

**Definition** (The  $\mathcal{U}$  monad). The  $\mathcal{U}$  monad on **DP** consists of:

1. The functor  $\text{Unc} : \mathbf{DP} \rightarrow \mathbf{DP}$ ;
2. The natural transformation  $\text{un}_{\mathcal{U}} : \text{Id}_{\mathbf{DP}} \Rightarrow \text{Unc}$ , specified as

$$\text{un}_{\mathcal{U}}^{\mathbf{A}} : \mathbf{A} \dashrightarrow \mathbf{Int}(\mathbf{A})$$

$$\langle a^*, [x, y] \rangle \mapsto a \leq x.$$

3. The natural transformation  $\text{mu}_{\mathcal{U}} : \text{UncUnc} \Rightarrow \text{Unc}$ , specified as:

$$\text{mu}_{\mathcal{U}}^{\mathbf{A}} : \mathbf{Int}(\mathbf{Int}(\mathbf{A}))^{\text{op}} \times \mathbf{Int}(\mathbf{A}) \rightarrow_{\mathbf{Pos}} \mathbf{Bool}$$

$$\langle [[a, b], [c, d]]^*, [e, f] \rangle \mapsto (a \leq e) \wedge (b \leq e) \wedge (c \leq f) \wedge (d \leq f).$$