Definition (Category $\mathbf{Pos}_{\mathcal{U}}$). The category $\mathbf{Pos}_{\mathcal{U}}$ consists of:

1. *Objects*: objects are posets;

2. *Morphisms*: given objects $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_{\mathcal{U}}}$, morphisms from $f : X \to Y$ are monotone maps of the form $f^{\star} : X \to_{\mathbf{Pos}} \langle \mathcal{U}\mathbf{Y}^{\mathrm{op}}, \supseteq \rangle$.

3. *Composition of morphisms*: Given morphisms $f : X \to Y$, $g : Y \to Z$, their composition $f \,\mathring{\,}\, g : X \to Z$ is given by

$$f^{\star} \,\mathring{\,}\, g^{\star} : X \to_{\mathbf{Pos}} \langle \mathcal{U}\mathbf{Z}^{\mathrm{op}}, \supseteq \rangle$$
$$x \mapsto \bigcup_{y \in f^{\star}(x)} g^{\star}(y);$$

4. *Identity morphism*: given an object $X \in \mathrm{Ob}_{\mathbf{Pos}_{\mathcal{U}}}$, the identity morphism is given by the application of the upper closure operator: $\mathrm{Id}_X(x) := \uparrow \{x\}$.