Definition (Category $\mathbf{Pos}_{\mathcal{U}}$). The category $\mathbf{Pos}_{\mathcal{U}}$ consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_{\mathcal{U}}}$, morphisms from X to Y are monotone maps of the form $f : X \to \mathcal{U}Y$.
3. *Composition of morphisms*: Given morphisms $f : X \to \mathcal{U}Y$ $g : Y \to \mathcal{U}Z$, their composition is given as

$$f \fatsemi g : X \to \mathcal{U}Z$$
$$x \mapsto \bigcup_{y \in f(x)} g(y);$$

4. *Identity morphism*: given an object $X \in \mathrm{Ob}_{\mathbf{Pos}_{\mathcal{U}}}$, the identity morphism is given by the application of the upper closure operator: $\mathrm{Id}_X(x) := \uparrow\{x\}$.