Week 3 Sequences and Series Lecture Note

Notebook: Computational Mathematics

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Cornell Notes

Topic:

Sequences and Series

Course: BSc Computer Science

Class: Computational Mathematics[Lecture]

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Essential Question:

What are sequences?

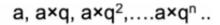
Questions/Cues:

- What is a geometric sequence?
- What is an arithmetic sequence?
- What is the formal definition of a sequence?
- What is the Fibonacci Sequence?

Notes

• Geometric Sequence = has the property that the ratio between a value in the sequence and its previous value is constant, it can any number q with a being the initial value of the sequence. The general form of a geometric sequence is as followed:

Geometric sequence





• Arithmetic sequence = has the property where the nth term in the sequence is the sum of the previous term before it and a constant number q. In other words, the difference between each element of the sequence and its previous is constant.

What is a sequence?

- Formal definition: given a set X a sequence is a function a: N→X
 i.e. a set a(0), a(1)...a(n)... denoted with {a_n}_{n∈N}
- Can be defined explicitly a_n=f(n)
 example: a_n=2n+1 →1,3,5,7,9...
- or by recursion i.e. a_n=f(a_{n-1},... a_{n-k})
 example 1: Arithmetic sequence a_n=q+a_{n-1}
 → a₀, a₀+q, a₀+2q,... → a_n=n×q+a₀
 example 2: Geometric sequence a_n=q×a_{n-1}
 → a₀, a₀q, a₀q², a₀q³... a₀qⁿ... → a_n=a₀×qⁿ
- A sequence is said to be convergent, when upon increasing n it approaches a finite constant value

$$lim_{n\to\infty}$$
 $a_n = L < \infty$

example: geometric seq. q<1 \rightarrow q=1/2 1/2,1/4,1/8.....1/64,.... converges to 0

- A sequence is said to be divergent when increasing n it never reaches a constant finite value (either goes to ∞ or oscillates) example: geometric seq. q>1 → q=2
 2,4,8,16,32,....256,...
 Examples:Fibonacci Sequence
- Definition by recursion: a₀=0, a₁=1, a_n=a_{n-1}+a_{n-2}
- 0,1,1,2,3,5,8,13,21,34,55,89,144....
- $\frac{a_n}{a_{n-1}} \rightarrow \varphi = \frac{1+\sqrt{5}}{2} = 1.618...$ Golden Ratio



Summary

called the Fibonacci Sequence.