Definition (Group). A *group* is a monoid together with an "inverse" operation. In more detail, a group **G** is

Constituents

1. a set **G**;
2. a binary operation $\mathbin{\text{⨾}} : \mathbf{G} \times \mathbf{G} \to \mathbf{G}$, called *composition*;
3. a specified element $\mathrm{id} \in \mathbf{G}$;
4. a map $\mathrm{inv} : \mathbf{G} \to \mathbf{G}$, called *inverse*.

Conditions

1. Associative law: $(x \mathbin{\text{⨾}} y) \mathbin{\text{⨾}} z = x \mathbin{\text{⨾}} (y \mathbin{\text{⨾}} z)$ $\hspace{2em} \forall\, x, y, z \in \mathbf{G}$;
2. Neutrality laws: $\mathrm{id} \mathbin{\text{⨾}} x = x = x \mathbin{\text{⨾}} \mathrm{id}$ $\hspace{3em} \forall\, x \in \mathbf{G}$;
3. Inverse laws:

$$\mathrm{inv}(x) \mathbin{\text{⨾}} x = \mathrm{id} = \mathrm{inv}(x) \mathbin{\text{⨾}} x \hspace{2em} \forall\, x \in \mathbf{G}.$$