

## Definition ( $M$ -algebra morphism)

Let  $\langle M, \mathbf{un}, \mathbf{mu} \rangle$  be a monad on a category  $\mathbf{C}$ , and let  $\langle X_1, a_1 \rangle$  and  $\langle X_2, a_2 \rangle$  be algebras of  $M$ . A morphism  $\langle X_1, a_1 \rangle \rightarrow \langle X_2, a_2 \rangle$  of  $M$ -algebras is specified by:

### Constituents

1. A morphism  $f : X_1 \rightarrow X_2$  in  $\mathbf{C}$ .

### Conditions

1. The diagram

$$\begin{array}{ccc} M(X_1) & \xrightarrow{Mf} & M(X_2) \\ a_1 \downarrow & & \downarrow a_2 \\ X_1 & \xrightarrow{f} & X_2 \end{array}$$

commutes.