

Definition (Subcategory). A subcategory \mathbf{D} of a category \mathbf{C} is a category for which:

1. All the objects in $\text{Ob}_{\mathbf{D}}$ are in $\text{Ob}_{\mathbf{C}}$;
2. For any objects $X, Y \in \text{Ob}_{\mathbf{D}}$, $\text{Hom}_{\mathbf{D}}(X; Y) \subseteq \text{Hom}_{\mathbf{C}}(X; Y)$;
3. If $X \in \text{Ob}_{\mathbf{D}}$, then $\text{Id}_X \in \text{Hom}_{\mathbf{C}}(X; X)$ is in $\text{Hom}_{\mathbf{D}}(X; X)$ and acts as its identity morphism;
4. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ in \mathbf{D} , then the composite $f \circ g$ in \mathbf{C} is in \mathbf{D} and represents the composite in \mathbf{D} .