Definition (Strong monoidal functor). Let $\langle \mathbf{C}, \boldsymbol{\otimes}_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}} \rangle$ and $\langle \mathbf{D}, \boldsymbol{\otimes}_{\mathbf{D}}, \mathbf{1}_{\mathbf{D}} \rangle$ be two monoidal categories. A *strong monoidal functor* between \mathbf{C} and \mathbf{D} is given by:

1. A functor

$$F: \mathbf{C} \to \mathbf{D};$$

2. An isomorphism

iso:
$$\mathbf{1}_{\mathbf{D}} \to F(\mathbf{1}_{\mathbf{C}});$$

3. A natural isomorphism μ

$$\mu_{X,Y}: F(X) \otimes_{\mathbf{D}} F(Y) \to F(X \otimes_{\mathbf{C}} Y), \quad \forall X, Y \in \mathbf{C},$$

satisfying the following conditions:

- a) Associativity: For all objects $X, Y, Z \in \mathbb{C}$, there are associators as and as such that the diagram in ?? commutes.
- b) Unitality: For all $X \in \mathbb{C}$, there exist left and right unitors $lu^{\mathbb{C}}$ and $ru^{\mathbb{C}}$, the diagram in ?? commutes.