Definition (Upper bounds in a poset). The *upper bounds* of a subset $\mathbf{A}$ of a poset $\mathbf{P}$ are, if they exist, the elements of $\mathbf{P}$ which dominate all elements in $\mathbf{A}$. In other words, the upper bounds of $\mathbf{A}$ are the elements of the set

$$\{y \in \mathbf{P} \mid \forall x \in \mathbf{A} : \ x \leq y\}.$$

A *least upper bound* of $\mathbf{A} \subseteq \mathbf{P}$, if it exists, is a least element among the upper bounds of $\mathbf{A}$. It is denoted $\bigvee \mathbf{A}$ or Sup $\mathbf{A}$, and also called the *join* or *supremum* of $\mathbf{A}$. So, given $\mathbf{A} \subseteq \mathbf{P}$ and $y \in \mathbf{P}$, $y = \bigvee \mathbf{A}$ if and only if

1. $x \leq y \ \forall x \in \mathbf{A}$, and
2. $x \leq y' \ \forall x \in \mathbf{A} \Rightarrow y \leq y'$.

If a least upper bound of a subset $\mathbf{A} \subseteq \mathbf{P}$ exists, it is unique (can you prove this?), so we speak of "the" least upper bound.