

**Definition** (Free category on a graph). Let  $G = (V, A, s, t)$  be a graph. The *free category on  $G$* , denoted  $\mathbf{Free}(G)$ , has as objects the vertices  $V$  of  $G$ , and given vertices  $x \in V$  and  $y \in V$ , the morphisms  $\mathbf{Free}(G)(x, y)$  are the paths from  $x$  to  $y$ . The composition of morphisms is given by concatenation of paths, and for any object  $x \in V$ , the associated identity morphism  $\text{id}_x$  is the trivial path which starts and ends at  $x$ .