**Definition** (Semicategory). A *semicategory* $\mathbf{C}$ is specified by:

Constituents

1. Objects: A collection $\mathrm{Ob}_{\mathbf{C}}$ whose elements are called *objects*.

2. Morphisms: For every pair of objects $X$, $Y$ in $\mathrm{Ob}_{\mathbf{C}}$, there is a set $\mathrm{Hom}_{\mathbf{C}}(X;Y)$, elements of which are called *morphisms*. We write

$$f : X \to_{\mathbf{C}} Y$$

to indicate

$$f \in \mathrm{Hom}_{\mathbf{C}}(X;Y).$$

3. Composition operations: For every three objects $X, Y, Z$ in $\mathrm{Ob}_{\mathbf{C}}$ there is a composition map

$$\mathbin{\overset{\circ}{,}}_{X,Y,Z} : \mathrm{Hom}_{\mathbf{C}}(X;Y) \times \mathrm{Hom}_{\mathbf{C}}(Y;Z) \to \mathrm{Hom}_{\mathbf{C}}(X;Z).$$

We usually just write $\mathbin{\overset{\circ}{,}}$ instead of $\mathbin{\overset{\circ}{,}}_{X,Y,Z}$:

$$\frac{f : X \to Y \quad g : Y \to Z}{f \mathbin{\overset{\circ}{,}} g : X \to Z}$$

The morphism $f \mathbin{\overset{\circ}{,}} g$ is called the *composition* of $f$ and $g$.

Conditions

1. Associativity: it holds that

$$\frac{f : X \to Y \quad g : Y \to Z \quad h : Z \to U}{(f \mathbin{\overset{\circ}{,}} g) \mathbin{\overset{\circ}{,}} h = f \mathbin{\overset{\circ}{,}} (g \mathbin{\overset{\circ}{,}} h)}$$