

**Definition** (Monad). Let  $\mathbf{C}$  be a category. A *monad* on  $\mathbf{C}$  consists of:

1. A functor  $M : \mathbf{C} \rightarrow \mathbf{C}$ .
2. A natural transformation  $\text{un} : \text{Id}_{\mathbf{C}} \Rightarrow M$  called *unit*.
3. A natural transformation  $\text{mu} : M \circ M \Rightarrow M$  called *composition* or *multiplication*.

The constituents must satisfy *left and right unitality*

$$\begin{array}{ccc}
 M & \xRightarrow{\text{un}M} & M \circ M \\
 \searrow \text{Id}_M & & \downarrow \text{mu} \\
 & & M
 \end{array}
 \qquad
 \begin{array}{ccc}
 M & \xRightarrow{M\text{un}} & M \circ M \\
 \searrow \text{Id}_M & & \downarrow \text{mu} \\
 & & M
 \end{array}$$

and *associativity*

$$\begin{array}{ccc}
 M \circ M \circ M & \xRightarrow{M\text{mu}} & M \circ M \\
 \downarrow \text{mu}M & & \downarrow \text{mu} \\
 M \circ M & \xRightarrow{\text{mu}} & M
 \end{array}$$