Optimal Robustification of Linear Quadratic Regulator

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***Abstract*— This paper presents a robust control design for linear systems with matched external disturbances based on linear-quadratic regulator (LQR) and sliding mode control (SMC). The design includes a quadratic integral sliding manifold for which LQR is equivalent control, so it does not force the system to change its optimal dynamics during the transient response like in other commonly used state-of-the-art SMC design procedures. The simulation study suggests that the proposed control design robustifies optimal LQR with minimal deterioration of optimality.**

I. INTRODUCTION

The solution of optimal control problems for linear systems with a quadratic cost is based on the solution of the Algebraic Riccati Equation (ARE) which is easy to solve. The obtained solution forms a well-known linear-quadratic regulator (LQR) (see, e.g. [1]). Although LQR possess a certain level of robustness to model uncertainties assuming the states are fully measurable, it is still vulnerable when some states are required to be estimated and to external disturbances as well.

There are many techniques evolved over the past three decades aiming to address system uncertainties and external disturbances in control problems both for linear and nonlinear systems. Some of the most frequently utilized control techniques include robust control [2], nonlinear adaptive control [3], [4], model predictive control [5], [6], backstepping [7] and sliding mode control (SMC) [8].

The SMC has become widely popular in control community mostly because its design is simple and it is insensitive to matched disturbances once the system is on the sliding manifold. The early works on the SMC include [9]–[11] and since then, it has become a standard tool in the field of robust control. Although the SMC can have some disadvantages such as, it may require high control input to reach the sliding manifold, it may experience chattering phenomena during reaching phase, and it may have a non-optimal dynamics along an arbitrarily selected sliding manifold, there are plethora of work that have addressed these problems, see e.g. [8], [12] and references therein.

In order to improve dynamics of the system along sliding manifold, the optimal control theory was merged with the SMC to give rise to a new field of robust control known as optimal SMC [13]–[15]. Namely, the optimal SMC is usually constructed based on an integral sliding manifold [16]–[18] as a result of linear-quadratic optimization. The most relevant paper to our work [14] extends the system dynamics in accordance with a conventional integral sliding manifold and then solves for a linear-quadratic optimal problem to obtain the integrand function used to construct the manifold. Although the optimal SMC [14] effectively robustifies the optimal LQR, it requires solving two optimizations in parallel, one for the integrand function under a linear-quadratic optimization and one for the parameters used in the switching control term under a stochastic optimization. In addition, the optimal SMC [14] assumes a conventional integral sliding mode manifold and a priory given control form while designing the optimization, which prevents the optimal LQR from being equivalent control for the derived manifold. This further means that the resulting dynamics may significantly deviate from the LQR-optimal dynamics during the transient response.

We propose a robust control design for linear systems with matched external disturbances based on SMC with a quadratic integral sliding mode manifold, which ensures an optimal LQR is equivalent control during the transient response. Unlike other state-of-the-art SMCs, such a design does not force the system to change its LQR-optimal dynamics during the transient response. Consequently, the proposed approach allows for an effective LQR robustification.

In Section II, we provide a short theoretical background for LQR and integral SMC design procedures relevant to this paper. In Section III, we explain the proposed robust control design by means of sliding manifold choice and related control law construction. Section IV gives an insight on the role of design parameters, a simulation example and a statistical comparison with optimal LQR, conventional and optimal SMC. Section V concludes the work.

II. THEORETICAL BACKGROUND

A. Linear-Quadratic Regulator

Consider a class of continuous-time linear systems described by equations of the form

$$\dot{x} = Ax + Bu, \tag{1}$$

with state $x(t) \in \mathbb{R}^n$, input $u(t) \in \mathbb{R}^m$, $A \in \mathbb{R}^{nxn}$ and $B \in \mathbb{R}^{nxm}$. Assume that the pair (A,B) is stabilizable and the pair $(Q^{1/2},A)$ is detectable, where the cost function is

$$V(x_0,u) = \int_0^\infty (x^T Qx + u^T Ru)dt, \tag{2}$$

with Q being a positive semidefinite, while R a symmetric positive definite matrix.

Assuming that the optimal value function is of the form

$$V^*(x) = x^T Px, \tag{3}$$

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where $P = P^T$ is a positive definite solution of the Algebraic Riccati Equation (ARE)

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \tag{4}$$

The ARE is easily solvable and has a unique positive definite solution P. The optimal control action can be computed as

$$u_{lqr}(x) = -R^{-1}B^T Px \tag{5}$$

which is well-known as linear-quadratic regulator (LQR).

