

**Definition** ( $\mathcal{U}$  monad). The  $\mathcal{U}$  monad on **Pos** consists of:

1. The  $\mathcal{U}$  endofunctor (??).
2. The unit natural transformation  $\text{un}_{\mathcal{U}} : \text{Id}_{\mathbf{Pos}} \Rightarrow \mathcal{U}$ , which associates to every object  $P \in \text{Ob}_{\mathbf{Pos}}$  a morphism in **Pos** given by:

$$\begin{aligned} \text{un}_{\mathcal{U}}^P : P &\rightarrow \mathcal{U}P \\ p &\mapsto \uparrow \{p\}. \end{aligned}$$

3. The compositiona natural transformation  $\text{mu}_{\mathcal{U}} : \mathcal{U} \circ \mathcal{U} \Rightarrow \mathcal{U}$ , which associates to every  $P \in \text{Ob}_{\mathbf{Pos}}$  the morphism in **Pos** given by:

$$\begin{aligned} \text{mu}_{\mathcal{U}}^P : \mathcal{U}(\mathcal{U}P) &\rightarrow \mathcal{U}P \\ P'' &\mapsto \bigcup_{P' \in P''} P'. \end{aligned}$$