

Definition (Category **LPos**). The category **LPos** consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects $X, Y \in \mathbf{Ob}_{\mathbf{LPos}}$, morphisms from X to Y are monotone maps of the form $f : X \rightarrow \mathcal{L}Y$.
3. *Composition of morphisms*: Given morphisms $f : X \rightarrow \mathcal{L}Y$, $g : Y \rightarrow \mathcal{L}Z$, their composition is given by

$$f \circ g : X \rightarrow \mathcal{L}Z$$
$$x \mapsto \bigcup_{y \in f(x)} g(y);$$

4. *Identity morphism*: given an object $X \in \mathbf{Ob}_{\mathbf{LPos}}$, the identity morphism is given by the application of the lower closure operator: $\text{Id}_X(x) := \downarrow \{x\}$.