

THIS PAPER IS NOT TO BE REMOVED FROM THE EXAMINATION HALL



**UNIVERSITY  
OF LONDON**

**CM1025**

**BSc EXAMINATION**

**COMPUTER SCIENCE**

**Fundamentals of Computer Science**

Mock

Time allowed: 2 hours

**DO NOT TURN OVER UNTIL TOLD TO BEGIN**

**INSTRUCTIONS TO CANDIDATES:**

This examination paper is in two parts: Part A and Part B. You should answer **ALL** of question 1 in Part A and **TWO** questions from Part B. Part A carries 40 marks, and each question from Part B carries 30 marks. If you answer more than **TWO** questions from **Part B** only your first **TWO** answers will be marked.

All answers must be written in the answer books, answers written on the question paper will not be marked. You may write notes in your answer book. Any notes or additional answers in the answer book(s) should be crossed out.

The marks for each part of a question are indicated at the end of the part in [.] brackets. There are 100 marks available on this paper.

Graph Paper is provided at the end of this question paper. If used, it must be detached and fastened securely inside the answer book.

A handheld calculator may be used when answering questions on this paper but it must not be pre-programmed or able to display graphics, text or algebraic equations. The make and type of machine must be stated clearly on the front cover of the answer book.

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## **PART A**

Candidates should answer **ALL** of Question 1 in Part A.

### Question 1

- (a) What is the contrapositive of  $p \rightarrow q$ ? [4]
- a)  $\neg p \rightarrow q$
  - b)  $\neg q \rightarrow p$
  - c)  $\neg p \rightarrow \neg q$
  - d)  $\neg q \rightarrow \neg p$
- (b) How many ways there are to choose 8 letters from a deck of 30 letters? [4]
- a)  $30^8$
  - b)  $\frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23}{8!}$
  - c)  $30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23$
  - d)  $8!$
- (c) What is  $\Sigma^5$ ? [4]
- a) all non-empty strings over  $\Sigma$
  - b) all strings over  $\Sigma$  of length 5
  - c) any language over  $\Sigma$
  - d) all strings up to length 5

(d) Which of the following is correct?

[4]

a)  $(A \cup B) \circ C = A \cup (B \circ C)$

b)  $A \cup \emptyset = \emptyset$

c)  $A \circ B = B \circ A$

d)  $A \circ \emptyset = \emptyset$

(e) Which of the following strings can not be generated from the context-free grammar G?

$$G : S \rightarrow bSa | V$$

$$V \rightarrow a | \epsilon$$

[4]

a)  $\epsilon$

b)  $ba$

c)  $bba$

d)  $baa$

(f) What is the average case time complexity for the Quick sort?

[4]

a)  $O(n)$

b)  $O(n \log n)$

c)  $O(n^2)$

d)  $O(n^3)$

(g) A Turing machine

Choose ONE option.

[4]

- a) Has an infinite set of states
- b) Has no empty cells
- c) Is a machine where every cell contains more than one character
- d) Is a finite automaton with random access memory

(h) What is the maximum number of comparisons made for sorting an array of 7 numbers, using the Bubble sort?

[4]

- a) 28
- b) 21
- c) 7
- d) 15

(i) What is the asymptotic upper bound (Big O) of  $f(n)$  if  $f(n) = 2n + 3n \log n + 5$ ?

[4]

- a)  $\log n$
- b)  $2n$
- c)  $n$
- d)  $n \log n$

(j)  $p \vee q$  is false when

[4]

- a)  $p$  is true,  $q$  is true
- b)  $p$  is true,  $q$  is false
- c)  $p$  is false,  $q$  is true
- d)  $p$  is false,  $q$  is false

## **PART B**

Candidates should answer any **TWO** questions from Part B.

## Question 2

- (a) Sort the following list using Quick sort. Write and explain every step.

9, 5, 16, 30, 19, 7, 1, 10

[5]

- (b) This is a pseudo code for a recursive algorithm, execute it for input  $n = 4$ , show your work step by step. Write the output at every step.

[6]

PSUM( $n$ )

```

1  if  $n = 1$  then
2      return 1
3  else  $x \leftarrow$  PSUM( $n - 1$ )
4      return  $n^2 + x$ 

```

- (c) What is the time complexity of the Bubble sort(worst, best, and average)?

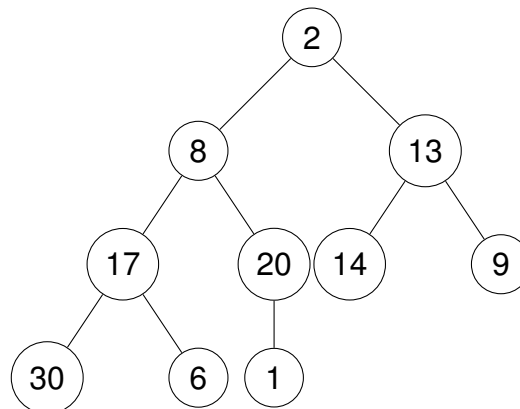
[3]

- (d) Using the Master theorem write the time complexity of  $T(n)$ ,  $T(n) = 4T(n/2) + O(n^2)$ ?

[5]

- (e) Heapify the following tree, make every step clear. (Min heap)

[5]



- (f) Write the asymptotic bound of the following functions?  $f(n) = 7n + 3n^2$ ,  $g(n) = n \log n + 8n$ ,  $h(n) = 2 \log n + 5n + 9$ .

[6]



### Question 3

(a) Using truth table show that  $(p \rightarrow q) \wedge (p \rightarrow r)$  not equivalent  $(p \wedge (q \vee r))$ .

[7]

(b) Write the negation of  $\neg p \wedge q \vee r$ .

[4]

(c) Prove the following statement by induction. For all

$$n \geq 1, 2n^3 + n \text{ is divisible by 3.}$$

[7]

(d) Prove by contrapositive that if  $n^2 + 3$  is odd if  $n$  is even.

[6]

(e) There are 6 blue, 7 green, 8 red, and 9 yellow balls in a bag. How many balls should be taken out of the bag to guarantee 5 balls have the same color?

[3]

(f) Each student has a password, which is 6 characters long and each character is either a digit or a lower case letter. Each password must contain at least ONE letter. How many possible passwords are there?

[3]

#### Question 4

- (a) If  $\Sigma = \{0, 1, 2\}$ . What is  $\Sigma^2$ ? What is the cardinality of  $\Sigma^4$ ? [5]
- (b) Build a finite automaton that accepts every binary string that contains 101. [7]
- (c) Write a regular expression for the language of all binary words that does not contain  $ab$ . [4]
- (d) What is the language of the following regular expression  $b^*a^*b^*a^*$ ? Give three words that are and three words that are not in this language. [8]
- (e) Give a context-free grammar  $G$  such that  $L(G) = \{0^i1^j | i \neq j\}$  [6]

END OF PAPER