

Examples of Categories: Currency Exchangers

Applied Compositional Thinking for Engineers

Gioele Zardini



Thinking formally about currency exchangers

Currency
Exchanger



Thinking formally about currency exchangers

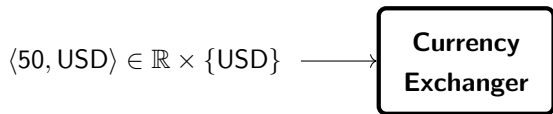
$\{\text{EUR}, \text{USD}, \text{CHF}, \text{SGD}, \dots\}$

**Currency
Exchanger**



Thinking formally about currency exchangers

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A currency exchanger from USD to CHF can be written as:

$$\begin{aligned} E_{a,b}: \mathbb{R} \times \{\text{USD}\} &\rightarrow \mathbb{R} \times \{\text{CHF}\} \\ \langle x, \text{USD} \rangle &\mapsto \langle ax - b, \text{CHF} \rangle, \end{aligned}$$

where a is the *exchange rate* and b is the *commission*.



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Assume you get 0.95 CHF for 1 USD and the commission is 2.0 CHF, i.e. $a = 0.95$, $b = 2.0$.



Thinking formally about currency exchangers

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Composing currency exchangers



Composing currency exchangers



Given $E_{a,b}: \mathbb{R} \times \{\text{USD}\} \rightarrow \mathbb{R} \times \{\text{CHF}\}$, and $E_{a',b'}: \mathbb{R} \times \{\text{CHF}\} \rightarrow \mathbb{R} \times \{\text{EUR}\}$, we have:

$$E_{a,b} \circ E_{a',b'}: \mathbb{R} \times \{\text{USD}\} \rightarrow \mathbb{R} \times \{\text{EUR}\}$$
$$\langle x, \text{USD} \rangle \mapsto \langle aa'x - a'b - b', \text{EUR} \rangle$$



Currency exchangers form a category

A currency category has:

- ▶ *Objects*: A collection of currencies.
- ▶ *Morphisms*: Currency exchangers.
- ▶ *Identity morphism*: Identity currency exchanger (exchange rate 1 and 0 commission).
- ▶ *Composition of morphisms*: Composition of currency exchangers.



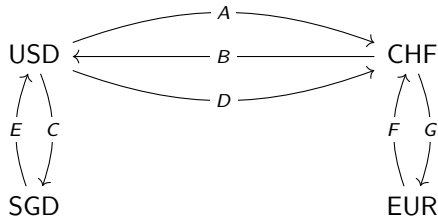
We can represent categories with graphs

Company name	Exchanger	Direction	exchange rate	fixed commission
ExchATM	<i>A</i>	USD to CHF	0.95 CHF/USD	2.0 CHF
ExchATM	<i>B</i>	CHF to USD	1.05 USD/CHF	1.5 USD
ExchATM	<i>C</i>	USD to SGD	1.40 SGD/USD	1.0 SGD
MoneyLah	<i>D</i>	USD to CHF	1.00 CHF/USD	1.0 CHF
MoneyLah	<i>E</i>	SGD to USD	0.72 USD/SGD	3.0 USD
Frankurrencies	<i>F</i>	EUR to CHF	1.20 CHF/EUR	0.0 CHF
Frankurrencies	<i>G</i>	CHF to EUR	1.00 EUR/CHF	1.0 EUR



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