Definition (Monad). Let **C** be a category. A *monad* on **C** is specified by:

Constituents

- 1. A functor $M: \mathbb{C} \to \mathbb{C}$;
- 2. A natural transformation mu : $M \, ; M \Rightarrow M$, called the *composition* or *multiplication*;
- 3. A natural transformation un : $Id_{\mathbb{C}} \Rightarrow M$, called the *unit*.

Conditions

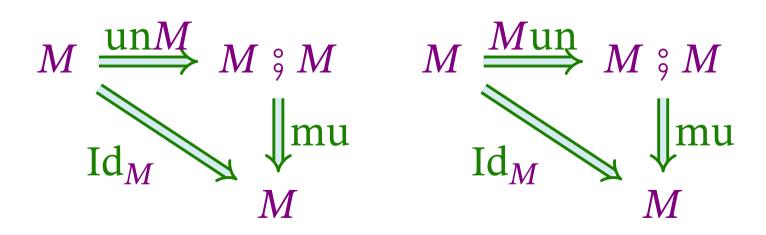
1. Associativity: the diagram

$$M \stackrel{\circ}{\circ} M \stackrel{\circ}{\circ} M \xrightarrow{M mu} M \stackrel{\circ}{\circ} M$$

$$\downarrow muM \qquad \qquad \downarrow mu$$
 $M \stackrel{\circ}{\circ} M \xrightarrow{mu} M$

must commute.

2. Left and right unitality: the diagrams



must commute.