

**Definition** (Functions as relations). Let  $\mathbf{A}$  and  $\mathbf{B}$  be sets. A relation  $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{B}$  is a *function* if it satisfies the following two conditions:

1.  $\forall x \in \mathbf{A} \quad \exists y \in \mathbf{B} : \langle x, y \rangle \in \mathbf{R}$
2.  $\forall \langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle \in \mathbf{R} \text{ holds} : x_1 = x_2 \Rightarrow y_1 = y_2.$