

Lemma. For $A, B, C, D \in \mathcal{AP}$, $A \leq_{AR} C$ and $B \leq_{AR} D$ implies $A \cup B \leq_{AR} C \cup D$.