

Lemma. Given $\mathbf{d}, \mathbf{e} \in \text{Hom}_{\mathbf{DP}} (\mathbf{R} \times \mathbf{P}; \mathbf{R} \times \mathbf{Q})$, one has:

$$\frac{\mathbf{d} \leq_{\mathbf{DP}} \mathbf{e}}{\text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}) \leq_{\mathbf{DP}} \text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{e})} .$$

In other words, trace preserves order on \mathbf{DP} .