

**Definition** (Trace of a design problem)

Given a design problem  $\mathbf{d} : \mathbf{P} \times \mathbf{R} \rightarrow \mathbf{Q} \times \mathbf{R}$ , we can define its *trace*  $\text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}) : \mathbf{P} \rightarrow \mathbf{Q}$  as follows:

$$\text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}) : \mathbf{P}^{\text{op}} \times \mathbf{Q} \rightarrow_{\text{Pos}} \mathbf{Bool}$$

$$\langle a^*, b \rangle \mapsto \bigvee_{c \in \mathbf{R}} \mathbf{d}(\langle a, c \rangle^*, \langle b, c \rangle).$$