Definition (DPI composition)

The series composition of two DPIs

$$\begin{aligned} \mathbf{d}_1 &= & \langle \mathbf{F}_1, \mathbf{R}_1, \mathbf{I}_1, \mathsf{prov}_1, \mathsf{req}_1 \rangle, \\ \mathbf{d}_2 &= & \langle \mathbf{F}_2, \mathbf{R}_2, \mathbf{I}_2, \mathsf{prov}_2, \mathsf{req}_2 \rangle, \end{aligned}$$

for which $\mathbf{F}_2 = \mathbf{R}_1$, is defined as

$$(\mathbf{d}_1 \stackrel{\circ}{,} \mathbf{d}_2) := \langle \mathbf{F}_1, \mathbf{R}_2, \mathbf{I}, \text{prov}, \text{req} \rangle,$$

where:

$$\begin{split} \mathbf{I} = \{ [i_1 \; ; \; i_2] \in (\mathbf{I}_1 \; ; \; \mathbf{I}_2) \; | \; \mathsf{req}_1(i_1) \preceq_{\mathbf{R}_1} \mathsf{prov}_2(i_2) \}, \\ \mathsf{prov} \; : \quad [i_1 \; ; \; i_2] \mapsto \mathsf{prov}_1(i_1), \\ \mathsf{req} \; : \quad [i_1 \; ; \; i_2] \mapsto \mathsf{req}_2(i_2). \end{split}$$