Definition (Enriched functor)

Given two categories $\mathbf{C}$ and $\mathbf{D}$ enriched in the same monoidal category $\mathbf{V}$, an enriched functor $F : \mathbf{C} \to \mathbf{D}$ consists of:

1. A map $F : \mathrm{Ob}_{\mathbf{C}} \to \mathrm{Ob}_{\mathbf{D}}$ that maps objects of $\mathbf{C}$ to objects of $\mathbf{D}$.
2. For each X, Y in $\mathrm{Ob}_{\mathbf{C}}$, there exists a morphism in $\mathbf{V}$ given by

   $$F_{X,Y} : \mathrm{Hom}_{\mathbf{C}}(X; Y) \to \mathrm{Hom}_{\mathbf{D}}(F(X); F(Y)),$$

   such that composing maps "across F" respects the composition in $\mathbf{C}$ and the unit in $\mathbf{V}$ in the obvious ways:

   $$\begin{tikzcd}
   \mathrm{Hom}_{\mathbf{C}}(X;Y) \otimes \mathrm{Hom}_{\mathbf{C}}(Y;Z) \arrow[r, "m_{X,Y,Z}"] \arrow[d, "F_{X,Y} \otimes F_{Y,Z}"'] & \mathrm{Hom}_{\mathbf{C}}(X;Z) \arrow[d, "F_{X,Z}"] \\
   \mathrm{Hom}_{\mathbf{D}}(F(Y);F(Z)) \otimes \mathrm{Hom}_{\mathbf{D}}(F(X);F(Y)) \arrow[r, "m_{F(X),F(Y),F(Z)}"'] & \mathrm{Hom}_{\mathbf{D}}(F(X);F(Z))
   \end{tikzcd}$$

   and

   $$\begin{tikzcd}
   \mathbf{1} \arrow[d, "j_X"] \arrow[dr, "j_{F(X)}"] & \\
   \mathrm{Hom}_{\mathbf{C}}(X;X) \arrow[r, "F_{X,X}"'] & \mathrm{Hom}_{\mathbf{D}}(F(X);F(X))
   \end{tikzcd}$$

   where $\otimes$ and $\mathbf{1}$ are the monoidal product and monoidal unit in $\mathbf{V}$.