

Definition (\mathcal{U} endofunctor). The \mathcal{U} *endofunctor* has the form $\mathcal{U} : \mathbf{Pos} \rightarrow \mathbf{Pos}$ and acts on objects and morphisms as follows:

1. *On objects*: Given a poset $\mathbf{P} \in \mathbf{Ob}_{\mathbf{Pos}}$, \mathcal{U} maps \mathbf{P} to its upper set. {bhf:1}
2. *On morphisms*: Given posets \mathbf{P}, \mathbf{Q} , and a monotone map $f : \mathbf{P} \rightarrow \mathbf{Q}$, the \mathcal{U} endofunctor acts as:

$$\mathcal{U}(f) : \mathcal{U}\mathbf{P} \rightarrow \mathcal{U}\mathbf{Q}$$

$$\mathbf{P}' \mapsto \uparrow \left(\bigcup_{p \in \mathbf{P}'} \{f(p)\} \right).$$

Recall that in ?? we proved that the upper set is itself an object of \mathbf{Pos} .