

Definition (Graph homomorphism). Given graphs $\mathcal{G} = \langle \mathbf{V}, \mathbf{A}, s, t \rangle$ and $\mathcal{G}' = \langle \mathbf{V}', \mathbf{A}', s', t' \rangle$, a graph homomorphism $f : \mathcal{G} \rightarrow \mathcal{G}'$ is given by maps $f_0 : \mathbf{V} \rightarrow \mathbf{V}'$ and $f_1 : \mathbf{A} \rightarrow \mathbf{A}'$, such that the following diagrams commute:

$$\begin{array}{ccc} \mathbf{A} & \xrightarrow{f_1} & \mathbf{A}' \\ \downarrow s & & \downarrow s' \\ \mathbf{V} & \xrightarrow{f_0} & \mathbf{V}' \end{array}$$

$$\begin{array}{ccc} \mathbf{A} & \xrightarrow{f_1} & \mathbf{A}' \\ \downarrow t & & \downarrow t' \\ \mathbf{V} & \xrightarrow{f_0} & \mathbf{V}' \end{array}$$