

Definition (Enriched category)

Let $\langle \mathbf{V}, \otimes, \mathbf{1}, \text{as}, \text{lu}, \text{ru} \rangle$ be a monoidal category. A category \mathbf{C} *enriched* in \mathbf{V} is composed of:

1. The set of objects $\text{Ob}_{\mathbf{C}}$;
2. For all $X, Y \in \text{Ob}_{\mathbf{C}}$, an object $\text{Hom}_{\mathbf{C}}(X; Y)$, called the *hom-object* from X to Y .
3. For all $X, Y, Z \in \text{Ob}_{\mathbf{C}}$, there exists a morphism $\circ_{X,Y,Z}$ in \mathbf{V} :

$$\circ_{X,Y,Z} : \text{Hom}_{\mathbf{C}}(X; Y) \otimes \text{Hom}_{\mathbf{C}}(Y; Z) \rightarrow \text{Hom}_{\mathbf{C}}(X; Z).$$

This is called *composition morphism*.

4. For each $X \in \text{Ob}_{\mathbf{C}}$, a morphism $\text{Id}_X : \mathbf{1} \rightarrow \text{Hom}_{\mathbf{C}}(X; X)$, called *identity element*.

Furthermore, for any $X, Y, Z, U \in \text{Ob}_{\mathbf{C}}$, the following diagrams must commute.