$\mathbf{M} = \langle \mathbf{M}, \S_{\mathbf{M}}, \mathrm{id}_{\mathbf{M}} \rangle$  and  $\mathbf{N} = \langle \mathbf{N}, \S_{\mathbf{N}}, \mathrm{id}_{\mathbf{N}} \rangle$ 

**Definition** (Monoid morphism). A morphism  $F: \mathbf{M} \to \mathbf{N}$  between monoids

$$\mathbf{W} - \mathbf{W}, \mathbf{g}_{\mathbf{M}}, \mathbf{W} / \mathbf{W} = \mathbf{W}, \mathbf{g}_{\mathbf{N}}, \mathbf{W}$$

is a function  $F: \mathbf{M} \to \mathbf{N}$  such that for all x, y in  $\mathbf{M}$ ,

$$F(x \circ_{\mathbf{M}} y) = F(x) \circ_{\mathbf{N}} F(y)$$

and

$$F(\mathrm{id}_{\mathbf{M}}) = \mathrm{id}_{\mathbf{N}}$$