**Lemma.** For $A, B \in \mathcal{A}\mathbf{P}$, and $S \subseteq P$, $A \preceq_{\mathcal{A}\mathbf{R}} B$ implies $A \cap S \preceq_{\mathcal{A}\mathbf{R}} B \cap S$.