$$S_1 \leq_{\mathcal{AP}} S_2 \equiv \uparrow S_1 \supseteq \uparrow S_2.$$