

Definition (Semicategory)

A *semicategory* \mathbf{C} is specified by:

Constituents

1. Objects: A collection $\mathbf{Ob}_{\mathbf{C}}$ whose elements are called *objects*.
2. Morphisms: For every pair of objects X, Y in $\mathbf{Ob}_{\mathbf{C}}$, there is a set $\mathbf{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms*. We write

$$f : X \rightarrow_{\mathbf{C}} Y$$

to indicate

$$f \in \mathbf{Hom}_{\mathbf{C}}(X; Y).$$

3. Composition operations: For every three objects X, Y, Z in $\mathbf{Ob}_{\mathbf{C}}$ there is a composition map

$$\circ_{X,Y,Z} : \mathbf{Hom}_{\mathbf{C}}(X; Y) \times \mathbf{Hom}_{\mathbf{C}}(Y; Z) \rightarrow \mathbf{Hom}_{\mathbf{C}}(X; Z).$$

We usually just write \circ instead of $\circ_{X,Y,Z}$:

$$\frac{f : X \rightarrow Y \quad g : Y \rightarrow Z}{f \circ g : X \rightarrow Z}.$$

The morphism $f \circ g$ is called the *composition* of f and g .

Conditions

1. Associativity: it holds that

$$\frac{f : X \rightarrow Y \quad g : Y \rightarrow Z \quad h : Z \rightarrow U}{(f \circ g) \circ h = f \circ (g \circ h)}.$$