

Vector math

$$\begin{cases} m = \sqrt{v_x^2 + v_y^2} \\ \sigma = \tan \frac{v_y}{v_x} \end{cases} \iff \begin{cases} v_x = m \cos \sigma \\ v_y = m \sin \sigma \end{cases}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y) \hat{i} - (a_x b_z - a_z b_x) \hat{j} + (a_x b_y - a_y b_x) \hat{k}$$

$$\vec{a} \cdot \vec{b} = \begin{cases} |a||b| \cos(\phi) \text{ -Mag/Angl} \\ a_x b_x + a_y b_y \text{ -Comp.} \end{cases} \quad \cos(\phi) = \frac{\vec{a} \cdot \vec{b}}{|a||b|}$$

Energy

$$K = \frac{1}{2}mv^2 \quad U = mad \quad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$W = \Delta K = -\Delta U = \int \vec{F} \cdot d\vec{r}$$

$$E_{\text{sys}} = \Delta U + \Delta K \iff \Delta U_i + \Delta K_i = \Delta U_f + \Delta K_f$$

Rotation

$$\theta = \frac{s}{r} \rightarrow \omega = \frac{d\theta}{dt} \rightarrow \alpha = \frac{d\omega}{dt} \quad \begin{matrix} s = \theta r \\ v = \omega r \\ a_t = \alpha r \end{matrix}$$

$$\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2 \quad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Gravitation

$$\vec{F}_{1 \rightarrow 2} = G \frac{m_1 m_2}{r^2} (-\hat{r}_{1 \rightarrow 2}) \quad U = -\frac{GMm}{r}$$

$$F_g = mg \implies g = a_c g - \omega^2 r$$

$$M_{\text{int}} = M \frac{r^3}{R^3} \quad v_e = \sqrt{\frac{2GM}{R}}$$

Heat and work

$$\Delta E_{\text{int}} = E_{\text{int},f} - E_{\text{int},i} = Q - W$$

$$\begin{cases} \text{Adiabatic: } Q = 0 \implies E_{\text{int}} = W \\ \text{Constant } V: W = 0 \implies E_{\text{int}} = Q \\ \text{Cycle: } E_{\text{int}} = 0 \implies Q_{\text{TOT}} = W_{\text{TOT}} \\ \text{Free: } \Delta E_{\text{int}} = 0 \end{cases}$$

$$P = \frac{Q}{t} = \begin{cases} \text{Conduction: } kA \frac{T_H - T_C}{L} \\ \text{Radiation: } \sigma \epsilon A T^4 \end{cases}$$

Magnetism

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad B = \frac{F_B}{|q|v}$$

$$r = \frac{mv}{|q|B} \text{ circulating particle}$$

$$\vec{F}_B = i\vec{L} \times \vec{B} \quad \int \vec{B} \cdot d\vec{S} = \mu_0 i_{\text{enc}}$$

$$B = \frac{\mu_0 i}{2\pi R} \text{ by a wire}$$

$$\vec{F}_{a,b} = \frac{\mu_0 L i_a i_b}{2\pi d} \hat{d} \text{ force between wires}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \vec{B} \cdot \vec{A}$$

$$\oint \vec{E} \cdot d\vec{S} = \mathcal{E} = -\frac{d\Phi_B}{dt}$$

General motion

$$\vec{d} = \vec{d}_0 + (\vec{v}_0 + \vec{v})t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{d} - \vec{d}_0)$$

Projectile motion

$$y = (\tan \theta_0)x - \frac{gx^2}{2v_{0x}^2}$$

$$R = \frac{v_0^2}{g} \sin(2\theta_0)$$

Spring

$$F_x = -kx \quad \vec{F}_s = -k\vec{d}$$

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$U_x = \frac{1}{2}kx^2$$

Circular motion

$$a_c = \frac{v^2}{r}$$

$$T = \frac{2\pi r}{v}$$

Friction

$$f_{s,\text{max}} = \mu_s F_N$$

$$f_k = \mu_k F_N$$

Center of Mass

$$\vec{r}_{\text{com}} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$M = \sum_i m_i$$

Linear momentum

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}} = \frac{d\vec{P}}{dt}$$

$$\vec{P} = M\vec{v}_{\text{com}}$$

$$\vec{J} = \int \vec{F}_{\text{net}} dt = \Delta \vec{P}$$

Collisions

$$\text{Inelastic: } \vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

$$\text{Elastic: } \begin{cases} \vec{v}_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} \vec{v}_{1,i} \\ \vec{v}_{2,f} = \frac{2m_1}{m_1 + m_2} \vec{v}_{1,i} \end{cases}$$

