

Definition (Series composition)

Let $\mathbf{d} : \mathbf{P} \rightarrow \mathbf{Q}$ and $\mathbf{e} : \mathbf{Q} \rightarrow \mathbf{R}$ be design problems. We define their *series composition* $(\mathbf{d} \circ \mathbf{e}) : \mathbf{P} \rightarrow \mathbf{R}$ as:

$$(\mathbf{d} \circ \mathbf{e}) : \mathbf{P}^{\text{op}} \times \mathbf{R} \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle p^*, r \rangle \mapsto \bigvee_{q \in \mathbf{Q}} \mathbf{d}(p^*, q) \wedge \mathbf{e}(q, r).$$

Alternatively, rather than taking the "or" (\vee) over an element q , we can iterate over pairs q_1, q_2 such that $q_1 \leq q_2$:

$$(\mathbf{d} \circ \mathbf{e}) : \mathbf{P}^{\text{op}} \times \mathbf{R} \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle p^*, r \rangle \mapsto \bigvee_{q_1 \leq q_2, q_1, q_2 \in \mathbf{Q}} \mathbf{d}(p^*, q_1) \wedge \mathbf{e}(q_2, r).$$