Definition (Order on morphisms in $\mathbf{Pos}_{\mathscr{L}}$)
Given any two morphisms $f, g : X \to Y$ in $\mathbf{Pos}_{\mathscr{L}}$, we define an order between them as

$$\frac{f \leq_{\mathbf{Pos}_{\mathscr{L}}} g}{f^{\star}(x) \leq_{\mathscr{L}Y} g^{\star}(x), \quad \forall x \in X}.$$