Deepak Kumar Singh.

Date 10/08/17

UNIT - I

Set theory, Relations and functions.

Set is a collection of element.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A - B = \{x : x \in A, x \notin B\}$

$B - A = \{x : x \in B, x \notin A\}$

Cartesian Product

$A \times B = \{(x, y) ; x \in A, y \in B\}$
of A & B

$A' \equiv A^c \equiv C(A)$
complement of A

* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
* $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Let $x \in A \cap (B \cup C) \Leftrightarrow x \in A$ and $x \in (B \cup C)$
$\Leftrightarrow x \in A$ and $x \in B$ or $x \in C$
$\Leftrightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \in C$
$\Leftrightarrow x \in (A \cap B)$ or $x \in (A \cap C)$
$\Leftrightarrow x \in (A \cap B) \cup (A \cap C)$

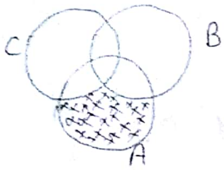
② $A-(B\cup C) = (A-B)\cap(A-C)$

Let $x \in A-(B\cup C) = x\in A;\ x\notin B\cup C'$

$\Rightarrow x\in A$ and $x\notin B$ or $x\notin C$

$\Rightarrow x\in (A-B)$ and $x\in A, x\notin C$

$\Rightarrow x\in (A-B)\cap(A-C)$



※ $A-(B\cup C)$

$A\times B = \{(x,y);\ x\in A, y\in B\}$

* $A\times(B\cup C) = (A\times B)\cup(A\times C)$

Proof – Let $x,y \in A\times(B\cup C)$

$\Rightarrow x\in A$ and $y\in(B\cup C)$

$\Rightarrow x\in A, y\in B$ or $y\in C$

$\Rightarrow x\in A, y\in B$ or $x\in A, y\in C$

$\Rightarrow (x,y)\in A\times B$ or $(x,y)\in A\times C$

$\Rightarrow (A\times B)\cup(A\times C)$

Cardial number of A

$n(A)$ = No. of elements in A

Power set of set A is denoted by $P(A)$.

$A = \{a,b,c\}$

$P(A) = [\phi, \{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},$
$\{a,b,c\}]$

$n(A) = m$

$n[P(A)] = 2^m$

$(1+x)^n = \{x^n + {}^nC_1 x^{n-1} + {}^xC_2 x^{n-2} + \dots + {}^nC_n \cdot 1^n\}$

$(1+1)^n = \{1+1+\dots (1)^n\}$

Qr If A and B are two finite sets having common number m_1 and m_2 respectively Then, If $n\{P(A)\}$ is 56 more than $n[P(B)]$ Then find the values of m_1 and m_2.

Sol:→ $n[P(A)] = m_1$, $n[P(B)] = m_2$

A/Q $n(A) = m_1$; $n(B) = m_2$

$\Rightarrow n[P(A)] = n[P(B)] + 56$

$\Rightarrow 2^{m_1} = 2^{m_2} + 56$

$\Rightarrow 2^{m_1} - 2^{m_2} = 56$

$\Rightarrow 2^{m_2}[2^{m_1-m_2}-1] = 56$

$\Rightarrow 2^{m_2}(2^{m_1-m_2}-1) = 2^3 \times 7$ $(\because 56 = 8\times 7)$

$\Rightarrow 2^{m_2} = 2^3,\ 2^{m_1-m_2}-1 = 7$

$\Rightarrow 2^{m_2} = 2^3$

$\because m_2 = 3$

$\Rightarrow 2^{m_1-m_2}-1 = 7$

$\Rightarrow 2^{m_1-3} = 7+1$

$\Rightarrow 2^{m_1-3} = 8$

$\Rightarrow 2^{m_1-3} = 2^3$

bases are same, So we can equate the power.

$m_1 - 3 = 3$

$m_1 = 3+3$

$m_1 = 6$ Ans

Here, $m_1 = 6$, $m_2 = 3$

Complement of set :-

Complement of a set A is denoted by A' and is defined by $A' = (U-A)$ {Universal set U}

$x \in (U-A) \Rightarrow x \in U, x \notin A$

Let $A = \{1,2,3,4\}$, Universal set is set of natural number N.

Then,

$A' = U - A$

$A' = N - A$

$A' = \{5,6,7,8,\ldots\}$

$5 \in (N-A)$

$\Rightarrow 5 \in N,\ 5 \notin A$

$P = \{x, y, z, u\}$, $Q = \{y, u\}$

$P - Q = \{x, z\}$

$Q - P = \phi$

* $(A\cup B)' = A' \cap B'$

proof → Let $x \in (A\cup B)' \Rightarrow x \notin (A\cup B)$

$\Leftrightarrow x \notin A$ or $x \notin B$

$\Leftrightarrow x \notin A$ and $x \notin B$

$\Leftrightarrow x \in A'$ and $x \in B'$

$\Leftrightarrow x \in A' \cap B'$

5 element, 4 — order pain = 5x4 = 20

$A \times B = \{(x;y) : x \in A, y \in B\}$

$n(A) = m_1$

$n(B) = m_2$

$n[P(A)] = 2^{m_1}$

$n[P(B)] = 2^{m_2}$

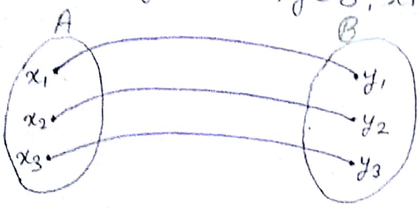
$n[A \times B] = m_1 \cdot m_2$

$n[P(A \times B)] = 2^{m_1 \cdot m_2}$

$(A \cup B)' = A' \cup B'$

$(A \cup B)' = A' \cap B'$

* $A \times B = \{(x, y) : x \in A, y \in B, xRy\}$



$R; A \to B$ $\qquad$ $R \subseteq A \times B$

↳ Subset

$\left.\begin{matrix} x_1 R y_1 \\ x_2 R y_2 \end{matrix}\right\}$ Subsets of $A \times B$

$R : A \to B$

1) Reflexive

2) Symmetric $\qquad$ $R \subseteq A \times B$

3) Transitive

① xRx

$\quad x = y \quad = \quad y = x$

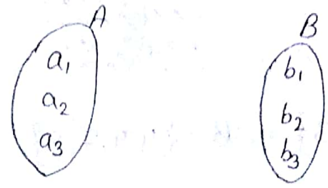
② $xRy \Rightarrow yRx$

③ $T \Rightarrow xRy, yRz$

$\Rightarrow xRz$

Reflexive, Symmetric, transitive all together called Equivalence Relation.

$R : A \to B$



Scanned by CamScanner

Q. If $A = \{p, q, r\}$ $B = \{7, 3, r\}$
find $\{A-B\} \cup \{B-A\}$

$\{A-B\} = \{p, q\}$

$\{B-A\} = \{7, 3\}$

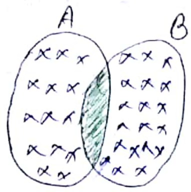
$\because \{p, q\} \cup \{7, 3\}$

$= \{p, q, 7, 3\}$

Symmetric Difference -

Symmetric difference between A and B is denoted by $A \Delta B$.

$$A \Delta B = (A \cup B) - (A \cap B)$$



$A = \{1, 1, 1, 2, 2, 3\}$, $B = \{4, 4, 7, 8\}$

$U(1) = 3$ $\quad$ $U(4) = 2$

Relations -

Let A and B be two non-empty sets, then any subject of $A \times B$ is called a relation from A to B.

$$R : A \to B$$

If Set A has m_1 elements and B has m_2 elements then,

$$A \times B = m_1 \cdot m_2$$

Ex:→ $A = \{1, 2, 5, 8\}$, $B = \{2, 4, 6\}$

$A \times B = \{(1,2), (1,4), (1,6), (2,2), (2,4), (2,6), (5,2), (5,4), (5,6), (8,2), (8,4), (8,6)\}$

$$\boxed{xRy \Rightarrow x < y}$$

$R \to$ '<'

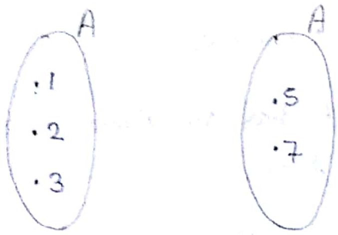
$R = \{(1,2), (1,4), (1,6), (2,4), (2,6), (5,6)\}$

Range of $R \overset{\to \text{Second}}{=} \{2, 4, 6\}$

Domain of R $d(R) = \{1, 2, 5\}$

Equivalence Relation :— A Relation of R from A to A " is less than or equal to"

$R : A \to A$



Reflexive $\longrightarrow$ $xRx \Rightarrow x \leq x$

Symmetric $\longrightarrow$ $xRy \Rightarrow yRx$

$x = y \not\Rightarrow y \leq x$

Transitive $\longrightarrow$ $xRy, yRz \Rightarrow xRz$

$x \leq y, y \leq z \Rightarrow x \leq z$

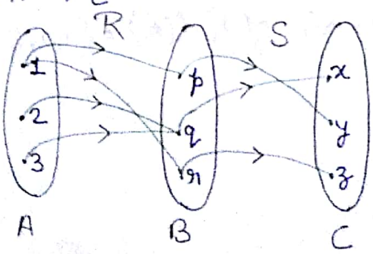
Composite Relation —

Let $R : A \to B$ and $S : B \to C$ be two relation then the composite of R and S, RoS then $RoS : A \longrightarrow C$

$A = \{1, 2, 3\}$

$B = \{p, q, r\}$

$C = \{x, y, z\}$



$RoS = \{(1,y), (1,z), (2,x), (3,x)\}$

Inverse Relation

$R = \{(a,b) ; a \in A, b \in B \text{ and } aRb\}$

$R^{-1} = \{(b,a) ; (a,b) \in R\}$

Ex $\rightarrow$

$A = \{1,2,3\}$

$B = \{7,8\}$

$R = \{(1,7), (1,8), (2,7), (2,8), (3,7), (3,8)\}$

$R^{-1} = \{(7,1), (8,1), (7,2), (8,2), (7,3), (8,3)\}$

Complement of a Relation $\longrightarrow$

Let $R : A \to B$ then $R = \{(x,y) : x \in A, y \in B\}$

Ex $\rightarrow$ If $A = \{a,b,c,d\}$, $B = \{7,8,9\}$

$A \times B = \{(a,7), (a,8), (a,9), (b,7), (b,8), (b,9), (c,7), (c,8), (c,9), (d,7), (d,8), (d,9)\}$

$R \subset A \times B$

$R \equiv A \times B$

$R \equiv R'$

$= \phi$

Scanned by CamScanner

If R is an equivalence Relation then R^{-1} is also an equivalence Relation.

