

$$\begin{aligned}
\text{Tr}_{U \times X, U \times Y}^Z(\text{Id}_U \times f)(u, x) &= \left\{ \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} \langle u, y, z \rangle \in (\text{Id}_U \times f)(u, x, z) \right\} \\
&= \left\{ \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} (u \in \text{Id}_U(u)) \wedge (\langle y, z \rangle \in f(x, z)) \right\} \\
&= \left\{ \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} (u \in \uparrow\{u\}) \wedge (\langle y, z \rangle \in f(x, z)) \right\} \\
&= \left\{ \langle u, y \rangle \in \uparrow\{u\} \times Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f(x, z) \right\} \\
&= \left\langle \uparrow\{u\}, \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f(x, z) \right\} \right\rangle
\end{aligned}$$