B. Integral Sliding Mode Manifold

For clarity and without loss of generality, consider the system (1) with a single input $u(t) \in \mathbb{R}$, that is

$$\dot{x} = Ax + Bu + \xi, \tag{6}$$

where $\Xi = [0 \ldots 0 \;\; \xi]^T$ is the matched external disturbance. Assume that the matrices A and B are given in a canonical form

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & \ldots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ a_1 & a_2 & \cdots & a_{n-1} & a_n \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \tag{7}$$

Then, the integral sliding mode control design is based on a dynamic sliding manifold

$$s(t) = x_n(t) - \int_0^t g(\tau)d\tau - x_n(0), \tag{8}$$

where $g(t)$ is a properly selected function of states. Unlike a conventional sliding model control, where the sliding manifold is given in a linear form of system states x, the integral sliding mode control avoids the reaching phase since the system is in sliding mode from the beginning.

The main idea behind the sliding mode control is to keep the system trajectories in ε-vicinity of the sliding manifold $s(t) = 0$, that is $|s(t)| < \varepsilon$, despite the presence of matched external disturbances. The control law is designed to consist two terms, the equivalent and switching control term. The equivalent control u_e ensures the nominal system trajectories stay along the sliding manifold when disturbance is not taken into account. The equivalent control satisfies $\frac{d}{dt}s(t;u_e) = 0$, $\forall t$, which guarantees s does not change in time. For the integral sliding mode control, the choice of the sliding manifold (8) ensures $s(0) = 0$, which provides the system is in sliding mode $\forall t$.

The role of switching control u_s is to force the system back onto the sliding manifold whenever the external disturbance moves the system from it. One choice of the switching control is $u_s = -M\sigma_\varepsilon(s)$, where

$$\sigma_\varepsilon(s) = \begin{cases} 1 & \text{if } s < \varepsilon \\ \frac{s}{\varepsilon} & \text{if } |s| \geq \varepsilon \\ -1 & \text{if } s > \varepsilon, \end{cases} \tag{9}$$

ε is the defined vicinity around the manifold, M is a constant such that $M > |\xi| + \rho$, and ρ is a small constant.

The role of parameter M is to ensure that the first derivative of a candidate Lyapunov function $V = \frac{1}{2}s^2$ is negative definite except for $|s| \leq \varepsilon$ which guarantees the system trajectories stay inside ε-vicinity of the manifold $s(t)$. For instance, a common form of integral sliding model control is

$$u_{ismc} = -\sum_{i=1}^{n} a_i x_i + g - M\sigma_\varepsilon(s), \tag{10}$$

which leads to

$$\dot{V} = s\dot{s} = s(\sum_{i=1}^{n} a_i x_i + u + \xi - g) = s(-M\sigma_\varepsilon + \xi) < -\rho s \sigma_\varepsilon \tag{11}$$

and guarantees $|s| \leq \varepsilon$. This means that the role of M is to predominate external disturbances whenever $|s| > \varepsilon$.

III. PROPOSED ROBUSTIFICATION OF LQR

A. LQR as Equivalent Control

Theorem 1: Consider the system without external disturbances (1) and its optimal control u_{lqr} (5) related to performance function (2). Then, u_{lqr} is equivalent control of the system (1) with respect to the dynamic sliding manifold

$$s(t) = x^T Px + \int_0^t (x^T Qx + u_{lqr}^T R u_{lqr} - M_2 s)dt - x_0^T Px_0 = \bar{s} \tag{12}$$

where $\bar{s} = 0$, P is the solution to the ARE (4) and M_2 a selected positive constant.

Proof: In accordance to Section II-B, the equivalent control u_e with respect to any given manifold $s(t)$ is the control for which $\frac{d}{dt}s(t;u_e) = 0$ holds $\forall t$. For the selected manifold, we have

$$\begin{aligned} \frac{d}{dt}s(t;u_e) &= \dot{x}^T Px + x^T P\dot{x} + x^T Qx + u_{lqr}^T R u_{lqr} - M_2 s = \\ &(A + Bu_{lqr})^T Px + x^T P(A + Bu_{lqr}) + x^T Qx + u_{lqr}^T R u_{lqr} - M_2 s. \end{aligned} \tag{13}$$

