Definition (Category $\mathbf{Pos}_{\mathscr{L}}$). The category $\mathbf{Pos}_{\mathscr{L}}$ consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_{\mathscr{L}}}$, morphisms from X to Y are monotone maps of the form $f : X \to \mathscr{L}Y$.
3. *Composition of morphisms*: Given morphisms $f : X \to \mathscr{L}Y$, $g : Y \to \mathscr{L}Z$, their composition is given by

$$f \fatsemi g : X \to \mathscr{L}Z$$
$$x \mapsto \bigcup_{y \in f(x)} g(y);$$

4. *Identity morphism*: given an object $X \in \mathrm{Ob}_{\mathbf{Pos}_{\mathscr{L}}}$, the identity morphism is given by the application of the lower closure operator: $\mathrm{Id}_X(x) := {\downarrow}\{x\}$.