Definition (Functor). Given categories $\mathbf{C}$ and $\mathbf{D}$, a *functor* $F : \mathbf{C} \to \mathbf{D}$ from $\mathbf{C}$ to $\mathbf{D}$ is defined by the following data, satisfying the following conditions.

Data:

i) For every object $X \in \mathrm{Ob}_{\mathbf{C}}$, an object $F(X) \in \mathrm{Ob}_{\mathbf{D}}$;
ii) For every morphism $f : X \to Y$ in $\mathbf{C}$, a morphism $F(f) : F(X) \to F(Y)$ in $\mathbf{D}$.

Conditions:

i) For every object $X \in \mathrm{Ob}_{\mathbf{C}}$, one has $F(\mathrm{Id}_X) = \mathrm{Id}_{F(X)}$;
ii) For every three objects $X, Y, Z \in \mathrm{Ob}_{\mathbf{C}}$ and two morphisms $f \in \mathrm{Hom}_{\mathbf{C}}(X; Y)$, $g \in \mathrm{Hom}_{\mathbf{C}}(Y; Z)$, the equation

$$F(f \mathbin{⨟} g) = F(f) \mathbin{⨟} F(g)$$

holds in $\mathbf{D}$.

This situation is graphically reported in **??**.