Definition (Group). A *group* is a monoid together with an "inverse" operation. In more detail, a group $\mathbf{M}$ is

Constituents

1. a set $\mathbf{M}$;
2. a binary operation $\fatsemi : \mathbf{M} \times \mathbf{M} \to \mathbf{M}$, called *composition*;
3. a specified element $\mathrm{id} \in \mathbf{M}$;
4. a map $\mathrm{inv} : \mathbf{M} \to \mathbf{M}$ called "inverse".

Conditions

1. Associative law: $(x \fatsemi y) \fatsemi z = x \fatsemi (y \fatsemi z)$;
2. Neutrality Laws: $\mathrm{id} \fatsemi x = x = x \fatsemi \mathrm{id}$.
3. Inverse law:

$$\mathrm{inv}(x) \fatsemi x = \mathrm{id} = \mathrm{inv}(x) \fatsemi x$$