Week 3 Sequences and Series Lecture Note

Notebook:	Computational Mathematics		
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Cornell Notes	**Topic:** Sequences and Series	Course: BSc Computer Science
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Essential Question:

What are sequences?

Questions/Cues:

- What is a geometric sequence?
- What is an arithmetic sequence?
- What is the formal definition of a sequence?
- What is the Fibonacci Sequence?

Notes

- Geometric Sequence = has the property that the ratio between a value in the sequence and its previous value is constant, it can any number q with a being the initial value of the sequence. The general form of a geometric sequence is as followed:

- Arithmetic sequence = has the property where the nth term in the sequence is the sum of the previous term before it and a constant number q. In other words, the difference between each element of the sequence and its previous is constant.

What is a sequence?

- Formal definition: given a set X a sequence is a function a: $\mathbb{N} \to X$
  i.e. a set $a(0), a(1) \ldots a(n) \ldots$ denoted with $\{a_n\}_{n \in N}$

- Can be defined explicitly $a_n = f(n)$
  example: $a_n = 2n+1 \quad \to 1,3,5,7,9 \ldots$

- or by recursion i.e. $a_n = f(a_{n-1}, .. a_{n-k})$
  example 1: *Arithmetic sequence* $a_n = q + a_{n-1}$
  $\to a_0, a_0+q, a_0+2q, \ldots \to a_n = n \times q + a_0$
  example 2: *Geometric sequence* $a_n = q \times a_{n-1}$
  $\to a_0, a_0 q, a_0 q^2, a_0 q^3 \ldots a_0 q^n \ldots \to a_n = a_0 \times q^n$

- A sequence is said to be convergent, when upon increasing n it approaches a finite constant value

$$lim_{n \to \infty} a_n = L < \infty$$

example: geometric seq. $q<1 \quad \to \quad q=1/2$
1/2,1/4,1/8……1/64,…. converges to 0

- A sequence is said to be divergent when increasing n it never reaches a constant finite value (either goes to ∞ or oscillates)
  example: geometric seq. $q>1 \quad \to \quad q=2$
  2,4,8,16,32,….256,…

Examples:Fibonacci Sequence

- Definition by recursion: $a_0=0$, $a_1=1$, $a_n = a_{n-1} + a_{n-2}$
- 0,1,1,2,3,5,8,13,21,34,55,89,144….
- $\frac{a_n}{a_{n-1}} \to \varphi = \frac{1+\sqrt{5}}{2} = 1.618 \ldots$ Golden Ratio

Summary

In this week, we learned about what a sequence is and the two different kinds of sequences, Geometric and Arithmetic. Finally, in the end we look at a famous example of a sequence

called the Fibonacci Sequence.