Definition (Lower closure operator)

The *lower closure operator* $\downarrow$ maps a subset to the smallest lower set that includes it:

$$\downarrow : \mathscr{P}\mathbf{P} \to \mathscr{L}\mathbf{P}$$

$$\mathbf{S} \mapsto \{y \in \mathbf{P} \mid \exists x \in \mathbf{S} : y \leq_{\mathbf{P}} x\}.$$