

Definition (Series composition). Let $\mathbf{f} : \mathbf{A} \multimap \mathbf{B}$ and $\mathbf{g} : \mathbf{B} \multimap \mathbf{C}$ be design problems. We define their *series composition* $(\mathbf{f} \circ \mathbf{g}) : \mathbf{A} \multimap \mathbf{C}$ as:

$$(\mathbf{f} \circ \mathbf{g}) : \mathbf{A}^{\text{op}} \times \mathbf{C} \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle a^*, c \rangle \mapsto \bigvee_{b \in \mathbf{B}} f(a^*, b) \wedge g(b^*, c).$$

Alternatively:

$$(\mathbf{f} \circ \mathbf{g}) : \mathbf{A}^{\text{op}} \times \mathbf{C} \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle a^*, c \rangle \mapsto \bigvee_{b_1 \leq b_2, b_1, b_2 \in \mathbf{B}} \mathbf{f}(a^*, b_1) \wedge \mathbf{g}(b_2^*, c).$$