

**Lemma.** Given a set  $\mathbf{A}$  and a family of subsets of  $\mathbf{A}$ , denoted  $\{\mathbf{A}_i\}_{i \in I}$  (with index set  $I$ ), one has:

$$\bigcup_{i \in I} \bigcup \{A_i\} = \bigcup_{i \in I} \{A_i\}.$$