

Definition (Semigroup left action (preliminary version)). A *semigroup **left** action* of a semigroup **S** onto a set **A** is a map

$$\text{Lact} : \mathbf{S} \times \mathbf{A} \rightarrow \mathbf{A}$$

such that, for all $a \in \mathbf{A}$,

$$\text{Lact}(x, \text{Lact}(y, a)) = \text{Lact}(x \circ_{\mathbf{S}} y, a).$$