

**Lemma.** Given posets  $\mathbf{P}, \mathbf{Q}$  and a monotone map  $f : \mathbf{P} \rightarrow \mathbf{Q}$ , we have:

$$\uparrow \left( \bigcup_{p' \in \uparrow \{p\}} \{f(p')\} \right) = \uparrow \{f(p)\}.$$