

Definition (Twisted arrow category). Given a category \mathbf{C} , we denote its *twisted arrow category* by $\mathbf{Tw}(\mathbf{C})$. This is a category which is composed of:

1. *Objects*: Arrows (morphisms) in \mathbf{C} .
2. *Morphisms*: A morphism between two arrows $f : X \rightarrow Y, g : Z \rightarrow U$ is given by a pair of arrows $\langle h, i \rangle$ such that the following diagram commutes:

$$\begin{array}{ccc} X & \xleftarrow{h} & Z \\ f \downarrow & & \downarrow g \\ Y & \xrightarrow{i} & U \end{array}$$

3. *Composition*: Composition in $\mathbf{Tw}(\mathbf{C})$ is given by playing commutative squares side by side.