Definition (Category $\mathbf{Pos}_{\mathscr{L}}$). The category $\mathbf{Pos}_{\mathscr{L}}$ consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_{\mathscr{L}}}$, morphisms $f : X \to Y$ are monotone maps of the form $f^\star : X \to_{\mathbf{Pos}} \mathscr{L}Y$.
3. *Composition of morphisms*: Given morphisms $f : X \to Y, g : Y \to Z$, their composition $f \fatsemi g : X \to Z$ is given by

$$(f \fatsemi g)^\star : X \to_{\mathbf{Pos}} \mathscr{L}Z$$
$$x \mapsto \bigcup_{y \in f^\star(x)} g^\star(y);$$

4. *Identity morphism*: given an object $X \in \mathrm{Ob}_{\mathbf{Pos}_{\mathscr{L}}}$, the identity morphism $\mathrm{Id}_X : X \to X$ is given by the application of the lower closure operator: $\mathrm{Id}_X{}^\star(x) := {\downarrow}\{x\}$.