Advanced Econometrics (PhD in Economics) and Advanced Topics in Econometrics (PhD in Mathematics Applied to Economics and Management) 2025/2026

Exercise Sheet 2 - Discrete Choice Models

(version 20/09/2025)

1. Consider a latent variable modeled by $Y_i^* = X_i'\beta + \varepsilon_i$, with $\varepsilon_i \sim N(0,1)$. Suppose we observe only $Y_i = 1$ if $Y_i^* < U_i$ and $Y_i = 0$ if $Y_i^* \geq U_i$, where the upper limit U_i is a known constant for each individual (i.e., data) and may differ over individuals.

   (a) Find $\mathcal{P}[Y_i = 1|X_i, U_i]$. [**Hint:** Note that this differs from the standard case both due to presence of U_i and because the equalities are reversed with $Y_i = 1$ if $Y_i^* < U_i$.]

   (b) Provide details on an estimation method to consistently estimate β.

2. Suppose we use an index formulation for a discrete choice model, but it is felt that the latent variable is strictly positive. This is accommodated by supposing that the latent variable Y^* has exponential density with parameter γ, so the density function of Y^* conditional on the vectors of exogenous variables X and Z is

$$f(y^*|X,Z) = \begin{cases} \gamma^{-1}\exp(-y^*/\gamma) \text{ if } y^* > 0 \\ 0 \text{ if } y^* \leq 0 \end{cases},$$

   with $\gamma = \exp(X'\beta)$ and the conditional distribution function is

$$F(y^*|X,Z) = \begin{cases} 1-\exp(-y^*/\gamma) \text{ if } y^* > 0 \\ 0 \text{ if } y^* \leq 0 \end{cases}.$$

   We observe $Y = 1$ if $Y^* > Z'\alpha$ and $Y = 0$ if $Y^* \leq Z'\alpha$. We would like to estimate β and γ. Assume additionally that a random sample is available $\left\{(Y_i, X_i', Z_i')'\right\}_{i=1}^n$.

   (a) Give the log-likelihood function for the observed data.

   (b) Let X_j be the element j of X, what is the effect of a one-unit change in X_j on $\mathcal{P}[Y = 1|X, Z]$?

   (c) Suppose that $Y = 1$ if $Y^* > \exp(Z'\alpha)$ and $Y = 0$ if $Y^* \leq \exp(Z'\alpha)$ and $X = Z$. Is there a problem identifying α and/or β? Explain your answer.

3. Consider the following latent variable model

$$y^* = x'\beta_0 + u,$$

   where y^* is not observed, x is a vector of exogenous regressors, β_0 is a vector of parameters and $u \sim \mathcal{N}(0,1)$, that is the cumulative distribution function of u is given by $\Phi(\cdot)$.

Assume that x and u are independent. Furthermore, suppose that the researcher observes

$$y = \begin{cases} 0 & \text{if} \quad y^* \leq 0 \\ 1 & \text{if} \quad y^* > 0 \end{cases} .$$

It is suspected that there is a probability τ of a value of $y^* \leq 0$ being reported as being strictly larger than zero. That is, the data may be "one-inflated". Assuming that a sample of n independent observations is available and the regressors are exogenous. write down the log-likelihood function for this problem.