Definition (Directed set). A set $S \subseteq \mathbb{P}$ is directed if each pair of elements in S has an upper bound: for all $a, b \in S$, there exists $c \in S$ such that $a \leq c$ and $b \leq c$.