

# Definition (Disjoint union category)

Given two categories  $\mathbf{C}$  and  $\mathbf{D}$ , their *disjoint union*  $\mathbf{C} + \mathbf{D}$  is the category specified as follows:

1. *Objects*: Objects are elements of  $\mathbf{Ob}_{\mathbf{C}} + \mathbf{Ob}_{\mathbf{D}}$ ; that is, objects are tuples of the form  $\langle X, i \rangle$ , with  $i = 1$  or  $i = 2$ , depending on whether  $X \in \mathbf{Ob}_{\mathbf{C}}$  or  $X \in \mathbf{Ob}_{\mathbf{D}}$ .
2. *Morphisms*: Given objects  $\langle X, i \rangle, \langle Y, j \rangle \in \mathbf{Ob}_{\mathbf{C} + \mathbf{D}}$ ,

$$\mathbf{Hom}_{\mathbf{C} + \mathbf{D}}(\langle X, i \rangle; \langle Y, j \rangle) := \begin{cases} \mathbf{Hom}_{\mathbf{C}}(X; Y) & \text{if } i = j = 1, \\ \mathbf{Hom}_{\mathbf{D}}(X; Y) & \text{if } i = j = 2, \\ \emptyset & \text{else.} \end{cases}$$

3. *Identity morphisms*: Given  $f : X \rightarrow Y \in \mathbf{Hom}_{\mathbf{C} + \mathbf{D}}(\langle X, i \rangle; \langle Y, i \rangle)$ , one has

$$\mathbf{Id}_{\mathbf{C} + \mathbf{D}} := \begin{cases} \mathbf{Id}_{\mathbf{C}} & \text{if } i = 1, \\ \mathbf{Id}_{\mathbf{D}} & \text{if } i = 2. \end{cases}$$

4. *Composition*: Given  $f : X \rightarrow Y \in \mathbf{Hom}_{\mathbf{C} + \mathbf{D}}(\langle X, i \rangle; \langle Y, i \rangle)$  and  $g : Y \rightarrow Z \in \mathbf{Hom}_{\mathbf{C} + \mathbf{D}}(\langle Y, j \rangle; \langle Z, j \rangle)$ , one has

$$f \circ_{\mathbf{C} + \mathbf{D}} g := \begin{cases} f \circ_{\mathbf{C}} g & \text{if } i = j = 1, \\ f \circ_{\mathbf{D}} g & \text{if } i = j = 2, \\ \text{does not exist} & \text{else.} \end{cases}$$