Lemma. Let $\langle \mathbf{S}, \fatsemi \rangle$ be a semigroup. If there exists elements $1 \in \mathbf{S}$ and $1' \in \mathbf{S}$ such that $\langle \mathbf{S}, \fatsemi, 1 \rangle$ and $\langle \mathbf{S}, \fatsemi, 1' \rangle$ are each monoids, then $1 = 1'$ must hold. In other words, the neutral element of a monoid is uniquely determined by the underlying semigroup structure.