Definition (Semigroup isomorphism)**.** Let $\mathbf{S}$ and $\mathbf{T}$ be semigroups. A homomorphism of semigroups $F : \mathbf{S} \to \mathbf{T}$ is called a *semigroup isomorphism* if there exists a homomorphism of semigroups $G : \mathbf{T} \to \mathbf{S}$ such that

$$F \fatsemi G = \mathrm{Id}_{\mathbf{S}} \text{ and } G \fatsemi F = \mathrm{Id}_{\mathbf{T}}.$$