Definition (Operad). An *operad* is defined by:

<ins>Constituents</ins>

1. *Objects:* A collection $\mathrm{Ob}_{\mathcal{O}}$;
2. *Morphisms:* Let $n \in \mathbb{N}$. For each finite string $[X_1, \dots, X_n]$ of objects and each object Y, one specifies a set $\mathrm{Hom}_{\mathcal{O}}([X_1, \dots, X_n]; Y)$, elements of which are morphisms $[X_1, \dots, X_n] \to Y$;
3. *Identity morphisms:* For each object X, a morphism $\mathrm{Id}_X \in \mathrm{Hom}_{\mathcal{O}}([X_1, \dots, X_n]; Y)$;
4. *Composition operations:*

$$\mathrm{Hom}_{\mathcal{O}}([X_1^1, \dots, X_{n_1}^1]; Y_1) \times \dots \times \mathrm{Hom}_{\mathcal{O}}\left([X_1^m, \dots, X_{n_m}^m]; Y_m\right) \times \mathrm{Hom}_{\mathcal{O}}([Y_1, \dots, Y_m]; Z) \to \mathrm{Hom}_{\mathcal{O}}\left([X_1^1, \dots, X_{n_m}^m]; Z\right)$$
$$\langle f_1, \dots, f_m, g\rangle \mapsto [f_1, \dots, f_m] \mathbin{⨟} g.$$

<ins>Conditions</ins>

1. *Associativity:*

$$[[f_1^1, \dots, f_{n_1}^1] \mathbin{⨟} g_1, [f_1^2, \dots, f_{n_2}^2] \mathbin{⨟} g_2, \dots, [f_1^m, \dots, f_{n_m}^m] \mathbin{⨟} g_m] \mathbin{⨟} h = [f_1^1, \dots, f_{n_m}^m] \mathbin{⨟} ([g_1, \dots, g_m] \mathbin{⨟} h).$$

2. *Unitality*:

$$[\mathrm{Id}_{X_1}, \dots, \mathrm{Id}_{X_n}] \mathbin{⨟} f = f = f \mathbin{⨟} \mathrm{Id}_Y, \quad \forall f : [X_1, \dots, X_n] \to Y.$$