

**Definition** (Power set as lattice). Given a set  $S$ , its power set  $\mathcal{P}S$  (the set of all subsets) is a lattice where, given  $A, B \in \mathcal{P}S$ :

▷ Order is given by inclusion:

$$A \leq B \doteq A \subseteq B;$$

▷ The join is given by the union of sets:

$$A \vee B \doteq A \cup B;$$

▷ The meet is given by the intersection of sets:

$$A \wedge B \doteq A \cap B;$$

▷ The top element is the set  $S$  itself:

$$\top = S;$$

▷ The bottom element is the empty set:

$$\perp = \emptyset.$$