Definition (Semi-category action)

A *semi-category action* of a semi-category $\mathbf{C}$ is defined by

- a map φ that associates, to each object $X \in \mathrm{Ob}_{\mathbf{C}}$, a set $\varphi(X)$:

$$\varphi : \mathrm{Ob}_{\mathbf{C}} \to \mathrm{Ob}_{\mathbf{Set}};$$

- a map γ that associates, to each morphism in $\mathbf{C}$, a function:

$$\gamma : \mathrm{Hom}_{\mathbf{C}}(X; Y) \to \mathrm{Hom}_{\mathbf{Set}}(\varphi(X); \varphi(Y));$$

Moreover, this condition must hold:

$$\gamma(f \fatsemi g) = \gamma(f) \fatsemi \gamma(g).$$