## **Definition** (Enriched functor)

Given two categories C and D enriched in the same monoidal category V, an enriched functor  $F: C \to D$  consists of:

- 1. A map  $F: Ob_{\mathbb{C}} \to Ob_{\mathbb{D}}$  that maps objects of  $\mathbb{C}$  to objects of  $\mathbb{D}$ .
- 2. For each X, Y in  $Ob_{\mathbf{C}}$ , there exists a morphism in  $\mathbf{V}$  given by

$$F_{X,Y}$$
:  $\operatorname{Hom}_{\mathbf{C}}(X;Y) \to \operatorname{Hom}_{\mathbf{D}}(F(X);F(Y))$ ,

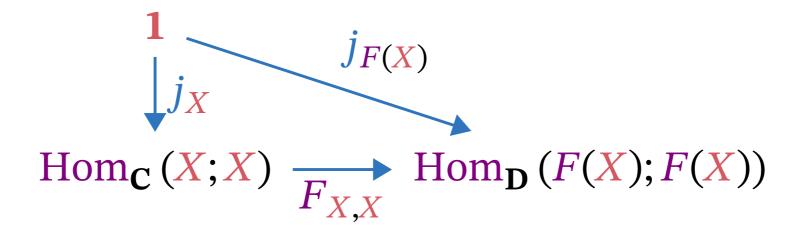
such that composing maps "across F" respects the composition in  ${\bf C}$  and the unit in  ${\bf V}$  in the obvious ways:

$$\operatorname{Hom}_{\mathbf{C}}(X;Y) \otimes \operatorname{Hom}_{\mathbf{C}}(Y;Z) \xrightarrow{m_{X,Y,Z}} \operatorname{Hom}_{\mathbf{C}}(X;Z)$$

$$\downarrow^{F_{X,Z}}$$

$$\operatorname{Hom}_{\mathbf{D}}(F(Y);F(Z)) \otimes \operatorname{Hom}_{\mathbf{D}}(F(X);F(Y)) \xrightarrow{m_{F(X),F(Y),F(Z)}} \operatorname{Hom}_{\mathbf{D}}(F(X);F(Z))$$

and



where  $\otimes$  and 1 are the monoidal product and monoidal unit in V.