Proof → Let R be an equivalence relation.

Then R is reflexive, Symmetric and transitive

1) Reflexive R: Let R be reflexive

$$xRx \Rightarrow (x,x) \in R$$

$$\Rightarrow (x,x) \in R^{-1}$$

$$\Rightarrow R^{-1} \text{ is reflexive.}$$

2) Symmetric R: Let R be Symmetric

$$\text{then } xRy = yRx$$

$$\Rightarrow (x,y) \in R = (y,x) \in R$$

$$\Rightarrow (y,x) \in R^{-1}$$

$$\Rightarrow (x,y) \in R^{-1}$$

$$\Rightarrow yR^{-1}x = xR^{-1}y$$

$\therefore R^{-1}$ is Symmetric.

3) Transitive R: Let R is transitive

then $\forall x,y,z \in X$, $xRy, yRz \Rightarrow xRz$

i.e... $yR^{-1}x, zR^{-1}y \Rightarrow zR^{-1}x$

i.e..., $zR^{-1}y, yR^{-1}x \Rightarrow zR^{-1}x$

i.e... $(z,y) \in R^{-1}, (y,x) \in R^{-1}$

$$\Rightarrow (z,y,z) \in R^{-1}$$

$\therefore R^{-1}$ is transitive.

from ①, ②, ③ Here we say that R^{-1} is equivalence Relation.

Theorem:—

If $R: A \to B$ and $S: B \to C$ be two relations then prove that

$$(SoR)^{-1} = R^{-1} o S^{-1}$$

proof:- Let $(c,a) \in (SoR)^{-1}$

$\Rightarrow (a,c) \in SoR$

$\Rightarrow$ $b \in B$ such that $(a,b) \in R, (b,c) \in S$

$\Rightarrow (b,a) \in R^{-1}, (c,b) \in S^{-1}$

$\Rightarrow (c,b) \in S^{-1}, (b,a) \in R^{-1}$

$\Rightarrow (c,a) \in S^{-1} o R^{-1}$

Qr If $X = \{1,2,3,4,5, \ldots\ldots\}$ and $R = \{(x,y): (x-y) \text{ is divisible by } 3\}$ then prove that R is an equivalence Relation.

xRy such that $(x-y)$ is divisible by 3.

① Reflexive property:— Let $R = \{(x,y): (x-y) \text{ is divisible by } 3\}$

Now, $x-x$ is obviously divisible by 3

$\Rightarrow xRx$

R is reflexive

Symmetric property:-

Let $xRy \Rightarrow (x-y)$ is divisible by 3.

$\Rightarrow (y-x)$ is divisible by 3.

$\Rightarrow yRx$

$\therefore$ R is symmetric.

Transitive property:-

Let $xRy, yRz \Rightarrow (x,y) \in R, (y,z) \in R, \forall$
$x,y,z \in X$

$\Rightarrow (x-y)$ is divisible by 3, $(y-z)$ is div. by 3.

$\Rightarrow x-y = 3K, y-z = 3l$

adding both,

$\Rightarrow x-\cancel{y}+\cancel{y}-z = 3K+3l$

$\Rightarrow x-z = 3(K+l)$

$\Rightarrow x-z$ is divisible by 3

$\Rightarrow xRz$

$\Rightarrow R$ is transitive

Therefore, R is an equivalence Relation

Function

$R = \{(x,y); x \in A, y \in B\}$

Polynomial function:-

If $f: R \to R$ is a function and

$f(x) = a_0 + a_1x + a_2x^2 + \text{------} + a_nx^n \quad n \in N, a \in R$

Rational function:-

$f: A \to R$ is said to be rational function

if $f(x) = \frac{P(x)}{x}$, $Q(x) \neq 0$ and

$P(x)$ and $Q(x)$ are polynomial function.

Real function:- A function $f: A \to B$ is said to be real valued function if f- image of $x \in A$ is real number.

Identify function:- If $f(x) = x, \forall x \in A$, then f is called Identify function.

Reciprocal function:- $f(x) = \frac{1}{x}, \forall x \in A, \frac{1}{x} \in B$

$D_f = R, \quad R_f = R - \{0\}$

Exponential function:-

$f(x) = e^x, x \in R, e^x \in R$

$D_f = R, R_f = (0, \infty)$

Greatest Integer function :—

$f(x) = [x]$, $[2] = 2$, $[-1] = -1$

$[2.1] = 3$ $[2.9] = 2$

$[x]$ = greatest integer not exceeding x.

Even function :— If $f(-x) = f(x)$, the f is even function.

Ex— $f(x) = x^2$, $f(-x) = (-x)^2 = x^2 = f(x)$

$f(x) = \cos x$

$f(x) = \cos(-x) = \cos x$

$f(-x) = f(x)$

Odd function :— $f(-x) = -f(x)$

Ex— 1. $f(x) = x^2$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

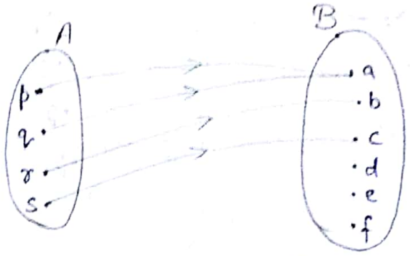
2. $f(x) = \sin x$

$= f(-x) = \sin(-x) = -\sin x$

$= -f(x)$

Into Mapping :—

Let $f: A \to B$ is a mapping then f is called into mapping. If at least one element of B is not f- image of any element of A.



$D_f = A$ $f: A \to B$

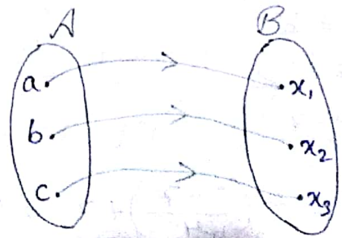
$R_f = \{a, b, c\}$

co-ordinate $f = B$

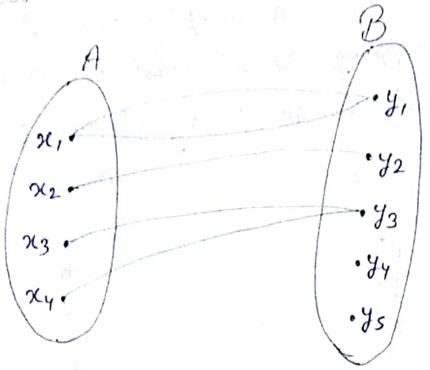
$R_f \subseteq B$

Onto Mapping :—

$f: A \to B$ is said to onto if every element of A has its distinct images in B.



Many-one into :-



Q: If $f: A \to B$ is such that $f(x) = 4x+7$ then prove that f is one-one and onto.

proof:- ① Let $f(x) = 4x+7$

Let $f(x_1) = f(x_2) \quad \forall x_1, x_2 \in A$

$\Rightarrow 4x_1 + \cancel{7} = 4x_2 + \cancel{7}$

$\Rightarrow \cancel{4}x_1 = \cancel{4}x_2$

$\Rightarrow x_1 = x_2$

f is one-one

f is $1-1$

② Let $f(x) = z$

$\Rightarrow 4x+7 = z$

$\Rightarrow x = \frac{z-7}{4}$

$f\left(\frac{z-7}{4}\right) = \cancel{4}\left(\frac{z-7}{\cancel{4}}\right)+7 = z$

f is onto.

Q: $f(x) = \sin x$

verify wheather f is one-one

Sol: $f(x) = \sin x$

then $f(x_1) = f(x_2) \Rightarrow \sin x_1 = \sin x_2$

$\Rightarrow \sin \pi = \sin 2\pi$

$\Rightarrow x_1 \neq x_2$

To find Range of a mapping

Q: Find range of mapping $f(x) = \frac{1}{2-\cos 3x}$

Sol: Let $f(x) = \frac{1}{2-\cos 3x} = y$

$\Rightarrow 2 - \cos 3x = \frac{1}{y}$

$\Rightarrow \cos 3x = 2 - \frac{1}{y}$ —— ①

$-1 \leq \cos 3x \leq 1$

$\Rightarrow -1 \leq 2 - \frac{1}{y} \leq 1$

$\Rightarrow -1 \leq 2 - \frac{1}{y}, 2 - \frac{1}{y} \leq 1$

$\Rightarrow \frac{1}{y} \leq 3, \ 1/y \geq 1$

$\Rightarrow y \geq 1/3, \ y \leq 1$

$y \in [\frac{1}{3}, 1]$

$R_f = [\frac{1}{3}, 1]$

π radian $= 180°$

$\frac{22}{7} r = 180°$

$r = \frac{180 \times 7}{22}$

$= \frac{630}{11}$

$= 57.27$

$\pi = \frac{\text{Circumference}}{D}$

Q. If $f(x) = x^2+3$, find $f^{-1}(19)$, $f^{-1}(39,52)$

Sol^n $f(x) = x^2+3 = y$

$\therefore x^2+3-19$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

Also, $x^2+3 = 39$

$x^2 = 36$

$x = \pm 6$

$x^2+3 = 52$

$\Rightarrow x^2 = 49$

$\Rightarrow x = \pm 7$

$f^{-1}(19) = \{-4, 4\}$

$f^{-1}(3,9,52) = \{-7,-6,6,7\}$

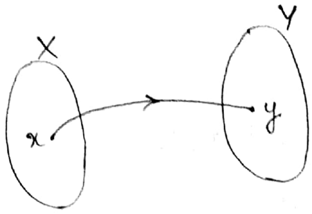
(1) If $f: R \to R$ such that $f(x) = x^2$ then find the Range R_f.

(2) If $X = \{-1, 1\}$ such that $f(x) = x^3$ then $\forall x \in$ find the range and domain of x.

(3) If $f(x) = -9x+19$ and find $f^{-1}(4,5,6)$.

