Lemma. Given $\mathbf{d}, (\mathbf{d} \mathbin{\raisebox{0.3ex}{\tiny 9}} \mathbf{e}) \in \mathrm{Hom}_{\mathbf{DP}}(\mathbf{R} \times \mathbf{P}; \mathbf{R} \times \mathbf{Q})$, one has:

$$\frac{\mathbf{d} \leq_{\mathbf{DP}} \mathbf{e}}{\mathrm{Tr}^{\mathbf{R}}_{\mathbf{P},\mathbf{Q}}(\mathbf{d}) \leq_{\mathbf{DP}} \mathrm{Tr}^{\mathbf{R}}_{\mathbf{P},\mathbf{Q}}(\mathbf{e})} .$$

In other words, trace preserves order on **DP**.