

**Definition** (Category  $\mathbf{Pos}_{\mathcal{L}}$ ). The category  $\mathbf{Pos}_{\mathcal{L}}$  consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects  $X, Y \in \mathbf{Ob}_{\mathbf{Pos}_{\mathcal{L}}}$ , morphisms  $f : X \rightarrow Y$  are monotone maps of the form  $f^{\star} : X \rightarrow_{\mathbf{Pos}} \mathcal{L} Y$ .
3. *Composition of morphisms*: Given morphisms  $f : X \rightarrow Y$ ,  $g : Y \rightarrow Z$ , their composition  $f \circ g : X \rightarrow Z$  is given by

$$(f \circ g)^{\star} : X \rightarrow_{\mathbf{Pos}} \mathcal{L} Z$$

$$x \mapsto \bigcup_{y \in f^{\star}(x)} g^{\star}(y);$$

4. *Identity morphism*: given an object  $X \in \mathbf{Ob}_{\mathbf{Pos}_{\mathcal{L}}}$ , the identity morphism  $\text{Id}_X : X \rightarrow X$  is given by the application of the lower closure operator:  $\text{Id}_X^{\star}(x) := \downarrow \{x\}$ .