By taking now u_{lqr} in the form (5), one can easily obtain

$$\frac{d}{dt}s(t;u_e) = x^T(A^T P + PA - PBR^{-1}B^T P + Q)x - M_2 s, \tag{14}$$

where the first term is equal to zero by the assumption that the matrix P is the solution to ARE (4). This leads to the expression of sliding manifold dynamics

$$\frac{d}{dt}s(t;u_e) = -M_2 s(t;u_{lqr}), \tag{15}$$

from where $\frac{d}{dt}s(t;u_e) = 0$ holds for $\forall t$ if the initial condition of (15) is $s(0;u_{lqr}) = 0$, meaning that the trajectories of the system stay on the manifold $s(t) = 0$ when $u_e = u_{lqr}$ for all t, which completes the proof. ■

Theorem 1 affords a conclusion that (12) is the inherent sliding manifold for the LQR and the system without external disturbances. This means that it is not necessary to force the system to slide a priory selected sliding manifold by reducing and/or changing system dynamics as it is usually the case with other state-of-the-art design procedures. It is also worth noting that the manifold is given in a quadratic form and it can be considered a quadratic integral sliding manifold.

Furthermore, the dynamics of the manifold s is stable for other initial conditions of (15) as well, and s will ultimately converge to zero letting u_{lqr} become the equivalent control. Even in case $M_2 = 0$, one obtains $\frac{d}{dt}s(t;u_{lqr}) = 0$ for $\forall t$. However, a role of the term $M_2 s$ is important in presence of disturbances and has a different influence on the sliding manifold than the constant M used for the switching part of the robustifying control.

B. Robustifying Switching Control

Theorem 2: Consider the system with matched disturbances (6) and its nominal optimal control u_{lqr} (5) related to performance function (2). Then, any form of the switching control law

$$u_s = \{sgn(u_{lqr})\frac{M_1}{\varepsilon}s,\; u_{lqr}\frac{M_1}{\varepsilon}s\} \tag{16}$$

robustifies u_{lqr} with respect to the quadratic integral sliding manifold (12), where the resulting robust control law is used in the form

$$u_r = u_{lqr} + u_s \tag{17}$$

and the constant M_1 is selected to be sufficiently large to predominate the matched disturbances.

Proof: Starting from (12), we obtain

$$\begin{aligned}&\frac{d}{dt}s(t;u_r) = \dot{x}^T Px + x^T P\dot{x} + x^T Qx + u_{lqr}^T Ru_{lqr} - M_2 s = \\ &(A + Bu_r + B\xi)^T Px + x^T P(A + Bu_r + B\xi) + x^T Qx + u_{lqr}^T Ru_{lqr} \\ &- M_2 s,\end{aligned} \tag{18}$$

which yields

$$\begin{aligned}&\frac{d}{dt}s(t;u_r) = (A + Bu_{lqr})^T Px + x^T P(A + Bu_{lqr}) + x^T Qx \\ &+ u_{lqr}^T Ru_{lqr} + (Bu_s)^T Px + x^T P(Bu_s) + (B\xi)^T Px + x^T PB\xi \\ &- M_2 s.\end{aligned} \tag{19}$$

The first four terms constitute the left-hand side of ARE (4) which is zero for all t, yielding

$$\begin{aligned}&\frac{d}{dt}s(t;u_r) = u_s^T RR^{-1}B^T Px + x^T PBR^{-1}Ru_s + (B\xi)^T Px + x^T PB\xi \\ &- M_2 s = -u_s Ru_{lqr} - u_{lqr}Ru_s - \xi Ru_{lqr} - u_{lqr}R\xi - M_2 s \\ &= -2(u_s + \xi)Ru_{lqr} - M_2 s.\end{aligned} \tag{20}$$

Let now a Lyapunov function candidate be $V = \frac{1}{2}s^2$. Then its time derivative is

$$\dot{V} = s\dot{s} = -2s(u_s + \xi)Ru_{lqr} - M_2 s^2, \tag{21}$$

which, by taking one form of the switching control (16), becomes

$$\dot{V} = -2s(sgn(u_{lqr})\frac{M_1}{\varepsilon}s + \xi)Ru_{lqr} - M_2 s^2 \tag{22}$$

and

$$\dot{V} = -2s(u_{lqr}\frac{M_1}{\varepsilon}s + \xi)Ru_{lqr} - M_2 s^2, \tag{23}$$

for the first and the second form of switching control, respectively.

To explain the roles of parameters ε, M_1 and M_2 in (22) and (23), we need to consider three stages separately, 1) the transient interval in which u_{lqr} has a dominant role and guides the system towards equilibrium state; 2) the period in which s manifold approaches a non-zero value which happens immediately after the end of transient response; and 3) the steady-state interval where the switching control u_s increases its role in suppressing the influence of external disturbances on the system states.

