

Lemma. Given a poset P and a family of upper sets of P , denoted $\{P_i\}_{i \in \mathbf{I}}$ (with index set \mathbf{I}), their union is also an upper set.