Definition (DPI composition)

The series composition of two DPIs

$$\mathbf{d}_1 = \langle \mathbf{F}_1, \mathbf{R}_1, \mathbf{I}_1, \mathsf{prov}_1, \mathsf{req}_1 \rangle,$$

$$\mathbf{d}_2 = \langle \mathbf{F}_2, \mathbf{R}_2, \mathbf{I}_2, \mathsf{prov}_2, \mathsf{req}_2 \rangle,$$

for which $\mathbf{F}_2 = \mathbf{R}_1$, is defined as

$$(\mathbf{d}_1 \fatsemi \mathbf{d}_2) := \langle \mathbf{F}_1, \mathbf{R}_2, \mathbf{I}, \mathsf{prov}, \mathsf{req} \rangle,$$

where:

$$\mathbf{I} = \{ [i_1 \,\mathring{,}\, i_2] \in (\mathbf{I}_1 \,\mathring{,}\, \mathbf{I}_2) \mid \mathsf{req}_1(i_1) \preceq_{\mathbf{R}_1} \mathsf{prov}_2(i_2) \},$$

$$\mathsf{prov} : \quad [i_1 \,\mathring{,}\, i_2] \mapsto \mathsf{prov}_1(i_1),$$

$$\mathsf{req} : \quad [i_1 \,\mathring{,}\, i_2] \mapsto \mathsf{req}_2(i_2).$$