

# Definition (Algebra of a monad)

Let  $\langle M, \mu, \eta \rangle$  be a monad on a category  $\mathbf{C}$ . An algebra of  $M$  (also called an  $M$ -algebra) is specified by:

## Constituents

1. an object  $X$  of  $\mathbf{C}$ ;
2. a morphism  $a : M(X) \rightarrow X$  of  $\mathbf{C}$ .

## Conditions

1. *Composition*: the diagram

$$\begin{array}{ccc} (M \circ M)(X) & \xrightarrow{Ma} & M(X) \\ \mu_X \downarrow & & \downarrow a \\ M(X) & \xrightarrow{a} & X \end{array}$$

commutes.

2. *Unit*: the diagram

$$\begin{array}{ccc} X & \xrightarrow{\eta_X} & M(X) \\ & \searrow \text{Id} & \downarrow a \\ & & X \end{array}$$

commutes.