Lemma. Let $\langle S, \mathring{,} \rangle$ be a semigroup. If there exists elements $1 \in M$ and $1' \in M$ such that $\langle S, \mathring{,}, 1 \rangle$ and $\langle S, \mathring{,}, 1' \rangle$ are each monoids, then $1 = 1'$ must hold. In other words, the neutral element of a monoid is uniquely determined by the underlying semigroup structure.