Definition

A category **C** is specified by:

Constituents

- 1. Objects: A set $Ob_{\mathbb{C}}$ whose elements are called *objects*.
- 2. Morphisms: For every pair of objects X, Y in $\mathsf{Ob}_{\mathbb{C}}$, there is a set $\mathsf{Hom}_{\mathbb{C}}(X;Y)$, elements of which are called *morphisms* from X to Y. A morphism $f \in \mathsf{Hom}_{\mathbb{C}}(X;Y)$ is often indicated by writing

$$f: X \to Y$$
.

- 3. Identity morphisms: for each object X, there is a morphism $\mathrm{Id}_X: X \to X$ called *the identity morphism of* X.
- 4. Composition operations: For every three objects X, Y, Z in $Ob_{\mathbb{C}}$ there is a composition function

$$\S_{X,Y,Z}$$
: $\operatorname{Hom}_{\mathbf{C}}(X;Y) \times \operatorname{Hom}_{\mathbf{C}}(Y;Z) \to \operatorname{Hom}_{\mathbf{C}}(X;Z)$

We denote the *composition* of composable morphisms f and g by $f \, \S \, g$. (Traditionally, the typical notation would be $g \circ f$).

Conditions

1. Associativity: for all composable morphisms f, g, h,

$$(f \circ g) \circ h = f \circ (g \circ h).$$

2. Unitality: for every morphism $f: X \to Y$,

$$\operatorname{Id}_{X} \circ f = f$$
 and $f \circ \operatorname{Id}_{Y} = f$.