Physics' notes

by and for the Sapienza's ACSAI 2020/21 students

1 Measurements

Changing units Based on where we are in the world or what task we are trying to accomplish there exist different units of measure for the same quantity, a fundamental thing to know is how to switch between them: some changes are fairly trivial, like going from kilometer to meter ($1\,\text{km} = 10^3\,\text{m}$), but others not quite so- an example may be converting minutes to seconds or square kilometers to square miles.
The process is usually the same:

1. Find/know the equivalence between two units of measure.
2. Manipulate the ratio such that the wanted final unit is on top of the fraction.
3. Apply the conversion.

Following on the previous examples, our procedure would look like this:

- $1\,\text{min} = 60\,\text{s} \rightarrow 1 = \frac{60\,\text{s}}{1\,\text{min}}$

$$\begin{aligned} t &= 13\,\text{min} \\ &= 1 \times 13\,\text{min} \\ &= \frac{60\,\text{s}}{1\,\text{min}} \times 13\,\text{min} = \boxed{7.8 \times 10^2\,\text{s}} \end{aligned}$$

- $1.61\,\text{km} = 1\,\text{mi} \rightarrow 1 = \frac{1\,\text{mi}}{1.61\,\text{km}} \rightarrow \frac{1\,\text{mi}^2}{2.59\,\text{km}^2}$

$$\begin{aligned} A &= 27.0\,\text{km}^2 \\ &= \frac{1\,\text{mi}^2}{2.59\,\text{km}^2} \times 27.0\,\text{km}^2 = \boxed{10.4\,\text{mi}^2} \end{aligned}$$

Significant figures The significant figures used to represent a quantity depend on the accuracy of the tool which took the survey: to count the amount of significant figures in a number just count all the digits which are **not** zero, all the zeroes (or groups of) which are in between non-zero figures and all of those zeroes which are deliberately left as decimal digits.
When displaying the result of a calculation, the number of significant figures to be chosen has to be equal to the lower amount of significant figures used by any value of the calculation.

$$1.22357894 \times 2.10 = 2.57$$

2 Vectors

Vector notations

Notation	Specs
Magnitude-Angle notation	$\vec{v} = \begin{cases} m \text{ - Magnitude} \\ \sigma \text{ - Angle} \end{cases} \equiv \langle m, \sigma \rangle$
Component notation	$\vec{v} = v_x\hat{i} + v_y\hat{j} \equiv \begin{bmatrix} v_x \\ v_y \end{bmatrix}$

Vector operations

Name	Equation
Changing vector notation	$\begin{cases} m = \sqrt{v_x^2 + v_y^2} \\ \sigma = \tan \frac{v_y}{v_x} \end{cases} \iff \begin{cases} v_x = m\cos\sigma \\ v_y = m\sin\sigma \end{cases}$
Unit vector	$\hat{v} = \begin{cases} \lvert v\rvert = 1 \text{ - Magnitude-Angle notation} \\ \frac{1}{\lvert v\rvert}\vec{v} \text{ - Component notation} \end{cases}$
Vector negation	$-\vec{v} = \begin{cases} \langle m, \sigma + \pi \rangle \text{ -Mag/Angl} \\ (-v_x, -v_y) \text{ -Comp.} \end{cases}$
Vector sum	$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$
Scalar multiplication	$a\vec{v} = \begin{cases} \langle \lvert am\rvert, \texttt{if } a \geq 0 : \sigma \texttt{ otherwise } \sigma + \pi \rangle \text{ -Mag/Angl} \\ (av_x, av_y) \text{ -Comp.} \end{cases}$
Dot product	$\vec{a} \cdot \vec{b} = \begin{cases} \lvert a\rvert\lvert b\rvert\cos(\phi) \text{ -Mag/Angl} \\ a_xb_x + a_yb_y \text{ -Comp.} \end{cases}$
Angle between two vectors	$\cos(\phi) = \dfrac{\vec{a} \cdot \vec{b}}{\lvert a\rvert\lvert b\rvert}$
Cross product	$\vec{a}\times\vec{b} = \begin{cases} \langle \lvert a\rvert\lvert b\rvert\sin(\phi), \sigma \text{ ortho. to inputs} \rangle \text{ -Mag/Angl} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_yb_z - a_zb_y)\hat{i} - (a_xb_z - a_zb_x)\hat{j} + (a_xb_y - a_yb_x)\hat{k} \end{cases}$

3 Motion in Two and Three dimensions

Basic definitions

Quantity	Equation	Units
Position	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$	m
Displacement	$\Delta\vec{r} = \begin{cases} \vec{r_2} - \vec{r_1} \\ (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \end{cases}$	m
Velocity	$\vec{v}_{\text{avg}} = \begin{cases} \frac{\Delta\vec{r}}{\Delta t} \\ \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k} \end{cases}$	m/s
Instantaneous velocity	$\vec{v} = \begin{cases} \frac{d\vec{r}}{dt} \\ v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt} \end{cases}$	m/s
Acceleration	$\vec{a}_{\text{avg}} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \dfrac{\Delta\vec{v}}{\Delta t}$	m/s²
Instantaneous acceleration	$\vec{a} = \begin{cases} \frac{d\vec{v}}{dt} \\ a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt} \end{cases}$	m/s²

Applications

Projectile motion

Name	Equation
Projectile motion	$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j} \ \leftarrow\ v_{0x} = v_0\cos\theta_0, \quad v_{0y} = v_0\sin\theta_0$
Horizontal motion	$x - x_0 = v_{0x}t$
Vertical motion	$y - y_0 = v_{0y}t - \dfrac{1}{2}gt^2$
Final velocity	$v_y = v_{0y} - gt \qquad v_y^2 = v_{0y}^2 - 2g(y - y_0)$
Path's equation	$y = (\tan\theta_0)x - \dfrac{gx^2}{2v_{0x}^2}$
Horizontal range	$R = \dfrac{v_0^2}{g}\sin(2\theta_0)$

Uniform circular motion

Name	Equation
Centripetal acceleration	$a_c = \frac{v^2}{r}$
Period	$T = \frac{2\pi r}{v}$

Relative motion

Name	Equation
Relative position	$\vec{r}_{\mathrm{PA}} = \vec{r}_{\mathrm{PB}} + \vec{r}_{\mathrm{BA}}$
Relative velocity	$\vec{v}_{\mathrm{PA}} = \vec{v}_{\mathrm{PB}} + \vec{v}_{\mathrm{BA}}$
Relative acceleration	$\vec{a}_{\mathrm{PA}} = \vec{a}_{\mathrm{PB}}$