

**Definition** (Trace of a generalized endomorphism). Let  $\langle \mathbf{C}, \otimes, \mathbf{1}, \mathbf{br} \rangle$  be a symmetric monoidal category. Let  $X \in \mathbf{Ob}_{\mathbf{C}}$  be dualizable and let  $f \in \mathbf{Hom}_{\mathbf{C}}(Y \otimes X; Z \otimes X)$ . The *trace over  $X$*  of  $f$  is the morphism  $\mathrm{Tr}_{Y,Z}^X(f) \in \mathbf{Hom}(Y, Z)$  defined by

$$\begin{array}{c}
 \text{id}_Y \otimes \text{coev}_X \\
 \text{Y} \xrightarrow{\quad} \text{Y} \otimes X \otimes X^\vee \xrightarrow{f \otimes \text{id}_{X^\vee}} Z \otimes X \otimes X^\vee \xrightarrow{\text{id}_Z \otimes \mathbf{br}} Z \otimes X^\vee \otimes X \xrightarrow{\text{id}_Z \otimes \text{ev}_X} Z
 \end{array}$$