

Lemma. For $A, B \in \mathcal{AP}$, and $S \subseteq P$, $A \leq_{AR} B$ implies $A \cap S \leq_{AR} B \cap S$.