

**Definition** (Enriched category). Let  $\langle \mathbf{B}, \otimes, \mathbf{1}, \text{as}, \text{lu}, \text{ru} \rangle$  be a monoidal category. A category  $\mathbf{C}$  *enriched* in  $\mathbf{B}$  is composed of:

1. The set of objects  $\text{Ob}_{\mathbf{C}}$ ;
2. For all  $X, Y \in \text{Ob}_{\mathbf{C}}$ , an object  $\text{Hom}_{\mathbf{C}}(X; Y)$ , called the *hom-object* from  $X$  to  $Y$ .
3. For all  $X, Y, Z \in \text{Ob}_{\mathbf{C}}$ , there exists a morphism  $m_{X,Y,Z}$  in  $\mathbf{B}$ :

$$m_{X,Y,Z} : \text{Hom}_{\mathbf{C}}(X; Y) \otimes \text{Hom}_{\mathbf{C}}(Y; Z) \rightarrow \text{Hom}_{\mathbf{C}}(X; Z).$$

This is called *composition morphism*.

4. For each  $X \in \text{Ob}_{\mathbf{C}}$ , a morphism  $j_X : \mathbf{1} \rightarrow \text{Hom}_{\mathbf{C}}(X; X)$ , called *identity element*.

Furthermore, for any  $X, Y, Z, W \in \text{Ob}_{\mathbf{C}}$ , the following diagrams must commute.