

Week 7 Angles, Triangles, Trigonometry Lecture Note

Notebook: Computational Mathematics

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Cornell Notes

Angles, triangles,
trigonometry

Course: BSc Computer Science

Class: Computational
Mathematics[Lecture]

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Essential Question:

What are angles and what is trigonometry and how are these related to the study of triangles?

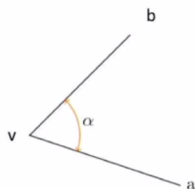
Questions/Cues:


- What is an angle?
- What are some special types of angles, how do we measure radians and how do we convert between radians/degrees?
- What are some properties of triangles?
- What are similar triangles?
- What are properties of right triangles?

Notes

What is an angle?

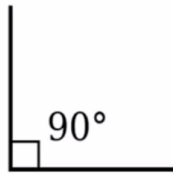
- It is a measure of the separation of two rays emanating from a vertex v



- It is measured in degrees (sexagesimal) or radians
- 1 Degree is $1/180$ of a flat angle  180°
- 1 min is $1/60$ of a degree, 1 sec is $1/60$ of min
- ex. $35^\circ 23' 12''$
 - A flat angle is just the separation between two rays that emanate from the vertex and they depart in opposite direction. A flat angle corresponds to 180°

- When working with degrees, we are working in base 60

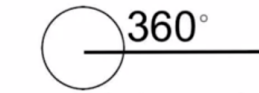
Types of angle



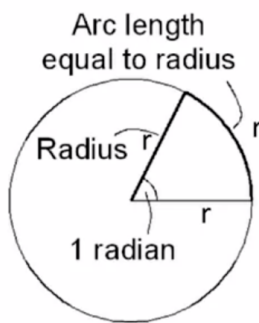
right angle



flat angle



complete angle



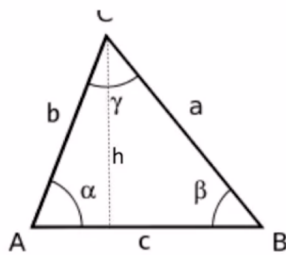
$$r \rightarrow 1 \text{ radian} \Rightarrow (\text{circ.}) 2\pi r \rightarrow 2\pi \text{ radians}$$

$$360^\circ = 2\pi \text{ radians}$$

$$\text{radians} = \text{degrees} \times \pi / 180^\circ$$

- The complete angle is the angle that is formed by two rays emanating from a vertex when they coincide
- Angles are periodic, in degree they periodical of 360°

Triangles: properties

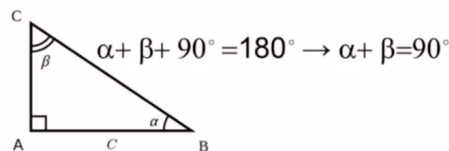


Triangle: Right trian.

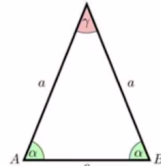
$$\alpha + \beta + \gamma = 180^\circ$$

$$S = c \times h / 2$$

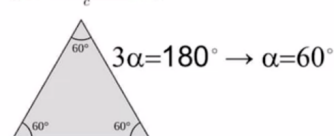
$$P = a + b + c$$



Isosceles

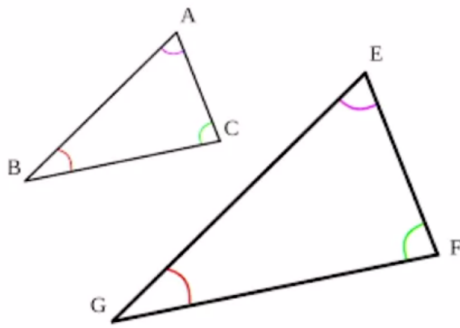


Equilateral



- In an isosceles triangle, two sides are the same length and the angles adjacent to the non-equal side are equal
- Similar Triangles = have the same angles but they have sides which are rescaled by the same rescaling factor; they have the same angles and proportional sides. This means the ratio between each side of one triangle and the corresponding side of the similar triangle are the same

Similar Triangles

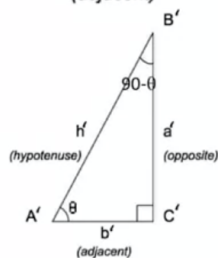
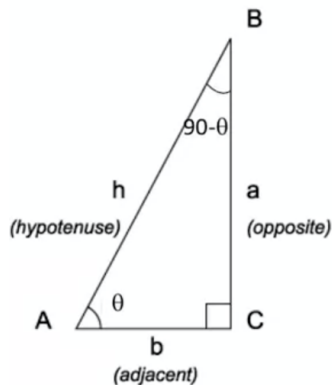


Similar Triangles
rescale one (zoom in or out) and
will coincide with the other

Same angles, proportional sides:

$$\rightarrow AB/EG = AC/EF = BC/GF$$

Right Triangles: properties



$$h/h' = b/b' \rightarrow b/h = b'/h' = \cos(\theta) = \sin(90-\theta)$$

$$h/h' = a/a' \rightarrow a/h = a'/h' = \sin(90-\theta) = \cos(\theta)$$

$$a/a' = b/b' \rightarrow a/b = a'/b' = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$

$$1) \sin(\theta) = a/h = \frac{\text{opposite}}{\text{hypotenuse}} \rightarrow a = h \sin(\theta)$$

$$2) \cos(\theta) = b/h = \frac{\text{adjacent}}{\text{hypotenuse}} \rightarrow b = h \cos(\theta)$$

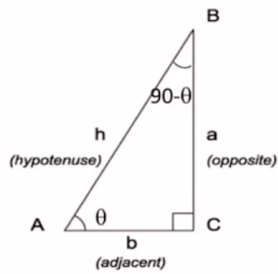
$$3) \tan(\theta) = a/b = \frac{\text{opposite}}{\text{adjacent}} \rightarrow a = b \tan(\theta)$$

$a^2 + b^2 = h^2$ Pithagora's theorem

From 1) and 2) it follows

$$h^2 \sin^2(\theta) + h^2 \cos^2(\theta) = h^2$$

$$\rightarrow \sin^2(\theta) + \cos^2(\theta) = 1$$



$$\rightarrow a = h \sin(\theta)$$

$$\rightarrow b = h \cos(\theta) = h \sin(90^\circ - \theta)$$

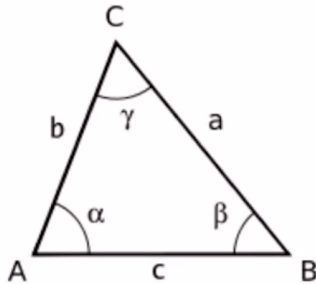
what if $\theta \rightarrow 0$? $h \rightarrow b$ and $a \rightarrow 0$

$$\rightarrow \cos(0^\circ) = \sin(90^\circ) = 1, \quad \sin(0^\circ) = \cos(90^\circ) = 0$$

$$\rightarrow \frac{a}{\sin(\theta)} = \frac{b}{\sin(90^\circ - \theta)} = h = \frac{h}{\sin(90^\circ)}$$

Sine rule

General triangle



$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

generalized *Pithagora's th.*

$$\rightarrow a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

(a.k.a. cosine rule)

Summary

In this week, we learned about what an angle is. Alongside this, we looked at properties of triangles, Pythagoras theorem, the laws of sines and cosines