Definition (Berg)

Let **Berg** be the category defined as follows:

- *Objects*: Objects are tuples $\langle p, v \rangle$, where
  - $p \in L$,
  - $v \in \mathbb{R}^3$ (we think of this as a tangent vector to L at p).
- *Morphisms*: A morphism $\langle p_1, v_1 \rangle \to \langle p_2, v_2 \rangle$ is $\langle \gamma, T \rangle$, where
  - $T \in \mathbb{R}_{\geq 0}$,
  - $\gamma : [0, T] \to L$ is a C^1 function with $\gamma(0) = p_1$ and $\gamma(T) = p_2$, as well as $\dot{\gamma}(0) = v_1$ and $\dot{\gamma}(T) = v_2$ (we take one-sided derivatives at the boundaries).
- *Identity morphism*: For any object $\langle p, v \rangle$, we define its identity morphism

$$\mathrm{Id}_{\langle p, v \rangle} = \langle \gamma, 0 \rangle$$

  formally: its path γ is defined on the closed interval $[0, 0]$, (with $T = 0$ and $\gamma(0) = p$). We declare this path to be C^1 by convention, and declare its derivative at 0 to be v.
- *Composition of morphisms*: Given morphisms

$$\langle \gamma_1, T_1 \rangle : \langle p_1, v_1 \rangle \to \langle p_2, v_2 \rangle$$

  and

$$\langle \gamma_2, T_2 \rangle : \langle p_2, v_2 \rangle \to \langle p_3, v_3 \rangle,$$

  their composition is $\langle \gamma, T \rangle$ with $T = T_1 + T_2$ and

$$\gamma(t) = \begin{cases} \gamma_1(t) & 0 \leq t \leq T_1 \\ \gamma_2(t - T_1) & T_1 \leq t \leq T_1 + T_2. \end{cases}$$