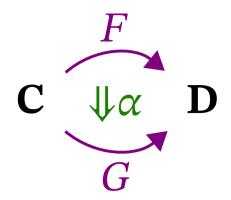
Definition (Natural transformation). Let **C** and **D** be categories, and let $F, G : \mathbf{C} \to \mathbf{D}$ be functors. A *natural transformation* $\alpha : F \Rightarrow G$



is defined by the following constituent data, satisfying the following condition. Constituents

1. For each object $X \in \mathbf{C}$, a morphism $\alpha_X : F(X) \to G(X)$ in \mathbf{D} , called the *X-component* of α .

Conditions

1. For every morphism $f: X \to Y$ in \mathbf{C} , the components of α must satisfy the *naturality condition*

$$F(f) \stackrel{\circ}{,} \alpha_Y = \alpha_X \stackrel{\circ}{,} G(f).$$

In other words, the following diagram must commute:

$$F(X) \xrightarrow{F(f)} F(Y)$$

$$\alpha_X \downarrow \qquad \qquad \downarrow \alpha_Y$$

$$G(X) \xrightarrow{G(f)} G(Y)$$

The situation is represented diagrammatically in ??.