**Definition** (Monoid homomorphism). Let  $\langle \mathbf{M}, \S_{\mathbf{M}}, 1_{\mathbf{M}} \rangle$  and  $\langle \mathbf{N}, \S_{\mathbf{N}}, 1_{\mathbf{N}} \rangle$  be monoids.

A homomorphism of monoids from M to N is a function  $f: M \to N$  such that

$$f(m_1 \, \beta_{\mathbf{M}} \, n_2) = f(m_1) \, \beta_{\mathbf{N}} \, f(n_2) \quad \forall m_1, n_2 \in \mathbf{M}$$

and

$$f(1_{\mathbf{M}}) = 1_{\mathbf{N}}$$