

$$\begin{aligned}
\mathrm{Tr}_{X',Y}^Z((g \times \mathrm{Id}_Z) \circ f)(x') &= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in ((g \times \mathrm{Id}_Z) \circ f)(x', z) \right\} \\
&= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \bigcup_{\langle x, z' \rangle \in g(x') \times \{z\}} f(x, z') \right\} \\
&= \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \bigcup_{x \in g(x')} f(x, z) \right\}.
\end{aligned}$$