Definition (Profunctor composition). Given two profunctors $F : \mathbf{C} \nrightarrow \mathbf{D}$ and $G : \mathbf{D} \nrightarrow \mathbf{E}$ we can define their composition $(F \mathbin{⨟} G) : \mathbf{C} \nrightarrow \mathbf{E}$ as follows:

$$
\begin{aligned}
(F \mathbin{⨟} G)_{\mathrm{ob}} : \mathrm{Ob}(\mathbf{C}^{\mathrm{op}} \times \mathbf{E}) &\to \mathrm{Ob}\,\mathbf{Set}, \\
\langle C^*, E \rangle &\mapsto \textstyle\coprod_{D \in \mathrm{Ob}\mathbf{D}} F_{\mathrm{ob}}(C^*, D) \times G_{\mathrm{ob}}(D^*, E) / \sim
\end{aligned}
$$

$$
\begin{aligned}
(F \mathbin{⨟} G)_{\mathrm{mor}} : \mathrm{Hom}_{(\mathbf{C}^{\mathrm{op}} \times \mathbf{E})}(\langle C_1^*, E_1 \rangle ; \langle C_2^*, E_2 \rangle) &\to \mathrm{Hom}_{\mathbf{Set}}((F \mathbin{⨟} G)_{\mathrm{ob}}(C_1^*, E_1) ; (F \mathbin{⨟} G)_{\mathrm{ob}}(C_2^*, E_2)) \\
\langle \alpha^*, \beta \rangle &\mapsto \begin{cases} (F \mathbin{⨟} G)_{\mathrm{ob}}(C_1^*, E_1) \to (F \mathbin{⨟} G)_{\mathrm{ob}}(C_2^*, E_2)) \\ \qquad \langle s, t \rangle \mapsto \langle F_{\mathrm{mor}}(\langle \alpha, \mathrm{Id}_D \rangle)(s), G_{\mathrm{mor}}(\langle \mathrm{Id}_{D^*}, \beta \rangle)(t) \rangle \end{cases}
\end{aligned}
$$

In the formulas:

$$
\alpha : C_2 \to C_1, \qquad \beta : E_1 \to E_2,
$$

and $\langle s, t \rangle$ is a pair of elements for which there exists a $D \in \mathrm{Ob}\mathbf{D}$ such that

$$
s \in F_{\mathrm{ob}}(C_1^*, D), \qquad t \in G_{\mathrm{ob}}(D^*, E).
$$