Definition (Group). A *group* is a monoid together with an "inverse" operation. In more detail, a group $\mathbf{G}$ is

Constituents

1. a set $\mathbf{G}$;
2. a binary operation $\fatsemi : \mathbf{G} \times \mathbf{G} \to \mathbf{G}$, called *composition*;
3. a specified element $\mathrm{id} \in \mathbf{G}$;
4. a map $\mathrm{inv} : \mathbf{G} \to \mathbf{G}$, called *inverse*.

Conditions

1. Associative law: $(x \fatsemi y) \fatsemi z = x \fatsemi (y \fatsemi z)$ $\qquad \forall\, x, y, z \in \mathbf{G}$;
2. Neutrality laws: $\mathrm{id} \fatsemi x = x = x \fatsemi \mathrm{id}$ $\qquad \forall\, x \in \mathbf{G}$;
3. Inverse laws:

$$\mathrm{inv}(x) \fatsemi x = \mathrm{id} = \mathrm{inv}(x) \fatsemi x \qquad \forall\, x \in \mathbf{G}.$$