

Definition (loop). Suppose dp is a DPI with factored functionality space $\mathbf{F}_1 \times \mathbf{R}$:

$$\text{dp} = \langle \mathbf{F}_1 \times \mathbf{R}, \mathbf{R}, \mathbf{I}, \langle \text{prov}_1, \text{prov}_2 \rangle, \text{req} \rangle.$$

Then we can define the DPI $\text{loop}(\text{dp})$ as

$$\text{loop}(\text{dp}) := \langle \mathbf{F}_1, \mathbf{R}, \mathbf{I}', \text{prov}_1, \text{req} \rangle,$$

where $\mathbf{I}' \subseteq \mathbf{I}$ limits the implementations to those that respect the additional constraint $\text{req}(i) \leq \text{prov}_2(i)$:

$$\mathbf{I}' = \{i \in \mathbf{I} : \text{req}(i) \leq \text{prov}_2(i)\}.$$

This is equivalent to “closing a loop” around dp with the constraint $f_2 \geq r$ (??).