Q. Find the range of the function

$$f(x) = \frac{x^2+2x+3}{x}$$



$y = f(x)$

Sol^n Let $y = \dfrac{x^2+2x+3}{x}$

$\Rightarrow x^2+2x+3 = xy$

$\Rightarrow x^2+(2-y)x+3 = 0$ ——— (1)

$\therefore (2-y)^2 - 4\times 3 \geq 0$

$\Rightarrow 4+y^2-4y-12 \geq 0$

$\Rightarrow y^2-4y-8 \geq 0$

$y = \dfrac{4 \pm \sqrt{16+32}}{2}$

$\Rightarrow \dfrac{4 \pm 4\sqrt{3}}{2}$

$\Rightarrow 2 \pm 2\sqrt{3}$

$y \in (-\infty, 2-\sqrt{3}) \cup (2+2\sqrt{3}, \infty)$

$R_f = R - (2-2\sqrt{3}) \cup (2+2\sqrt{3})$

Q. Find domain and range of

i) $f(x) = \frac{1}{x-3}$ (ii) $g(x) = \sqrt{9-x^2}$

iii) $f(x) = \frac{1}{1-x^2}$

Sol. $f(x) = \frac{1}{x-3}$ is not defined at $x = 3$

$D_f = R - \{3\}$

$R_f = ?$

Let $\frac{1}{x-3} = y$

$\Rightarrow x - 3 = 1/y$

$\Rightarrow x = 3 + 1/y$

$\Rightarrow x = \frac{3y+1}{y}$

$R_f = R - \{0\}$

(ii) $g(x) = \sqrt{9-x^2}$

$9 - x^2$ must not be −ve

$\Rightarrow 9 - x^2 \geq 0$

$\Rightarrow x^2 \leq 9$

$\Rightarrow x \leq 3$

$\Rightarrow x \geq -3$

$\Rightarrow x \in [-3, 3]$

$D_g = [-3, 3]$

$R_g = [0, 3]$

(iii) $n(x) = \frac{1}{1-x^2}$

Sol. $n(x) = \frac{1}{1-x^2}$ is not defined at $x^2 = 1$

i.e. not defined at $x = \pm 1$

$D_n = R - [-1, +1]$

$R_n = ?$

$y = \frac{1}{1-x^2} \Rightarrow 1 - x^2 = 1/y$

$\Rightarrow x^2 = 1 - \frac{1}{y}$

$\Rightarrow x = \sqrt{1 - 1/y}$, $y = 0$

$\Rightarrow 1 - \frac{1}{y} > 0$ $\{1 - 1/y \not< 0\}$

$\Rightarrow 1/y < 1$

$\Rightarrow y > 1$

$R_n = \{y : 1 \leq y < \infty\}$

Theorem :—

If $f: x \to y$ and $g: y \to z$ be one-one and onto then gof is also one-one and onto and $(gof)^{-1} = f^{-1}og^{-1}$

proof :— Let f and g be one-one and onto

$\Rightarrow$ f^{-1} and g^{-1} are also 1-1 and onto, Also $y = fx$, $z = g(y)$, $\forall\ x \in X, y \in Y; z \in Z$.

To proof :— gof is 1-1 :—

Let $(gof)(x_1) = (gof)(x_2)$

$g[f(x_1)] = g[f(x_2)]$

$f(x_1) = f(x_2)$

$x_1 = x_2$

($\because$ f is one-one)

Hence, gof is one-one

$\Rightarrow g(y_1) = g(y_2)$

$\Rightarrow y_1 = y_2$

($\because$ g is one-one)

To prove :— gof is onto :—

$(gof)(x) = g[f(x)] = g(y) = z$

$\forall\ x \in X, y \in Y, z \in Z$.

$\therefore$ gof is onto.

To prove :— $(gof)^{-1} = f^{-1}og^{-1}$

$y = f(x)$, $g(y) = z$

$f^{-1}(y) = x$, $g^{-1}(z) = y$

$(gof)(x) = g[f(x) = g(y)] = z$

$\Rightarrow (gof)^{-1}(z) = x$ —— (1)

$(f^{-1}og^{-1})(z) = f^{-1}[g^{-1}(z)] = f^{-1}(y) = x$ —— (2)

$(gof)^{-1}(z) = (f^{-1}og^{-1})(z)$

$\Rightarrow (gof)^{-1} = f^{-1}og^{-1}$

UNIT - II

Date
11/8/17

Algebric Structure
(Groups, Rings and field)

Algebric Structure:— A non-empty set G equipped with one ore more binary operator is called an algebraic structure.

$(G,*)$ $(G*,0)$ $(R,+,.)$ etc.

1. **Groupoid:**— A non empty set G with b.o. '$*$' is said to be a group of Groupoid G is closed under b.o. '$*$'.

i.e. closure prop. holdsgood.

i.e. $\forall a,b \in G, a*b \in G$

Algebric Structure parts:—

1. Groupoid
2. Semi-group
3. Monoid
4. Group
5. Com. group
6. Ring
7. Field

2. **Semi Group:**— A non-empty set G with b.o.'$*$' is said to be a semi-group $(G,*)$

If $\forall a,b \in G, a*b \in G$

i.e. closure property holds good.

$\forall a,b,c \in G$ $(a*b)*c = a*(b*c)$

i.e. associative P. holds good.

Ex: $(N,+)$ is a semi group.

3. **Monoid:**— A semi group $(M,*)$ is said to be a monoid if $\exists$ an identity $e \in M$ s.t.

$a*e = e*a = a, \forall a \in M$

or

An Algebric Structure (A.S) $(M.*)$ is called a monoid if following proctulete all salisfying→

i) CP $\forall a,b \in M, a*b \in M$

ii) Associativity:— $\forall a,b,c \in M, (a*b)*c = a*(b*c)$

iii) $\exists$ an element $e \in M$ s.t. $e*a = a*e = a$, $\forall a \in M$, e is called identity element.

4. **Group:**— An A.S. $(G,*)$ is said to be a group (with respect to binary operation) w.r.t. b.o. '$*$' if following postuates are satisfied:—

i) C.P. → $\forall a,b \in G, a*b \in G$

ii) A.P. $\forall a,b,c \in G$

$(a*b)*c = a*(b*c)$

iii) $\exists$ an element e s.t.

$a * e = e * a = a, \forall a \in G$

iv) $\exists$ an element a^{-1} in G s.t.

$a * a^{-1} = a^{-1} * a = e, \forall a \in G$

5. Comutative Group:— If in a group the b.o. $*$ satisfies the condition

$a*b = b*a$

$\forall a, b \in G$, then

then, it is called a C.G.

Ex:- $(P(S), \cup)$ is a monoid.

(Power set union)

1. C.P. :— $\forall A, B \in P(S)$

$A \cup B \in P(S)$

2. A.P. :— $\forall A, B, C \in P(S)$

$(A \cup B) \cup C = A \cup (B \cup C)$

3. $\exists$ an element E in $P(S)$ such that

$A \cup \phi = \phi \cup A = A, \forall A \in (S)$

Ex:- $(Q, +)$ is a group, where Q is set of rational number.

CP (Closure Property)

i) CP, $\forall \frac{p_1}{q_1}, \frac{p_2}{q_2} \in Q \in Q$

ii) A.P. $\forall \frac{p_1}{q_1}, \frac{p_2}{q_2}, \frac{p_3}{q_3} \in Q$

$$\left(\frac{p_1}{q_1} + \frac{p_2}{q_2}\right) + \frac{p_3}{q_3} = \frac{p_1}{q_1} + \left(\frac{p_2}{q_2} + \frac{p_3}{q_3}\right)$$

iii) $\exists$ identity element, $\frac{0}{1}$ such that

$\forall \frac{p}{q} \in Q, \frac{p}{q} + \frac{0}{1} = \frac{0}{1} + \frac{p}{q} = \frac{p}{q}$

iv) $\exists -\sigma = \frac{-p}{q}$ s.t. $\forall p/q \in Q$

$p/q + (p/q) = (-p/q) + p/q = \frac{0}{1}$

Q:- $<\{1,-1\}, \cdot>$ is a group.

i) CP:- $\forall$ $x, y \in (-1,1) = G$

$x.y \in G$

ii) Asso. :- Z elements are there in G.

Associativity holds good.

iii) Existance of Identity Element :-

$1 \in G$ is identity element in G

$1.1 = 1$

$1.(-1) = -1$

iv) Inverse :- $1.1 = 1 \Rightarrow a.a^{-1} = e$

is inverse of 1.

$(-1)(-1) = 1$

inverse element $= e$

-1 is inverse of -1

$1.(-1) = (-1)1 = -1$ $(G, \cdot)$ is an obelive group.

v) Communicative :-

$G = \{1, \omega, \omega^2\}$

$\{G, \cdot\}$ is a group

$1, \omega, \omega^2$ are three cube roots of 1.

CP: $\forall$ $x, y \in G$, $x.y \in G$

$1.\omega = \omega \in G$

$1.\omega^2 = \omega^2 \in G$

$\omega^2.\omega = \omega^3 = 1 \in G$

Identity Element :-

$e = 1$ is identy element of G.

$\forall$ $x \in G$, $x.1 = x$

Inverse Element x inverse $= e$

$\omega^2.(\omega) = 1$

$x = \omega^2$

$x.1 = \omega^2.1$

$= \omega^2$

$\forall x \in G \exists$ inverse of x s.t.

x (div x) $= e = 1$

$\omega^2(\omega) = 1$

$\Rightarrow$ ω^2 is inverse of ω.

Associative :- $(1.\omega)\omega^2 = 1$ $(\omega.\omega^2)$

Order of an element in a group G :-

Let G be a group w.r.t. b.o. multiplication then order of $x \in G$ is least +ve integer, n st. $a^n = e$, $e \in G$, e is identity element.

In case of $(G, +)$ is a group,

$\forall$ $x \in G$, $n.a = e$, $e \in G$ id w.r.t. '+'

$G = \{1, \omega, \omega^2\}$ is a group. w.r.t. '.'

$\omega^2 = 1$

$a \in G$

$a^n = e$

$(\omega)^3 = 1$

then,

order of → $o(\omega) = 3$

$(\omega^2)^3 = \omega^6 = 1$

$= (\omega^3)^2 = 1$

$o(\omega^2) = 3$

* $o(1) = ?$

$1^1 =$

$G = \{-1, +1, i, -i\}$, $(G, \cdot)$ is a group

$(1)^{1/4} = x$

$1 = x^4$

$\Rightarrow x^4 - 1 = 0$

$\Rightarrow (x^2+1)(x^2+1) = 0$

$\Rightarrow x^2+1 = 0, x = +1, -1$

$\Rightarrow x^2 = -1$

$\Rightarrow x = \pm i.$

$x^2 + 1 = 0$

$(x+i)(x-i) = 0$

$x = -i, i$

C.P. (Closer Property) $\forall x, y \in G, x.y \in G$

$i \times (-i) = -i^2 = i$

$1 \times i = i$

$-1 \times i = -i$

$(-1)(1) = -1$

In a group $\langle G, \cdot \rangle$, and a and $a^{-1} \in G$ s.t.

