

## **Definition** (Companion and conjoint)

Let  $\mathbf{P}$  and  $\mathbf{Q}$  be posets, and suppose that  $f : \mathbf{P} \rightarrow_{\mathbf{Pos}} \mathbf{Q}$  is a monotone map. We define its *companion* in  $\mathbf{DP}$ , denoted  $\hat{f} : \mathbf{P} \rightarrowtail \mathbf{Q}$ , and its *conjoint*, denoted  $\check{f} : \mathbf{Q} \rightarrowtail \mathbf{P}$  as

$$\hat{f}(p^*, q) := f(p) \leq_{\mathbf{Q}} q \quad \text{and} \quad \check{f}(q^*, p) := q \leq_{\mathbf{P}} f(p).$$