$S_1 \leq_{\mathcal{A}\mathbf{P}} S_2 \equiv \uparrow S_1 \supseteq \uparrow S_2.$