

**Lemma.** The metrics  $P_{\text{track}}$  and  $P_{\text{effort}}$  can be written as

$$\lim_{t \rightarrow \infty} \mathbb{E}\{\mathbf{x}_t^\top \mathbf{Q}_0 \mathbf{x}_t\} = \text{Tr}(\mathbf{Q}_0 (\mathbf{\Sigma} + \mathbf{F})),$$

$$\lim_{t \rightarrow \infty} \mathbb{E}\{\mathbf{u}_t^\top \mathbf{R}_0 \mathbf{u}_t\} = \text{Tr}(\mathbf{S} \mathbf{B}^* \mathbf{R}^{-1} \mathbf{R}_0 \mathbf{R}^{-1} \mathbf{B} \mathbf{S} \mathbf{F}),$$

where  $\mathbf{\Sigma}$  solves the Riccati equation for estimation,  $\mathbf{F}$  solves the Lyapunov equation

$$(\mathbf{A} - \mathbf{B} \mathbf{K}) \mathbf{F} + \mathbf{F} (\mathbf{A} - \mathbf{B} \mathbf{K})^* + \mathbf{L} \mathbf{V} \mathbf{L}^* = \mathbf{0},$$

$\mathbf{S}$  solves the Riccati equation for control, and  $\mathbf{L} = \mathbf{\Sigma} \mathbf{C}^* \mathbf{V}^{-1}$  is the Kalman gain.