

Definition (Functions as relations). Let \mathbf{A} and \mathbf{B} be sets. A relation $\mathbf{R} \subseteq \mathbf{A} \times \mathbf{B}$ is a *function* if it satisfies the following two conditions:

1. $\forall x \in \mathbf{A} \quad \exists y \in \mathbf{B} : \langle x, y \rangle \in \mathbf{R}$
2. $\forall \langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle \in \mathbf{R} \text{ holds} : x_1 = x_2 \Rightarrow y_1 = y_2.$