**Definition** (Order on morphisms in **Pos** )

 $f \leq_{\mathbf{Pos}_{\mathbf{\mathcal{Y}}}} g$ 

 $f^*(x) \leq_{YY} g^*(x), \quad \forall x \in X$ 

Given any two morphisms  $f, g: X \to Y$  in  $\mathbf{Pos}_{\varphi}$ , we define an order between







