

Definition (Free category on a graph)

Let $\mathcal{G} = \langle \mathbf{V}, \mathbf{A}, \text{src}, \text{tgt} \rangle$ be a graph. The *free category on \mathcal{G}* , denoted $\mathbf{Free}(\mathcal{G})$, has as objects the vertices \mathbf{V} of \mathcal{G} , and given vertices $u \in \mathbf{V}$ and $w \in \mathbf{V}$, the morphisms $\mathbf{Hom}_{\mathbf{Free}(\mathcal{G})}(u; w)$ are the paths from u to w . The composition of morphisms is given by concatenation of paths, and for any object $u \in \mathbf{V}$, the associated identity morphism id_u is the trivial path which starts and ends at u .