

Lemma. Given any $X, Y \in \mathbf{Ob}_{\mathbf{Pos}_{\mathcal{U}}}$, $\mathbf{Hom}_{\mathbf{Pos}_{\mathcal{L}}}(X; Y)$ is a bounded lattice with intersection \wedge of morphisms in $\mathbf{Pos}_{\mathcal{L}}$ as meet, union \vee of morphisms in $\mathbf{Pos}_{\mathcal{L}}$ as join, least upper bound $\top_{\mathbf{Hom}_{\mathbf{Pos}_{\mathcal{L}}}(X; Y)} : X \rightarrow Y$ given by

$$\begin{aligned} \top_{\mathbf{Hom}_{\mathbf{Pos}_{\mathcal{L}}}(X; Y)}^{\star} : X &\rightarrow \mathcal{L}Y \\ x &\mapsto Y, \end{aligned}$$

and greatest lower bound $\perp_{\mathbf{Hom}_{\mathbf{Pos}_{\mathcal{U}}}(X; Y)} : X \rightarrow Y$ given by

$$\begin{aligned} \perp_{\mathbf{Hom}_{\mathbf{Pos}_{\mathcal{L}}}(X; Y)}^{\star} : X &\rightarrow \mathcal{L}Y \\ x &\mapsto \emptyset. \end{aligned}$$