

# Exhalatory dynamic interactions between patients connected to a shared ventilation device

## Analytical Solution

$$\begin{pmatrix} \frac{1}{\bar{C}_1} & 0 \\ 0 & \frac{1}{\bar{C}_2} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} R_1 + R_v & R_v \\ R_v & R_2 + R_v \end{pmatrix} \begin{pmatrix} \dot{V}_1 \\ \dot{V}_2 \end{pmatrix} = \begin{pmatrix} PEEP_1 \\ PEEP_2 \end{pmatrix} \quad (1)$$

$\bar{C} \qquad \bar{V} \qquad \bar{R} \qquad \dot{\bar{V}} \qquad P\bar{E}\bar{E}P$

The analytical solution of the ODE system presented on equation (1) takes the form of,

$$\begin{pmatrix} V_1(t) \\ V_2(t) \end{pmatrix} = \alpha \bar{A}_1 e^{-\frac{1}{|\lambda_1|}t} + \beta \bar{A}_2 e^{-\frac{1}{|\lambda_2|}t} + \begin{pmatrix} C_1 PEEP_1 \\ C_2 PEEP_2 \end{pmatrix} \quad (2)$$

The scalar  $\lambda_i$  and vector  $\bar{A}_i$  are associated with the homogeneous solution and depend on resistances and compliances. The scalar  $\alpha$  and  $\beta$  depend on the values of tidal volumes at the end of inspiration.

The eigenvalues  $\lambda_i$  can be computed as  $\lambda_i = \frac{1}{2} \cdot (b \pm \sqrt{b^2 - 4c})$ , where  $b = C_1(R_1 + R_v) + C_2(R_2 + R_v)$  and  $c = C_1 C_2 \frac{R_1 + R_v}{R_2 + R_v} - C_1 C_2 R_v^2$ .

The general expression of eigenvectors  $\bar{A}_i$  are presented on equation (3) and (4),

$$\bar{A}_1 = \begin{pmatrix} 1 \\ -\frac{1}{2} \frac{C_1 R_1 + C_1 R_v - C_2 R_2 - C_2 R_v + \sqrt{C_1^2 R_1^2 + 2C_1^2 R_1 R_v + C_1^2 R_v^2 - 2C_1 C_2 R_1 R_2 - 2C_1 C_2 R_1 R_v - 2C_1 C_2 R_2 R_v + 2C_1 C_2 R_v^2 + C_2^2 R_2^2 + 2C_2^2 R_2 R_v + C_2^2 R_v^2}}{C_1 R_v} \end{pmatrix} \quad (3)$$

$$\bar{A}_2 = \begin{pmatrix} 1 \\ -\frac{1}{2} \frac{C_1 R_1 + C_1 R_v - C_2 R_2 - C_2 R_v - \sqrt{C_1^2 R_1^2 + 2C_1^2 R_1 R_v + C_1^2 R_v^2 - 2C_1 C_2 R_1 R_2 - 2C_1 C_2 R_1 R_v - 2C_1 C_2 R_2 R_v + 2C_1 C_2 R_v^2 + C_2^2 R_2^2 + 2C_2^2 R_2 R_v + C_2^2 R_v^2}}{C_1 R_v} \end{pmatrix} \quad (4)$$

Using the initial conditions, scalars  $\alpha$  and  $\beta$  can be determined.