Definition (Monoid homomorphism). Let $\langle \mathbf{M}, \fatsemi_{\mathbf{M}}, 1_{\mathbf{M}} \rangle$ and $\langle \mathbf{N}, \fatsemi_{\mathbf{N}}, 1_{\mathbf{N}} \rangle$ be monoids. A homomorphism of monoids from $\mathbf{M}$ to $\mathbf{N}$ is a function $f : \mathbf{M} \to \mathbf{N}$ such that

$$f(m_1 \fatsemi_{\mathbf{M}} n_2) = f(m_1) \fatsemi_{\mathbf{N}} f(n_2) \quad \forall m_1, n_2 \in \mathbf{M}$$

and

$$f(1_{\mathbf{M}}) = 1_{\mathbf{N}}$$