Definition (Adjunction, Version 2)**.** Let $\mathbf{C}$ and $\mathbf{D}$ be categories. An *adjunction* from $\mathbf{C}$ to $\mathbf{D}$ is given by the following data, satisfying the following conditions.

<u>Data:</u>

1. A functor $L : \mathbf{C} \to \mathbf{D}$ (the *left adjoint*);
2. A functor $R : \mathbf{D} \to \mathbf{C}$ (the *right adjoint*);
3. Natural transformations $\mathrm{un} : \mathrm{Id}_{\mathbf{C}} \Rightarrow L \fatsemi R$ and $\mathrm{co} : R \fatsemi L \Rightarrow \mathrm{Id}_{\mathbf{D}}$

<u>Conditions:</u>

1. For all objects X of $\mathbf{C}$, it holds that

$$L\mathrm{un}_X \fatsemi \mathrm{co}_{LX} = \mathrm{Id}_{LX} \text{ and } \mathrm{un}_{RY} \fatsemi R\mathrm{co}_Y = \mathrm{Id}_{RY}$$

i.e. that the following diagrams commute:

$$\begin{tikzcd} LX \arrow[r, "L\mathrm{un}_X"] \arrow[dr, "\mathrm{Id}_{LX}"'] & LRLX \arrow[d, "\mathrm{co}_{LX}"] \\ & LX \end{tikzcd} \qquad \begin{tikzcd} RY \arrow[r, "\mathrm{un}_{RY}"] \arrow[dr, "\mathrm{Id}_{RY}"'] & RLRY \arrow[d, "R\mathrm{co}_Y"] \\ & RY \end{tikzcd}$$

The 2-morphisms un and co are called the *unit* and *counit* of the adjunction. An adjunction is called an *adjoint equivalence* if the unit and counit are natural isomorphisms.