$o(a) = o(a^{-1})$

Let $o(a) = n$, $o(a^{-1}) = m$

then $o(a) = n \Rightarrow a^n = e$, $e \in G$, e is identity $\left\{\begin{array}{l} a^n = e \\ o(a) = n \end{array}\right\}$

$\Rightarrow (a^n)^{-1} = e^{-1}$

$\Rightarrow (a^{-1})^n = e \quad (\because e^{-1} = e)$

$\Rightarrow o(a^{-1}) \leq n \Rightarrow m \leq n$ —— ①

Again,

$o(a^{-1}) = m$

$\Rightarrow (a^{-1})^m = e$ (by default)

$\Rightarrow (a^m)^{-1} = e^{-1}$

$\Rightarrow (a^m) = e$

$\Rightarrow o(a) \leq m$

$\Rightarrow n \leq m$ —— ②

$\boxed{(a)^4 = e}$

eq^n ① + ②

$(a)^2 = e + n \leq m$

$o(a) = o(a^{-1})$

Theorem:— A non-empty select H of a group $(G, \cdot)$ is a subgroup of G if $\forall$ $a, b \in H \Rightarrow a.b^{-1} \in H$

$(H, \cdot)$ is a subgroup of G

Proof Let H be a subgroup of G

Then, $a, b \in H \Rightarrow a.b^{-1} \in H$

$\Rightarrow a, b^{-1} \in H$

$(\because H$ is a subgroup of $G)$

A subset of group G is a subgroup of G. if H is itself a group.

$(G, \cdot)$ $(H, \cdot)$

To prove that H is a subgroup of G

Take $a.b^{-1} \in H$

C.P.:— $a, b \in H \Rightarrow ab^{-1} \in H$

$\Rightarrow a.b \in H$

Asso. $\forall$ $a, b, c, \in H$

$\Rightarrow a, b, c \in G$

$\Rightarrow a.(b.c) = (a.b).c$

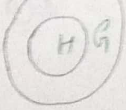
Identify.— $e \in G \Rightarrow e \in H$

$a^{-1} \in G \Rightarrow a^{-1} \in H$

$e, a \in H \Rightarrow e.a = a$

$\forall$ $a \in H$

$e^{-1} = e$



Inverse

$\forall$ $a \in H$

$\Rightarrow a.a^{-1} \in H$

$\Rightarrow a.a^{-1} = e \in H$

$\therefore$ H is a subgroup of G.

Let H be a subgroup of G.

e' be identity H

e be identity is G.

$\forall$ $a \in H \Rightarrow e'.a = a$ —— (1)

$\forall$ $a \in G \Rightarrow e.a = a$ —— (2)

$e'a = ea$

$\Rightarrow e' = e$

$a^{-1} \in G, a^{-1} \in H$

$a.a^{-1} = e$

Let $a \in H$

$\exists a^{-1}$ in H s.t. $a.a^{-1} = e$

and also $\exists$ $b \in G$ s.t. $e.b. = b$.

Cyclic Group

Let G be a group w.r.t. '.'

$\{a\} = G = \{a^n : n \in Z\}$

$\exists\, n$ (integer) s.t. $a^n = x \in G$

a is called the generator of G.

$G = \{-1, 1, i, -i\}$

$G = \{a^n : n \in z\}$

$G = \{i^n : n \in Z\} = \{i\}$

$G = \{i\}$

$= \{i^{-1}, i^{-2}, i^{-3}, i^{-4}\}$

$= \{i, -1, -i, 1\}$

$G = \{i^2\} = \{i^2 (i^2)^2, (i^2)^3, (i^2)^4\}$

$= [-1, i, -i, i]$

$G = \{-i\}$

$= \{(-i)^1, (-i)^2, (-i)^3, (-i)^4\}$

$= \{-i, -1, i, 1\}$

Let $<G, \cdot>$ be a group. For $a \in G$. if Every $x \in G$ is of the form a^n then G is called cyclic group.

$G = \{a^n : n \in Z\}$

$G = \{a\}$, a is generator of G.

Theorem:- If a is generator of G (group) then a^{-1} is also the generator of G.

Proof Let a be generator of G

i.e. $G = \{a\} = \{a^n : n \in Z\}$

If $x \in G \Rightarrow x = a^n$

$\boxed{a^n = (a^{-1})^{-n}}$

$i, -i$ are generator of $G = \{a^n : n \in z\}$

$G = \{i, -i, -1, +1\}$

We have $(i) = G$, $G = \{i\}$

$i^{-1} = \frac{1}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$

Scanned by CamScanner

Theorem :- Every Cyclic group is an abelian group.

Proof Let G be the cyclic group

$\Rightarrow G = \{a\}$, a is generator of G.

$\forall\ x, y \in G$ we have $x = a^r, y = a^s, r, s \in Z$

Now, $x.y = a^r . a^s = a^{r+s} = a^{s+r}$

$= a^s . a^r = y.x.$

Cyclic Group is abelian.

If $G = \{-i, +i, 1, -1\}$

then G is cyclic Group.

$G = \{i\} = \{i^1, i^2, i^3, i^4\}$

Cosets :- Let $<G, .>$ be a group and H be its subgroup.

$Ha = \{ha : a \in G, h \in H\}$

$G = \{g_1, g_2 - - - - g_n\}$, $H = \{h_1, h_2, - - - - - - - h_n\}$

then,

Ha is called Right cosets of H in G.

$aH = \{ah : a \in G, h \in H\}$

Let $<G, +>$ be a group and H be its subgroup.

$H + a = \{h + a : h \in H, a \in G\}$

$= \{h_1 + a, h_2 + a, - - - - - - h_n + a\}$

$a + H = \{a + h : h \in H, a \in G\}$

$= \{a + h_1, a + h_2 - - - - - - a + h_n\}$

Q: Let $G = \{- - - - -3, -2, -1, 0, 1, 2, 3 - - - -\}$ be a group w.r.t. addition binary operation.

and $H = \{- - - - -6, -4, -2, 0, 2, 4, 6 - - -\}$

Sol: $H + 0 = H$

$H + 1 = \{- - - -5, -3, -1, 1, 3, 5 - - -\}$

$H + 2 = \{- -4, -2, 0, 2, 4, 6 - - -\} = H$

$H + 3 = \{- - - -3, -1, 1, 3, 5, 7 - - -\} = H + 1$

$H + 4 = \{- - - -4, -2, 0, 2, 4, 6 - - -\} = H$

$G = H \cup \{H + 1\}$

$= \{g_1, g_2 - - - - g_n\}$

Scanned by CamScanner

Normal Subgroup:— A subgroup H of a group G is said to be normal of $x \in H$, $xbx^{-1} \in H$ or $xHx^{-1} \subseteq H$

Q:- A subgroup H of a group G is normal if $xHx^{-1} = H$

Proof:- Let $xHx^{-1} = H$

$\Rightarrow xHx^{-1} \subseteq H$

$\Rightarrow$ H is normal subgroup of G.

Again, let H is normal subgroup of G.

$\Rightarrow xHx^{-1} \subseteq H$ —— ①

$x \in G = x^{-1} \in G$

$\therefore x^{-1}H(x^{-1})^{-1} \subseteq H$

$\Rightarrow x^{-1}Hx \subseteq H$

$\Rightarrow x(x^{-1}Hx)x^{-1} \subseteq xHx^{-1}$

$\Rightarrow xx^{-1}Hx^{-1}x \subseteq xHx^{-1}$

$\Rightarrow H \subseteq xHx^{-1}$ —— ②

① & ② $\Rightarrow xHx^{-1} = H$

Lagrange's Theorem:—

The order of a subgroup H of G is divisor of order of group G.

Proof:- Let H be a subgroup of G s.t. $O(G) = n$, $O(H) = m$.

Then, $G = \{a_1, a_2, a_3 \ldots\ldots a_n\}$

and $H = \{h_1, h_2, h_3, \ldots\ldots h_m\}$

For $a_1 \in G$, $Ha_1 = \{ha_1 : h \in H\}$

$Ha_1 = \{h_1a_1, h_2a_1, h_3a_1, \ldots\ldots h_ma_1\}$

$Ha_2, Ha_3 \ldots\ldots Ha_n$.

There m cosets

Let Ha_K be the distinct cosets of H in G.

$\Rightarrow n = m + m + m + \ldots\ldots$ (K time)

$\Rightarrow n = mK$

$K = \frac{n}{m} = K = \frac{O(G)}{O(H)}$

$\Rightarrow O(H)$ is divisor of $O(G)$

$\Rightarrow$ The order of subgroup H is division of order of G.

⇒ Factor Group (or Quotient group)

Let S be the set of n elements.

$S = \{a_1, a_2 \ldots\ldots a_n\}$

$G/H = \{ha : h \in H, a \in G\}$

then G/H is factor group.

The Product of two right cosets of H in G is a right coset of H in G.

Let Ha, Hb are two right cosets of H in G.

Then $HaHb = H(aH)b$

$= H(Ha)b$

$= Hab$

The right cosets of H in G.

Permutation Group :— Let S be a finite set of having n elements.

The set of all 1-1 and onto mappings from S to S, $f : S \to S$ forms a group w.r.t. composition of mappings. This group is called the P.G. (Permutation Group).

i.e. $S = \{a_1, a_2, a_3, \ldots\ldots a_n\}$

$S_n = \{f : f \text{ is 1-1 and onto}\}$

$$f = \begin{pmatrix} a_1 & a_2 & a_n \\ f(a_1) & f(a_2) & f(a_n) \end{pmatrix}$$

A 1-1 mapping of $f : S \to S$ is defined as

$$f = \begin{pmatrix} a_1 & a_2 & a_n \\ f(a_1) & f(a_2) & f(a_3) \end{pmatrix}$$

is a permutation.

