

Definition (Semi-functor)

Given two semi-categories \mathbf{C} and \mathbf{D} , a *semi-functor* $F : \mathbf{C} \rightarrow \mathbf{D}$ from \mathbf{C} to \mathbf{D} is defined by the following constituents and conditions.

Constituents:

i) A map

$$F_{\bullet} : \text{Ob}_{\mathbf{C}} \rightarrow \text{Ob}_{\mathbf{D}}.$$

ii) For every pair of objects $X, Y \in \text{Ob}_{\mathbf{C}}$ a map

$$F_{\rightarrow} : \text{Hom}_{\mathbf{C}}(X; Y) \rightarrow \text{Hom}_{\mathbf{D}}(F_{\bullet}(X); F_{\bullet}(Y)).$$

Conditions:

1. It holds that the functor application to morphisms commutes with category composition:

$$\frac{f : X \rightarrow_{\mathbf{C}} Y \quad g : Y \rightarrow_{\mathbf{C}} Z}{F_{\rightarrow}(f \circ g) = F_{\rightarrow}(f) \circ F_{\rightarrow}(g)}.$$