

**Lemma.** Given  $\mathbf{d}_1, \mathbf{d}_2 \in \text{Hom}_{\mathbf{DP}}(\mathbf{R} \times \mathbf{P}; \mathbf{R} \times \mathbf{Q})$ , one has:

$$\mathbf{d}_1 \preceq_{\mathbf{DP}} \mathbf{d}_2$$


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$$\text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}_1) \preceq_{\mathbf{DP}} \text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}_2)$$

In other words, trace preserves order on  $\mathbf{DP}$ .