

**Definition** (Category **LPos**). The category **LPos** consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects  $X, Y \in \mathbf{Ob}_{\mathbf{LPos}}$ , morphisms from  $X$  to  $Y$  are monotone maps of the form  $f : X \rightarrow \mathcal{L}Y$ .
3. *Composition of morphisms*: Given morphisms  $f : X \rightarrow \mathcal{L}Y$ ,  $g : Y \rightarrow \mathcal{L}Z$ , their composition is given by

$$f \circ g : X \rightarrow \mathcal{L}Z$$
$$x \mapsto \bigcup_{y \in f(x)} g(y);$$

4. *Identity morphism*: given an object  $X \in \mathbf{Ob}_{\mathbf{LPos}}$ , the identity morphism is given by the application of the lower closure operator:  $\text{Id}_X(x) := \downarrow \{x\}$ .