Definition (The $\mathcal{U}$ monad)**.** The $\mathcal{U}$ monad on **DP** consists of:

1. The functor $\mathsf{Unc} : \mathbf{DP} \to \mathbf{DP}$;
2. The natural transformation $\mathrm{un}_{\mathcal{U}} : \mathrm{Id}_{\mathbf{DP}} \Rightarrow \mathsf{Unc}$, specified as

$$\mathrm{un}_{\mathcal{U}}^{\mathbf{A}} : \mathbf{A} \nrightarrow \mathbf{Int}(\mathbf{A})$$
$$\langle a^*, [x, y] \rangle \mapsto a \leq x.$$

3. The natural transformation $\mathrm{mu}_{\mathcal{U}} : \mathsf{UncUnc} \Rightarrow \mathsf{Unc}$, specified as:

$$\mathrm{mu}_{\mathcal{U}}^{\mathbf{A}} : \mathbf{Int}(\mathbf{Int}(\mathbf{A}))^{\mathrm{op}} \times \mathbf{Int}(\mathbf{A}) \to_{\mathbf{Pos}} \mathbf{Bool}$$
$$\left\langle [[a, b], [c, d]]^*, [e, f] \right\rangle \mapsto (a \leq e) \wedge (b \leq e) \wedge (c \leq f) \wedge (d \leq f).$$