

**Definition** (Identity morphism). Let  $\mathbf{S}$  be a semigroup. The identity function  $\text{Id}_{\mathbf{S}} : \mathbf{S} \rightarrow \mathbf{S}$  is always a morphism of semigroups. We can easily check that ?? is satisfied:

$$\text{Id}_{\mathbf{S}}(x \circ_{\mathbf{S}} y) = x \circ_{\mathbf{S}} y = \text{Id}_{\mathbf{S}}(x) \circ_{\mathbf{S}} \text{Id}_{\mathbf{S}}(y).$$

We call this the *identity morphism* of  $\mathbf{S}$ .