Definition (Moo). The *semi-category of Moore machines* **Moo** is given by:

1. *Objects:* sets.
2. *Morphisms:* A morphism is a tuple

$$f = \langle \mathbf{U}_f, \mathbf{X}_f, \mathbf{Y}_f, \mathrm{dyn}_f, \mathrm{ro}_f, \mathrm{start}_f \rangle,$$

   where:
   - $\mathbf{U}$, $\mathbf{X}$, $\mathbf{Y}$ are sets;
   - $\mathrm{dyn} : \mathbf{U} \to \mathbf{End}(\mathbf{X})$;
   - $\mathrm{ro} : \mathbf{X} \to \mathbf{Y}$.
3. *Composition of morphisms:* Composition is given by:

$$\begin{aligned} \mathbf{U}_{f \mathbin{;} g} &= \mathbf{X}_f \\ \mathbf{X}_{f \mathbin{;} g} &= \mathbf{X}_f \mathbin{;} \mathbf{X}_g \\ \mathrm{start}_{f \mathbin{;} g} &= [\, \mathrm{start}_f \mathbin{;} \mathrm{start}_g \,] \\ \mathbf{Y}_{f \mathbin{;} g} &= \mathbf{Y}_g, \end{aligned}$$

   with

$$\begin{aligned} \mathrm{dyn}_{f \mathbin{;} g} : \mathbf{U}_f \times (\mathbf{X}_f \mathbin{;} \mathbf{X}_g) &\longrightarrow (\mathbf{X}_f \mathbin{;} \mathbf{X}_g) \\ \langle u, [x_f \mathbin{;} x_g] \rangle &\longmapsto [\mathrm{dyn}_f(u, x_f) \mathbin{;} \mathrm{dyn}_g(\mathrm{ro}_f(x_f), x_g)], \end{aligned}$$

   and

$$\begin{aligned} \mathrm{ro}_{f \mathbin{;} g} : (\mathbf{X}_f \mathbin{;} \mathbf{X}_g) &\longrightarrow \mathbf{Y}_g \\ [x_f \mathbin{;} x_g] &\longmapsto \mathrm{ro}_g(x_g) \end{aligned}$$