

**Definition** (Monoid morphism). A morphism  $F : \mathbf{M} \rightarrow \mathbf{N}$  between monoids

$$\mathbf{M} = \langle \mathbf{M}, \circ_{\mathbf{M}}, \text{id}_{\mathbf{M}} \rangle \quad \text{and} \quad \mathbf{N} = \langle \mathbf{N}, \circ_{\mathbf{N}}, \text{id}_{\mathbf{N}} \rangle$$

is a function  $F : \mathbf{M} \rightarrow \mathbf{N}$  such that for all  $x, y$  in  $\mathbf{M}$ ,

$$F(x \circ_{\mathbf{M}} y) = F(x) \circ_{\mathbf{N}} F(y)$$

and

$$F(\text{id}_{\mathbf{M}}) = \text{id}_{\mathbf{N}}$$