Definition (Strong monoidal functor). Let $\langle \mathbf{C}, \otimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}} \rangle$ and $\langle \mathbf{D}, \otimes_{\mathbf{D}}, \mathbf{1}_{\mathbf{D}} \rangle$ be two monoidal categories. A *strong monoidal functor* between $\mathbf{C}$ and $\mathbf{D}$ is given by:

1. A functor
$$F : \mathbf{C} \to \mathbf{D};$$

2. An isomorphism
$$\mathrm{iso} : \mathbf{1}_{\mathbf{D}} \to F(\mathbf{1}_{\mathbf{C}});$$

3. A natural isomorphism μ
$$\mu_{X,Y} : F(X) \otimes_{\mathbf{D}} F(Y) \to F(X \otimes_{\mathbf{C}} Y), \quad \forall X, Y \in \mathbf{C},$$

satisfying the following conditions:

a) *Associativity*: For all objects $X, Y, Z \in \mathbf{C}$, the following diagram commutes.

$$\begin{tikzcd}
(F(X) \otimes_{\mathbf{D}} F(Y)) \otimes_{\mathbf{D}} F(Z) \arrow[r, "\mathrm{as}^{\mathbf{D}}_{F(X),F(Y),F(Z)}"] \arrow[d, "\mu_{X,Y} \otimes_{\mathbf{D}} \mathrm{Id}(F(Z))"'] & F(X) \otimes_{\mathbf{D}} (F(Y) \otimes_{\mathbf{D}} F(Z)) \arrow[d, "\mathrm{Id}(F(X)) \otimes_{\mathbf{D}} \mu_{Y,Z}"] \\
F(X \otimes_{\mathbf{C}} Y) \otimes_{\mathbf{D}} F(Z) \arrow[d, "\mu_{X \otimes_{\mathbf{D}} Y, Z}"'] & F(X) \otimes_{\mathbf{D}} F(Y \otimes_{\mathbf{C}} Z) \arrow[d, "\mu_{X, Y \otimes_{\mathbf{D}} Z}"] \\
F((X \otimes_{\mathbf{C}} Y) \otimes_{\mathbf{C}} Z) \arrow[r, "F(\mathrm{as}^{\mathbf{C}}_{X,Y,Z})"'] & F(X \otimes_{\mathbf{C}} (Y \otimes_{\mathbf{C}} Z))
\end{tikzcd}$$

where $\mathrm{as}^{\mathbf{C}}$ and $\mathrm{as}^{\mathbf{D}}$ are called *associators*.

b) *Unitality*: For all $X \in \mathbf{C}$, the following diagrams commute:

$$\begin{tikzcd}
\mathbf{1}_{\mathbf{D}} \otimes_{\mathbf{D}} F(X) \arrow[r, "\mathrm{iso} \otimes_{\mathbf{D}} \mathrm{Id}_{F(X)}"] \arrow[d, "\mathrm{lu}_{\mathbf{D}}"'] & F(\mathbf{1}_{\mathbf{C}}) \otimes_{\mathbf{D}} F(X) \arrow[d, "\mu_{\mathbf{1}_{\mathbf{C}},X}"] \\
F(X) & F(\mathbf{1}_{\mathbf{C}} \otimes_{\mathbf{C}} X) \arrow[l, "F(\mathrm{lu}^{\mathbf{C}}_X)"]
\end{tikzcd}$$

$$\begin{tikzcd}
F(X) \otimes_{\mathbf{D}} \mathbf{1}_{\mathbf{D}} \arrow[r, "\mathrm{Id}_{F(X)} \otimes_{\mathbf{D}} \mathrm{iso}"] \arrow[d, "\mathrm{ru}_{\mathbf{D}}"'] & F(X) \otimes_{\mathbf{D}} F(\mathbf{1}_{\mathbf{C}}) \arrow[d, "\mu_{X,\mathbf{1}_{\mathbf{C}}}"] \\
F(X) & F(X \otimes_{\mathbf{C}} \mathbf{1}_{\mathbf{C}}) \arrow[l, "F(\mathrm{ru}^{\mathbf{C}}_X)"]
\end{tikzcd}$$

where $\mathrm{lu}^{\mathbf{C}}$ and $\mathrm{ru}^{\mathbf{C}}$ represent the left and right *unitors*.