

Definition (Traced monoidal category). A symmetric monoidal category $\langle \mathbf{C}, \otimes, \{\bullet\}, \sigma \rangle$ is said to be *traced* if equipped with a family of functions

$$\mathrm{Tr}_{A,B}^X : \mathbf{C}(A \otimes X, B \otimes X) \rightarrow \mathbf{C}(A, B),$$

satisfying the following axioms:

1. *Vanishing*: For all morphisms $f : A \rightarrow B$ in \mathbf{C} ,

$$\mathrm{Tr}_{A,B}^1(f) = f.$$

Furthermore, for all morphisms $f : A \otimes X \otimes Y \rightarrow B \otimes X \otimes Y$ in \mathbf{C} :

$$\mathrm{Tr}_{A,B}^{X \otimes Y}(f) = \mathrm{Tr}_{A,B}^X \left(\mathrm{Tr}_{A \otimes X, B \otimes X}^Y(f) \right).$$

2. *Superposing*: For all morphisms $f : A \otimes X \rightarrow B \otimes X$ in \mathbf{C} :

$$\mathrm{Tr}_{C \otimes A, C \otimes B}^X(\mathrm{id}_C \otimes f) = \mathrm{id}_C \otimes \mathrm{Tr}_{A,B}^X(f).$$

3. *Yanking*:

$$\mathrm{Tr}_{X,X}^X(\sigma_{X,X}) = \mathrm{id}_X.$$