Definition (Traced monoidal category)

A symmetric monoidal category $\langle \mathbf{C}, \otimes, \mathbf{1}, \mathbf{br} \rangle$ is said to be *traced* if it is equipped with a family of functions

$$\operatorname{Tr}_{X,Y}^{Z}$$
: $\operatorname{Hom}_{\mathbb{C}}(X \otimes Z; Y \otimes Z) \to \operatorname{Hom}_{\mathbb{C}}(X; Y)$,

satisfying the following axioms:

1. *Naturality in X*: For all morphisms $f: X \otimes Z \to Y \otimes Z$ and $g: X' \to X$,

$$\operatorname{Tr}_{X',Y}^{Z}((g \otimes \operatorname{Id}_{Z}) \circ f) = g \circ \operatorname{Tr}_{X,Y}^{Z}(f)$$

2. *Naturality in Y*: For all morphisms $f: X \otimes Z \to Y \otimes Z$ and $g: Y \to Y'$,

$$\operatorname{Tr}_{X,Y'}^{Z}(f \circ (g \otimes \operatorname{Id}_{Z})) = \operatorname{Tr}_{X,Y}^{Z}(f) \circ g$$

3. *Vanishing*: For all morphisms $f: X \to Y$ in \mathbb{C} ,

$$\operatorname{Tr}^{\mathbf{1}}_{X,Y}(f) = f.$$

Furthermore, for all morphisms $f: X \otimes Z \otimes U \to Y \otimes Z \otimes U$ in **C**,

$$\operatorname{Tr}_{X,Y}^{Z \otimes U}(f) = \operatorname{Tr}_{X,Y}^{Z} \left(\operatorname{Tr}_{X \otimes Z,Y \otimes Z}^{U}(f) \right).$$

4. Superposing: For all morphisms $f: X \otimes Z \to Y \otimes Z$ in \mathbb{C} ,

$$\operatorname{Tr}^{Z}_{V \otimes X, V \otimes Y}(\operatorname{Id}_{V} \otimes f) = \operatorname{Id}_{V} \otimes \operatorname{Tr}^{Z}_{X, Y}(f).$$

5. Yanking:

$$\operatorname{Tr}_{Z,Z}^{Z}(\operatorname{br}_{Z,Z}) = \operatorname{Id}_{Z}.$$