

**Lemma.** **UPos** and **LPos** are equivalent: there exists a pair of functors

$$\swarrow : \mathbf{UPos} \rightarrow \mathbf{LPos},$$

$$\nearrow : \mathbf{LPos} \rightarrow \mathbf{UPos},$$

such that  $\swarrow \circ \nearrow = \text{Id}_{\mathbf{UPos}}$  and  $\nearrow \circ \swarrow = \text{Id}_{\mathbf{LPos}}$ , where  $\text{Id}_{\mathbf{UPos}}$  and  $\text{Id}_{\mathbf{LPos}}$  are the identity functors on **UPos** and **LPos**, respectively.