Lemma. The product of antichains $\times : \mathcal{A}R_1 \times \mathcal{A}R_2 \to \mathcal{A}(R_1 \times R_2)$ is monotone.