Definition (Arrow category). Given any category $\mathbf{C}$, its *arrow category* $\mathbf{Arr}(\mathbf{C})$ is the category in which:

1. *Objects:* An object $X \in \mathbf{Arr}(\mathbf{A})$ is a morphism $X : X_0 \to X_1$ of $\mathbf{C}$;

2. *Morphisms:* A morphism $f : X \to Y$ in $\mathbf{Arr}(\mathbf{C})$ is a commutative square

$$
\begin{array}{ccc}
X_0 & \xrightarrow{f_0} & Y_0 \\
{\scriptstyle X}\downarrow & & \downarrow{\scriptstyle Y} \\
X_1 & \xrightarrow[f_1]{} & Y_1
\end{array}
$$

in $\mathbf{C}$;

3. *Composition:* Composition in $\mathbf{Arr}(\mathbf{C})$ is given by playing commutative squares side by side.