

**Lemma** (Kleene's fixed-point theorem [davey02]). Assume  $\mathbf{P}$  is a CPO, and  $f : \mathbf{P} \rightarrow \mathbf{P}$  is Scott continuous. Then the least fixed point of  $f$  is the supremum of the Kleene ascent chain

$$\perp \leq f(\perp) \leq f(f(\perp)) \leq \cdots \leq f^{(n)}(\perp) \leq \cdots.$$