$$\operatorname{Tr}_{U \times X, U \times Y}^{Z}(\operatorname{Id}_{U} \times f)(u, x) = \begin{cases} \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} \langle u, y, z \rangle \in (\operatorname{Id}_{U} \times f)(u, x, z) \end{cases}$$

$$= \begin{cases} \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} (u \in \operatorname{Id}_{U}(u)) \wedge (\langle y, z \rangle \in f(x, z)) \end{cases}$$

$$= \begin{cases} \langle u, y \rangle \in U \times Y \mid \bigvee_{z \in Z} (u \in \uparrow \{u\}) \wedge (\langle y, z \rangle \in f(x, z)) \end{cases}$$

$$= \begin{cases} \langle u, y \rangle \in \uparrow \{u\} \times Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f(x, z) \end{cases}$$

$$= \langle \uparrow \{u\}, \begin{cases} y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f(x, z) \end{cases} \rangle$$