

$$S_1 \leq_{AP} S_2 \equiv \uparrow S_1 \sqsubseteq \uparrow S_2.$$