

Definition (U endofunctor)

The U *endofunctor* has the form $U : \mathbf{Pos} \rightarrow \mathbf{Pos}$ and acts on objects and morphisms as follows:

- 1. *On objects*: Given a poset $\mathbf{P} \in \mathbf{Ob}_{\mathbf{Pos}}$, U maps \mathbf{P} to its upper set. {bhn:1}
- 2. *On morphisms*: Given posets \mathbf{P}, \mathbf{Q} , and a monotone map $f : \mathbf{P} \rightarrow \mathbf{Q}$, the U endofunctor acts as:

$$U(f) : \mathcal{U}\mathbf{P} \rightarrow \mathcal{U}\mathbf{Q}$$
$$\mathbf{P}' \mapsto \uparrow \left(\bigcup_{p \in \mathbf{P}'} \{f(p)\} \right).$$

Recall that in ?? we proved that the upper set is itself an object of \mathbf{Pos} .