

**Definition** (Design problems with implementation as monotone functions)

Suppose that  $\mathbf{F}$ ,  $\mathbf{R}$  are posets. A *design problem with implementation* is a monotone map (a **Set**-enriched profunctor)  $\langle \mathbf{I}_d, \text{prov}, \text{req} \rangle : \mathbf{F} \multimap \mathbf{R}$ , where  $\mathbf{I}_d$  is a set,  $\text{prov}$  and  $\text{req}$  are functions from  $\mathbf{I}_d$  to  $\mathbf{F}$  and  $\mathbf{R}$ , respectively

$$\mathbf{F} \xleftarrow{\text{prov}} I \xrightarrow{\text{req}} \mathbf{R},$$

and  $\langle \mathbf{I}_d, \text{prov}, \text{req} \rangle : \mathbf{F} \multimap \mathbf{R}$  is given by

$$\langle \mathbf{I}_d, \text{prov}, \text{req} \rangle : \mathbf{F} \multimap \mathbf{R} : \mathbf{F}^{\text{op}} \times \mathbf{R} \rightarrow_{\mathbf{Pos}} \mathcal{P}(\mathbf{I}_d)$$

$$\langle f^*, r \rangle \mapsto \{i \in \mathbf{I}_d : (f \leq_{\mathbf{F}} \text{prov}(i)) \wedge (\text{req}(i) \leq_{\mathbf{R}} r)\},$$

where the partial order on  $\mathcal{P}\mathbf{I}$  is given by subset inclusion.