

$$\underbrace{\uparrow \left( \bigcup_{\substack{p \in \uparrow \\ i \in I}} S_i \right) \{f(p)\}}_{\star} \subseteq \uparrow \underbrace{\left( \bigcup_{i \in I} \{f(s_i)\} \right)}_{\diamond}$$