Lemma. $\mathbf{Pos}_{\mathcal{U}}$ is a monoidal category with the following additional structure:

1. *Tensor product* $⨾$: On objects, the tensor product corresponds to the product of posets. Given two morphisms $f : X \to \mathcal{U}Y$ and $g : Z \to \mathcal{U}U$, we have:

$$\begin{aligned} f ⨾ g : X \times Z &\to \mathcal{U}(Y \times U) \\ \langle x, z \rangle &\mapsto f(x) \times g(z). \end{aligned}$$

   Note that the Cartesian product of upper sets is an upper set.
2. *Unit*: The unit is the identity poset.
3. *Left unitor*: The left unitor is given by the pair of morphisms

$$\begin{aligned} \mathrm{lu}_{\mathbf{P}} : \{\bullet\} ⨾ X &\to \mathcal{U}X \\ \langle \bullet, x \rangle &\mapsto \uparrow \{x\}, \end{aligned}$$

   and

$$\begin{aligned} {\mathrm{lu}_{\mathbf{P}}}^{-1} : X &\to \mathcal{U}(\{\bullet\} ⨾ X) \\ x &\mapsto \{\bullet\} \times \uparrow \{x\}. \end{aligned}$$

4. *Right unitor*: The right unitor is given by the pair of morphisms

$$\begin{aligned} \mathrm{ru}_{\mathbf{P}} : X ⨾ \{\bullet\} &\to \mathcal{U}X \\ \langle x, \bullet \rangle &\mapsto \uparrow \{x\}, \end{aligned}$$

   and

$$\begin{aligned} {\mathrm{ru}_{\mathbf{P}}}^{-1} : X &\to \mathcal{U}(X ⨾ \{\bullet\}) \\ x &\mapsto \uparrow \{x\} \times \{\bullet\}. \end{aligned}$$

5. *Associator*: The associator is given by the pair of morphisms:

$$\begin{aligned} \mathrm{as}_{XY,Z} : (X ⨾ Y) ⨾ Z &\to \mathcal{U}X \times (\mathcal{U}Y \times \mathcal{U}Z) \\ \langle \langle x, y \rangle, z \rangle &\mapsto \uparrow \{x\} \times (\uparrow \{y\} \times \uparrow \{z\}), \end{aligned}$$

   and

$$\begin{aligned} \mathrm{as}_{X,YZ} : X ⨾ (Y ⨾ Z) &\to (\mathcal{U}X \times \mathcal{U}Y) \times \mathcal{U}Z \\ \langle x, \langle y, z \rangle \rangle &\mapsto (\uparrow \{x\} \times \uparrow \{y\}) \times \uparrow \{z\}. \end{aligned}$$