**Definition** (Category). A *category* C is specified by four components:

- 1. **Objects**: a collection  $Ob_{\mathbb{C}}$ , whose elements are called *objects*.
- 2. **Morphisms**: for every pair of objects  $X, Y \in \mathrm{Ob}_{\mathbb{C}}$ , there is a set  $\mathrm{Hom}_{\mathbb{C}}(X; Y)$ , elements of which are called *morphisms* from X to Y. The set is called the "hom-set from X to Y".
- 3. **Identity morphisms**: for each object X, there is an element  $\mathrm{Id}_X \in \mathrm{Hom}_{\mathbb{C}}(X;X)$  which is called *the identity morphism of* X.

Furthermore, the constituents are required to satisfy the following conditions:

a) Unitality: for any morphism  $f \in \operatorname{Hom}_{\mathbb{C}}(X;Y)$ :

$$\operatorname{Id}_{\mathbf{X}} \circ f = f = f \circ \operatorname{Id}_{\mathbf{Y}}.$$

b) Associativity: for morphisms  $f \in \operatorname{Hom}_{\mathbb{C}}(X;Y)$ ,  $g \in \operatorname{Hom}_{\mathbb{C}}(Y;Z)$ , and  $h \in \operatorname{Hom}_{\mathbb{C}}(Z;W)$ ,

$$(f \circ g) \circ h = f \circ (g \circ h).$$