

**Definition** (Twisted arrow category). Given a category  $\mathbf{C}$ , we denote its *twisted arrow category* by  $\mathbf{Tw}(\mathbf{C})$ . This is a category which is composed of:

1. *Objects*: Arrows (morphisms) in  $\mathbf{C}$ .
2. *Morphisms*: A morphism between two arrows  $f : X \rightarrow Y, g : Z \rightarrow U$  is given by a pair of arrows  $\langle h, i \rangle$  such that the following diagram commutes:

$$\begin{array}{ccc} X & \xleftarrow{h} & Z \\ f \downarrow & & \downarrow g \\ Y & \xrightarrow{i} & U \end{array}$$

3. *Composition*: Composition in  $\mathbf{Tw}(\mathbf{C})$  is given by playing commutative squares side by side.