**Definition** (Monoid morphism). A morphism $F : \mathbf{M} \to \mathbf{N}$ between monoids

$$\mathbf{M} = \langle \mathbf{M}, \fatsemi_{\mathbf{M}}, \mathrm{id}_{\mathbf{M}} \rangle \qquad \text{and} \qquad \mathbf{N} = \langle \mathbf{N}, \fatsemi_{\mathbf{N}}, \mathrm{id}_{\mathbf{N}} \rangle$$

is a function $F : \mathbf{M} \to \mathbf{N}$ such that for all $x, y$ in $\mathbf{M}$,

$$F(x \fatsemi_{\mathbf{M}} y) = F(x) \fatsemi_{\mathbf{N}} F(y)$$

and

$$F(\mathrm{id}_{\mathbf{M}}) = \mathrm{id}_{\mathbf{N}}$$