

**Definition** (Arrow category). Given any category  $\mathbf{C}$ , its *arrow category*  $\mathbf{Arr}(\mathbf{C})$  is the category in which:

1. *Objects*: An object  $X \in \mathbf{Arr}(\mathbf{A})$  is a morphism  $X : X_0 \rightarrow X_1$  of  $\mathbf{C}$ ;
2. *Morphisms*: A morphism  $f : X \rightarrow Y$  in  $\mathbf{Arr}(\mathbf{C})$  is a commutative square

$$\begin{array}{ccc} X_0 & \xrightarrow{f_0} & Y_0 \\ X \downarrow & & \downarrow Y \\ X_1 & \xrightarrow{f_1} & Y_1 \end{array}$$

in  $\mathbf{C}$ ;

3. *Composition*: Composition in  $\mathbf{Arr}(\mathbf{C})$  is given by playing commutative squares side by side.