

**Lemma.** The product of antichains  $\times : \mathcal{A}\mathbf{R}_1 \times \mathcal{A}\mathbf{R}_2 \rightarrow \mathcal{A}(\mathbf{R}_1 \times \mathbf{R}_2)$  is monotone.