$$\underbrace{\uparrow \left(\bigcup_{p \in \uparrow \bigcup_{i \in I} S_i} \{f(p)\} \right)}_{\star} \subseteq \underbrace{\uparrow \left(\bigcup_{i \in I} \{f(s_i)\} \right)}_{\diamond}$$