

Definition (Monoid isomorphism). A homomorphism of semigroups $f : \mathbf{M} \rightarrow \mathbf{N}$ is called a *monoid isomorphism* if there exists a homomorphism of monoids $g : \mathbf{N} \rightarrow \mathbf{M}$ such that

$$f \circ g = \text{id}_{\mathbf{M}} \quad \text{and} \quad g \circ f = \text{id}_{\mathbf{N}} .$$