Definition (Order on morphisms in $\mathbf{Pos}_{\mathscr{L}}$). Given any two morphisms $f, g \colon X \to Y$ in $\mathbf{Pos}_{\mathscr{L}}$, we define an order between them as

$$\frac{f \preceq_{\mathbf{Pos}_{\mathscr{L}}} g}{f^{\star}(x) \preceq_{\mathscr{L} Y} g^{\star}(x), \quad \forall x \in X}.$$