

**Definition** (Semigroup homomorphism). Let  $\mathbf{S}$  and  $\mathbf{T}$  be semigroups. A homomorphism of semigroups from  $\mathbf{S}$  to  $\mathbf{T}$  is a function  $F : \mathbf{S} \rightarrow \mathbf{T}$  such that for all  $x, y \in \mathbf{S}$ ,

$$F(x \circ_{\mathbf{S}} y) = F(x) \circ_{\mathbf{T}} F(y)$$