Definition (Traced monoidal category). A symmetric monoidal category $\langle \mathbf{C}, \otimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}}, \mathrm{br} \rangle$ is said to be *traced* if it is equipped with a family of functions

$$\mathrm{Tr}^{Z}_{X,Y} : \mathbf{C}(X \otimes_{\mathbf{C}} Z, Y \otimes_{\mathbf{C}} Z) \to \mathbf{C}(X, Y),$$

satisfying the following axioms:

1. *Vanishing:* For all morphisms $f : X \to Y$ in $\mathbf{C}$,

$$\mathrm{Tr}^{1}_{X,Y}(f) = f.$$

   Furthermore, for all morphisms $f : X \otimes_{\mathbf{C}} Z \otimes_{\mathbf{C}} W \to Y \otimes_{\mathbf{C}} Z \otimes_{\mathbf{C}} W$ in $\mathbf{C}$:

$$\mathrm{Tr}^{Z \otimes_{\mathbf{C}} W}_{X,Y}(f) = \mathrm{Tr}^{Z}_{X,Y}\left(\mathrm{Tr}^{W}_{X \otimes_{\mathbf{C}} Z, Y \otimes_{\mathbf{C}} Z}(f)\right).$$

2. *Superposing:* For all morphisms $f : X \otimes_{\mathbf{C}} Z \to Y \otimes_{\mathbf{C}} Z$ in $\mathbf{C}$:

$$\mathrm{Tr}^{Z}_{V \otimes_{\mathbf{C}} X, V \otimes_{\mathbf{C}} Y}(\mathrm{Id}_V \otimes_{\mathbf{C}} f) = \mathrm{Id}_V \otimes_{\mathbf{C}} \mathrm{Tr}^{Z}_{X,Y}(f).$$

3. *Yanking:*

$$\mathrm{Tr}^{Z}_{Z,Z}\left(\mathrm{br}_{Z,Z}\right) = \mathrm{Id}_Z.$$