Definition (Disjoint union category). Given two categories $\mathbf{C}$ and $\mathbf{D}$, their *disjoint union* $\mathbf{C} + \mathbf{D}$ is the category specified as follows:

1. *Objects*: Objects are elements of $\mathrm{Ob}_{\mathbf{C}} + \mathrm{Ob}_{\mathbf{D}}$; that is, objects are tuples of the form $\langle X, i \rangle$, with $i = 1$ or $i = 2$, depending on whether $X \in \mathrm{Ob}_{\mathbf{C}}$ or $X \in \mathrm{Ob}_{\mathbf{D}}$.
2. *Morphisms*: Given objects $\langle X, i \rangle, \langle Y, j \rangle \in \mathrm{Ob}_{\mathbf{C}+\mathbf{D}}$,

$$\mathrm{Hom}_{\mathbf{C}+\mathbf{D}}(\langle X, i \rangle, \langle Y, j \rangle) := \begin{cases} \mathrm{Hom}_{\mathbf{C}}(X, Y) & \text{if } i = j = 1, \\ \mathrm{Hom}_{\mathbf{D}}(X, Y) & \text{if } i = j = 2, \\ \emptyset & \text{else.} \end{cases}$$

3. *Identity morphisms*:
4. *Composition of morphisms*: