**Lemma** ( $\varphi$  is monotone in the valuation). Suppose that  $\mathbf{v}_1, \mathbf{v}_2 : \mathcal{A} \to \mathbf{DP}$  are

two valuations for which it holds that  $\mathbf{v}_1(a) \leq_{\mathbf{DP}} \mathbf{v}_2(a)$ . Then  $\varphi[\langle \mathcal{A}, \mathsf{T}, \mathbf{v}_1 \rangle] \leq_{\mathbf{DP}} \mathbf{v}_2(a)$ 

 $\varphi[\![\langle \mathcal{A}, \mathsf{T}, \boldsymbol{v}_2 \rangle]\!].$