Definition (Trace in $\mathbf{Pos}_{\mathcal{U}}$)**.** Given a morphism $f : X \times Z \to Y \times Z$ in $\mathbf{Pos}_{\mathcal{U}}$, its trace in is defined as a morphism $\mathrm{Tr}^{Z}_{X,Y}(f) : X \to Y$, given by

$$\mathrm{Tr}^{Z}_{X,Y}(f)^{\star} : X \to \mathcal{U}Y$$

$$x \mapsto \{y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f^{\star}(x, z)\}.$$