Stage 1: During the transient period where $u_{lqr} \neq 0$ and $|u_{lqr}| \gg 0$ can be most of the time, the idea of using M_1 and ε is the same as in classical sliding mode control where the switching control is supposed to predominate disturbance ξ. Namely, since the aim of the robustification is to keep the trajectories of the system in ε-vicinity of the sliding manifold (12), it is then important to ensure the first term of (22) (or 23) is negative for $|s| \geq \varepsilon$. The idea is to select M_1 to be a positive constant such that

$$sgn(u_{lqr})M_1 Ru_{lqr} \geq |\xi Ru_{lqr}| + \rho, \tag{24}$$

and

$$u_{lqr}M_1 Ru_{lqr} \geq |\xi Ru_{lqr}| + \rho, \tag{25}$$

for the first and the second form of switching law, respectively, where ρ is a small positive constant, which leads to

$$\dot{V} = -2\rho\frac{s^2}{\varepsilon} - M_2 s^2 \leq 0. \tag{26}$$

Note for $u_{lqr} \neq 0$, the condition (24) can be written in the form $M_1 \geq |\xi| + \rho$ as in conventional integral SMC. Knowing that the system trajectories start from $s(0) = 0$, this means that they will never leave the ε-vicinity of the manifold $s(t) = 0$ during the transient period. It is worth noting that M_2 is selected to be a small constant in order to allow the first term of (22) (or 23) being dominate for this part of response. However, the term $-M_2 s$ additionally supports the dominance over disturbance ξ in (22) and (23).

Stage 2: Immediately after the transient period is completed, neither of the terms in (22) (or 23) dominates since $s \approx 0$ and $u_{lqr} \approx 0$ holds. During this stage, it is not possible to conclude negative definiteness of $\dot{V}$ even when $|s| > \varepsilon$, so it will not be possible to guarantee the systems trajectories stay around $s(t) = 0$. However, one can see from (12) that $s(t)$ increases until the balance between $x^T Qx + u_{lqr}^T Ru_{lqr}$ and $-M_2 s$ is achieved. Namely, at the beginning of the second stage when s is still close to zero, the second term in (12) has already achieved the value of $x_0^T Px_0$ which is then canceled out by the last term. Moreover, the second term continues to increase due to its quadratic shape and variation of x caused by disturbances. The stage 2 is completed when $s(t)$ reaches the new value $\bar{s}$ for which the balance is achieved in accordance to the stable linear dynamics (20). The larger M_2, the faster transient manifold dynamics is. On the other hand, it is desirable to select M_2 to be sufficiently small to decrease its influence on the switching control during the transient response (stage 1).

Stage 3: When $s(t)$ is converged to a new value $\bar{s}$, the first term of (22) (or 23) dominates again, ensuring $\dot{V} \leq 0$ and keeping trajectories of the system within ε-vicinity of the shifted manifold $s(t) = \bar{s}$. ■

IV. SIMULATION RESULTS

We provide simulation results by considering a double integrator system (27) and comparing the results obtained with the proposed control law (u_{rlqr}) with optimal LQR (u_{opt}), conventional SMC (u_{smc}) and optimal SMC (u_{osmc}) as in [14].

$$\dot{x}_1 = x_2, \;\; \dot{x}_2 = u + \xi, \;\; x_1(0) = 3, \;\; x_2(0) = 0 \qquad (27)$$

The cost function is

$$J = \frac{1}{2}\int_0^{\infty}(x^T Q x + Ru^2)dt, \qquad (28)$$

where $Q = diag(100, 100)$ and $R = 1$. The disturbance is a band-limited white noise whose maximum magnitude is 15. As stated in [14], the form of optimal, conventional SMC and optimal SMC controllers are respectively as follows

$$u_{opt} = -10x_1 - 10.9545x_2, \qquad (29)$$

$$u_{smc} = -2x_2 - 15\sigma_{\varepsilon}(2x_1 + x_2), \qquad (30)$$

$$\begin{aligned} u_{osmc} &= g - \frac{M}{\varepsilon}s, \qquad s = x_2 - \int_0^t g(\tau)d\tau \\ \dot{g} &= -1411.6577x_1 - 1556.3935x_2 - 152.1202g, \end{aligned} \qquad (31)$$

where $M = 18.6$ and $\varepsilon = 0.1$. As in [14], we perform the comparison within 20 sec simulation against the cost $I_1 = \frac{1}{2}\int_0^{20}(x^T Q x + Ru^2)dt$, which reflects how well a selected controller is suboptimal during the transient response, and the cost $I_2 = \frac{1}{2}\int_{10}^{20}(x^T Q x + Ru^2)dt$, which reflects the robustness level of steady-state behavior of the system.

