

$$\alpha \geq_{AR} \text{Min} \bigcup_{\substack{\leq_R \\ r \in \alpha}} h_{f_1}(r) \cap \uparrow r.$$