Physics' formulary

by and for the Sapienza's ACSAI 2020/21 students

1 Measurements

Changing units Based on where we are in the world or what task we are trying to accomplish there exist different units of measure for the same quantity, a fundamental thing to know is how to switch between them: some changes are fairly trivial, like going from kilometer to meter ($1\,\text{km} = 10^3\,\text{m}$), but others not quite so- an example may be converting minutes to seconds or square kilometers to square miles.
The process is usually the same:

1. Find/know the equivalence between two units of measure.
2. Manipulate the ratio such that the wanted final unit is on top of the fraction.
3. Apply the conversion.

Following on the previous examples, our procedure would look like this:

- $1\,\text{min} = 60\,\text{s} \rightarrow 1 = \frac{60\,\text{s}}{1\,\text{min}}$

$$\begin{aligned} t &= 13\,\text{min} \\ &= 1 \times 13\,\text{min} \\ &= \frac{60\,\text{s}}{1\,\text{min}} \times 13\,\text{min} = \boxed{7.8 \times 10^2\,\text{s}} \end{aligned}$$

- $1.61\,\text{km} = 1\,\text{mi} \rightarrow 1 = \frac{1\,\text{mi}}{1.61\,\text{km}} \rightarrow \frac{1\,\text{mi}^2}{2.59\,\text{km}^2}$

$$\begin{aligned} A &= 27.0\,\text{km}^2 \\ &= \frac{1\,\text{mi}^2}{2.59\,\text{km}^2} \times 27.0\,\text{km}^2 = \boxed{10.4\,\text{mi}^2} \end{aligned}$$

Significant figures The significant figures used to represent a quantity depend on the accuracy of the tool which took the survey: to count the amount of significant figures in a number just count all the digits which are **not** zero, all the zeroes (or groups of) which are in between non-zero figures and all of those zeroes which are deliberately left as decimal digits.
When displaying the result of a calculation, the number of significant figures to be chosen has to be equal to the lower amount of significant figures used by any value of the calculation.

$$1.22357894 \times 2.10 = 2.57$$

2 Vectors

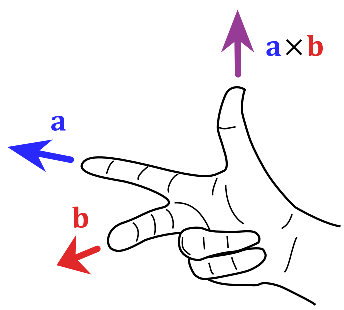
Vector notations

Notation	Specs
Magnitude-Angle notation	$\vec{v} = \begin{cases} m \text{ - Magnitude} \\ \sigma \text{ - Angle} \end{cases} \equiv \langle m, \sigma \rangle$
Component notation	$\vec{v} = v_x\hat{i} + v_y\hat{j} \equiv \begin{bmatrix} v_x \\ v_y \end{bmatrix}$

Vector operations

Name	Equation
Changing vector notation	$\begin{cases} m = \sqrt{v_x^2 + v_y^2} \\ \sigma = \tan \frac{v_y}{v_x} \end{cases} \iff \begin{cases} v_x = m \cos \sigma \\ v_y = m \sin \sigma \end{cases}$
Unit vector	$\hat{v} = \begin{cases} \|v\| = 1 \text{ - Magnitude-Angle notation} \\ \frac{1}{\|v\|}\vec{v} \text{ - Component notation} \end{cases}$
Vector negation	$-\vec{v} = \begin{cases} \langle m, \sigma + \pi \rangle \text{ -Mag/Angl} \\ (-v_x, -v_y) \text{ -Comp.} \end{cases}$
Vector sum	$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$
Scalar multiplication	$a\vec{v} = \begin{cases} \langle \|am\|, \texttt{if } a \geq 0 : \sigma \texttt{ otherwise } \sigma + \pi \rangle \text{ -Mag/Angl} \\ (av_x, av_y) \text{ -Comp.} \end{cases}$
Dot product	$\vec{a} \cdot \vec{b} = \begin{cases} \|a\|\|b\| \cos(\phi) \text{ -Mag/Angl} \\ a_x b_x + a_y b_y \text{ -Comp.} \end{cases}$
Angle between two vectors	$\cos(\phi) = \frac{\vec{a} \cdot \vec{b}}{\|a\|\|b\|}$
Cross product	$\vec{a} \times \vec{b} = \begin{cases} \langle \|a\|\|b\| \sin(\phi), \sigma \text{ ortho. to inputs} \rangle \text{ -Mag/Angl} \\ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k} \end{cases}$

Right-hand rule The right-hand rule is a simple way to imagine the direction of the vector resulting off a cross product, indeed it is not easy to find it through the Magnitude-



Angle notation, nor it is so through Component notation (even though by crunching the numbers it is possible to do so). If done well it is easy to visualize how impossible it is to process the same result by switching the arguments: spoiler it would be of the opposite direction.

