

**Definition** (Category). A *category* **C** is:

### Constituents

1. Objects: a collection  $\text{Ob}_{\mathbf{C}}$ , whose elements are called *objects*.
2. Morphisms: for every pair of objects  $X, Y \in \text{Ob}_{\mathbf{C}}$ , there is a set  $\text{Hom}_{\mathbf{C}}(X; Y)$ , elements of which are called *morphisms* from  $X$  to  $Y$ . The set is called the “hom-set from  $X$  to  $Y$ ”.
3. Identity morphisms: for each object  $X$ , there is an element  $\text{Id}_X \in \text{Hom}_{\mathbf{C}}(X; X)$  which is called *the identity morphism of  $X$* .
4. Composition operations: given any morphism  $f \in \text{Hom}_{\mathbf{C}}(X; Y)$  and any morphism  $g \in \text{Hom}_{\mathbf{C}}(Y; Z)$ , there exists a morphism  $f \circ g \in \text{Hom}_{\mathbf{C}}(X; Z)$  which is the *composition of  $f$  and  $g$* .

### Conditions

1. Unitality: It holds that:

$$\frac{\text{Id}_X : X \rightarrow X \quad f : X \rightarrow Y \quad \text{Id}_Y : Y \rightarrow Y}{\text{Id}_X \circ f = f = f \circ \text{Id}_Y}$$

2. Associativity: it holds that

$$\frac{f : X \rightarrow Y \quad g : Y \rightarrow Z \quad h : Z \rightarrow W}{(f \circ g) \circ h = f \circ (g \circ h)}$$