

Definition (Opposite category). Given a category \mathbf{C} , its *opposite category* \mathbf{C}^{op} is specified by:

1. *Objects*: $\text{Ob}_{\mathbf{C}^{\text{op}}} = \text{Ob}_{\mathbf{C}}$.

Given $X \in \text{Ob}_{\mathbf{C}}$, we will sometimes (though not always) write X^{op} to signify when we are thinking of X as an object of $\text{Ob}_{\mathbf{C}^{\text{op}}}$.

2. *Morphisms*: Given objects $X^{\text{op}}, Y^{\text{op}} \in \text{Ob}_{\mathbf{C}^{\text{op}}} = \text{Ob}_{\mathbf{C}}$,

$$\text{Hom}_{\mathbf{C}^{\text{op}}}(X^{\text{op}}, Y^{\text{op}}) := \text{Hom}_{\mathbf{C}}(Y, X).$$

Given $f \in \text{Hom}_{\mathbf{C}}(Y, X)$, when we are thinking of it as an element of $\text{Hom}_{\mathbf{C}^{\text{op}}}(X^{\text{op}}, Y^{\text{op}})$, we will sometimes write f^{op} .

3. *Identity morphisms*: Given $X^{\text{op}} \in \text{Ob}_{\mathbf{C}^{\text{op}}}$, its identity morphism is

$$\text{Id}_{X^{\text{op}}} := \text{Id}_X^{\text{op}}.$$

4. *Composition*: Let $f^{\text{op}} \in \text{Hom}_{\mathbf{C}^{\text{op}}}(X^{\text{op}}, Y^{\text{op}})$ and $g^{\text{op}} \in \text{Hom}_{\mathbf{C}^{\text{op}}}(Y^{\text{op}}, Z^{\text{op}})$, then

$$f^{\text{op}} \circ_{\mathbf{C}^{\text{op}}} g^{\text{op}} := (g \circ_{\mathbf{C}} f)^{\text{op}}.$$