Definition (Category). A *category* $\mathbf{C}$ is:

Constituents

1. Objects: a collection $\mathrm{Ob}_{\mathbf{C}}$, whose elements are called *objects*.
2. Morphisms: for every pair of objects $X, Y \in \mathrm{Ob}_{\mathbf{C}}$, there is a set $\mathrm{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms* from X to Y. The set is called the "hom-set from X to Y".
3. Identity morphisms: for each object X, there is an element $\mathrm{Id}_X \in \mathrm{Hom}_{\mathbf{C}}(X; X)$ which is called *the identity morphism of* X.
4. Composition operations: given any morphism $f \in \mathrm{Hom}_{\mathbf{C}}(X; Y)$ and any morphism $g \in \mathrm{Hom}_{\mathbf{C}}(Y; Z)$, there exists a morphism $f \mathbin{⨟} g \in \mathrm{Hom}_{\mathbf{C}}(X; Z)$ which is the *composition of* f *and* g.

Conditions

1. Unitality: It holds that:

$$\frac{\mathrm{Id}_X : X \to X \quad f : X \to Y \quad \mathrm{Id}_Y : Y \to Y}{\mathrm{Id}_X \mathbin{⨟} f = f = f \mathbin{⨟} \mathrm{Id}_Y}$$

2. Associativity: it holds that

$$\frac{f : X \to Y \quad g : Y \to Z \quad h : Z \to U}{(f \mathbin{⨟} g) \mathbin{⨟} h = f \mathbin{⨟} (g \mathbin{⨟} h)}$$