

**Definition** (Lattice homomorphism). Given two bounded lattices  $\mathbf{P}$ ,  $\mathbf{Q}$ , a *lattice homomorphism* is a map  $f : \mathbf{P} \rightarrow \mathbf{Q}$  which preserves meets, joins, top, and bottom:

$$f(p \wedge_{\mathbf{P}} q) = f(p) \wedge_{\mathbf{Q}} f(q)$$

$$f(p \vee_{\mathbf{P}} q) = f(p) \vee_{\mathbf{Q}} f(q)$$

$$f(\perp_{\mathbf{P}}) = \perp_{\mathbf{Q}}$$

$$f(\top_{\mathbf{P}}) = \top_{\mathbf{Q}}$$