For our approach, we obtain a slightly better results with the second form of switching control law (16). However, we provide the relevant results of what can be achieved with the first form as well. For clarity, we denote the two variants of the proposed control as u^I_{rlqr} and u^{II}_{rlqr}, respectively. We now give an insight how the parameters M_1 and M_2 can be selected.

We first select M_2 and examine the cost function for different values of M_1 when the second form of switching control (16) is used. Fig. (1) shows how the costs I_1 and I_2 change with $M_\varepsilon = \frac{M_1}{\varepsilon}$ for a fixed value $M_2 = 0.2$ and one realization of disturbance. We compare the results obtained with u_{osmc} (31). One can see that there exists an interval $M_\varepsilon \in (60, 80)$ for which the proposed approach is better than u_{osmc} with respect to both I_1 and I_2. However, the interval where I_1 is better is even larger, while I_2 is still acceptable especially in comparison with the level of robustness obtained with u_{opt} for which $I_2 = 2.76$.

Fig. (2) shows the same two diagrams when the first form of switching control (16) is used with arbitrarily selected $M_2 = 0.1$. Although there is no interval in this case for M_ε for which the proposed control is better in both costs, there

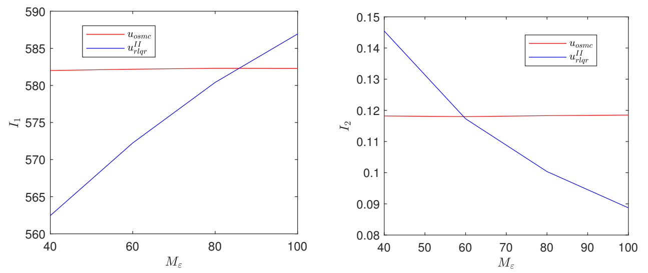

Fig. 1: Change of the costs I_1 (left) and I_2 (right) with respect to M_ε and comparison with u_{osmc}. All results are obtained with one realization of disturbance ξ, $\varepsilon = 0.1$, $M_2 = 0.2$ using u^{II}_{rlqr} (16).

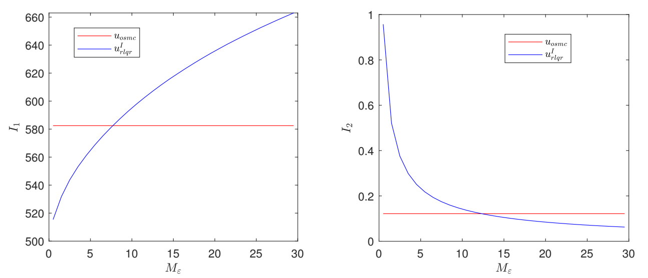

Fig. 2: Change of the costs I_1 (left) and I_2 (right) with respect to M_ε and comparison with u_{osmc}. All results are obtained with one realization of disturbance ξ, $\varepsilon = 0.1$, $M_2 = 0.1$ using u^I_{rlqr} (16).

is a large interval for M_ε for which the suboptimality level is much better (Fig. 2-left), while robustness level is slightly worse (Fig. 2-right) than in case u_{osmc} (31) is used. However, robustness still significantly improves with respect to the case when u_{opt} is used for which $I_2 = 2.76$.

In order to illustrate the influence of the parameter M_2 on the final result, we show the change of the costs I_1 and I_2 with respect to M_2 when M_1 is fixed ($M_1 = 1$) and the first form of switching control law (16) is used. Fig. (3) suggests that for a significant small value M_2, the cost I_1 worsen relative to u_{osmc}. For a larger value $M_2 \leq 1$, the cost I_2 can become larger that the one obtained with u_{osmc}, but it is still significantly smaller than the cost I_2 obtained with u_{opt}. Fig. (3) also indicates that any value $M_2 > 0.3$ will be a good choice to obtain a superior suboptimality level with respect to u_{osmc}, together with an acceptable robustness potential with respect to the optimal control.

From the conducted simulations based on both forms of switching control (17), the results suggest that for M_2, which is selected to be sufficiently small, one can easily find M_1 to form the final control law which provides good suboptimality level and significantly improved robustness with respect to optimal control as well. For illustration purposes, we select $M_1 = 7$ and $M_2 = 0.2$ for u^{II}_{rlqr}, while $M_1 = 1$ and $M_2 = 1$ for u^I_{rlqr}.

