

**Definition** (The  $\mathcal{U}$  monad). The  $\mathcal{U}$  monad on **DP** consists of:

1. The functor  $\text{Unc} : \mathbf{DP} \rightarrow \mathbf{DP}$ ;
2. The natural transformation  $\text{un}_{\mathcal{U}} : \text{Id}_{\mathbf{DP}} \Rightarrow \text{Unc}$ , specified as

$$\begin{aligned} \text{un}_{\mathcal{U}}^{\mathbf{A}} : \mathbf{A} &\dashrightarrow \text{Int}(\mathbf{A}) \\ \langle a^*, [x, y] \rangle &\mapsto a \leq x. \end{aligned}$$

3. The natural transformation  $\text{mu}_{\mathcal{U}} : \text{UncUnc} \Rightarrow \text{Unc}$ , specified as:

$$\begin{aligned} \text{mu}_{\mathcal{U}}^{\mathbf{A}} : \text{Int}(\text{Int}(\mathbf{A}))^{\text{op}} \times \text{Int}(\mathbf{A}) &\rightarrow_{\mathbf{Pos}} \mathbf{Bool} \\ \langle [[a, b], [c, d]]^*, [e, f] \rangle &\mapsto (a \leq e) \wedge (b \leq e) \wedge (c \leq f) \wedge (d \leq f). \end{aligned}$$