

**Definition** (Subcategory). A subcategory  $\mathbf{D}$  of a category  $\mathbf{C}$  is a category for which:

1. All the objects in  $\text{Ob}_{\mathbf{D}}$  are in  $\text{Ob}_{\mathbf{C}}$ ;
2. For any objects  $X, Y \in \text{Ob}_{\mathbf{D}}$ ,  $\text{Hom}_{\mathbf{D}}(X; Y) \subseteq \text{Hom}_{\mathbf{C}}(X; Y)$ ;
3. If  $X \in \text{Ob}_{\mathbf{D}}$ , then  $\text{Id}_X \in \text{Hom}_{\mathbf{C}}(X; X)$  is in  $\text{Hom}_{\mathbf{D}}(X; X)$  and acts as its identity morphism;
4. If  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  in  $\mathbf{D}$ , then the composite  $f \circ g$  in  $\mathbf{C}$  is in  $\mathbf{D}$  and represents the composite in  $\mathbf{D}$ .