

Lemma. Given posets \mathbf{P}, \mathbf{Q} and a monotone map $f : \mathbf{P} \rightarrow \mathbf{Q}$, we have:

$$\uparrow \left(\bigcup_{p' \in \uparrow \{p\}} \{f(p')\} \right) = \uparrow \{f(p)\}.$$