30 & -8 \end{bmatrix} \begin{array}{l} \\ R_2 = R_2 - 5R_1 \\ R_3 = R_3 - 15R_1 \end{array} \quad \begin{bmatrix} 2 & 5 & 3 & -1 \\ 0 & 5 & -5 & 6 \\ 0 & -65 & -15 & 7 \end{bmatrix} \begin{array}{l} R_1 = R_1 - R_2 \\ \\ R_3 = R_3 + 13R_2 \end{array}$$

$$\begin{bmatrix} 2 & 0 & 8 & -7 \\ 0 & 5 & -5 & 6 \\ 0 & 0 & -80 & 85 \end{bmatrix} \begin{array}{l} R_1 = R_1 + \frac{R_3}{10} \\ R_2 = R_2 - \frac{R_3}{16} \\ \\ \end{array} \begin{bmatrix} 2 & 0 & 0 & \frac{3}{2} \\ 0 & 5 & 0 & \frac{11}{16} \\ 0 & 0 & -80 & 85 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{11}{80} \\ 0 & 0 & 1 & -\frac{17}{16} \end{bmatrix}$$

$$\boxed{x_1 = 0.75 \quad x_2 = 11/80 \quad x_3 = -\frac{17}{16}}$$

c) Given: $-3x_1 + x_2 + 3x_3 = -6$

$3x_1 + 4x_2 + 2x_3 = 13$

$2x_1 + 3x_2 + 4x_3 = 10$

Find: Solution x_1, x_2, x_3

$$\begin{bmatrix} -3 & 1 & 3 & -6 \\ 3 & 4 & 2 & 13 \\ 2 & 3 & 4 & 10 \end{bmatrix} \begin{matrix} \\ R_2 = R_2 + R_1 \\ R_3 = 3R_3 + 2R_1 \end{matrix} \quad \begin{bmatrix} -3 & 1 & 3 & -6 \\ 0 & 5 & 5 & 7 \\ 0 & 11 & 18 & 18 \end{bmatrix} \begin{matrix} R_1 = 5R_1 - R_2 \\ \\ R_3 = 5R_3 - 11R_2 \end{matrix}$$

$$\begin{bmatrix} -15 & 0 & 10 & -37 \\ 0 & 5 & 5 & 7 \\ 0 & 0 & 35 & 13 \end{bmatrix} \begin{matrix} R_1 = R_1 - \frac{2}{7}R_3 \\ R_2 = R_2 - \frac{1}{7}R_3 \\ \end{matrix} \quad \begin{bmatrix} -15 & 0 & 0 & -\frac{285}{7} \\ 0 & 5 & 0 & \frac{36}{7} \\ 0 & 0 & 35 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{19}{7} \\ 0 & 1 & 0 & \frac{36}{35} \\ 0 & 0 & 1 & \frac{13}{35} \end{bmatrix} \quad \boxed{\begin{matrix} x_1 = \frac{19}{7} \\ x_2 = \frac{36}{35} \\ x_3 = \frac{13}{35} \end{matrix}}$$