Definition (Semicategory). A *semicategory* **C** is specified by:

Constituents

1. Objects: A collection $\mathrm{Ob}_{\mathbf{C}}$ whose elements are called *objects*.
2. Morphisms: For every pair of objects X, Y in $\mathrm{Ob}_{\mathbf{C}}$, there is a set $\mathrm{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms*. We write

$$f : X \to_{\mathbf{C}} Y$$

to indicate

$$f \in \mathrm{Hom}_{\mathbf{C}}(X; Y).$$

3. Composition operations: For every three objects X, Y, Z in $\mathrm{Ob}_{\mathbf{C}}$ there is a composition map

$$\fatsemi_{X,Y,Z} : \mathrm{Hom}_{\mathbf{C}}(X; Y) \times \mathrm{Hom}_{\mathbf{C}}(Y; Z) \to \mathrm{Hom}_{\mathbf{C}}(X; Z).$$

We usually just write $\fatsemi$ instead of $\fatsemi_{X,Y,Z}$:

$$\frac{f : X \to Y \quad g : Y \to Z}{f \fatsemi g : X \to Z}$$

The morphism $f \fatsemi g$ is called the *composition* of f and g.

Conditions

1. Associativity: it holds that

$$\frac{f : X \to Y \quad g : Y \to Z \quad h : Z \to U}{(f \fatsemi g) \fatsemi h = f \fatsemi (g \fatsemi h)}$$