

$$\begin{aligned}
& \text{Tr}_{U \times X, U \times Y}^Z (\text{Id}_U \otimes f)^\star(u, x) \\
&= \left\{ \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} \langle u, y, z \rangle \in (\text{Id}_U \otimes f)^\star(u, x, z) \right\} \\
&= \left\{ \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} (u \in \text{Id}_U^\star(u)) \wedge (\langle y, z \rangle \in f^\star(x, z)) \right\} \\
&= \left\{ \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} (u \in \uparrow\{u\}) \wedge (\langle y, z \rangle \in f^\star(x, z)) \right\} \\
&= \left\{ \langle u, y \rangle \in \uparrow\{u\} \times Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f^\star(x, z) \right\} \\
&= \uparrow\{u\} \times \left\{ y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f^\star(x, z) \right\}
\end{aligned}$$