

**Definition.** A symmetric matrix  $\mathbf{M} \in \mathbb{R}^{n \times n}$  is *positive semi-definite* if  $x^\top \mathbf{M} x \geq 0$  for all non-zero  $x \in \mathbb{R}^n$ . We call the set of all such matrices  $\mathcal{P}^n$ .