Definition (Opposite category)**.** Given a category $\mathbf{C}$, its *opposite category* $\mathbf{C}^{\mathrm{op}}$ is specified by:

1. *Objects*: $\mathrm{Ob}_{\mathbf{C}^{\mathrm{op}}} = \mathrm{Ob}_{\mathbf{C}}$.
   Given $X \in \mathrm{Ob}_{\mathbf{C}}$, we will sometimes (though not always) write X^{op} to signify when we are thinking of X as an object of $\mathrm{Ob}_{\mathbf{C}}^{\mathrm{op}}$.
2. *Morphisms*: Given objects $X^{\mathrm{op}}, Y^{\mathrm{op}} \in \mathrm{Ob}_{\mathbf{C}^{\mathrm{op}}} = \mathrm{Ob}_{\mathbf{C}}$,

   $$\mathrm{Hom}_{\mathbf{C}^{\mathrm{op}}}\left(X^{\mathrm{op}}; Y^{\mathrm{op}}\right) := \mathrm{Hom}_{\mathbf{C}}\left(Y; X\right).$$

   Given $f \in \mathrm{Hom}_{\mathbf{C}}\left(Y; X\right)$, when we are thinking of it as an element of $\mathrm{Hom}_{\mathbf{C}^{\mathrm{op}}}\left(X^{\mathrm{op}}; Y^{\mathrm{op}}\right)$, we will sometimes write f^{op}.
3. *Identity morphisms*: Given $X^{\mathrm{op}} \in \mathrm{Ob}_{\mathbf{C}^{\mathrm{op}}}$, its identity morphism is

   $$\mathrm{Id}_{X^{\mathrm{op}}} := \mathrm{Id}_{X}^{\mathrm{op}}.$$

4. *Composition*: Let $f^{\mathrm{op}} \in \mathrm{Hom}_{\mathbf{C}^{\mathrm{op}}}\left(X^{\mathrm{op}}; Y^{\mathrm{op}}\right)$ and $g^{\mathrm{op}} \in \mathrm{Hom}_{\mathbf{C}^{\mathrm{op}}}\left(Y^{\mathrm{op}}; Z^{\mathrm{op}}\right)$, then

   $$f^{\mathrm{op}} \mathbin{⨟}_{\mathbf{C}^{\mathrm{op}}} g^{\mathrm{op}} := \left(g \mathbin{⨟}_{\mathbf{C}} f\right)^{\mathrm{op}}.$$