Definition (Pre-ordered set)

A *pre-ordered* set is a tuple $\mathbf{P} = \langle \mathbf{P}, \leq_{\mathbf{P}} \rangle$, where $\mathbf{P}$ is a set (also called the *carrier set* or *underlying set*), together with a relation $\leq_{\mathbf{P}}$ that is:

▷ *Reflexive:*

$$\frac{\top}{p \leq_{\mathbf{P}} p}.$$

▷ *Transitive:*

$$\frac{p \leq_{\mathbf{P}} q \quad q \leq_{\mathbf{P}} r}{p \leq_{\mathbf{P}} r}.$$