Definition (Category **Rel**)

The category of relations **Rel** is given by:

- 1. Objects: The objects of this category are all sets.
- 2. *Morphisms*: Given sets X, Y, the homset $\operatorname{Hom}_{\operatorname{Rel}}(X; Y)$ consists of all relations $R \subseteq X \times Y$.
- 3. *Identity morphisms*: Given a set X, its identity morphism is

$$Id_X := \{\langle x, y \rangle \in X \times X \mid x = y\}.$$

4. *Composition*: Given relations $R: X \to Y, S: Y \to Z$, their composition is given by

$$R \ \stackrel{\circ}{,} S := \{\langle x, z \rangle \in X \times Z \mid \exists y \in Y : (xRy) \land (ySz)\}.$$