Figs. (4-8) illustrate the shapes of system states x_1, x_2, control and manifold s over time for all five types of

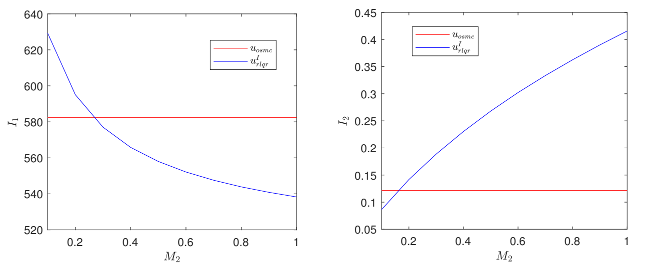

Fig. 3: Left: Change of the cost I_1 (left) and I_2 (right) with respect to M_2 and comparison with u_{osmc}. All results are obtained with one realization of disturbance ξ, $\varepsilon = 0.1$, $M_1 = 1$ and using u^I_{rlqr} (16).

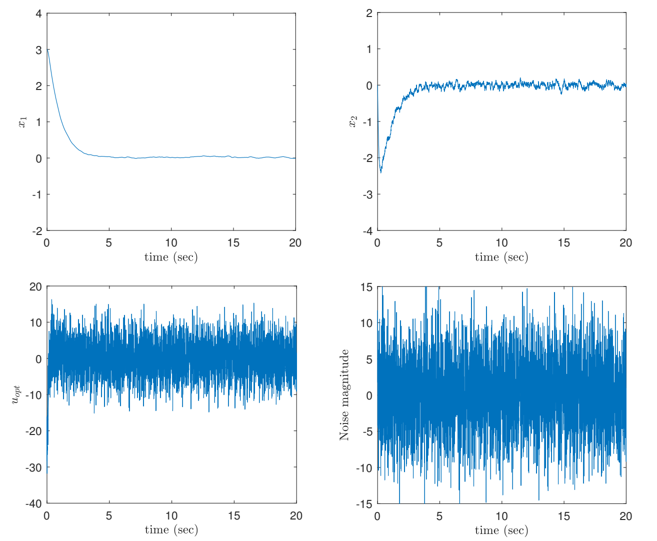

Fig. 4: System states x_1, x_2 and control signal u_{opt} generated by optimal control. The last diagram represents one realization of disturbance ξ.

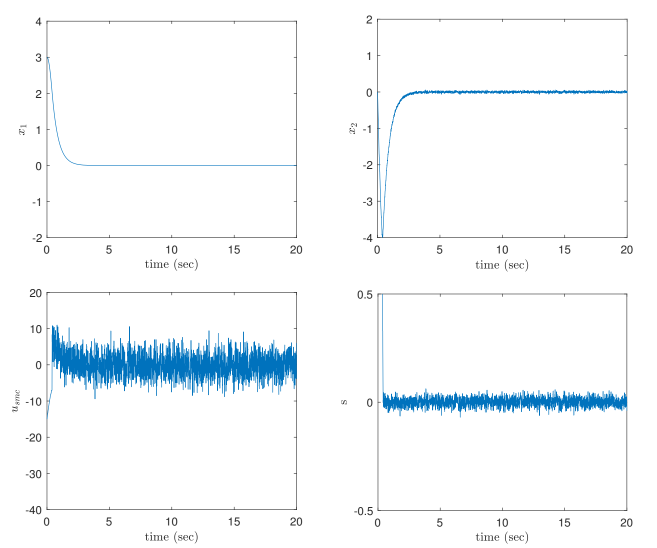

Fig. 5: System states x_1, x_2, control signal u_{smc} and sliding manifold s related to conventional SMC.

