Definition (U endofunctor)**.** The *U endofunctor* has the form $U : \mathbf{Pos} \to \mathbf{Pos}$ and acts on objects and morphisms as follows:

1. *On objects*: Given a poset $\mathbf{P} \in \mathrm{Ob}_{\mathbf{Pos}}$, U maps $\mathbf{P}$ to its upper set. {bhfn:1}
2. *On morphisms*: Given posets $\mathbf{P}, \mathbf{Q}$, and a monotone map $f : \mathbf{P} \to \mathbf{Q}$, the U endofunctor acts as:

$$U(f) : \mathcal{U}\mathbf{P} \to \mathcal{U}\mathbf{Q}$$

$$\mathbf{P}' \mapsto \uparrow \left(\bigcup_{p \in \mathbf{P}'} \{f(p)\} \right).$$

Recall that in **??** we proved that the upper set is itself an object of **Pos**.