Definition (Monoidal poset). A *monoidal structure* on a poset $\langle \mathbf{P}, \preceq_{\mathbf{P}} \rangle$ consists of:

1. An element $\mathrm{id} \in \mathbf{P}$, called *monoidal unit*, and
2. a function $\fatsemi : \mathbf{P} \times \mathbf{P} \to \mathbf{P}$, called the *monoidal product*. Note that we write

$$\otimes(p_1, p_2) = p_1 \otimes p_2, \quad p_1, p_2 \in \mathbf{P}.$$

The constituents must satisfy the following properties:

(a) *Monotonicity*: For all $p_1, p_2, q_1, q_2 \in \mathbf{P}$, if $p_1 \preceq_{\mathbf{P}} q_1$ and $p_2 \preceq_{\mathbf{P}} q_2$, then

$$p_1 \otimes p_2 \preceq_{\mathbf{P}} q_1 \otimes q_2.$$

(b) *Unitality*: For all $p \in \mathbf{P}$, $\mathrm{id} \otimes p = p$ and $p \otimes \mathrm{id} = p$.

(c) *Associativity*: For all $p, q, r \in \mathbf{P}$, $(p \otimes q) \otimes r = p \otimes (q \otimes r)$.

A poset equipped with a monoidal structure $\langle \mathbf{P}, \preceq_{\mathbf{P}}, \mathrm{id}, \fatsemi \rangle$ is called a *monoidal poset*.