Lemma. The product of antichains $\times : \mathcal{A}\mathbf{R}_1 \times \mathcal{A}\mathbf{R}_2 \to \mathcal{A}(\mathbf{R}_1 \times \mathbf{R}_2)$ is monotone.