

**Definition** ( $\mathcal{U}$  endofunctor). The  $\mathcal{U}$  *endofunctor* has the form  $\mathcal{U} : \mathbf{Pos} \rightarrow \mathbf{Pos}$  and acts on objects and morphisms as follows:

1. *On objects*: Given a poset  $P \in \mathbf{Ob}_{\mathbf{Pos}}$ ,  $\mathcal{U}$  maps  $P$  to its upper set.
2. *On morphisms*: Given posets  $P, Q$ , and a monotone map  $f : P \rightarrow Q$ , the  $\mathcal{U}$  endofunctor acts as:

$$\mathcal{U}(f) : \mathcal{U}P \rightarrow \mathcal{U}Q$$

$$P' \mapsto \uparrow \left( \bigcup_{p \in P'} \{f(p)\} \right).$$

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Recall that in ?? we proved that the upper set is itself an object of  $\mathbf{Pos}$ .