Definition (Profunctor composition). Given two profunctors $F : \mathbf{C} \nrightarrow \mathbf{D}$ and $G : \mathbf{D} \nrightarrow \mathbf{E}$ we can define their composition $(F \mathbin{⨟} G) : \mathbf{C} \nrightarrow \mathbf{E}$ as follows:

$$
\begin{aligned}
(F \mathbin{⨟} G)_{\mathrm{ob}} :\ & \mathrm{Ob}(\mathbf{C}^{\mathrm{op}} \times \mathbf{E}) \to \mathrm{Ob}\,\mathbf{Set}, \\
& \langle C^*, E\rangle \mapsto \textstyle\coprod_{D \in \mathrm{Ob}\mathbf{D}} F_{\mathrm{ob}}(C^*, D) \times G_{\mathrm{ob}}(D^*, E)/\sim
\end{aligned}
$$

$$
\begin{aligned}
(F \mathbin{⨟} G)_{\mathrm{mor}} :\ & \mathrm{Hom}_{(\mathbf{C}^{\mathrm{op}} \times \mathbf{E})}(\langle C_1^*, E_1\rangle ; \langle C_2^*, E_2\rangle) \to \mathrm{Hom}_{\mathbf{Set}}((F \mathbin{⨟} G)_{\mathrm{ob}}(C_1^*, E_1); (F \mathbin{⨟} G)_{\mathrm{ob}}(C_2^*, E_2)) \\
& \langle \alpha^*, \beta\rangle \mapsto \begin{cases} (F \mathbin{⨟} G)_{\mathrm{ob}}(C_1^*, E_1) \to (F \mathbin{⨟} G)_{\mathrm{ob}}(C_2^*, E_2)) \\ \qquad \langle s, t\rangle \mapsto \langle F_{\mathrm{mor}}(\langle \alpha, \mathrm{Id}_D\rangle)(s), G_{\mathrm{mor}}(\langle \mathrm{Id}_{D^*}, \beta\rangle)(t)\rangle \end{cases}
\end{aligned}
$$

In the formulas:

$$
\alpha : C_2 \to C_1, \qquad \beta : E_1 \to E_2,
$$

and $\langle s, t\rangle$ is a pair of elements for which there exists a $D \in \mathrm{Ob}\mathbf{D}$ such that

$$
s \in F_{\mathrm{ob}}(C_1^*, D), \qquad t \in G_{\mathrm{ob}}(D^*, E).
$$