ton sets  $\{S_i\}_{i\in I}$ , with  $S_i = \{s_i\}, s_i \in \mathbf{P}$ , the following equality holds:

**Lemma.** Given posets  $\mathbf{P}$ ,  $\mathbf{Q}$ , a monotone map  $f: \mathbf{P} \to \mathbf{Q}$ , and a family of single-

 $\uparrow \left(\bigcup_{p \in \uparrow} \{f(p)\}\right) = \uparrow \left(\bigcup_{i \in I} \{f(s_i)\}\right).$