Definition (Order on morphisms in $\mathbf{Pos}_{\mathcal{U}}$)
Given any two morphisms $f, g : X \to Y$ in $\mathbf{Pos}_{\mathcal{U}}$, we define an order between them as

$$\frac{f \leq_{\mathbf{Pos}_{\mathcal{U}}} g}{f^{\star}(x) \leq_{\mathcal{U}Y} g^{\star}(x), \quad \forall x \in X}.$$