

**Lemma.** Let  $\langle S, \circ \rangle$  be a semigroup. If there exists elements  $1 \in M$  and  $1' \in M$  such that  $\langle S, \circ, 1 \rangle$  and  $\langle S, \circ, 1' \rangle$  are each monoids, then  $1 = 1'$  must hold. In other words, the neutral element of a monoid is uniquely determined by the underlying semigroup structure.