

Definition (Monoid)

A *monoid* \mathbf{M} is:

Constituents

1. A set \mathbf{M} ;
2. A binary operation $\circ : \mathbf{M} \times \mathbf{M} \rightarrow \mathbf{M}$;
3. A specified element $\text{id} \in \mathbf{M}$, called *neutral element*.

Conditions

1. Associative law: $(x \circ y) \circ z = x \circ (y \circ z) \quad \forall x, y, z \in \mathbf{M}$;
2. Neutrality Laws: $\text{id} \circ x = x = x \circ \text{id} \quad \forall x \in \mathbf{M}$.