Definition (Semigroup). A *semigroup* S is a set S, together with a binary operation:

$${}_{9}^{\circ}: \mathbf{S} \times \mathbf{S} \to \mathbf{S},$$

called *composition*, which satisfies the *associative* law:

$$(x \circ y) \circ z = x \circ (y \circ z)$$

for all $x, y, z \in S$.