$$\left(\operatorname{Tr}_{X,Y}^{Z}\left(\operatorname{Tr}_{X\times Z,Y\times Z}^{U}(f)\right)\right)^{\star}(x) = \begin{cases} y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \operatorname{Tr}_{X\times Z,Y\times Z}^{U}(f)^{\star}(x,z) \end{cases} \\
= \begin{cases} y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in \left\{ \langle y', z' \rangle \in Y \times Z \mid \bigvee_{u \in U} \langle y', z', U \rangle \in f^{\star}(x,z',u) \right\} \end{cases}$$

$$= \left\{ y \in Y \mid \bigvee z \in Z \left(\bigvee_{u \in U} \langle y, z, u \rangle \in f^{\star}(x, z, u) \right) \right\}$$

 $= \left\{ y \in Y \mid \bigvee_{\langle z, u \rangle \in Z \times U} \langle y, z, u \rangle \in f^{\star}(x, z, u) \right\}.$