Lemma. Let $\mathcal{A}\mathbf{P}$ be the set of antichains of $\mathbf{P}$. $\mathcal{A}\mathbf{P}$ is a poset itself, with the partial order $\preceq_{\mathcal{A}\mathbf{P}}$ defined as

$$S_1 \preceq_{\mathcal{A}\mathbf{P}} S_2 \equiv \uparrow S_1 \supseteq \uparrow S_2.$$