Exhalatory dynamic interactions between patients connected to a shared ventilation device

Analytical Solution

$$\begin{pmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} R_1 + R_v & R_v \\ R_v & R_2 + R_v \end{pmatrix} \begin{pmatrix} \dot{V_1} \\ \dot{V_2} \end{pmatrix} = \begin{pmatrix} PEEP_1 \\ PEEP_2 \end{pmatrix} \tag{1}$$

$$\bar{\bar{C}} \qquad \bar{V} \qquad\qquad \bar{\bar{R}} \qquad\qquad \dot{\bar{V}} \qquad P\bar{E}EP$$

The analytical solution of the ODE system presented on equation (1) takes the form of,

$$\begin{pmatrix} V_1(t) \\ V_2(t) \end{pmatrix} = \alpha \bar{A}_1 e^{-\frac{1}{|\lambda_1|}t} + \beta \bar{A}_2 e^{-\frac{1}{|\lambda_2|}t} + \begin{pmatrix} C_1 PEEP_1 \\ C_2 PEEP_2 \end{pmatrix} \tag{2}$$

The scalar λ_i and vector $\bar{A}_i$ are associated with the homogeneous solution and depend on resistances and compliances. The scalar α and β depend on the values of tidal volumes at the end of inspiration.

The eigenvalues λ_i can be computed as $\lambda_i = \frac{1}{2}.(b \pm \sqrt{b^2 - 4c}\,)$, where $b = C_1(R_1 + R_v) + C_2(R_1 + R_v)$ and $c = C_1 C_2 \frac{R_1 + R_V}{R_2 + R_V} - C_1 C_2 R_v^2$.

The general expression of eigenvectors $\bar{A}_i$ are presented on equation (3) and (4),

$$\bar{A}_1 = \begin{pmatrix} 1 \\ -\frac{1}{2} \frac{C_1 R_1 + C_1 R_v - C_2 R_2 - C_2 R_v + \sqrt{C_1{}^2 R_1{}^2 + 2C_1{}^2 R_1 R_v + C_1{}^2 R_v{}^2 - 2C_1 C_2 R_1 R_2 - 2C_1 C_2 R_1 R_v - 2C_1 C_2 R_2 R_v + 2C_1 C_2 R_v{}^2 + C_2{}^2 R_2{}^2 + 2C_2{}^2 R_2 R_v + C_2{}^2 R_v{}^2}}{C_1 R_V} \end{pmatrix} \tag{3}$$

$$\bar{A}_2 = \begin{pmatrix} 1 \\ -\frac{1}{2} \frac{C_1 R_1 + C_1 R_v - C_2 R_2 - C_2 R_v - \sqrt{C_1{}^2 R_1{}^2 + 2C_1{}^2 R_1 R_v + C_1{}^2 R_v{}^2 - 2C_1 C_2 R_1 R_2 - 2C_1 C_2 R_1 R_v - 2C_1 C_2 R_2 R_v + 2C_1 C_2 R_v{}^2 + C_2{}^2 R_2{}^2 + 2C_2{}^2 R_2 R_v + C_2{}^2 R_v{}^2}}{C_1 R_V} \end{pmatrix} \tag{4}$$

Using the initial conditions, scalars α and β can be determined.