Lemma. There is a *contravariant functor* $\Pi_r : \mathbf{DP} \to \mathbf{Pos}_{\mathscr{L}}$ which maps:

1. An object (poset) of **DP** to the same object (poset) in $\mathbf{Pos}_{\mathscr{L}}$.
2. A morphism $\mathsf{dp} \in \mathrm{Hom}_{\mathbf{DP}}(\mathbf{F}; \mathbf{R})$ to the morphism $g \in \mathrm{Hom}_{\mathbf{Pos}_{\mathscr{L}}}(\mathbf{R}; \mathbf{F})$, where:

$$g : \mathbf{R} \to \mathscr{L}\mathbf{F}$$

$$r \mapsto \{f \in \mathbf{F} \mid \mathsf{dp}(f, r) = \top\}.$$