

Definition (Semcategory). A *semicategory* \mathbf{C} is:

Constituents

1. A collection $\mathbf{Ob}_{\mathbf{C}}$ whose elements are called *objects*.
2. For every pair of objects X, Y in $\mathbf{Ob}_{\mathbf{C}}$, there is a set $\mathbf{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms*. We write

$$f : X \rightarrow_{\mathbf{C}} Y$$

to indicate

$$f \in \mathbf{Hom}_{\mathbf{C}}(X; Y).$$

3. For every three objects X, Y, Z in $\mathbf{Ob}_{\mathbf{C}}$ there is a composition map

$$\circ_{X,Y,Z} : \mathbf{Hom}_{\mathbf{C}}(X; Y) \times \mathbf{Hom}_{\mathbf{C}}(Y; Z) \rightarrow \mathbf{Hom}_{\mathbf{C}}(X; Z)$$

Given any morphism $f \in \mathbf{Hom}_{\mathbf{C}}(X; Y)$ and any morphism $g \in \mathbf{Hom}_{\mathbf{C}}(Y; Z)$, there exists a morphism $f \circ g \in \mathbf{Hom}_{\mathbf{C}}(X; Z)$ which is the *composition of f and g* .

Conditions

1. Associativity: it holds that

$$\frac{f : X \rightarrow Y \quad g : Y \rightarrow Z \quad h : Z \rightarrow W}{(f \circ g) \circ h = f \circ (g \circ h)}$$