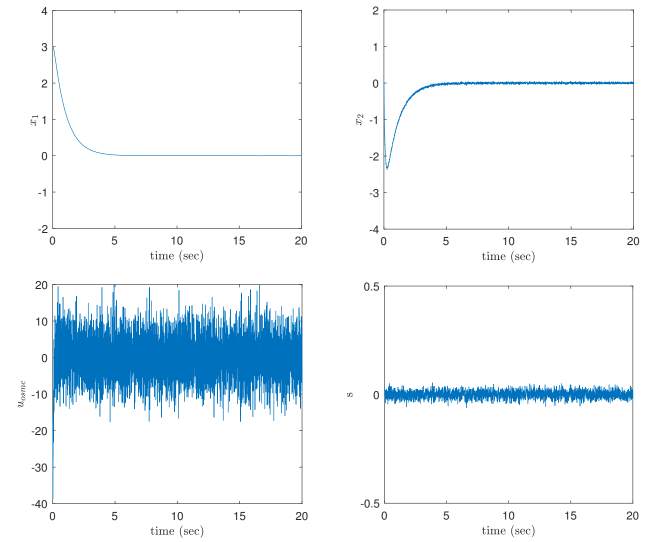

Fig. 6: System states x_1, x_2, control signal u_{osmc} and sliding manifold s related to optimal SMC.

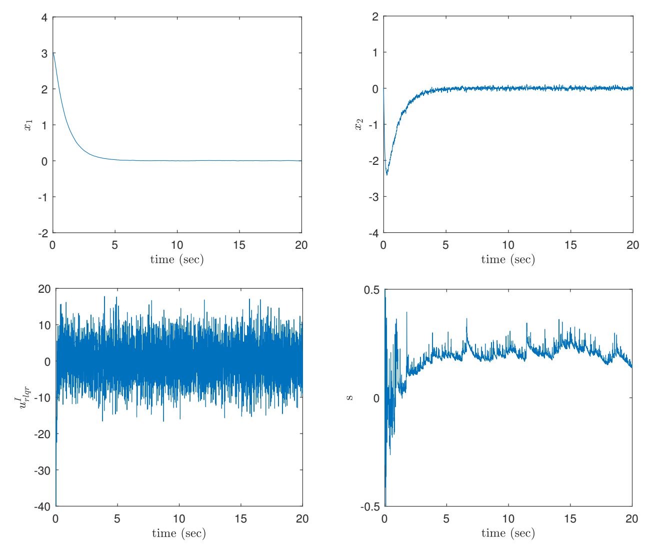

Fig. 7: System states x_1, x_2, control signal u^I_{rlqr} and sliding manifold s related to the proposed approach.

considered control laws and a realization of disturbance ξ as well. Unlike the control u_{smc}, it can be seen that states and control signal of the control laws u_{osmc}, u^I_{rlqr} and u^{II}_{rlqr} are similar to u_{opt} during the transient time. One can also observe that u_{opt} generates the largest variations in states x_1 and x_2, especially during the steady-state period. Although u^I_{lqr} has slightly worse variations with respect to u_{smc}, u_{osmc} and u^{II}_{rlqr}, we will see that it generates much better cost I_1 than these three control laws. As stated in Section III, one can see from the shape of s of the proposed approach (Figs. 7 and 8), that s is shifted from the manifold generated during the transient period, that is $s = 0$, to the manifold generated during the steady-state period, $s = \bar{s}$.

As in [14], we conduct several realizations of external

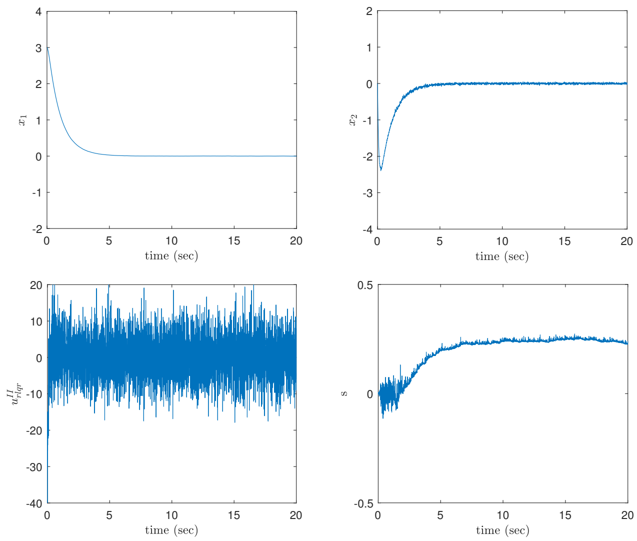

Fig. 8: System states x_1, x_2, control signal u^{II}_{rlqr} and sliding manifold s related to the proposed approach.

