

Definition (Power set as lattice)

Given a set **S**, its power set $\mathcal{P} \mathbf{S}$ (the set of all subsets) is a lattice where, given **A**, **B** $\in \mathcal{P} \mathbf{S}$:

▷ Order is given by inclusion:

$$\mathbf{A} \leq \mathbf{B} := \mathbf{A} \subseteq \mathbf{B};$$

▷ The join is given by the union of sets:

$$\mathbf{A} \vee \mathbf{B} := \mathbf{A} \cup \mathbf{B};$$

▷ The meet is given by the intersection of sets:

$$\mathbf{A} \wedge \mathbf{B} := \mathbf{A} \cap \mathbf{B};$$

▷ The top element is the set **S** itself:

$$\top = \mathbf{S};$$

▷ The bottom element is the empty set:

$$\perp = \emptyset.$$