Definition (Profunctor composition). Given two profunctors $F: \mathbb{C} \to \mathbb{D}$ and $G: \mathbb{D} \to \mathbb{E}$ we can define their composition $(F ; G): \mathbb{C} \to \mathbb{E}$ as follows:

$$(F \circ G)_{ob} : Ob(\mathbf{C}^{op} \times \mathbf{E}) \to Ob \operatorname{\mathbf{Set}},$$

$$\langle C^*, E \rangle \mapsto \coprod_{D \in ObD} F_{ob}(C^*, D) \times G_{ob}(D^*, E) / \sim$$

$$(F \circ G)_{mor} : \operatorname{Hom}_{(\mathbf{C}^{op} \times \mathbf{E})} (\langle C_1^*, E_1 \rangle; \langle C_2^*, E_2 \rangle) \to \operatorname{Hom}_{\operatorname{\mathbf{Set}}} ((F \circ G)_{ob}(C_1^*, E_1); (F \circ G)_{ob}(C_2^*, E_2))$$

 $\langle \alpha^*, \beta \rangle \mapsto \begin{cases} (F \circ G)_{\text{ob}}(C_1^*, E_1) \to (F \circ G)_{\text{ob}}(C_2^*, E_2)) \\ \langle s, t \rangle \mapsto \langle F_{\text{mor}}(\langle \alpha, \text{Id}_D \rangle)(s), G_{\text{mor}}(\langle \text{Id}_{D^*}, \beta \rangle)(t) \rangle \end{cases}$

In the formulas:

$$\alpha: C_2 \to C_1, \qquad \beta: E_1 \to E_2,$$

and $\langle s, t \rangle$ is a pair of elements for which there exists a $D \in Ob\mathbf{D}$ such that

$$s \in F_{ob}(C_1^*, D), \qquad t \in G_{ob}(D^*, E).$$