Definition (Semigroup). A *semigroup* $\mathbf{S}$ is a set $\mathbf{S}$, together with a binary operation

$$\fatsemi : \mathbf{S} \times \mathbf{S} \to \mathbf{S},$$

called *composition*, which satisfies the *associative* law:

$$(x \fatsemi y) \fatsemi z = x \fatsemi (y \fatsemi z)$$

for all $x, y, z \in \mathbf{S}$.