

**Definition** (Lower closure operator). The *lower closure operator*  $\downarrow$  maps a subset to the smallest lower set that includes it:

$$\downarrow : \mathcal{P}\mathbf{P} \rightarrow \mathcal{L}\mathbf{P}$$

$$S \mapsto \{y \in \mathbf{P} \mid \exists x \in S : y \leq_{\mathbf{P}} x\}.$$