Definition (Monoid morphism)

A morphism $F: \mathbf{M} \to \mathbf{N}$ between monoids

$$\mathbf{M} = \langle \mathbf{M}, \S_{\mathbf{M}}, \mathrm{id}_{\mathbf{M}} \rangle$$
 and $\mathbf{N} = \langle \mathbf{N}, \S_{\mathbf{N}}, \mathrm{id}_{\mathbf{N}} \rangle$

is a function $F: \mathbf{M} \to \mathbf{N}$ such that for all x, y in \mathbf{M} ,

 $F(\mathrm{id}_{\mathbf{M}}) = \mathrm{id}_{\mathbf{N}}.$

$$F(x \, _{\mathbf{M}}^{\circ} y) = F(x) \, _{\mathbf{N}}^{\circ} F(y),$$

and

$$F(x \, _{9\mathbf{M}} \, y) = F(x) \, _{9\mathbf{N}}^{\circ}$$