

Definition (Monad). Let \mathbf{C} be a category. A *monad* on \mathbf{C} is specified by:

Constituents

1. A functor $M : \mathbf{C} \rightarrow \mathbf{C}$;
2. A natural transformation $\mu : M \circ M \Rightarrow M$, called the *composition* or *multiplication*;
3. A natural transformation $\eta : \text{Id}_{\mathbf{C}} \Rightarrow M$, called the *unit*.

Conditions

1. *Associativity*: the diagram

$$\begin{array}{ccc}
 M \circ M \circ M & \xrightarrow{M\mu} & M \circ M \\
 \downarrow \mu M & & \downarrow \mu \\
 M \circ M & \xrightarrow{\mu} & M
 \end{array}$$

must commute.

2. *Left and right unitality*: the diagrams

$$\begin{array}{ccc}
 M & \xrightarrow{\eta M} & M \circ M \\
 \searrow \text{Id}_M & & \downarrow \mu \\
 & & M
 \end{array}
 \qquad
 \begin{array}{ccc}
 M & \xrightarrow{M\eta} & M \circ M \\
 \searrow \text{Id}_M & & \downarrow \mu \\
 & & M
 \end{array}$$

must commute.