Definition (Enriched functor). Given two categories C and D enriched in the same monoidal category V, an enriched functor $F: C \to D$ consists of:

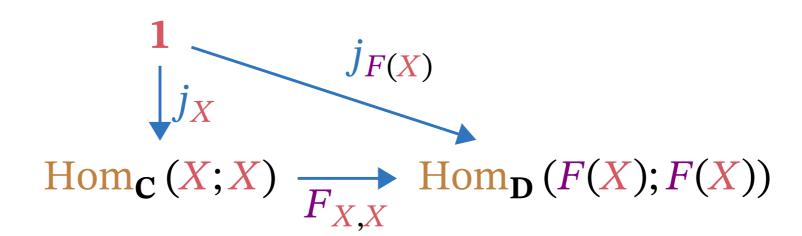
- 1. A map $F: Ob_{\mathbb{C}} \to Ob_{\mathbb{D}}$ that maps objects of \mathbb{C} to objects of \mathbb{D} .
- 2. For each X, Y in $Ob_{\mathbb{C}}$, there exists a morphism in \mathbb{V} given by

$$F_{X,Y}: \operatorname{Hom}_{\mathbf{C}}(X;Y) \to \operatorname{Hom}_{\mathbf{D}}(F(X);F(Y)),$$

such that composing maps "across F" respects the composition in \mathbf{C} and the unit in \mathbf{V} in the obvious ways:

$$\begin{array}{c} \operatorname{Hom}_{\mathbf{C}}(X;Y) \otimes \operatorname{Hom}_{\mathbf{C}}(Y;Z) & \longrightarrow \operatorname{Hom}_{\mathbf{C}}(X;Z) \\ F_{X,Y} \otimes F_{Y,Z} \downarrow & \downarrow F_{X,Z} \\ \operatorname{Hom}_{\mathbf{D}}(F(Y);F(Z)) \otimes \operatorname{Hom}_{\mathbf{D}}(F(X);F(Y)) & \xrightarrow{\operatorname{Hom}_{\mathbf{D}}(F(X);F(Z))} & \operatorname{Hom}_{\mathbf{D}}(F(X);F(Z)) \end{array}$$

and



where \otimes and 1 are the monoidal product and monoidal unit in V.