

Definition (Trace in $\mathbf{Pos}_{\mathcal{U}}$)

Given a morphism $f : X \times Z \rightarrow Y \times Z$ in $\mathbf{Pos}_{\mathcal{U}}$, its trace in is defined as a morphism $\mathrm{Tr}_{X,Y}^Z(f) : X \rightarrow Y$, given by

$$\mathrm{Tr}_{X,Y}^Z(f)^{\star} : X \rightarrow \mathcal{U}Y$$

$$x \mapsto \{y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f^{\star}(x, z)\}.$$