Definition (Enriched functor)

Given two categories C and D enriched in the same monoidal category V, an enriched functor $F: C \to D$ consists of:

- 1. A map $F: Ob_{\mathbb{C}} \to Ob_{\mathbb{D}}$ that maps objects of \mathbb{C} to objects of \mathbb{D} .
- 2. For each X, Y in $Ob_{\mathbb{C}}$, there exists a morphism in \mathbb{V} given by

$$F_{X,Y}$$
: $\operatorname{Hom}_{\mathbf{C}}(X;Y) \to \operatorname{Hom}_{\mathbf{D}}(F(X);F(Y))$,

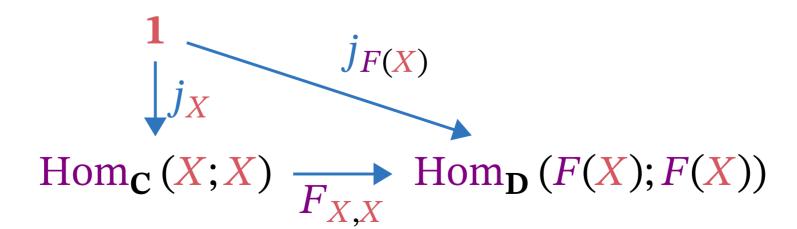
such that composing maps "across F" respects the composition in ${\bf C}$ and the unit in ${\bf V}$ in the obvious ways:

$$\operatorname{Hom}_{\mathbf{C}}(X;Y) \otimes \operatorname{Hom}_{\mathbf{C}}(Y;Z) \xrightarrow{m_{X,Y,Z}} \operatorname{Hom}_{\mathbf{C}}(X;Z)$$

$$F_{X,Y} \otimes F_{Y,Z} \downarrow \qquad \qquad \downarrow F_{X,Z}$$

$$\operatorname{Hom}_{\mathbf{D}}(F(Y);F(Z)) \otimes \operatorname{Hom}_{\mathbf{D}}(F(X);F(Y)) \xrightarrow{m_{F(X),F(Y),F(Z)}} \operatorname{Hom}_{\mathbf{D}}(F(X);F(Z))$$

and



where \otimes and 1 are the monoidal product and monoidal unit in V.