

# Definition (Strong monoidal functor)

Let  $\langle \mathbf{C}, \otimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}} \rangle$  and  $\langle \mathbf{D}, \otimes_{\mathbf{D}}, \mathbf{1}_{\mathbf{D}} \rangle$  be two monoidal categories.

A *strong monoidal functor* between  $\mathbf{C}$  and  $\mathbf{D}$  is given by:

1. A functor

$$F : \mathbf{C} \rightarrow \mathbf{D};$$

2. An isomorphism

$$\text{iso} : \mathbf{1}_{\mathbf{D}} \rightarrow F(\mathbf{1}_{\mathbf{C}});$$

3. A natural isomorphism  $\mu$

$$\mu_{X,Y} : F(X) \otimes_{\mathbf{D}} F(Y) \rightarrow F(X \otimes_{\mathbf{C}} Y), \quad \forall X, Y \in \mathbf{C},$$

satisfying the following conditions:

- a) *Associativity*: For all objects  $X, Y, Z \in \mathbf{C}$ , there are *associators*  $\text{as}^{\mathbf{C}}$  and  $\text{as}^{\mathbf{D}}$  such that the diagram in ?? commutes.
- b) *Unitality*: For all  $X \in \mathbf{C}$ , there exist left and right *unitors*  $\text{lu}^{\mathbf{C}}$  and  $\text{ru}^{\mathbf{C}}$ , the diagram in ?? commutes.