

$$\begin{aligned}
\mathrm{Tr}_{X',Y}^Z((g \otimes \mathrm{Id}_Z) \circ f)^\star(x') &= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in ((g \otimes \mathrm{Id}_Z) \circ f)^\star(x', z) \right\} \\
&= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \bigcup_{\langle x, z' \rangle \in g^\star(x') \times \uparrow\{z\}} f^\star(x, z') \right\} \\
&= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \bigcup_{x \in g^\star(x')} f^\star(x, z) \right\}.
\end{aligned}$$