

Definition (\mathcal{U} monad). The \mathcal{U} monad on **Pos** consists of:

1. The U endofunctor (??).
2. The unit natural transformation $\text{un}_{\mathcal{U}} : \text{Id}_{\mathbf{Pos}} \Rightarrow U$, which associates to every object $\mathbf{P} \in \text{Ob}_{\mathbf{Pos}}$ a morphism in **Pos** given by:

$$\text{un}_{\mathcal{U}}^{\mathbf{P}} : \mathbf{P} \rightarrow \mathcal{U}\mathbf{P}$$

$$p \mapsto \uparrow \{p\}.$$

3. The compositional natural transformation $\text{mu}_{\mathcal{U}} : U \circ U \Rightarrow U$, which associates to every $\mathbf{P} \in \text{Ob}_{\mathbf{Pos}}$ the morphism in **Pos** given by:

$$\text{mu}_{\mathcal{U}}^{\mathbf{P}} : \mathcal{U}(\mathcal{U}\mathbf{P}) \rightarrow \mathcal{U}\mathbf{P}$$

$$\mathbf{P}'' \mapsto \bigcup_{\mathbf{P}' \in \mathbf{P}''} \mathbf{P}'.$$