

Definition (Pre-ordered set)

A *pre-ordered* set is a tuple $\mathbf{P} = \langle \mathbf{P}, \leq_{\mathbf{P}} \rangle$, where \mathbf{P} is a set (also called the *carrier set* or *underlying set*), together with a relation $\leq_{\mathbf{P}}$ that is:

▷ *Reflexive*:

$$\frac{\top}{p \leq_{\mathbf{P}} p}.$$

▷ *Transitive*:

$$\frac{p \leq_{\mathbf{P}} q \quad q \leq_{\mathbf{P}} r}{p \leq_{\mathbf{P}} r}.$$