

**Definition** (Monad in functional programming). A monad

$$\langle \text{return}, \text{join}, \text{fmap}, \text{bind}, \text{fish}, \text{lift} \rangle$$

is a set of operations with the following signature:

$$\text{return} : X \rightarrow MX$$

$$\text{lift} : (X \rightarrow Y) \rightarrow (X \rightarrow MY)$$

$$\text{fish} : (X \rightarrow MY) \rightarrow (Y \rightarrow MZ) \rightarrow (X \rightarrow MZ)$$

$$\text{join} : MMX \rightarrow MX$$

$$\text{fmap} : (X \rightarrow Y) \rightarrow (MX \rightarrow MY)$$

$$\text{bind} : MX \rightarrow (X \rightarrow MY) \rightarrow MY$$

These maps satisfy the equivalent axioms of unitality and associativity:

- ▷ *return* is a left identity for *bind*;
- ▷ *return* is a right identity for *bind*;
- ▷ *bind* is associative.