

Lemma. For $A, B, C, D \in \mathcal{AP}$, $A \leq_{\mathcal{AR}} C$ and $B \leq_{\mathcal{AR}} D$ implies $A \cup B \leq_{\mathcal{AR}} C \cup D$.