

**Definition** (Monoid). A *monoid*  $\mathbf{M}$  is:

### Constituents

1. a set  $\mathbf{M}$ ;
2. a binary operation  $\circ : \mathbf{M} \times \mathbf{M} \rightarrow \mathbf{M}$ ;
3. a specified element  $\text{id} \in \mathbf{M}$ , called *neutral element*.

### Conditions

1. Associative law:  $(x \circ y) \circ z = x \circ (y \circ z) \quad \forall x, y, z \in \mathbf{M}$ ;
2. Neutrality Laws:  $\text{id} \circ x = x = x \circ \text{id} \quad \forall x \in \mathbf{M}$ .