Advanced Topics in Statistics 2020/2021
Homework set 5

1. Investigate the existence of minimal sufficient statistics and of sufficient and complete statistics in the context of a random sample of size n from the following statistical models:

   (a) $\{N(\mu, \mu^2) : \ \mu \in \mathbb{R}\}$

   (b) Log-normal: $f(x \mid \mu, \sigma^2) = \exp[-(\ln x - \mu)^2/(2\sigma^2)]/[2\pi\sigma^2 x]$, $x \geq 0$, $\mu \in \mathbb{R}$, $\sigma > 0$

   (c) Beta: $f(x \mid \alpha, \beta) = x^{\alpha-1}(1-x)^{\beta-1}/B(\alpha, \beta)$, $x \in (0, 1)$, $\alpha > 0$, $\beta > 0$

   (d) Weibull (shape parameter $= 2$): $f(x \mid \lambda) = 2\lambda^{-2} x \exp(-x^2/\lambda^2)$, $x \geq 0$, $\lambda > 0$

2. Show (for instance, in the continuous case) that $I_{X_1,\ldots,X_n}(\theta) = I_T(\theta)$ if T is sufficient for θ.

3. Let $X_1, \ldots, X_n$ be a random sample from a $\mathrm{N}(\mu, \sigma^2)$ population.

   (a) Compute the Fisher information about $\theta = (\mu, \sigma^2)$ in the random sample.

   (b) Consider the alternative parametrization $\phi = (\mu, \sigma)$. Compute the Fisher information about ϕ in the random sample.

   (c) Let $T = (\bar{X}, S^2)$. Determine $I_T(\phi)$.