For each of the three circles, i:

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = r_i^2$$

Subtracting circle 1 from circle 3:

$$(x_3 - x_1)x + (y_3 - y_1)y + (z_3 - z_1)z = f_1$$

where:

$$f_1 = \frac{(x_3^2 + y_3^2 + z_3^2 - r_3^2) - (x_1^2 + y_1^2 + z_1^2 - r_1^2)}{2}$$

Subtracting circle 2 from circle 3:

$$(x_3 - x_2)x + (y_3 - y_2)y + (z_3 - z_2)z = f_2$$

where:

$$f_2 = \frac{(x_3^2 + y_3^2 + z_3^2 - r_3^2) - (x_2^2 + y_2^2 + z_2^2 - r_2^2)}{2}$$

Subtracting circle 1 from circle 2:

$$(x_2 - x_1)x + (y_2 - y_1)y + (z_2 - z_1)z = f_3$$

where:

$$f_3 = \frac{(x_2^2 + y_2^2 + z_2^2 - r_2^2) - (x_1^2 + y_1^2 + z_1^2 - r_1^2)}{2}$$

We can rewrite these three equations as:

$$\begin{aligned}
a_1x + b_1y + c_1z &= f_1 \\
a_2x + b_2y + c_2z &= f_2 \\
a_3x + b_3y + c_3z &= f_3
\end{aligned}$$

where:

$$\begin{aligned}
a_1 &= (x_3 - x_1); b_1 = (y_3 - y_1); c_1 = (z_3 - z_1) \\
a_2 &= (x_3 - x_2); b_2 = (y_3 - y_2); c_2 = (z_3 - z_2) \\
a_3 &= (x_2 - x_1); b_3 = (y_2 - y_1); c_3 = (z_2 - z_1)
\end{aligned}$$

In matrix notation, we can write:

$$\mathbf{A} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

Given that $\mathbf{Ax} = \mathbf{b}$, we can write:

$$\mathbf{A^{-1}Ax} = \mathbf{A^{-1}b}$$

$$\mathbf{x} = \mathbf{A^{-1}b}$$

i.e. we take the inverse of the matrix $\mathbf{A}$ and multiply it by the vector $\mathbf{b}$ to obtain values for x, y, z