

## Definition (Adjunction, Version 2)

Let  $\mathbf{C}$  and  $\mathbf{D}$  be categories. An *adjunction* from  $\mathbf{C}$  to  $\mathbf{D}$  is given by the following data, satisfying the following conditions.

Data:

1. A functor  $L : \mathbf{C} \rightarrow \mathbf{D}$  (the *left adjoint*);
2. A functor  $R : \mathbf{D} \rightarrow \mathbf{C}$  (the *right adjoint*);
3. Natural transformations  $\text{un} : \text{Id}_{\mathbf{C}} \Rightarrow L \circ R$  and  $\text{co} : R \circ L \Rightarrow \text{Id}_{\mathbf{D}}$

Conditions:

1. For all objects  $X$  of  $\mathbf{C}$ , it holds that

$$L\text{un}_X \circ \text{co}_{LX} = \text{Id}_{LX} \text{ and } \text{un}_{RY} \circ R\text{co}_Y = \text{Id}_{RY}$$

**i.e.** that the following diagrams commute:



The 2-morphisms  $\text{un}$  and  $\text{co}$  are called the *unit* and *counit* of the adjunction. An adjunction is called an *adjoint equivalence* if the unit and counit are natural isomorphisms.