Lemma. Given posets $\mathbf{P}, \mathbf{Q}$, a monotone map $f : \mathbf{P} \to \mathbf{Q}$, and a family of singleton sets $\{S_i\}_{i \in I}$, with $S_i = \{s_i\}$, $s_i \in \mathbf{P}$, the following equality holds:

$$\uparrow \left(\bigcup_{p \in \uparrow \bigcup_{i \in I} S_i} \{f(p)\} \right) = \uparrow \left(\bigcup_{i \in I} \{f(s_i)\} \right).$$