Lemma. $\mathbf{Pos}_{\mathcal{U}}$ is a monoidal category with the following additional structure:

1. *Tensor product* $\otimes$: On objects, the tensor product corresponds to the product of posets. Given two morphisms $f : X \to \mathcal{U}Y$ and $g : Z \to \mathcal{U}U$, we have:

$$
\begin{aligned}
f \otimes g : X \times Z &\to \mathcal{U}(Y \times U) \\
\langle x, z \rangle &\mapsto f(x) \times g(z).
\end{aligned}
$$

   Note that the Cartesian product of upper sets is an upper set.
2. *Unit*: The unit is the identity poset: the poset with a singleton carrier set and only the identity relation. We denote this by $\{\bullet\}$.
3. *Left unitor*: The left unitor is given by the pair of morphisms

$$
\begin{aligned}
\mathrm{lu}_{\mathbf{P}} : \{\bullet\} \otimes X &\to \mathcal{U}X \\
\langle \bullet, x \rangle &\mapsto \uparrow\{x\},
\end{aligned}
$$

   and

$$
\begin{aligned}
\mathrm{lu}_{\mathbf{P}}{}^{-1} : X &\to \mathcal{U}(\{\bullet\} \otimes X) \\
x &\mapsto \{\bullet\} \times \uparrow\{x\}.
\end{aligned}
$$

4. *Right unitor*: The right unitor is given by the pair of morphisms

$$
\begin{aligned}
\mathrm{ru}_{\mathbf{P}} : X \otimes \{\bullet\} &\to \mathcal{U}X \\
\langle x, \bullet \rangle &\mapsto \uparrow\{x\},
\end{aligned}
$$

   and

$$
\begin{aligned}
\mathrm{ru}_{\mathbf{P}}{}^{-1} : X &\to \mathcal{U}(X \otimes \{\bullet\}) \\
x &\mapsto \uparrow\{x\} \times \{\bullet\}.
\end{aligned}
$$

5. *Associator*: The associator is given by the pair of morphisms:

$$
\begin{aligned}
\mathrm{as}_{XY,Z} : (X \otimes Y) \otimes Z &\to \mathcal{U}X \times (\mathcal{U}Y \times \mathcal{U}Z) \\
\langle \langle x, y \rangle, z \rangle &\mapsto \uparrow\{x\} \times (\uparrow\{y\} \times \uparrow\{z\}),
\end{aligned}
$$

   and

$$
\begin{aligned}
\mathrm{as}_{X,YZ} : X \otimes (Y \otimes Z) &\to (\mathcal{U}X \times \mathcal{U}Y) \times \mathcal{U}Z \\
\langle x, \langle y, z \rangle \rangle &\mapsto (\uparrow\{x\} \times \uparrow\{y\}) \times \uparrow\{z\}.
\end{aligned}
$$