**Definition** (Chain in a poset) Given a poset  $P = \langle P, \leq_P \rangle$ , a *chain* is a subset  $S \subseteq P$  such that any two elements of S are comparable. If  $x, y \in S$ , then  $x \leq_{\mathbf{p}} y$  or  $y \leq_{\mathbf{p}} x$ .