

**Definition** (Coproduct of DPIs). Given two DPIs with same functionality and resources  $\mathbf{f} = \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_1, \text{prov}_1, \text{req}_1 \rangle$  and  $\mathbf{g} = \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_2, \text{prov}_2, \text{req}_2 \rangle$ , define their co-product as

$$\mathbf{f} \sqcup \mathbf{g} := \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_1 \sqcup \mathbf{I}_2, \text{prov}, \text{req} \rangle,$$

where

$$\begin{aligned} \text{prov} &: i \mapsto \begin{cases} \text{prov}_1(i), & \text{if } i \in \mathbf{I}_1, \\ \text{prov}_2(i), & \text{if } i \in \mathbf{I}_2, \end{cases} \\ \text{req} &: i \mapsto \begin{cases} \text{req}_1(i), & \text{if } i \in \mathbf{I}_1, \\ \text{req}_2(i), & \text{if } i \in \mathbf{I}_2. \end{cases} \end{aligned} \tag{0.1}$$