Definition (Braided monoidal category). A *braided monoidal category* is a monoidal category $\langle \mathbf{C}, \otimes, \mathbf{1} \rangle$, cf. **??**, equipped with a *braiding*, which is specified by

<u>Constituents</u>

1. A natural isomorphism br, called the braiding, whose components are of the type

$$\mathrm{br}_{X,Y} : (X \otimes Y) \xrightarrow{\cong} (X \otimes Y), \quad X, Y \in \mathrm{Ob}_{\mathbf{C}}.$$

   Explicitly, this means that for any morphisms $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$, the following diagram

$$\begin{tikzcd}
X_1 \otimes X_2 \arrow[r, "f_1 \otimes f_2"] \arrow[d, "\mathrm{br}_{X_1,X_2}"'] & Y_1 \otimes Y_2 \arrow[d, "\mathrm{br}_{Y_1,Y_2}"] \\
X_2 \otimes X_1 \arrow[r, "f_2 \otimes f_1"'] & Y_2 \otimes Y_1
\end{tikzcd}$$

   commutes.

<u>Conditions</u>

1. *Hexagon identites:* Given any objects $X, Y, Z \in \mathrm{Ob}_{\mathbf{C}}$, the following diagrams must commute.

$$\begin{tikzcd}
(X \otimes Y) \otimes Z \arrow[r, "\mathrm{br}_{X,Y} \otimes \mathrm{Id}_Z"] \arrow[d, "\mathrm{as}_{X,Y,Z}"'] & (Y \otimes X) \otimes Z \arrow[r, "\mathrm{as}_{Y,X,Z}"] & Y \otimes (X \otimes Z) \arrow[d, "\mathrm{Id}_Y \otimes \mathrm{br}_{X,Z}"] \\
X \otimes (Y \otimes Z) \arrow[r, "\mathrm{br}_{X,Y \otimes Z}"'] & (Y \otimes Z) \otimes X \arrow[r, "\mathrm{as}_{Y,Z,X}"'] & Y \otimes (Z \otimes X)
\end{tikzcd}$$

$$\begin{tikzcd}
X \otimes (Y \otimes Z) \arrow[r, "\mathrm{Id}_X \otimes \mathrm{br}_{Y,Z}"] \arrow[d, "\mathrm{as}^{-1}_{X,Y,Z}"'] & X \otimes (Z \otimes Y) \arrow[r, "\mathrm{as}^{-1}_{Y,X,Z}"] & (X \otimes Z) \otimes Y \arrow[d, "\mathrm{br}_{X,Z} \otimes \mathrm{Id}_Y"] \\
(X \otimes Y) \otimes Z \arrow[r, "\mathrm{br}_{X \otimes Y,Z}"'] & Z \otimes (X \otimes Y) \arrow[r, "\mathrm{as}^{-1}_{Y,Z,X}"'] & (Z \otimes X) \otimes Y
\end{tikzcd}$$