called a *monoid isomorphism* if there is a morphism of monoids $g: \mathbb{N} \to \mathbb{M}$ such that

Definition (Monoid isomorphism). A morphism of semigroups $f: \mathbf{M} \to \mathbf{N}$ is

$$f \circ g = id_{\mathbf{M}}$$
 and $g \circ f = id_{\mathbf{N}}$.