Definition (Natural transformation)**.** Let **C** and **D** be categories, and let $F, G : \mathbf{C} \to \mathbf{D}$ be functors. A *natural transformation* $\alpha : F \Rightarrow G$ is specified by:

<u>Constituents</u>

1. For each object $X \in \mathbf{C}$, a morphism $\alpha_X : F(X) \to G(X)$ in **D**, called the X*-component* of α.

<u>Conditions</u>

1. For every morphism $f : X \to Y$ in **C**, the components of α must satisfy the *naturality condition*

$$F(f) ⨟ \alpha_Y = \alpha_X ⨟ G(f).$$

   In other words, the following diagram must commute:

$$\begin{tikzcd}
F(X) \arrow[r, "F(f)"] \arrow[d, "\alpha_X"'] & F(Y) \arrow[d, "\alpha_Y"] \\
G(X) \arrow[r, "G(f)"'] & G(Y)
\end{tikzcd}$$

A natural transformation $\alpha : F \Rightarrow G$ is denoted visually as follows:

$$\begin{tikzcd}
\mathbf{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\Downarrow\alpha"] & \mathbf{D}
\end{tikzcd}$$