

**Definition (Berg).** Let **Berg** be the category defined as follows:

- ▷ Objects are tuples  $\langle p, v \rangle$ , where
  - $p \in L$ ,
  - $v \in \mathbb{R}^3$  (we think of this as a tangent vector to  $L$  at  $p$ ).
- ▷ A morphism  $\langle p_1, v_1 \rangle \rightarrow \langle p_2, v_2 \rangle$  is  $\langle \gamma, T \rangle$ , where
  - $T \in \mathbb{R}_{\geq 0}$ ,
  - $\gamma : [0, T] \rightarrow L$  is a  $C^1$  function with  $\gamma(0) = p_1$  and  $\gamma(T) = p_2$ , as well as  $\dot{\gamma}(0) = v_1$  and  $\dot{\gamma}(T) = v_2$  (we take one-sided derivatives at the boundaries).
- ▷ For any object  $\langle p, v \rangle$ , we define its identity morphism  $\text{Id}_{\langle p, v \rangle} = \langle \gamma, 0 \rangle$  formally: its path  $\gamma$  is defined on the closed interval  $[0, 0]$ , (with  $T = 0$  and  $\gamma(0) = p$ ). We declare this path to be  $C^1$  by convention, and declare its derivative at 0 to be  $v$ .
- ▷ Given morphisms  $\langle \gamma_1, T_1 \rangle : \langle p_1, v_1 \rangle \rightarrow \langle p_2, v_2 \rangle$  and  $\langle \gamma_2, T_2 \rangle : \langle p_2, v_2 \rangle \rightarrow \langle p_3, v_3 \rangle$ , their composition is  $\langle \gamma, T \rangle$  with  $T = T_1 + T_2$  and

$$\gamma(t) = \begin{cases} \gamma_1(t) & 0 \leq t \leq T_1 \\ \gamma_2(t - T_1) & T_1 \leq t \leq T_1 + T_2. \end{cases}$$