

$$\begin{aligned}
\mathrm{Tr}_{X,Y}^Z \left(\mathrm{Tr}_{X \times Z, Y \times Z}^U (f) \right) (x) &= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \mathrm{Tr}_{X \times Z, Y \times Z}^U (f)(x, z) \right\} \\
&= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \left\{ \langle y', z' \rangle \in Y \times Z \mid \bigvee_{u \in U} \langle y', z', u \rangle \in f(x, z', u) \right\} \right\} \\
&= \left\{ y \in Y \mid \bigvee z \in Z \left(\bigvee_{u \in U} \langle y, z, u \rangle \in f(x, z, u) \right) \right\} \\
&= \left\{ y \in Y \mid \bigvee_{\langle z, u \rangle \in Z \times U} \langle y, z, u \rangle \in f(x, z, u) \right\}.
\end{aligned}$$