Definition (Dualizable object). Let $\langle \mathbf{C}, \otimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}} \rangle$ be a monoidal category, and let $X \in \mathrm{Ob}_{\mathbf{C}}$. A *right dual object* of X is specified by:

Constituents

1. an object $X^{\vee} \in \mathrm{Ob}_{\mathbf{C}}$;
2. an evaluation map $\epsilon_X : X^{\vee} \otimes X \to \mathbf{1}$;
3. a coevaluation map $\eta_X : \mathbf{1} \to X \otimes X^{\vee}$;

Conditions

1. $\mathrm{lu}_X^{-1} ⨟ (\eta_X \otimes \mathrm{Id}_X) ⨟ \mathrm{as}_{X,X^{\vee},X} ⨟ (\mathrm{Id}_X \otimes \epsilon_X) ⨟ \mathrm{ru}_X = \mathrm{Id}_X$;
2. $\mathrm{ru}_{X^{\vee}}^{-1} ⨟ (\mathrm{Id}_{X^{\vee}} \otimes \eta_X) ⨟ \mathrm{as}_{X^{\vee},X,X^{\vee}}^{-1} ⨟ (\epsilon_X \otimes \mathrm{Id}_{X^{\vee}}) ⨟ \mathrm{lu}_{X^{\vee}} = \mathrm{Id}_{X^{\vee}}$.