

Definition (Algebra of a monad)

Let $\langle M, \mu, \eta \rangle$ be a monad on a category \mathbf{C} . An algebra of M (also called an M -algebra) is specified by:

Constituents

1. an object X of \mathbf{C} ;
2. a morphism $a : M(X) \rightarrow X$ of \mathbf{C} .

Conditions

1. *Composition*: the diagram

$$\begin{array}{ccc} (M \circ M)(X) & \xrightarrow{Ma} & M(X) \\ \mu_X \downarrow & & \downarrow a \\ M(X) & \xrightarrow{a} & X \end{array}$$

commutes.

2. *Unit*: the diagram

$$\begin{array}{ccc} X & \xrightarrow{\eta_X} & M(X) \\ & \searrow \text{Id} & \downarrow a \\ & & X \end{array}$$

commutes.