Definition (Monad)

Let $\mathbf{C}$ be a category. A *monad* on $\mathbf{C}$ is specified by:

Constituents

1. A functor $M : \mathbf{C} \to \mathbf{C}$;
2. A natural transformation $\mathrm{mu} : M \fatsemi M \Rightarrow M$, called the *composition* or *multiplication*;
3. A natural transformation $\mathrm{un} : \mathrm{Id}_{\mathbf{C}} \Rightarrow M$, called the *unit*.

Conditions

1. *Associativity*: the diagram

$$\begin{array}{ccc} M \fatsemi M \fatsemi M & \xRightarrow{M\mathrm{mu}} & M \fatsemi M \\ \Big\Downarrow{\scriptstyle \mathrm{mu}M} & & \Big\Downarrow{\scriptstyle \mathrm{mu}} \\ M \fatsemi M & \xRightarrow{\mathrm{mu}} & M \end{array}$$

must commute.

2. *Left and right unitality*: the diagrams

$$\begin{array}{ccc} M & \xRightarrow{\mathrm{un}M} & M \fatsemi M \\ & \searrow^{\mathrm{Id}_M} & \Big\Downarrow{\scriptstyle \mathrm{mu}} \\ & & M \end{array} \qquad \begin{array}{ccc} M & \xRightarrow{M\mathrm{un}} & M \fatsemi M \\ & \searrow^{\mathrm{Id}_M} & \Big\Downarrow{\scriptstyle \mathrm{mu}} \\ & & M \end{array}$$

must commute.