Definition (Dualizable object)

Let $\langle \mathbf{C}, \bigotimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}} \rangle$ be a monoidal category, and let $X \in \mathrm{Ob}_{\mathbf{C}}$. A *right dual object* of X is specified by:

Constituents

- 1. an object $X^{\vee} \in Ob_{\mathbb{C}}$;
- 2. an evaluation map $ev_X : X^{\vee} \otimes X \to 1$;
- 3. a coevaluation map $coev_X$: $1 \rightarrow X \otimes X^{\vee}$;

Conditions

- 1. $lu_X^{-1} \ \ (coev_X \otimes Id_X) \ \ as_{X,X^\vee,X} \ \ (Id_X \otimes ev_X) \ \ ru_X = Id_X;$
- 2. $\operatorname{ru}_{X^{\vee}}^{-1} \circ (\operatorname{Id}_{X^{\vee}} \otimes \operatorname{coev}_{X}) \circ \operatorname{as}_{X^{\vee},X,X^{\vee}}^{-1} \circ (\operatorname{ev}_{X} \otimes \operatorname{Id}_{X^{\vee}}) \circ \operatorname{lu}_{X^{\vee}} = \operatorname{Id}_{X^{\vee}}.$