

## Definition (Categorical Coproduct)

Let  $\mathbf{C}$  be a category and let  $X, Y \in \mathbf{Ob}_{\mathbf{C}}$  be objects. The *coproduct* of  $X$  and  $Y$  is:

### Constituents

1. an object  $Z \in \mathbf{Ob}_{\mathbf{C}}$  (“the coproduct of  $X$  and  $Y$ ”)
2. *injection morphisms*  $\text{in}_1 : X \rightarrow Z$  and  $\text{in}_2 : Y \rightarrow Z$

### Conditions

1. For any  $T \in \mathbf{Ob}_{\mathbf{C}}$  and any morphisms  $f : X \rightarrow T, g : Y \rightarrow T$ , there exists a *unique* morphism  $\psi_{f,g} : Z \rightarrow T$  such that  $f = \text{in}_1 \circ \psi_{f,g}$  and  $g = \text{in}_2 \circ \psi_{f,g}$ .