

$$\begin{aligned}
& ((\text{br}_{X,Y} \otimes \text{Id}_Z) \circ \text{as}_{Y,X,Z} \circ (\text{Id}_Y \otimes \text{br}_{X,Z}))^{\star}(\langle x, y \rangle, z) \\
&= \bigcup_{\langle y', \langle x', z' \rangle \rangle \in \uparrow\{y\} \times (\uparrow\{x\} \times \uparrow\{z\})} (\text{Id}_Y \otimes \text{br}_{X,Z})^{\star}(y', \langle x', z' \rangle) \\
&= \bigcup_{\langle y', \langle x', z' \rangle \rangle \in \uparrow\{y\} \times (\uparrow\{x\} \times \uparrow\{z\})} \uparrow\{y'\} \times (\uparrow\{z'\} \times \uparrow\{x'\}) \\
&= \uparrow\{y\} \times (\uparrow\{z\} \times \uparrow\{x\}).
\end{aligned}$$