

Lemma. Given $\mathbf{d}_1, \mathbf{d}_2 \in \text{Hom}_{\mathbf{DP}} (\mathbf{R} \times \mathbf{P}; \mathbf{R} \times \mathbf{Q})$, one has:

$$\frac{\mathbf{d}_1 \leq_{\mathbf{DP}} \mathbf{d}_2}{\text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}_1) \leq_{\mathbf{DP}} \text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}_2)}$$

In other words, trace preserves order on \mathbf{DP} .