$$\uparrow\left(\bigcup_{p\in\uparrow\bigcup_{i\in I}S_i}\{f(p)\}\right)\supseteq\uparrow\left(\bigcup_{i\in I}\{f(s_i)\}\right)$$