**Definition** (Union of design problems). Given two design problems  $\mathbf{f}: \mathbf{A} \to \mathbf{B}$  and  $\mathbf{g}: \mathbf{A} \to \mathbf{B}$ , their *union*  $\mathbf{f} \vee \mathbf{g}: \mathbf{A} \to \mathbf{B}$  is defined by

$$(\mathbf{f} \vee \mathbf{g}) : \mathbf{A}^{\mathrm{op}} \times \mathbf{B} \rightarrow_{\mathbf{Pos}} \mathbf{Bool}$$

 $\langle a^*,b\rangle\mapsto \mathbf{f}(a^*,b)\vee\mathbf{g}(a^*,b).$