Composite of Permutations :-

$$f = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$$

$$g = \begin{pmatrix} a_1 & a_2 & a_3 \\ d_1 & d_2 & d_3 \end{pmatrix}$$

$$fog = -g$$

$$f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \quad S = \{1,2,3\}$$

$$g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \quad fog = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

II^{nd}

$$f = \begin{pmatrix} a_1 & a_2 & a_n \\ b_1 & b_2 & b_n \end{pmatrix}$$

$$f^{-1} = \begin{pmatrix} b_1 & b_2 & b_n \\ a_1 & a_2 & a_n \end{pmatrix}$$

$$fof^{-1} = \begin{pmatrix} a_1 & a_2 & a_n \\ a_1 & a_2 & a_n \end{pmatrix} = I$$

Identify Mapping I.

Q. Let G be the set of all real no's and G' be set of all non-zero real no.

Sol. Then If $f: G \to G'$ is defined as

$$f(a) = 5^a$$

P.t. $f: G \to G'$ is a hemomorphism

$$f(a+b) = 5^{a+b} \quad (G,+), (G',+)$$

$$= 5^a . 5^b \quad (G,+) \; (G',\cdot)$$

$$= f(a).f(b)$$

Ex. Let $(G,+)$ be a group of integers then $f: G \to G'$ is defined by $f(x) = 7x$

Sol. Then p.t.f. is a homomorphism

$$f(x+y) = 7(x+y) = 7x+7y$$

$$= f(x) + f(y)$$

f is a homomorphism.

Kernel of Homomorphism:- Let G and G' be two groups having identity elements e and e' respective. Then,

$f: G \to G'$ is a mapping.

The set, K of all those elements of G s.t.

$$f(x) = e'$$

$$K = \{x : f(x) = e', \text{ identity } e' \in G'\}$$

e' is a identity of a group.

Q: Let G and G' are groups.

Sol: s.t. $f: G \to G'$ having K as kernal of the $f: G \to G'$

P.t. K is a normal subgroup of G.

Then, $f = G \to G'$ s.t. $K = \{x : f(x) = e', e' \in G\}$

Let $a, b \in K \Rightarrow f(a) = e', f(b) = e'$

$f(ab^{-1}) = e'$

$ab^{-1} \in K$

$\therefore$ subset

$$\left\{\begin{array}{l} a, b \in K \\ a, b^{-1} \in K \\ a, b^{-1} \in K \end{array}\right\}$$

$a, b \in K$

$\Rightarrow a, b^{-1} \in K$

$f(ab^{-1}) = f(a) \cdot f(b^{-1})$

$= f(a) \cdot [f(b)]^{-1}$

$= e' \cdot (e')^{-1}$

$\forall\, ab \in K$

$ab^{-1} \in K$

$\therefore K$ is a subgroup

Subgroup K is normal

Let $k_1 \in K$

Then $f(gk_1 g^{-1})$

$= f(g) \cdot f(k_1) f(g^{-1})$

$= f(g) \cdot e' [f(g)]^{-1}$

$= e' \cdot f(g) [f(g)]^{-1}$

$= e' \cdot e' = e'$

Ring:- An algebric structure $\langle R, +, \cdot \rangle$ is said to be a Ring of binary operation '+' and '.' satisfy the following properties.

1) $(R, +)$ is an abelion group.

2) $(R, \cdot)$ is a semi group.

3) Multiplication is distribute over addition.

* Prove that $\langle Z, +, \cdot \rangle$ is a ring.

Sol^n:- 1) $(Z, +)$ is a commutative group.

i) CP:- $\forall\, a, b \in Z,\ a+b \in Z$

ii) Asso.:- $\forall\, a, b, c \in Z,\ a+(b+c) = (a+b)+c$

iii) $\forall\, a \in Z\ \exists\, 0 \in Z$ s.t. $a+0 = 0+a = a$

i.e. 0 is called identity element.

iv) Ex of Inverse element :-

$\forall\, a \in Z\ \exists\, a^{-1} \in Z$ s.t.

$a + a^{-1} = a^{-1} + a = 0$

i.e. $a + (-a) = (-a) + a = 0$

v) Commutative Group:-

$\forall\, a, b \in Z \Rightarrow a+b = b+a$

2. $(Z, \cdot)$ in a semigroup

i) CP:- $\forall\, a, b \in Z$

$a \cdot b \in Z$

ii) Asso. Property:- $\forall\, a, b, c \in Z$

$a(bc) = (ab)c$

Scanned by CamScanner

3. '·' is distributive over addition

$\forall\, a, b, c \in Z$

$a\cdot(b+c) = a\cdot b + ac$

$(b+c)a = ba + ca$

Commutative Ring :- A ring $(R,+,\cdot)$ is said to be commutative Ring.

If '·' commites is it.

i.e. $a, b \in R$, $a.b = b.a$

Ring With Unit Element :- A ring is said to be a ring with unity s.t.

$\forall\, a \in R$, $\exists\, 1 \in R$ such that

$a.1 = 1.a = a$.

Zero Divisors :- A non-zero element, $a \in R$, then $\forall\, a \in R$ $\exists\, b \neq 0$ in R such that either

$ab = 0$ or $ba = 0$.

Q. Let M be a set of all 2×2 matrices with 2 binary operation 'addition' and 'multiplication' then $(M,+,\cdot)$ is a ring with zero divisors.

Sol. 1) CP :- $\forall\, A, B \in M \Rightarrow AB \in M$

2) Asso. :- $\forall\, A, B, C \in M$, $A+(B+C) = (A+B)+C$

3) $\forall\, A \in M$, $\exists\, -A \in M$ s.t.

$A + (-A) = 0$

Existence of Inverse

4) Exist of Identity :- 0 is identity element

s.t. $A+0 = 0+A$

$0 \rightarrow$ (Null Matrix)

Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \in M$

$\therefore AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

$\langle M,+,\cdot\rangle$ has zero divisors.

Integral Domain:- A ring R is said to be integral domain if it satisfies the following properties:

1) R should be commutative ring.
2) It has unit element.
3) It has no zero divisors.

Field → A ring $(R,+,.)$ is said to be Field (having at least two finite elements) if it satisfies the following properties:

1) R is commutative ring.
2) R has unit element.
3) It has multiplicative inverse $\forall a \in R$

Theorem → Every field is an integral domain.

Proof:- Let $\langle F,+,.\rangle$ is a field having at least 2 elements.

Let $a \neq 0 \in F \Rightarrow a^{-1}$ exists.

Let $ab = 0 \Rightarrow a^{-1}(ab) = a^{-1}0$

$\Rightarrow (a^{-1}a)b = 0$

$\Rightarrow 1.b = 0$

$\Rightarrow b = 0$

II^{nd} → Let $b \neq 0 \in F \Rightarrow b^{-1}$ exists.

Let $ab = 0 \Rightarrow (ab)b^{-1} = 0(b^{-1})$

$\Rightarrow a(bb^{-1}) = 0$

$\Rightarrow a.1 = 0$

$\Rightarrow a = 0$

$\therefore$ F has no zero divisors.

$\Rightarrow$ Field F is an integral domain.

Theorem:- A finite integral domain is a field.

Proof:- Let D be a finite integral domain having n elements.

i.e. $D = \{a_1, a_2, a_3, \ldots\ldots a_n\}$

If $a \in D \Rightarrow aa_1, aa_2, aa_3 \ldots\ldots aa_n$ are n distinct elements.

If possible let $a_i = a_j \; (i \neq j)$

$\Rightarrow aa_i = aa_j$

$\Rightarrow aa_i - aa_j = 0$

$\Rightarrow a(a_i - a_j) = 0 \quad (a \neq 0)$

$a_i = a_j$

$\Rightarrow i = j$

$\therefore$ then, which is contradition

Scanned by CamScanner

To prove that D (integral Domain) is a field.

i.e $\forall\, a \in D \Rightarrow a.1 = 1.a = a$

and $\forall\, b \neq 0 \Rightarrow ab = ba = 1$

$\Rightarrow \exists\, b$ (multiplicative Inverse)

s.t. $a.b = 1$.

Let $a \neq 0 \in D \Rightarrow \exists\, b \in D \Rightarrow a.b = b.a = 1$

($\because$ C.P hold)

$\Rightarrow b$ is multiplicative Inverse of a.

and $y \in D \Rightarrow y = ax = xa, \forall\, a \neq 0 \in D$

$1.y = 1(ax) = 1(xa) = (1x)a = (xa)1 = y.1$

$\Rightarrow 1$ is multiplicative Identity in D.

Therefore, D is a field.

Division Ring :— $(R, \cdot)$ is common semi group.

Date:- 8/9/17

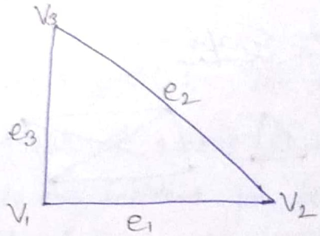
Unit-3

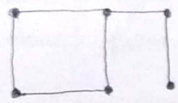
Graph

Let $V = \{v_1, v_2 \ldots\ldots v_n\}$ and $E = \{e_1, e_2, e_3 \ldots e_l\}$ are respectively sets of vertices v_i and edges e_i.

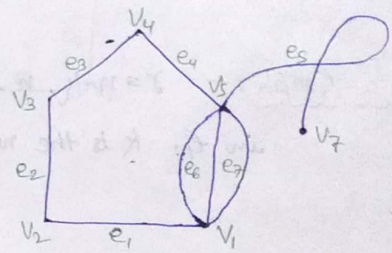
Then the geometric structure, $G = (V, E)$ is called the Graph.

Simple Graph :—





Multigraph :—

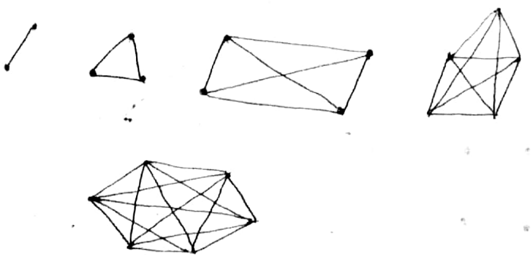


Component of a graph :— Let $G = (V, E)$ is a graph.

Then component of graph G are the subgraphs of G such that every subgraph is connected.

1. Disconnected Graph. 2. Connected Graph.

Complete Graph :-



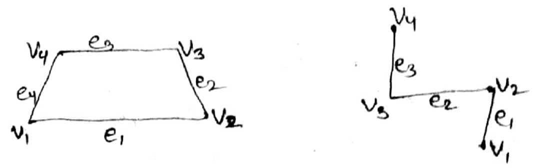
Rank of a Graph :- $r = n-k$, n is no. of vertices in G. K is the no. of components of G.

Rank of Graph G,

n is no. of vertices in G.

K is the no. of components of G.