3 Motion in Two and Three dimensions

Basic definitions

Quantity	Equation	Units
Position	$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$	m
Displacement	$\Delta\vec{r} = \begin{cases} \vec{r_2} - \vec{r_1} \\ (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k} \end{cases}$	m
Velocity	$\vec{v}_{\text{avg}} = \begin{cases} \frac{\Delta\vec{r}}{\Delta t} \\ \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} + \frac{\Delta z}{\Delta t}\hat{k} \end{cases}$	$^{\text{m}}/_{\text{s}}$
Instantaneous velocity	$\vec{v} = \begin{cases} \frac{d\vec{r}}{dt} \\ v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt} \end{cases}$	$^{\text{m}}/_{\text{s}}$
Acceleration	$\vec{a}_{\text{avg}} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \dfrac{\Delta\vec{v}}{\Delta t}$	$^{\text{m}}/_{\text{s}^2}$
Instantaneous acceleration	$\vec{a} = \begin{cases} \frac{d\vec{v}}{dt} \\ a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt} \end{cases}$	$^{\text{m}}/_{\text{s}^2}$

Applications

Projectile motion

Name	Equation
Projectile motion	$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j} \;\leftarrow\; v_{0x} = v_0 \cos\theta_0, \quad v_{0y} = v_0 \sin\theta_0$
Horizontal motion	$x - x_0 = v_{0x}t$
Vertical motion	$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$
Final velocity	$v_y = v_{0y} - gt \qquad v_y^2 = v_{0y}^2 - 2g(y - y_0)$
Path's equation	$y = (\tan\theta_0)x - \frac{gx^2}{2v_{0x}^2}$
Horizontal range	$R = \frac{v_0^2}{g}\sin(2\theta_0)$

Uniform circular motion

Name	Equation
Centripetal acceleration	$a_c = \frac{v^2}{r}$
Period	$T = \frac{2\pi r}{v}$

Relative motion

Name	Equation
Relative position	$\vec{r}_{\text{PA}} = \vec{r}_{\text{PB}} + \vec{r}_{\text{BA}}$
Relative velocity	$\vec{v}_{\text{PA}} = \vec{v}_{\text{PB}} + \vec{v}_{\text{BA}}$
Relative acceleration	$\vec{a}_{\text{PA}} = \vec{a}_{\text{PB}}$

4 Rotation

Basic definitions

Quantity	Equation	Units
Angular Position	$\theta = \frac{s}{r},\ \text{where } \begin{cases} s \text{ is portion of circumference} \\ r \text{ is radius} \end{cases}$	rad
Angular displacement	$\Delta\theta = \theta_2 - \theta_1$	rad
Angular velocity	$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}$	rad/s
Instantaneous angular velocity	$\omega = \frac{d\theta}{dt}$	rad/s
Angular speed	$\lvert\omega\rvert$	rad/s
Average angular acceleration	$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t}$	$\mathrm{rad}/\mathrm{s}^2$
Instantaneous angular acceleration	$\alpha = \frac{d\omega}{dt}$	$\mathrm{rad}/\mathrm{s}^2$

Derivations

Name	Equation
Angular velocity I	$\omega = \omega_0 + \alpha t$
Angular position	$(\theta - \theta_0) = \omega t + \frac{1}{2}\alpha t^2$
Angular velocity II	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
Speed	$v = \omega r$
Tangential acceleration	$a_t = \alpha r$

Applications

Name	Equation
Rotational inertia	$I = \sum_i m_i r_i^2$
Rot. inertia continuous bodies	$I = \int r^2 \; dm$
Kinetic energy	$K = \frac{1}{2} I \omega^2$

5 Gravitation

Constants

Constant	Value
Gravitational constant	$G = \begin{cases} 6.67 \times 10^{-11}\,\mathrm{N\,m}/\mathrm{kg}^2 \\ 6.67 \times 10^{-11}\,\mathrm{m}^3/\mathrm{kg\,s}^2 \end{cases}$

Equations

Notation	Equation
Gravitational force's magnitude	$F = G\frac{m_1 m_2}{r^2}$
Gravitational force from multiple objects (superposition)	$\vec{F}_{1,\text{net}} = \vec{F}_{1,2} + \vec{F}_{1,3} + \cdots + \vec{F}_{1,n} = \sum_{i=2}^{n} \vec{F}_{1,i}$
Superposition on an extended real object	$\vec{F}_1 = \int \vec{F}(x)\ d\vec{F}$
Gravitational acceleration from a (celestial) body	$a_g = \frac{GM}{r^2}$
Newton's second law for forces along r-axis	$F_N - ma_g = -m\omega^2 R$
Free-fall acceleration (near Earth's surface)	$g = a_g - \omega^2 R$
Gravitational force inside Earth	$F = \frac{GMm}{R^3} r$
Gravitational potential energy between two particles	$U = -\frac{Gm_1 m_2}{r}$
Gravitational potential energy between multiple particles	$U_{\text{TOT}} = U_{1,2}+U_{1,3}+\cdots+U_{1,n}+U_{2,3}+\cdots+U_{2,n}+\ldots$
Change of potential gravitational energy (path indep.)	$\Delta U = U_f - U_i = -W$
Escape velocity	$v = \sqrt{\frac{2GM}{R}}$