$$\mathrm{Tr}^{Z}_{U\times X,U\times Y}(\mathrm{Id}_U \times f)^{\star}(u,x) = \left\{ \langle u,y\rangle \in U\times Y \mid \bigvee_{z\in Z} \langle u,y,z\rangle \in (\mathrm{Id}_U \times f)^{\star}(u,x,z) \right\}$$

$$= \left\{ \langle u,y\rangle \in U\times Y \mid \bigvee_{z\in Z} (u \in \mathrm{Id}_U{}^{\star}(u)) \wedge (\langle y,z\rangle \in f^{\star}(x,z)) \right\}$$

$$= \left\{ \langle u,y\rangle \in U\times Y \mid \bigvee_{z\in Z} (u \in \uparrow\{u\}) \wedge (\langle y,z\rangle \in f^{\star}(x,z)) \right\}$$

$$= \left\{ \langle u,y\rangle \in \uparrow\{u\}\times Y \mid \bigvee_{z\in Z} \langle y,z\rangle \in f^{\star}(x,z) \right\}$$

$$= \uparrow\{u\} \times \left\{ y \in Y \mid \bigvee_{z\in Z} \langle y,z\rangle \in f^{\star}(x,z) \right\}$$