

Lemma. Given posets P, Q and a monotone map $f : P \rightarrow Q$, we have:

$$\uparrow \left(\bigcup_{p' \in \uparrow \{p\}} \{f(p')\} \right) = \uparrow \{f(p)\}.$$