Definition (Monoidal category). A *monoidal structure* on a category $\mathbf{C}$ consists of:

1. An object $\mathbf{1} \in \mathrm{Ob}_{\mathbf{C}}$ called the *monoidal unit*.
2. A functor $\otimes : \mathbf{C} \times \mathbf{C} \to \mathbf{C}$, called the *monoidal product*.

The two constituents are subject to the natural isomorphisms:

a) (Left unitor) $\mathrm{lu}_X : \mathbf{1} \otimes X \xrightarrow{\cong} X$ for every $X \in \mathrm{Ob}_{\mathbf{C}}$,

b) (Right unitor) $\mathrm{ru}_X : Z \otimes \mathbf{1} \xrightarrow{\cong} X$ for every $X \in \mathrm{Ob}_{\mathbf{C}}$,

c) (Associator) $\mathrm{as}_{X,Y,Z} : (X \otimes Y) \otimes Z \xrightarrow{\cong} X \otimes (Y \otimes Z)$ for every $X, Y, Z \in \mathrm{Ob}_{\mathbf{C}}$.

These isomorphisms are themselves required to satisfy the triangle identity

$$
\begin{tikzcd}
(X \otimes \mathbf{1}) \otimes Y \arrow[rr, "\mathrm{as}_{X,\mathbf{1},Y}"] \arrow[dr, "\mathrm{ru}_X \otimes \mathbf{1}"'] & & X \otimes (\mathbf{1} \otimes Y) \arrow[dl, "\mathbf{1} \otimes \mathrm{lu}_Y"] \\
& X \otimes Y &
\end{tikzcd}
$$

and the pentagon identity

$$
\begin{tikzcd}
& (X \otimes Y) \otimes (Z \otimes W) \arrow[dr, "\mathrm{as}_{X,Y,Z \otimes W}"] & \\
((X \otimes Y) \otimes Z) \otimes W \arrow[ur, "\mathrm{as}_{X \otimes Y,Z,W}"] \arrow[d, "\mathrm{as}_{X,Y,Z} \otimes \mathrm{Id}_W"'] & & (X \otimes (Y \otimes (Z \otimes W))) \\
(X \otimes (Y \otimes Z)) \otimes W \arrow[rr, "\mathrm{as}_{X,Y \otimes Z,W}"'] & & X \otimes ((Y \otimes Z) \otimes W) \arrow[u, "\mathrm{Id}_X \otimes \mathrm{as}_{Y,Z,W}"']
\end{tikzcd}
$$

for $X, Y, Z, W \in \mathrm{Ob}_{\mathbf{C}}$. A category equipped with a monoidal structure is called a *monoidal category*. If the isomorphisms in a), b), and c) are equivalences, one calls the category *strict* monoidal.