# **Chapter 1**

# **Probability**

#### 1.1 Basic concepts and results

- · Random experiment
- Sample space
- Event
- Probability measure and its properties
- · Conditional probability
- · Total probability theorem
- · Bayes' theorem
- Random variable
- Distribution function of a random variable
- Discrete random variable
- Continuous random variable
- Functions of a random variable
  - 1. Expected value
  - 2. Raw moment of order k
  - 3. Central moment of order k
  - 4. Moment generating function
- Bivariate random variables
  - 1. Marginal distributions
  - 2. Expectation of Z = h(X, Y)
  - 3. Conditional distributions
  - 4. Conditional expectations
  - 5. Raw moment of order (r, s)
  - 6. Central moment of order (r, s)
  - 7. Covariance
- · Law of iterated expectation
- Stochastic independence

- Other forms of independence
  - 1. Mean independence
  - 2. Uncorrelatedness

### 1.2 Convergence of sequences of random variables

- Notions of Convergence
  - 1. Pointwise convergence
  - 2. Uniform convergence
  - 3. Convergence in  $L^P$
  - 4. Convergence in measure
- Convergence for random variables
  - 1. Almost surely
  - 2. In the rth mean
  - 3. In probability
  - 4. In distribution
- Skorokhod representation theorem
- Continuous mapping theorem
- Slutsky theorem

### 1.3 Important asymptotic results

- Weak law of large numbers
- Strong law of large numbers
- Central limit theorem
- · Lévy's continuity theorem
- Applications
  - 1. Bernoulli
  - 2. Simple Monte Carlo
- Delta method and its applications
  - 1. Log odds
  - 2. Variance stabalizing

# **Chapter 2**

## **Classical Statistical Model**

### 2.1 Probability versus statistical inference

- · Probability theory
- Statistical inference

#### 2.2 Model specification

- Random sample
- · Sampling
- IID random sampling

#### 2.3 Statistics

• Statistic definition

### 2.4 Sampling distribution

- Definition
- Methods to obtain the sampling distribution of a statistic
  - Monte Carlo simulation
- Sample distribution of the sample moments
  - Sample moments
  - Properties of the sample mean
  - Properties of the sample variance
  - Properties of the bias-corrected sample variance
  - Properties of central sample moments
  - Asymptotic distribution of  $\bar{X}$
- Order statistics