

**Definition** (Monoid isomorphism). A morphism of semigroups  $f : \mathbf{M} \rightarrow \mathbf{N}$  is called a *monoid isomorphism* if there is a morphism of monoids  $g : \mathbf{N} \rightarrow \mathbf{M}$  such that

$$f \circ g = \text{id}_{\mathbf{M}} \quad \text{and} \quad g \circ f = \text{id}_{\mathbf{N}} .$$