

Definition (Category **Rel**)

The category of relations **Rel** is given by:

1. *Objects*: The objects of this category are all sets.
2. *Morphisms*: Given sets X, Y , the homset $\mathbf{Hom}_{\mathbf{Rel}}(X; Y)$ consists of all relations $R \subseteq X \times Y$.
3. *Identity morphisms*: Given a set X , its identity morphism is

$$\mathbf{Id}_X := \{\langle x, y \rangle \in X \times X \mid x = y\}.$$

4. *Composition*: Given relations $R : X \rightarrow Y, S : Y \rightarrow Z$, their composition is given by

$$R \circ S := \{\langle x, z \rangle \in X \times Z \mid \exists y \in Y : (x R y) \wedge (y S z)\}.$$