TABLE I: Statistical comparison.

	mean I_1	std I_1	mean I_2	std I_2
u_{opt}	505,04	2,38	2,88	0,422
u_{smc}	647,54	4,18	0,15	0,008
u_{osmc}	579,20	3,86	0,12	0,006
u^{I}_{rlqr}	544,07	3,68	0,37	0,018
u^{II}_{rlqr}	575,21	4,30	0,11	0,003

disturbances in order to compare costs I_1 and I_2 for all considered control strategies. Table I shows the statistical results based on 30 simulations. It can be concluded from the statistics related to I_1 that the proposed control strategy perform better than u_{smc} and u_{osmc} during the transient response, especially u^{I}_{lqr} which is significantly superior. During the steady-state period, the results indicate that the proposed approach is also capable to effectively robustify the optimal LQR. The proposed control approach u^{II}_{lqr} provides the highest robustness potential, making it the best control choice in terms of both costs I_1 and I_2, while u^{I}_{lqr} is slightly worse in I_2 in comparison with u_{smc} and u_{osmc}, but still much superior to u_{opt}.

V. CONCLUSIONS

In this paper, a new robust control design for linear systems with matched external disturbances is proposed. The design is based on SMC, where the sliding manifold is in a quadratic integral form which ensures the optimal LQR is equivalent control. This means that the proposed strategy does not change the optimal system dynamics during the transient response as it is the case with other SMC laws.

For the future work, we plan to utilize our recently published result on constructing an optimal control law for nonlinear systems based on policy iteration algorithm [19], [20]. The proposed strategy does not require solving the Hamilton–Jacobi–Bellman equation, i.e., a nonlinear partial differential equation, which is known to be hard or impossible to solve, and it solves optimal nonlinear control problems in an asymptotically exact, yet still linear-like manner. Similarly, it will be possible to extend the present work on finding a dynamic sliding manifold for which LQR is equivalent control for linear systems, to a linear-like procedure for finding a sliding mode control of nonlinear systems based on policy-iteration algorithm.

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REFERENCES

[1] D. Bertsekas, Dynamic programming and optimal control. Vol. 1. No. 2. Belmont, MA: Athena scientific, 1995.
[2] K. Zhou and J.C. Doyle. Essentials of robust control. Vol. 104. Upper Saddle River, NJ: Prentice hall, 1998.
[3] K.J. Aström and B. Wittenmark. Adaptive control. Courier Corporation, 2013.
[4] A. Astolfi, D. Karagiannis and R. Ortega, Nonlinear and adaptive control with applications. Springer Science and Business Media, 2007.
[5] E.F. Camacho and C.B. Alba, Model predictive control. Springer science and business media, 2013.
[6] A. Bemporad and M. Morari. "Robust model predictive control: A survey". Robustness in identification and control. Springer, London, 1999.
[7] F. Ikhouane and M. Krstic. "Robustness of the tuning functions adaptive backstepping design for linear systems". IEEE Transactions on Automatic Control, 1998
[8] V. Utkin, Sliding modes in control and optimization. Springer Science and Business Media, 2013.
[9] B. Drazenovic (Perunicic), "The invariance conditions in variable structure systems". Automatica, 1969
[10] S.V. Emel'yanov, "Theory of systems with variable structure". Nauka (In Russian), 1970
[11] V. Utkin, "Variable structure systems with sliding modes". IEEE Transactions on Automatic control, 1977
[12] M. Das and C. Mahanta. "Optimal second order sliding mode control for linear uncertain systems". ISA transactions, 2014
[13] K. D. Young and U. Ozguner. "Sliding-mode design for robust linear optimal control". Automatica, 1997
[14] R. Xu and Umit Ozguner. "Optimal sliding mode control for linear systems". IEEE International Workshop on Variable Structure Systems, 2006.
[15] HP. Pang and GY Tang. "Global robust optimal sliding mode control for a class of uncertain linear systems". IEEE Chinese Control and Decision Conference, 2008.
[16] Y. Niu, D.W.C Ho, and J. Lam. "Robust integral sliding mode control for uncertain stochastic systems with time-varying delay". Automatica, 2005
[17] F. Castaños and L. Fridman. "Analysis and design of integral sliding manifolds for systems with unmatched perturbations". IEEE Transactions on Automatic Control, 2006
[18] A. Ferrara and G. P. Incremona. "Design of an integral suboptimal second-order sliding mode controller for the robust motion control of robot manipulators". IEEE Transactions on Control Systems Technology, 2015
[19] A. Tahirović and A. Astolfi. "Linear-like policy iteration based optimal control for continuous-time nonlinear systems". IEEE Transactions on Automatic Control, 2022.
[20] A. Tahirović and A. Astolfi. "Optimal control for continuous-time nonlinear systems based on a linear-like policy iteration". IEEE 58th Conference on Decision and Control (CDC), 2019.