Definition (Semigroup isomorphism). A morphism of semigroups $F: \mathbf{S} \to \mathbf{T}$ is called a *semigroup isomorphism* if there exists a morphism of semigroups $G: \mathbf{T} \to \mathbf{S}$ such that

$$F \mathbin{\overset{\circ}{_\circ}} G = \mathrm{Id}_{\mathbf{S}} \quad \text{and} \quad G \mathbin{\overset{\circ}{_\circ}} F = \mathrm{Id}_{\mathbf{T}}.$$