

**Definition** (Traced monoidal category). A symmetric monoidal category  $\langle \mathbf{C}, \otimes, \mathbf{1}, \text{br} \rangle$  is said to be *traced* if it is equipped with a family of functions

$$\text{Tr}_{X,Y}^Z : \text{Hom}_{\mathbf{C}}(X \otimes Z; Y \otimes Z) \rightarrow \text{Hom}_{\mathbf{C}}(X; Y),$$

satisfying the following axioms:

1. *Naturality in  $X$* : For all morphisms  $f : X \otimes Z \rightarrow Y \otimes Z$  and  $g : X' \rightarrow X$ ,

$$\text{Tr}_{X',Y}^Z((g \otimes \text{Id}_Z) \circ f) = g \circ \text{Tr}_{X,Y}^Z(f)$$

2. *Naturality in  $Y$* : For all morphisms  $f : X \otimes Z \rightarrow Y \otimes Z$  and  $g : Y \rightarrow Y'$ ,

$$\text{Tr}_{X,Y'}^Z(f \circ (g \otimes \text{Id}_Z)) = \text{Tr}_{X,Y}^Z(f) \circ g$$

3. *Vanishing*: For all morphisms  $f : X \rightarrow Y$  in  $\mathbf{C}$ ,

$$\text{Tr}_{X,Y}^{\mathbf{1}}(f) = f.$$

Furthermore, for all morphisms  $f : X \otimes Z \otimes U \rightarrow Y \otimes Z \otimes U$  in  $\mathbf{C}$ ,

$$\text{Tr}_{X,Y}^{Z \otimes U}(f) = \text{Tr}_{X,Y}^Z \left( \text{Tr}_{X \otimes Z, Y \otimes Z}^U(f) \right).$$

4. *Superposing*: For all morphisms  $f : X \otimes Z \rightarrow Y \otimes Z$  in  $\mathbf{C}$ ,

$$\text{Tr}_{V \otimes X, V \otimes Y}^Z(\text{Id}_V \otimes f) = \text{Id}_V \otimes \text{Tr}_{X,Y}^Z(f).$$

5. *Yanking*:

$$\text{Tr}_{Z,Z}^Z(\text{br}_{Z,Z}) = \text{Id}_Z.$$