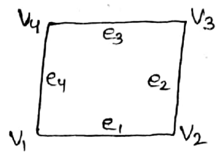
Path a group $G(V,E)$ is paid to be a path which either open or closed path.



$e_1 = [v_1, v_2]$

path is $v_1 e_1 v_2 e_2 v_3 e_3 v_4$.

Deletion of vertex from a graph :- The deletion of a vertex from graph G means that vertex and the edges which are incident on it are removed from graph G.



Date - 13/03/17

Theorem :- A completly connected graph G is said to have Hameltonion (Path) circuits if $m > \frac{1}{2}(n^2 - 3n + 6)$, where m is the no of edges in G and n is the no. of vertices.

Proof :- Let $G = (V, E)$ be completed a connected graph with m edges and n vertices.

Delete two vertices from graph G.

There is n-2 no. of vertices.

and $m - (d(v_1) + d(v_2))$ edges.

$$d(v_1) + d(v_2) \geq n$$

$$^{n-2}C_2 = m - [d(v_1) + d_2(v_2)]$$

$$^{n-2}C_2 = m - n$$

$$\Rightarrow \frac{(n-2)(n-3)}{\lfloor 2} \leq m - n$$

$$\frac{n^2 - 5n + 6}{2} \leq m - n$$

$$n^2 - 5n + 6 \leq 2m - 2n$$

$$n^2 - 3n + 6 \leq 2m$$

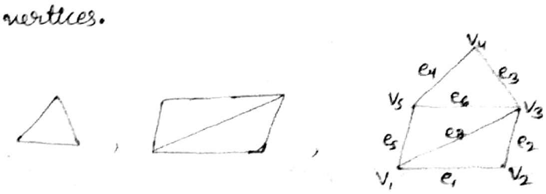
$$\frac{1}{2}(n^2 - 3n + 6) \leq m$$

Pendant Vertex

This one edges and this one vertex is known as Pendant vertex.

Null Graph :- A Graph G is said to be null Graph if it does not conoid of edges. The collection of isolated points is called a Null Graph.

Planar Graph :- A graph $G = (V, E)$ is said to be planar. If it is drawn in a plane without intersecting the edges between any pair of vertices.



Hand Shaking Lemma :— In a connected graph having n vertices, e edges

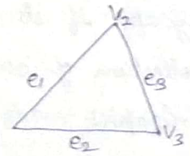
we have $\sum_{i=1}^{n} d(v_i) = 2e$

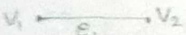
$d(v_1) + d(v_2) + \ldots\ldots + d(v_n) = 2 \times e$

Proof:- Since,

every edge is incident on 2 vertices in a graph.

$\therefore \sum_{i=1}^{n} d(v_i) = 2e$





$2+2+2 = 2 \times 3$

$V = \{v_1, v_2\} = G$

$E = \{G_1\}$

$d(v_1) + d(v_2) = 1 + 1 = 2$

$e = 1$

$E = 2.1 = 2$

Theorem :— In a complete connected graph G having n vertices, e edges and r regions (or faces).

$$\boxed{r \leq \frac{2}{3} e}$$

Euler's Theorem :— In a graph G having n vertices, r regions and e edges.

$n - e + r = 2.$

Proof:— Case I Graph G is  $n = 2,\ e = 1.$

$r = e - n + 2$

$r = 1 - 2 + 2$

$r = 1$

which is true.

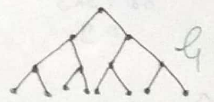
Case II  $n = 1,\ e = 1.$

$r = e - n + 2$

$r = 1 - 1 + 2$

$r = 2$

which is true.

Case III  G $n = 15$ $e = 14$

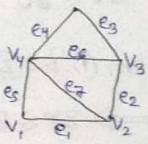
$r = e - n + 2$

$r = 14 - 15 + 2$

$r = 16 - 15$

$r = 1$

which is true.

Case IV 

$r = e - n + 2$

$G' = G - \{e_1\}$

$G,\ r = e - n + 2$

$G',\ r - 1 = (e - 1) + 2 - n$

$\Rightarrow r = e - n + 2$

$\Rightarrow n - e + r = 2.$

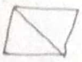
Scanned by CamScanner

Eular's formula :- $n-e+r=2$

Theorem :- In a connected planar graph. $G=(V,E)$ having n vertices, e edges and r regions.

$$r \leq \frac{2}{3}e.$$

Proof :- Let S be sum of all edges touching the regions.



$e=5$
$r=3$

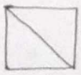
$S=10$

$S \leq 2e$ —— ①

Also one region includes at least 3 edges.

$3r \leq S$ —— ②

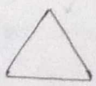
from ① & ②



$S=10$
$r=3$
$3r=3\times3$
$=9$

$\Rightarrow 3r \leq 2e$

$\Rightarrow \boxed{r \leq \frac{2}{3}e}$



$n=3,\ e=3$
$r=2$
$3\times2 \leq 2\times3$

Imp

Theorem :- In a connected planar graph having n vertices & e edges and r regions.

$$3n-e \geq 6$$

Proof :- By Eular's formula, $n-e+r=2$

$\Rightarrow r=e-n+2$ —— ①

Also, $r \leq \frac{2}{3}e$ —— ②

① & ② $\Rightarrow e-n+2 \leq \frac{2}{3}e$

$\Rightarrow 3e-3n+6 \leq 2e$

$\Rightarrow e-3n+6 \leq 0$

$\Rightarrow 6 \leq 3n-e$

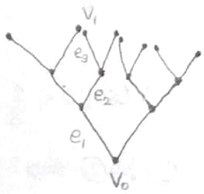
$\Rightarrow \boxed{3n-e \geq 6}$

Date- 03/10/2017

Properties of a tree :-

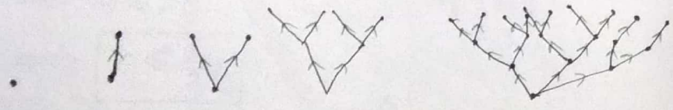
1. There is one and only one path between any pair of vertices in a tree.
2. In a tree, the number of edges is one less than the number of vertices.

   $\boxed{e = v - 1}$



3. A tree is a circuitless graph having no multiple edges.

Degree :— The degree of root in a tree, is either zero or one or two or more.



T_0 T_1 T_2 T_3 T_4

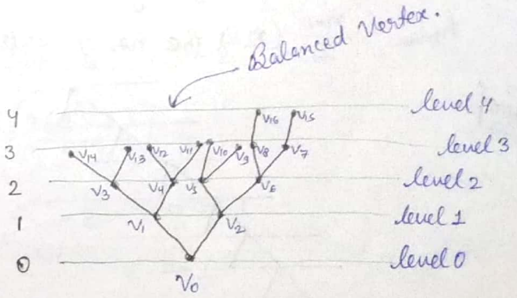
(∧) Outgoing degree (Out Degree)

(∨) Incoming degree (Indegree)

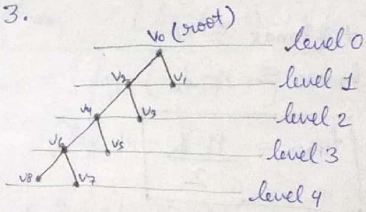
Subtree :— A tree is said to be subtree if it is obtained by deleting root v_0 from the tree.

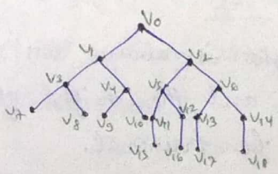
T_4' T_4'' T_4'''

Binary Tree :—

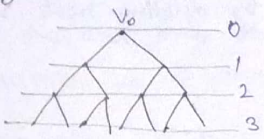


In a binary tree one vertex in of degree equal to 2 and other vertex are of degree either 1 or 3.





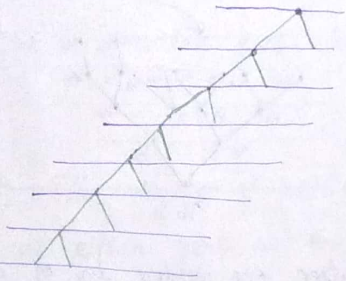
Full Binary Tree :-



Theorem-1

The maximum height of a Binary Tree by $h_{max} = \frac{n-1}{2}$ (n is the no. of vertices).

Proof



$1 + [2+2+ \dots\dots +2] = n$

$\implies 1 + 2h_{max} = n$

$2h_{max} = n-1$

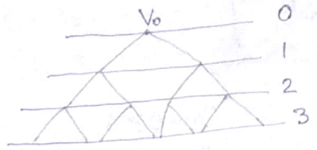
$$\boxed{h_{max} = \frac{n-1}{2}}$$

Note:- * Full binary height is always less than Not full binary and the height of any tree can never be fractional.

Theorem-2

The maximum height of a Binary Tree is given by $h_{min} = \lceil \log_2 (n+1) - 1 \rceil$

Proof



$2^0 + 2^1 + 2^2 + 2^3 + \dots\dots + 2^{h_{min}} = n$

$1 + \left[\frac{2(2^h - 1)}{2-1}\right] = n$

$2^{h+1} - 2 = n-1$

$2^{h+1} = n+1$

$\implies \log_2 2^{h+1} = \log_2 (n+1)$

$\implies (h+1)\log_2 2 = \log_2 (n+1)$

$\implies \boxed{h_{min} = \lceil \log_2 (n+1) - 1 \rceil}$

Ex- $h_{min} = \lceil \log_2 (13+1) - 1 \rceil$

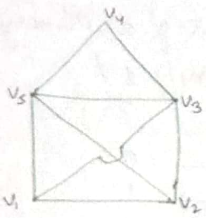
$= \lceil \log_2 14 - 1 \rceil$

$= 3.6 - 1 = 2.6$

It is not a binary tree because it is in fractional form.

