

Definition (\mathcal{U} endofunctor). The \mathcal{U} *endofunctor* has the form $\mathcal{U} : \mathbf{Pos} \rightarrow \mathbf{Pos}$ and acts on objects and morphisms as follows:

1. *On objects*: Given a poset $P \in \mathbf{Ob}_{\mathbf{Pos}}$, \mathcal{U} maps P to its upper set.
2. *On morphisms*: Given posets P, Q , and a monotone map $f : P \rightarrow Q$, the \mathcal{U} endofunctor acts as:

$$\mathcal{U}(f) : \mathcal{U}P \rightarrow \mathcal{U}Q$$

$$P' \mapsto \uparrow \left(\bigcup_{p \in P'} \{f(p)\} \right).$$

Recall that in ?? we proved that the upper set is itself an object of \mathbf{Pos} .