Lemma. $\mathbf{Pos}_{\mathcal{U}}$ is a monoidal category with the following additional structure:

1. *Tensor product* $\otimes$: On objects, the tensor product corresponds to the product of posets. Given two morphisms $f : X \to Y$ and $g : Z \to U$, we have $f \otimes g : X \times Z \to Y \times U$, with

$$\begin{aligned}(f \otimes g)^\star : X \times Z &\to_{\mathbf{Pos}} \mathcal{U}(Y \times U)\\ \langle x, z\rangle &\mapsto f^\star(x) \times g^\star(z).\end{aligned}$$

   Note that the Cartesian product of upper sets is an upper set.

2. *Unit*: The unit is the identity poset: the poset with a singleton carrier set and only the identity relation. We denote this by $\{\bullet\}$.
3. *Left unitor*: The left unitor is given by the pair of morphisms $\mathrm{lu}_X : \{\bullet\} \otimes X \to X$ and $\mathrm{lu}_X{}^{-1} : X \to \{\bullet\} \otimes X$, with

$$\begin{aligned}\mathrm{lu}_X{}^\star : \{\bullet\} \otimes X &\to_{\mathbf{Pos}} \mathcal{U}X\\ \langle \bullet, x\rangle &\mapsto \uparrow\{x\},\end{aligned}$$

   and

$$\begin{aligned}\mathrm{lu}_X{}^{-1\star} : X &\to_{\mathbf{Pos}} \mathcal{U}(\{\bullet\} \otimes X)\\ x &\mapsto \{\bullet\} \times \uparrow\{x\},\end{aligned}$$

   respectively.

4. *Right unitor*: The right unitor is given by the pair of morphisms $\mathrm{ru}_X : X \otimes \{\bullet\} \to X$ and $\mathrm{ru}_X{}^{-1} : X \to X \otimes \{\bullet\}$, with

$$\begin{aligned}\mathrm{ru}_X{}^\star : X \otimes \{\bullet\} &\to_{\mathbf{Pos}} \mathcal{U}X\\ \langle x, \bullet\rangle &\mapsto \uparrow\{x\},\end{aligned}$$

   and

$$\begin{aligned}\mathrm{ru}_X{}^{-1\star} : X &\to_{\mathbf{Pos}} \mathcal{U}(X \otimes \{\bullet\})\\ x &\mapsto \uparrow\{x\} \times \{\bullet\},\end{aligned}$$

   respectively.

5. *Associator*: The associator is given by the pair of morphisms $\mathrm{as}_{XY,Z} : (X \otimes Y) \otimes Z \to X \times (Y \times Z)$ and $\mathrm{as}_{X,YZ} : X \otimes (Y \otimes Z) \to (X \times Y) \times Z$, given by

$$\begin{aligned}\mathrm{as}_{XY,Z}{}^\star : (X \otimes Y) \otimes Z &\to_{\mathbf{Pos}} \mathcal{U}X \times (\mathcal{U}Y \times \mathcal{U}Z)\\ \langle\langle x, y\rangle, z\rangle &\mapsto \uparrow\{x\} \times (\uparrow\{y\} \times \uparrow\{z\}),\end{aligned}$$

   and

$$\begin{aligned}\mathrm{as}_{X,YZ}{}^\star : X \otimes (Y \otimes Z) &\to_{\mathbf{Pos}} (\mathcal{U}X \times \mathcal{U}Y) \times \mathcal{U}Z\\ \langle x, \langle y, z\rangle\rangle &\mapsto (\uparrow\{x\} \times \uparrow\{y\}) \times \uparrow\{z\}.\end{aligned}$$