Date: 22/8/17

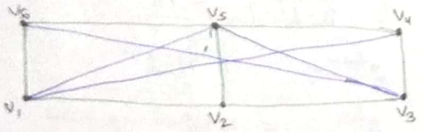
i) Kurotowski First Graph :—



$n=5$
$e=10$

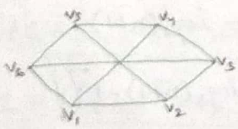
$3n-e \geq 6$
$3n-e = 3\times5-10$
$3n-e = 5$

ii) Kurotowski Second Kind :—



$n=6$
$e=11$

$n-e+r = 3n-e$
$= 3\times6-11 = 7$



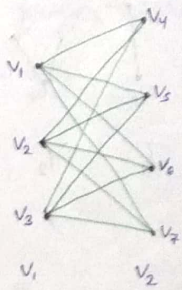
Planar graph

$4r \leq 2e$
$n-e+r=2$
$r = 2-n+e$
$= 2-6+11$
$= 7$

$n-e+r = 2$
$\Longrightarrow 6-11+7 = 2$
$4r \leq 2e$
$4\times7 = 2\times11$
$28 \leq 22$

Bipartile Graph :— Let $G=(V,E)$ be the graph such the partation of V, say V_1 and V_2 are such that no vertices of same set are joined by edges.

$V = \{v_1, v_2 ------ v_m\}$
$E = \{e_1, e_2 ------ e_l\}$

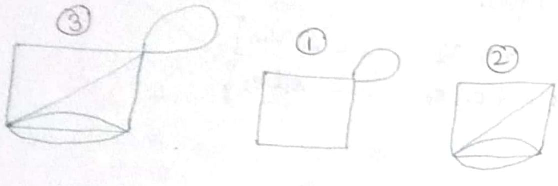


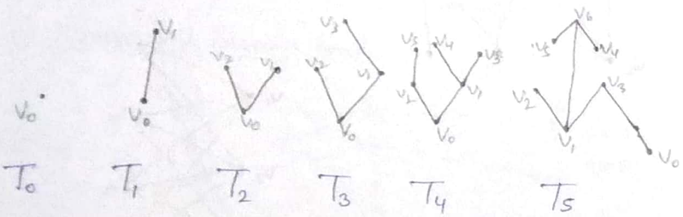
Bipartile Graph

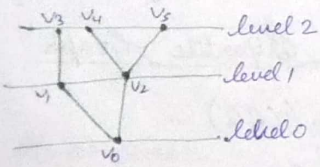
Complete Bipartile Graph (CBG)

$\forall \; V_i, V_j \in V$
$\exists$ an edge $[V_i, V_j] \in E$
$e_1 = [V_i, V_j]$

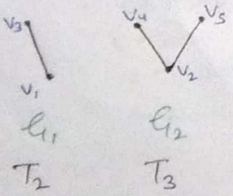
Tree:- A connected Graph $G=(V,E)$ is said to be a tree if it contains neither self loops nor circuits nor parallel edges.







when, $G \longrightarrow T$



Date - 06/10/2017

Coloring of Graph:- Let $G=(V,E,\psi)$ be a connected graph having n vertices such that it has no multiple edges.

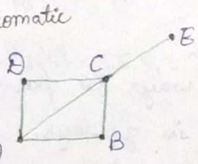
Let $C=\{C_1, C_2, C_3, \ldots\ldots C_p\}$.

Then $f: V \to C$ is called coloring of graph G.

The vertices of graph G are coloured in such a way that no two adjacent vertices are of same color. ~~The proper colouring of a Graph G is such that~~. This is called proper coloring.

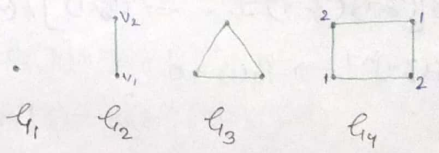
Proper coloring is very diffucult.

G.r. G is 3-chromatic



G_1.

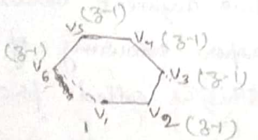
To Color graph G with 5, 4, 3 colors.



Chromatic Number:— The chromatic number, n is the minimum no. of ~~class~~ colors which are used to color the graph G properly.

Chromatic Polynomial:—

i) Coloring of open polygon.



Let z be the no. of colors used to color the polygon.

The no. of ways to paint the graph with z colors in z ways.

/* $P_G(z)$ = Polonomial Graph.
and z Total number of colours. */

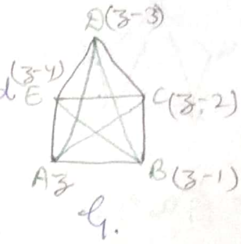
$P_G(z) = z[(z-1)(z-1)\cdots\cdots(z-1)]$ //(n-1) lines

$P_G(z) = z(z-1)^{n-1} \longrightarrow P_G(1) = 0$

Chromatic polynomial for a complete graph:—

Using z colors the vertex. A can be painted in z different ways.

Vertex B can be colored with (z-1) colors. in (z-1) different ways.



$P_G(z) = z(z-1)(z-2)(z-3)\cdots\cdots(z-n+1)$

$P_G(1) = 0,\ P_G(2) = 0$

$P_G(3) = 0,\ P_G(4) = 0$

$P_G(5) = 1,\ P_G(6) = 6,5,4,3,2,1$

$P_G(5) = \lfloor 5$

1) Chromatic polynomial for open polygon.
2) Chromatic polynomial for complete graph.
3) Chromatic polynomial for a graph other than the open polygon or complete graph.

1) $P_G(z) = z(z-1)^{n-1}$

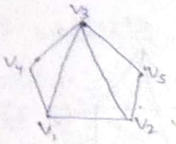
2) $P_G(z) = z[(z-1)(z-2)(z-3)\cdots\cdots(z-n+1)]$

$P_4(1) = 0$

$P_4(2) = 2$

$P_4(3) = 3\times 2^{n-1}$

2) $P_G(1) = 0,\ P_G(2) = 0,\ P_G(3) = 0,\ P_G(4) = 0,\ P_G(5) = 5!$



→ Incomplete Graph.

↳ $P_G(z) = \sum_{i=1}^{n} a_i\, {}^zC_i$

$= a_1\,{}^zC_1 + a_2\,{}^zC_2 + a_3\,{}^zC_3 + \ldots + a_n\,{}^zC_n$

$= a_1 z + a_2 \frac{z(z-1)}{2} + a_3 \frac{z(z-1)(z-2)}{\lfloor 3} \ldots$

$\ldots + a_n \frac{z(z-1)(z-2)\ldots(z-n+1)}{\lfloor n}$

If, $a_1 = 0 \ldots a_n = \lfloor n$

$P_G(z) = 0 + 0 + \cancel{a_3}^{\lfloor 3} \frac{z(z-1)(z-2)}{\cancel{\lfloor 3}} + a_4 \frac{z(z-1)(z-2)(z-3)}{\cancel{\lfloor 4}} +$ $\cancel{2\times 4 \times \lfloor 3}$

$\cancel{a_5}^{\lfloor 5} \frac{z(z-1)(z-2)(z-3)(z-4)}{\cancel{\lfloor 5}}$

$P_G(z) = z(z-1)(z-2)[1 + 2(z-3) + (z-3)(z-4)]$

$= z(z-1)(z-2)[1 + 2z - 6 + z^2 + 7z + 12]$

$P_G(z) = z(z-1)(z-2)[z^2 - 5z + 7]$

$P_G(1) = 0,\ P_G(2) = 0,\ P_G(3) = 6$

Date- 10/10/2017

Discrete Numeric Function :— The function $a: N \to R$ is said to be D.N.F.

$a \equiv (a_0, a_1, a_2, a_3 \ldots a_r, \ldots) \equiv a_r$

The value of a is $a_0, a_1, a_2 \ldots a_r \ldots$ at $r = 0, 1, 2, 3, \ldots r$

Ex:- $a = (1, 1, 1, \ldots 1, \ldots)$

$a = (1, 4, 16, 64 \ldots 4^r \ldots)$

G.F., $A(z)$ is defined as

$A(z) = \sum_{r=0}^{\infty} a_r z^r$

$= a_0 + a_1 z^1 + a_2 z^2 + \ldots + a_r z^r + \ldots$

$A(z)$ is G.F. for $a = (a_0, a_1, a_2, \ldots a_r, \ldots)$, $a = a_r$

Ex:- Amount A after r years for principal 1000 at the annual interest of R% then.

$a = P\left(1 + \frac{R}{100}\right)^r$

$a = 1000\left(1 + \frac{9}{100}\right)^r$

$\boxed{a = 1000 \times (1.09)^r} = a_r$

$a \equiv (1000, 1090, 1188.1, \ldots, 1000 \times (1.09)^r, \ldots)$

D.N.F.

$a = (a_0, a_1, a_2, a_3 \ldots\ldots a_r, \ldots\ldots)$

$\therefore$ G.F. $A(z) = a_0 \cdot z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \ldots + a_r z^r \ldots$

Ex-1 Find G.F. for D.N.F., $a = (1, 4, 16, 64, \ldots 4^r, \ldots)$

$A(z) = 4^0 z^0 + 4^1 z^1 + 4^2 z^2 + 4^3 z^3 + \ldots + 4^r z^r + \ldots$

$= 1 + (4z) + (4z)^2 + (4z)^3 + \ldots + (4z)^r + \ldots$

$= (1-4z)^{-1}$

$= \frac{1}{1-4z}$

$A(z) = \frac{1}{1-az}$

$= (1-az)^{-1}$

$= 1 + az + a^2 z^2 + \ldots + a^r z^r + \ldots$

Q:- Find the DNF for G.F. $A(z) = \frac{1}{z^2 - 5z + 6}$

Sol^n $\frac{1}{(z-2)(z-3)}$

$= \frac{1}{z-3} - \frac{1}{z-2}$

$= \frac{1}{-3(1-\frac{z}{3})} - \frac{1}{-2(1-z/2)}$

$A(z) = \frac{1}{2(1-z/2)} - \frac{1}{3(1-z/3)}$

$= \left(\frac{1}{2}\right)(1-z/2)^{-1} - \left(\frac{1}{3}\right) \cdot (1-z/3)^{-1}$

$a = \frac{1}{2}\left(\frac{1}{2}\right)^r - \frac{1}{3} \cdot \left(\frac{1}{3}\right)^r$

DNF $\rightarrow$ $a = \left(\frac{1}{2}\right)^{r+1} - \left(\frac{1}{3}\right)^{r+1}$

