

$$\alpha \geq_{AR} \operatorname{Min}_{\leq_R} \bigcup_{r \in \alpha} h_{f_1}(r) \cap \uparrow r.$$