Torque and angular momentum

$$\vec{\tau} = \vec{r} \times \vec{F} = rF_{\perp} = I\alpha \quad \vec{L} = m(\vec{r} \times \vec{p}) = \int \vec{\tau} dt$$

$$\text{Analogous: } \begin{cases} W = \tau \cdot \theta \\ P = \tau \cdot \omega \\ \dots \end{cases} \quad \vec{L} = \sum_i \vec{l}_i = I\omega$$

Inertia

$$I = \sum_i m_i r_i^2 = \oint r^2 dm$$

$$K = \frac{1}{2}I\omega^2$$

Thermodynamics

$$\Delta L = \alpha L \Delta T \quad \beta = 3\alpha$$

$$\Delta V = \beta V \Delta T$$

$$Q = C \Delta T = cm \Delta T$$

$$Q_T = \begin{cases} k_V m - \text{Vaporiz.} \\ k_F m - \text{Fusion} \end{cases}$$

Fluids

$$\rho = \frac{m}{V} \quad p = \frac{\vec{F}}{A} \quad p = p_0 + \rho gh \quad |F_g| = |F_b| = m_f g$$

$$\Delta V = \Delta V_1 = \Delta V_2 \implies A_1 v_1 = A_2 v_2$$

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o} \rightarrow V = d_i A_i = d_o A_o \quad W = \int_{V_i}^{V_f} p dV$$

Kinetic theory of gases

$$n = \frac{N}{N_A} = \frac{M_{\text{sample}}}{M} = \frac{M_{\text{sample}}}{m N_A}$$

$$pV = nRT = Nk_B T$$

$$W = \begin{cases} nRT \ln \frac{V_f}{V_i} \text{ constant } T \\ 0 \text{ constant } V \\ p \Delta V \text{ constant } p \end{cases}$$

$$\text{Free expansion: } p_i V_i = p_f V_f$$

Electricity

$$\vec{F}_{1 \rightarrow 2} = k \frac{q_1 q_2}{r^2} \hat{r}_{1 \rightarrow 2} \quad k = \frac{1}{4\pi\epsilon_0}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = k \frac{q}{r^2} \hat{r}$$

$$\vec{E} = \frac{q}{2\pi\epsilon_0 z^3} \frac{d}{[1 + (d/2z)^2]^2} \text{ across dipole's z axis}$$

$$\Phi_E = \vec{E} \cdot \Delta \vec{A} = \int \vec{E} \cdot d\vec{A} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$\vec{E} = \begin{cases} \frac{\sigma}{\epsilon_0} \text{ surface} & U = qV \\ \frac{\lambda}{2\pi\epsilon_0 r} \text{ line} & \Delta V = - \int_i^f \vec{E} \cdot d\vec{S} = -\vec{E} \cdot \Delta \vec{x} \\ \frac{\sigma}{2\epsilon_0} \text{ sheet} \end{cases}$$

$$\begin{cases} \text{Parallel: } V_1 = \dots = V_n \implies C_{\text{eq}} = \sum_i^n C_i \\ \text{Serial: } q_1 = \dots = q_n \implies C_{\text{eq}}^{-1} = \sum_i^n C_i^{-1} \end{cases}$$

$$\begin{cases} \text{Parallel: } V_1 = \dots = V_n \implies R_{\text{eq}}^{-1} = \sum_i^n R_i^{-1} \\ \text{Serial: } i_1 = \dots = i_n \implies R_{\text{eq}} = \sum_i^n R_i \end{cases}$$

$$\text{Kirchhoff } \Delta V = \begin{cases} -iR \text{ through resistor} \\ +\mathcal{E} \text{ through battery} \\ 0 \text{ through cable} \end{cases}$$

$$C = \frac{q}{V} = \begin{cases} \frac{\epsilon_0 A}{d} \text{ plates} \\ \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)} \text{ cylinder} \\ 4\pi\epsilon_0 \frac{ab}{b-a} \text{ sphere} \\ 4\pi\epsilon_0 \cdot a \text{ isolated sphere} \end{cases}$$

$$i = \frac{dq}{dt} = \vec{J} \cdot \vec{A} \quad \vec{E} = \rho \vec{J}$$

$$R = \frac{V}{i} = \rho \frac{L}{A} \quad \sigma = \rho^{-1}$$

$$P = Vi \quad \mathcal{E} = \frac{dW}{dq}$$

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

Maxwell's equations

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0} \quad \oint \vec{E} \cdot d\vec{S} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \quad \oint \vec{B} \cdot d\vec{S} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$