Lemma. Given $\mathbf{f}_1, \mathbf{f}_2 \in \mathrm{Hom}_{\mathbf{DP}}(\mathbf{C} \times \mathbf{A}; \mathbf{C} \times \mathbf{B})$, one has:

$$\frac{\mathbf{f}_1 \preceq_{\mathbf{DP}} \mathbf{f}_2}{\mathrm{Tr}^{\mathbf{C}}_{\mathbf{A},\mathbf{B}}(\mathbf{f}_1) \preceq_{\mathbf{DP}} \mathrm{Tr}^{\mathbf{C}}_{\mathbf{A},\mathbf{B}}(\mathbf{f}_2)}$$

In other words, trace preserves order on **DP**.