Definition (Monoidal poset). A *monoidal structure* on a poset $\mathbf{P} = \langle P, \preceq_{\mathbf{P}} \rangle$ is specified by:

<u>Constituents</u>

1. A monotone map $\otimes : \mathbf{P} \times \mathbf{P} \to \mathbf{P}$, called the *monoidal product*.
   Note that here we are implicitly assuming $\mathbf{P} \times \mathbf{P}$ as having the product order.
   In detail, monotonicity means that, for all $x_1, x_2, y_1, y_2 \in \mathbf{P}$:

$$x_1 \preceq_{\mathbf{P}} y_1 \text{ and } x_2 \preceq_{\mathbf{P}} y_2 \implies (x_1 \otimes x_2) \preceq_{\mathbf{P}} (y_1 \otimes y_2).$$

2. An element $\mathbf{1} \in \mathbf{P}$, called the *monoidal unit*.

<u>Conditions</u>

1. Associativity: for all $x, y, z \in \mathbf{P}$:

$$(x \otimes y) \otimes z = x(\otimes y \otimes z).$$

2. Left and right unitality: for all $x \in \mathbf{P}$:

$$\mathbf{1} \otimes x = x \qquad \text{and} \qquad x \otimes \mathbf{1} = x.$$

A poset equipped with a monoidal structure is called a *monoidal poset*.