Definition (Category **Rel**). The category of relations **Rel** is given by:

1. *Objects*: The objects of this category are all sets.
2. *Morphisms*: Given sets X, Y, the homset $\mathrm{Hom}_{\mathbf{Rel}}(X; Y)$ consists of all relations $\mathbf{R} \subseteq X \times Y$.
3. *Identity morphisms*: Given a set X, its identity morphism is

$$\mathrm{Id}_X := \{\langle x, x\rangle \mid x \in X\}.$$

4. *Composition*: Given relations $\mathbf{R} : X \to Y$, $\mathbf{S} : Y \to Z$, their composition is given by

$$\mathbf{R} \fatsemi \mathbf{S} := \{\langle x, z\rangle \mid \exists y \in Y : \quad (\langle x, y\rangle \in \mathbf{R}) \wedge (\langle y, z\rangle \in \mathbf{S})\}.$$