# Advanced Econometrics: Exercise #9

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*Professor Paulo Parente*

**Amy Qian**

# Problem 1

Obtain the 95% confidence interval for the **mean** of lhhexp1 using:

**i. Standard asymptotic theory**

## Solution

sum lhhexp1

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     lhhexp1 |      5,999    9.341561    .6877458   6.543108   12.20242
```

ci means lhhexp1

```
    Variable |        Obs        Mean    Std. err.       [95% conf. interval]
-------------+---------------------------------------------------------------
     lhhexp1 |      5,999    9.341561    .0088795        9.324154    9.358968
```

The 95% confidence interval is [9.324154, 9.358968]

**ii. Standard asymptotic theory using the boostrap mean and the boostrap standard errors**

## Solution

bootstrap r(mean), reps(499): sum lhhexp1

First generate bootstrap sample with 499 repetitions at the mean then showing the summary.

```
Bootstrap results                                        Number of obs = 5,999
                                                         Replications  =   499

      Command: summarize lhhexp1
        _bs_1: r(mean)

------------------------------------------------------------------------------
             |   Observed   Bootstrap                         Normal-based
             | coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _bs_1 |   9.341561   .0089509  1043.65   0.000     9.324018    9.359105
------------------------------------------------------------------------------
```

The 95% confidence interval is [9.324018, 9.359105].

**iii. The boostrap percentile method**

## Solution

estat bootstrap, all

This is asking STATA to show the information of all bootstrap confidence interval construction methods. The middle line is the bootstrap confidence interval using the percentile method.

```
Bootstrap results                               Number of obs     =      5,999
                                                Replications      =        499

      Command: summarize lhhexp1
        _bs_1: r(mean)

------------------------------------------------------------------------------
             |   Observed               Bootstrap
             | coefficient       Bias    std. err.  [95% conf. interval]
-------------+----------------------------------------------------------------
       _bs_1 |   9.3415612   .0001288   .00895086    9.324018   9.359105   (N)
             |                                        9.32274   9.359476   (P)
             |                                       9.323156   9.359813  (BC)
------------------------------------------------------------------------------
Key:  N: Normal
      P: Percentile
     BC: Bias-corrected
```

The 95% confidence interval is [9.32274, 9.359476]

**iv. The boostrap t-statistics with both methods**

## Solution

Minimizing $S(b) = \frac{1}{N}\sum_{i=1}^{N}(Y - b)^2 \rightarrow$ taking the derivative and setting to zero

$$\frac{2}{N}\sum_{i=1}^{N}(Y_i - \hat{b}) = 0$$

$$\frac{2}{N}\sum_{i=1}^{N}Y_i - \frac{2}{N}\sum_{i=1}^{N}\hat{b} = 0$$

$$\frac{2}{N}\sum_{i=1}^{N}Y_i - \frac{2}{N}N\hat{b} = 0$$

$$2\hat{b} = 2\frac{1}{N}\sum_{i=1}^{N}Y_i$$

$$\hat{b} = \frac{1}{N}\sum_{i=1}^{N}Y_i$$

which is just the sample mean.

The first method is the **non-symmetric test** with the critical values of $cv_{boot}(\frac{\alpha}{2})$ and $cv_{boot}(1 - \frac{\alpha}{2})$ for $(\hat{t}_1, \ldots, \hat{t}_B)$.

The second method is the **symmetric test** with the critical value of $cv_{boot}^{a}(1 - \alpha)$ for $(|\hat{t}_1|, \ldots, |\hat{t}_B|)$.

# Problem 2

Suppose $X_1, \ldots, X_n$ are i.i.d. continuous r.v. from distribution with cumulative distribution function $F_X(\cdot)$ and probability density function $f_X(\cdot)$. Let $M_n$ be the **sample median** and $m_0$ be the **population median**. In this case $\sqrt{n}(M_n - m_0) \xrightarrow{d} N\left(0, \frac{1}{4(f_X(m_0))^2}\right)$.

**i. Propose a consistent estimator for the asymptotic variance of $\sqrt{n}(M_n - m_0)$**

## Solution

A consistent estimator for the density is $\dfrac{1}{4\hat{f}_x^2(M_N)}$ where $\hat{f}_x(M_N) = \dfrac{1}{Nh}\sum_{i=1}^{N} K\left(\dfrac{y_i - M_N}{h}\right)$, a kernal density estimator.

**ii. Obtain a 95% CI for the median of lhhexp1**

**A. using standard asymptotic theory with the consistent estimator proposed previously**

## Solution

$SE(M_N) = \frac{1}{\sqrt{n}\sqrt{4f_X^2(M_N)}}$

$(M_N - Z_{\frac{\alpha}{2}}\, SE(M_N),\, M_N + Z_{\frac{\alpha}{2}}\, SE(M_N))$

$P(Z > Z_{\frac{\alpha}{2}}) = \frac{\alpha}{2}$, $Z \sim N(0,1)$

$h = 0.09904 \rightarrow$ obtained from Exercise sheet 8 with **Silverman's rule of thumb**

Steps to do:

1. Find the median
2. Generate the sequence
3. Find the mean of sequence

**B. using standard asymptotic theory with the bootstrap mean and bootstrap standard error**

## Solution

**C. using the boostrap percentile method**

## Solution