

Definition

A *category* \mathbf{C} is specified by:

Constituents

1. Objects: A set $\mathbf{Ob}_{\mathbf{C}}$ whose elements are called *objects*.
2. Morphisms: For every pair of objects X, Y in $\mathbf{Ob}_{\mathbf{C}}$, there is a set $\mathbf{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms* from X to Y . A morphism $f \in \mathbf{Hom}_{\mathbf{C}}(X; Y)$ is often indicated by writing

$$f : X \rightarrow Y.$$

3. Identity morphisms: for each object X , there is a morphism $\text{Id}_X : X \rightarrow X$ called *the identity morphism of* X .
4. Composition operations: For every three objects X, Y, Z in $\mathbf{Ob}_{\mathbf{C}}$ there is a composition function

$$\circ_{X,Y,Z} : \mathbf{Hom}_{\mathbf{C}}(X; Y) \times \mathbf{Hom}_{\mathbf{C}}(Y; Z) \rightarrow \mathbf{Hom}_{\mathbf{C}}(X; Z)$$

We denote the *composition* of composable morphisms f and g by $f \circ g$. (Traditionally, the typical notation would be $g \circ f$).

Conditions

1. Associativity: for all composable morphisms f, g, h ,

$$(f \circ g) \circ h = f \circ (g \circ h).$$

2. Unitality: for every morphism $f : X \rightarrow Y$,

$$\text{Id}_X \circ f = f \quad \text{and} \quad f \circ \text{Id}_Y = f.$$