Definition (Coproduct). Given two DPIs with same functionality and resources $dp_1 = \langle F, R, I_1, \text{prov}_1, \text{req}_1 \rangle$ and $dp_2 = \langle F, R, I_2, \text{prov}_2, \text{req}_2 \rangle$, define their co-product as

$$dp_1 \sqcup dp_2 := \langle F, R, I_1 \sqcup I_2, prov, req \rangle$$
,

where

$$\begin{array}{ll} \mathsf{prov} & : & i \mapsto \begin{cases} \mathsf{prov}_1(i), & \text{if } i \in I_1, \\ \mathsf{prov}_2(i), & \text{if } i \in I_2, \end{cases} \\ \\ \mathsf{req} & : & i \mapsto \begin{cases} \mathsf{req}_1(i), & \text{if } i \in I_1, \\ \mathsf{req}_2(i), & \text{if } i \in I_2. \end{cases} \end{array} \tag{0.1}$$