Lemma. Pos $_{\mathcal{U}}$ and Pos $_{\mathcal{Y}}$ are categories.