

Definition (Group). A *group* is a monoid together with an “inverse” operation. In more detail, a group \mathbf{G} is

Constituents

1. a set \mathbf{G} ;
2. a binary operation $\circ : \mathbf{G} \times \mathbf{G} \rightarrow \mathbf{G}$, called *composition*;
3. a specified element $\text{id} \in \mathbf{G}$;
4. a map $\text{inv} : \mathbf{G} \rightarrow \mathbf{G}$, called *inverse*.

Conditions

1. Associative law: $(x \circ y) \circ z = x \circ (y \circ z), \quad \forall x, y, z \in \mathbf{G}$;
2. Neutrality laws: $\text{id} \circ x = x = x \circ \text{id}, \quad \forall x \in \mathbf{G}$;
3. Inverse laws:

$$\text{inv}(x) \circ x = \text{id} = x \circ \text{inv}(x), \quad \forall x \in \mathbf{G}.$$