

**Definition** (Semigroup). A *semigroup*  $\mathbf{S}$  is a set  $\mathbf{S}$ , together with a binary operation

$$\circ : \mathbf{S} \times \mathbf{S} \rightarrow \mathbf{S},$$

called *composition*, which satisfies the *associative* law:

$$(x \circ y) \circ z = x \circ (y \circ z)$$

for all  $x, y, z \in \mathbf{S}$ .