## **Definition** (Monad)

Let **C** be a category. A monad on **C** is specified by:

## Constituents

- 1. A functor  $M : \mathbb{C} \to \mathbb{C}$ ;
- 2. A natural transformation mu :  $M \, \, \, \, \, \, M \Rightarrow M$ , called the *composition* or *multiplication*;
- 3. A natural transformation un :  $Id_{\mathbb{C}} \Rightarrow M$ , called the *unit*.

## Conditions

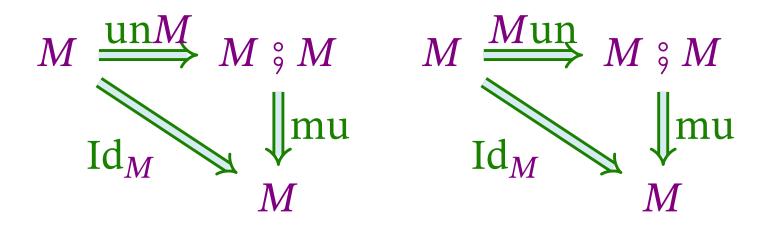
1. Associativity: the diagram

$$M \stackrel{\circ}{\circ} M \stackrel{\circ}{\circ} M \xrightarrow{M mu} M \stackrel{\circ}{\circ} M$$

$$\downarrow muM \qquad \qquad \downarrow mu$$
 $M \stackrel{\circ}{\circ} M \xrightarrow{mu} M$ 

must commute.

2. Left and right unitality: the diagrams



must commute.