

**Lemma.** Given posets  $\mathbf{P}, \mathbf{Q}$ , a monotone map  $f : \mathbf{P} \rightarrow \mathbf{Q}$ , and a family of singleton sets  $\{S_i\}_{i \in I}$ , with  $S_i = \{s_i\}$ ,  $s_i \in \mathbf{P}$ , the following equality holds:

$$\uparrow \left( \bigcup_{p \in \uparrow \bigcup_{i \in I} S_i} \{f(p)\} \right) = \uparrow \left( \bigcup_{i \in I} \{f(s_i)\} \right).$$