Definition (Monad in functional programming). A monad

$$\langle return, join, fmap, bind, fish, lift \rangle$$

is a set of operations with the following signature:

$$return : X \to MX$$
$$lift : (X \to Y) \to (X \to MY)$$
$$fish : (X \to MY) \to (Y \to MZ) \to (X \to MZ)$$
$$join : MMX \to MX$$
$$fmap : (X \to Y) \to (MX \to MY)$$
$$bind : MX \to (X \to MY) \to MY$$

These maps satisfy the equivalent axioms of unitality and associativity:

- ▷ *return* is a left identity for *bind*;
- ▷ *return* is a right identity for *bind*;
- ▷ *bind* is associative.