Exhalatory dynamic interactions between patients connected to a shared ventilation device

Analytical Solution

$$\begin{pmatrix} \frac{1}{C_1} & 0 \\ 0 & \frac{1}{C_2} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} R_1 + R_v & R_v \\ R_v & R_2 + R_v \end{pmatrix} \begin{pmatrix} \dot{V_1} \\ \dot{V_2} \end{pmatrix} = \begin{pmatrix} PEEP_1 \\ PEEP_2 \end{pmatrix} \tag{1}$$

$$\bar{\bar{C}} \qquad \bar{V} \qquad \bar{\bar{R}} \qquad \dot{\bar{V}} \qquad P\bar{E}EP$$

The analytical solution of the ODE system presented on equation (1) takes the form of,

$$\begin{pmatrix} V_1(t) \\ V_2(t) \end{pmatrix} = \alpha \bar{A}_1 e^{-|\lambda_1|t} + \beta \bar{A}_2 e^{-|\lambda_2|t} + \begin{pmatrix} C_1 PEEP_1 \\ C_2 PEEP_2 \end{pmatrix} \tag{2}$$

The scalar λ_i and vector $\bar{A}_i$ are associated with the homogeneous solution and depend on resistances and compliances. The scalar α and β depend on the values of tidal volumes at the end of inspiration.

The general expression of eigenvalues λ_i are presented on equation (3) and (4),

$$\lambda_1 = \frac{1}{2} \frac{C_1R_1+C_1R_v+C_2R_2+C_2R_v}{C_1C_2(R_1R_2+R_1R_v+R_2R_v)} + \frac{1}{2} \frac{\sqrt{{C_1}^2{R_1}^2+2{C_1}^2R_1R_v+{C_1}^2{R_v}^2-2C_1C_2R_1R_2-2C_1C_2R_1R_v-2C_1C_2R_2R_v+2C_1C_2{R_v}^2+{C_2}^2{R_2}^2+2{C_2}^2R_2R_v+{C_2}^2{R_v}^2}}{C_1C_2(R_1R_2+R_1R_v+R_2R_v)} \tag{3}$$

$$\lambda_2 = \frac{1}{2} \frac{C_1R_1+C_1R_v+C_2R_2+C_2R_v}{C_1C_2(R_1R_2+R_1R_v+R_2R_v)} - \frac{1}{2} \frac{\sqrt{{C_1}^2{R_1}^2+2{C_1}^2R_1R_v+{C_1}^2{R_v}^2-2C_1C_2R_1R_2-2C_1C_2R_1R_v-2C_1C_2R_2R_v+2C_1C_2{R_v}^2+{C_2}^2{R_2}^2+2{C_2}^2R_2R_v+{C_2}^2{R_v}^2}}{C_1C_2(R_1R_2+R_1R_v+R_2R_v)} \tag{4}$$

The general expression of eigenvectors $\bar{A}_i$ are presented on equation (5) and (6),

$$\bar{A_1} = \begin{pmatrix} 1 \\ -\frac{1}{2} \frac{C_1R_1+C_1R_v-C_2R_2-C_2R_v+\sqrt{{C_1}^2{R_1}^2+2{C_1}^2R_1R_v+{C_1}^2{R_v}^2-2C_1C_2R_1R_2-2C_1C_2R_1R_v-2C_1C_2R_2R_v+2C_1C_2{R_v}^2+{C_2}^2{R_2}^2+2{C_2}^2R_2R_v+{C_2}^2{R_v}^2}}{C_1R_V} \end{pmatrix} \tag{5}$$

$$\bar{A_2} = \begin{pmatrix} 1 \\ -\frac{1}{2} \frac{C_1R_1+C_1R_v-C_2R_2-C_2R_v-\sqrt{{C_1}^2{R_1}^2+2{C_1}^2R_1R_v+{C_1}^2{R_v}^2-2C_1C_2R_1R_2-2C_1C_2R_1R_v-2C_1C_2R_2R_v+2C_1C_2{R_v}^2+{C_2}^2{R_2}^2+2{C_2}^2R_2R_v+{C_2}^2{R_v}^2}}{C_1R_V} \end{pmatrix} \tag{6}$$

Using the initial conditions, scalars α and β can be determined.