Definition (Profunctor composition). Given two profunctors $F : \mathbf{C} \nrightarrow \mathbf{D}$ and $G : \mathbf{D} \nrightarrow \mathbf{E}$ we can define their composition $(F \mathbin{\fatsemi} G) : \mathbf{C} \nrightarrow \mathbf{E}$ as follows:

$$(F \mathbin{\fatsemi} G)_{\mathrm{ob}} : \mathrm{Ob}(\mathbf{C}^{\mathrm{op}} \times \mathbf{E}) \to \mathrm{Ob}\,\mathbf{Set},$$

$$\langle C^*, E \rangle \mapsto \amalg_{D \in \mathrm{Ob}\mathbf{D}} F_{\mathrm{ob}}(C^*, D) \times G_{\mathrm{ob}}(D^*, E)/\sim$$

$$(F \mathbin{\fatsemi} G)_{\mathrm{mor}} : \mathrm{Hom}_{(\mathbf{C}^{\mathrm{op}} \times \mathbf{E})}(\langle C_1^*, E_1 \rangle; \langle C_2^*, E_2 \rangle) \to \mathrm{Hom}_{\mathbf{Set}}((F \mathbin{\fatsemi} G)_{\mathrm{ob}}(C_1^*, E_1); (F \mathbin{\fatsemi} G)_{\mathrm{ob}}(C_2^*, E_2))$$

$$\langle \alpha^*, \beta \rangle \mapsto \begin{cases} (F \mathbin{\fatsemi} G)_{\mathrm{ob}}(C_1^*, E_1) \to (F \mathbin{\fatsemi} G)_{\mathrm{ob}}(C_2^*, E_2)) \\ \langle s, t \rangle \mapsto \langle F_{\mathrm{mor}}(\langle \alpha, \mathrm{Id}_D \rangle)(s), G_{\mathrm{mor}}(\langle \mathrm{Id}_{D*}, \beta \rangle)(t) \rangle \end{cases}$$

In the formulas:

$$\alpha : C_2 \to C_1, \qquad \beta : E_1 \to E_2,$$

and $\langle s, t \rangle$ is a pair of elements for which there exists a $D \in \mathrm{Ob}\mathbf{D}$ such that

$$s \in F_{\mathrm{ob}}(C_1^*, D), \qquad t \in G_{\mathrm{ob}}(D^*, E).$$