

**Definition** (Traced monoidal category). A symmetric monoidal category  $\langle \mathbf{C}, \otimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}}, \text{br} \rangle$  is said to be *traced* if it is equipped with a family of functions

$$\text{Tr}_{X,Y}^Z : \mathbf{C}(X \otimes_{\mathbf{C}} Z, Y \otimes_{\mathbf{C}} Z) \rightarrow \mathbf{C}(X, Y),$$

satisfying the following axioms:

1. *Vanishing*: For all morphisms  $f : X \rightarrow Y$  in  $\mathbf{C}$ ,

$$\text{Tr}_{X,Y}^1(f) = f.$$

Furthermore, for all morphisms  $f : X \otimes_{\mathbf{C}} Z \otimes_{\mathbf{C}} W \rightarrow Y \otimes_{\mathbf{C}} Z \otimes_{\mathbf{C}} W$  in  $\mathbf{C}$ :

$$\text{Tr}_{X,Y}^{Z \otimes_{\mathbf{C}} W}(f) = \text{Tr}_{X,Y}^Z \left( \text{Tr}_{X \otimes_{\mathbf{C}} Z, Y \otimes_{\mathbf{C}} Z}^W(f) \right).$$

2. *Superposing*: For all morphisms  $f : X \otimes_{\mathbf{C}} Z \rightarrow Y \otimes_{\mathbf{C}} Z$  in  $\mathbf{C}$ :

$$\text{Tr}_{V \otimes_{\mathbf{C}} X, V \otimes_{\mathbf{C}} Y}^Z(\text{Id}_V \otimes_{\mathbf{C}} f) = \text{Id}_V \otimes_{\mathbf{C}} \text{Tr}_{X,Y}^Z(f).$$

3. *Yanking*:

$$\text{Tr}_{Z,Z}^Z(\text{br}_{Z,Z}) = \text{Id}_Z.$$