$a \equiv a_r = {}^nC_r$

$A(z) = \sum_{r=0}^{\infty} {}^nC_r \cdot z^r$

$= {}^nC_0 \cdot z^0 + {}^nC_1 z^1 + {}^nC_2 z^2 + \ldots + {}^nC_n z^n$

$= 1 + nz + \frac{n(n-1)}{\lfloor 2} z^2 + \ldots +$

$\boxed{A(z) = (1+z)}$

Recurrence Relation

$a \equiv (a_0, a_1, a_2 \ldots a_r \ldots)$

$a \equiv a_r = (a_0, a_1, a_2 \ldots a_r \ldots)$

$a_r = (2^r + 3 \cdot 5^r)$

$a_{r-1} = \frac{(2^r)}{2} + \frac{3}{5}(5^r)$

$a_{r-2} = \frac{1}{4}(2)^r + \frac{3}{25} \cdot (5^r)$

$\therefore$ $\boxed{a_r + C_1 a_{r-1} + C_2 a_{r-2} \equiv 0}$

Scanned by CamScanner

Solution of Recurrence (Relations) Equations :—

$\boxed{a_r = f(r)}$

D.N.F. $a = \{a_r\} = (a_0, a_1, a_2, a_3 \ldots a_r \ldots)$

G.F. $A(z) \to$ G.F. $\to$ function of z

$a_r = f(r) = 3(4)^r + 2.(7)^r$

$= 3.4^r + 2.7^r$

$a_{r-1} = 3.4^{r-1} + 2\,7^{r-1}$

$= \frac{3}{4}.4^r + \frac{2}{7}.7^r$

$a_{r-2} = 3.4^{r-2} + 2.7^{r-2}$

$= \frac{3}{16}4^r + \frac{2}{49}.7^r$

Then,

$\boxed{a_r + k_1 a_{r-1} + k_2 a_{r-2} = 0}$

$a = \{a_r\} = \{a_0, a_1, a_2 \ldots a_r \ldots\}$

$A(z) = a_0 z^0 + a_1 z^1 + a_2 z^2 + \ldots + a_r . z^r + \ldots$

Terms of these equations :—

$$\begin{vmatrix} a^r & a^{r-1} & a^{r-2} \\ 3 & 3/4 & 3/16 \\ 2 & 2/7 & 2/49 \end{vmatrix}$$

* Solve the R.E., $a_r - 5a_{r-1} + 6a_{r-2} = 0, r \geq 2$.

(Characteristics) Ch. Eqn

$\alpha^2 - 5\alpha + 6 = 0$

$\Rightarrow \alpha = 2, 3$

$\therefore a_r = A_1(2)^r + A_2(3)^r$ —— ①

① $\Rightarrow a_0 = A_1 + A_2$

$\Rightarrow A_1 + A_2 = 8$ —— ②

① $\Rightarrow 21 = 2A_1 + 3A_2$ —— ③

from eqn ① & ②

$2A_1 + 2A_2 = 16$

$2A_1 + 3A_2 = 21$

$5A_2 =$

$A_2 = 5$

$A_1 = 3$

$\therefore$ ① $\Rightarrow a_r = 3.(2)^r + 5.(3)^r$

$a_r = 3(2)^r + 5.(3)^r$

$a_{r-1} = \frac{3}{2}2^r + \frac{5}{3}(3)^r$

$a_{r-2} = \frac{3}{4}.2^r + \frac{5}{9}(3)^r$

$$\begin{vmatrix} a_r & a_{r-1} & a_{r-2} \\ 3 & 3/2 & 3/4 \\ 5 & 5/3 & 5/9 \end{vmatrix} = 0$$

<u>Case I</u> → If roots of characteristics eqⁿ are equal.

$$\alpha^2 + K_1\alpha + K_2 = 0$$

<u>Solve</u> $a_r - 4a_{r-1} + 4a_{r-2} = 0, \quad a_0 = 5, \quad a_1 = 16$

characteristics eqⁿ of given R.E. $a_r - 4a_{r-1} + 4a_{r-2} = 0$

is given by $\alpha^2 - 4\alpha + 4 = 0$

$\implies (\alpha-2)^2 = 0$

$\implies \alpha = 2, 2$

∴ Solⁿ is given,

$a_r = (\lambda_1 + \lambda_2 r)(2)^r$ —— (1)

(1) $\implies a_r = (5 + 3 \cdot r)(2)^r$ —— (2)

$a_0 = \lambda_1$

$a_1 = (\lambda_1 + \lambda_2) * 2$

$a_1 = (5+3) \cdot 2$

$a_1 = 16$

<u>Q.</u> Find sol. of R.E. $a_r + 4a_{r-1} + 4a_{r-2} = r$

Ch. eqⁿ in $\alpha^2 + 4\alpha + 4 = 0$

$\implies (\alpha+2)^2 = 0$

$\implies \alpha = -2, -2$

$a_r^{(h)} = (\lambda_1 + \lambda_2 r)(-2) = \mu_1 r + \mu_2$

$a_r^{(p)} = \mu r_1 + \mu_2$

$\mu_1 r + \mu_2 + 4[\mu_1(r-1) + \mu_2] + 4[\mu_1(r-2) + \mu_2]$

$= \mu_1 r + \mu_2$

$r(\mu_1 + 4\mu_1 + 4\mu_1) + (\mu_1, \mu_2) = 1 \cdot r + 0$

$r(9\mu_1) + (-4\mu_1 - 8\mu_1 + 8\mu_2) = 1 \cdot r + 0$

$\mu_1 = \frac{1}{9},$

$a_r = a_r^{(h)} + a_r^{(p)}$

$a_r = (\lambda_1 + \lambda_2 r)(-2)^r + \frac{1}{9} r + \mu_2$

<u>Q.</u> Solve the R.R $a_r + 6a_{r-1} + 9a_{r-2} = 0, \quad r \geq 2$

$a_{r+2} + 3a_{r+1} + 9a_r$ —— (1)

$\alpha^2 + 6\alpha + 9 = 0$ —— (2)

$(\alpha+3)^2 = 0$

$\implies \alpha = -3, -3$

$a_r = (A_1 + A_2 r)(-3)^r$

$a_r = (\lambda_1 + \lambda_2 r - \lambda_3 r^2)(-3)^r$

$a_r = (\lambda_1 + \lambda_2 r)(-3)^r$ —— (3)

$(a_0 = \lambda_1 = 2)$

$a_1 = (\lambda_1 + \lambda_2 \cdot 1)(-3)$

$a_1 = (2 + \lambda_2)(-3) = (2 + \lambda_2)(-3)$

$a_1 = (\lambda_1 + \lambda_2 r)(-3)^r$

Scanned by CamScanner

$a_r^{(h)} = (d_1 + d_2 r)(-3)^r$

$a_r^{(h)} = u$

$a_{r-1} = u$

$a_{r-2} = u$

Then,

$u + 6u + 9u = 3$

$16u = 3$

$u = \frac{3}{16}$

Q:- Find the C.S. of R.R using G.F. method

$a_r - 2a_r + a_{r-2} = 2^r, r \geq 2$

Sol:- $\sum_{r=2}^{\infty} a_i z^r - 2\sum_{r=2}^{\infty} a_{r-1} z^r + \sum_{r=2}^{\infty} a_{r-2} \cdot z^r = \sum_{2}^{\infty} 2^r z^r$

$\Rightarrow (a_2 z^2 + a_3 z^3 + \ldots) - 2(a_1 z^2 + a_2 z^3 + \ldots)$

$+ (a_2 z^2 + a_3 z^3 + \ldots)$

$= 2^2 \cdot z^2 + 2^3 \cdot z^3 + \ldots$

$\Rightarrow (A(z) - a_0 - a_1 z) - 2z(a_1 z + a_2 z^2 + \ldots)$

$+ z^2 A(z) = 4z^2(1 + 2z + 4z + \ldots)$

$\Rightarrow (A(z) - a_0 - a_1 z) - 2z(A(z) - a_0)$

$+ z^2 A(z) = \left(4z^3\left(\frac{1}{1-2z}\right)\right)$

$(1 - 2z + z^2) A(z) - 2 - z + 4z = \frac{4z^2}{1-2z}$

$\Rightarrow (1-z)^2 A(z) = 2 - 3z + \frac{4z^2}{1-2z}$

$A(z) = \frac{6z^2 - 7z + 2 + 4z^2}{(1-z)^2(1-2z)}$

$= \frac{10z^2 - 7z + 2}{(1-z)^2(1-2z)}$

$\frac{A}{(1-z)} + \frac{B}{(1-z)^2} + \frac{C}{(1-2z)}$

$10z^2 - 7z + 2 = A(1-z)(1-2z) + B(1-2z) + C(1-z)^2$

$z = 1, \; 5 = B(1-2)$

$\Rightarrow B = -5$

$1 = C \times \frac{1}{4} \Rightarrow C = 4$

$\frac{10z^2 - 7z + 2}{(1-z)^2(1-2z)} = \frac{3}{(1-z)} - \frac{5}{(1-z)^2} + \frac{4}{(1-2z)}$

$\Rightarrow a_r = 3(1)^r - 5(1)^r + 4(2)^r + 2^{r+2}$

$a^r = r + 1$

$\left\{ \begin{array}{l} a_0 = 1, a_1 = 2, a_2 = 3 \ldots \\ 1 + 2z + 3z^2 + \ldots \end{array} \right\}$

Generating function:-

$a_r - 5a_{r-1} + 6a_{r-2} = 5^r$

Pigeon hole principle:- In n pigeons are assigned to m pigeonhole such ~~m~~ that $m<n$, then at lest one pigeon hole ~~m<n~~ contains at least two or more pigeons.

Proof:- Let P_1, P_2, P_3 ---- P_n be no. of pigeons. and H_1, H_2, H_3 ---- H_m are m pigeon. holes. st. $(m<n)$.

Let P_1 is assigned to H_1

P_2 to H_2, P_3 to H_3 --- P_n to H_m

then $n-m$ pigeons are left without getting pigeonhole.

∴ At least one pigeon hole contains two or more pigeons.

$\boxed{n = Km+1}$

Q:- Find the least min. no. of students of a class such that five students are born in the same month.

Sol:- $m=12$, $n=?$ $K=4$

Then, $n = mK+1$

$n = 12 \times 4 + 1$

$n = 49$

$n = (m-1) \times (K-1) + 5$

$n = (12-1) \times (5-1) + 5$

$n = 11 \times 4 + 5$

$n = 49$

Q:- If six colors are used to paint 50 cars then How many cars will have the same color.

Sol:- $\frac{50}{6} = \frac{n}{m}$

~~$50m = 6n$~~

$n = mK+1$ = $49 = 6 \times 8 + 1$

~~$49 = 6 \times 8$~~ $N = 8+1 = 9$ $\left[N = \frac{n-1}{m} + 1\right]$

$50 = 6 \times K + 1$

$49 = 6K$

$K = \frac{49}{6} = 8.2$