

**Definition (Moo).** The *semi-category of Moore machines* **Moo** is given by:

1. *Objects*: sets.
2. *Morphisms*: A morphism is a tuple

$$f = \langle \mathbf{U}_f, \mathbf{X}_f, \mathbf{Y}_f, \text{dyn}_f, \text{ro}_f, \text{start}_f \rangle$$

where

- ▷  $\mathbf{U}, \mathbf{X}, \mathbf{Y}$  are sets;
- ▷  $\text{dyn} : \mathbf{U} \rightarrow \mathbf{End}(\mathbf{X})$ ;
- ▷  $\text{ro} : \mathbf{X} \rightarrow \mathbf{Y}$ .

3. *Composition of morphisms*: Composition is given by

$$\mathbf{U}_{f \circ g} = \mathbf{X}_f$$

$$\mathbf{X}_{f \circ g} = \mathbf{X}_f \circ \mathbf{X}_g$$

$$\text{start}_{f \circ g} = [\text{start}_f ; \text{start}_g]$$

$$\mathbf{Y}_{f \circ g} = \mathbf{Y}_g,$$

$$\text{dyn}_{f \circ g} : \mathbf{U}_f \times (\mathbf{X}_f \circ \mathbf{X}_g) \longrightarrow (\mathbf{X}_f \circ \mathbf{X}_g)$$

$$\langle u, [x_f ; x_g] \rangle \longmapsto [\text{dyn}_f(u, x_f) ; \text{dyn}_g(\text{ro}_f(x_f), x_g)],$$

and

$$\text{ro}_{f \circ g} : (\mathbf{X}_f \circ \mathbf{X}_g) \longrightarrow \mathbf{Y}_g$$

$$[x_f ; x_g] \longmapsto \text{ro}_g(x_g)$$