

Definition (Functor). Given categories \mathbf{C} and \mathbf{D} , a *functor* $F : \mathbf{C} \rightarrow \mathbf{D}$ from \mathbf{C} to \mathbf{D} is defined by the following data, satisfying the following conditions.

Data:

- i) For every object $X \in \mathbf{Ob}_{\mathbf{C}}$, an object $F(X) \in \mathbf{Ob}_{\mathbf{D}}$;
- ii) For every morphism $f : X \rightarrow Y$ in \mathbf{C} , a morphism $F(f) : F(X) \rightarrow F(Y)$ in \mathbf{D} .

Conditions:

- i) For every object $X \in \mathbf{Ob}_{\mathbf{C}}$, one has $F(\text{Id}_X) = \text{Id}_{F(X)}$;
- ii) For every three objects $X, Y, Z \in \mathbf{Ob}_{\mathbf{C}}$ and two morphisms $f \in \text{Hom}_{\mathbf{C}}(X; Y)$, $g \in \text{Hom}_{\mathbf{C}}(Y; Z)$, the equation

$$F(f \circ g) = F(f) \circ F(g)$$

holds in \mathbf{D} .

This situation is graphically reported in ??.