Definition (Profunctor composition). Given two profunctors $F : \mathbf{C} \nrightarrow \mathbf{D}$ and $G : \mathbf{D} \nrightarrow \mathbf{E}$ we can define their composition $(F \mathbin{⨟} G) : \mathbf{C} \nrightarrow \mathbf{E}$ as follows:

$$\begin{aligned}(F \mathbin{⨟} G)_{\mathrm{ob}} :\ & \mathrm{Ob}(\mathbf{C}^{\mathrm{op}} \times \mathbf{E}) \to \mathrm{Ob}\,\mathbf{Set}, \\ & \langle C^*, E \rangle \mapsto \textstyle\coprod_{D \in \mathrm{Ob}\mathbf{D}} F_{\mathrm{ob}}(C^*, D) \times G_{\mathrm{ob}}(D^*, E) / \sim\end{aligned}$$

$$\begin{aligned}(F \mathbin{⨟} G)_{\mathrm{mor}} :\ & \mathrm{Hom}_{(\mathbf{C}^{\mathrm{op}} \times \mathbf{E})}(\langle C_1^*, E_1 \rangle; \langle C_2^*, E_2 \rangle) \to \mathrm{Hom}_{\mathbf{Set}}((F \mathbin{⨟} G)_{\mathrm{ob}}(C_1^*, E_1); (F \mathbin{⨟} G)_{\mathrm{ob}}(C_2^*, E_2)) \\ & \langle \alpha^*, \beta \rangle \mapsto \begin{cases} (F \mathbin{⨟} G)_{\mathrm{ob}}(C_1^*, E_1) \to (F \mathbin{⨟} G)_{\mathrm{ob}}(C_2^*, E_2)) \\ \qquad \langle s, t \rangle \mapsto \langle F_{\mathrm{mor}}(\langle \alpha, \mathrm{Id}_D \rangle)(s), G_{\mathrm{mor}}(\langle \mathrm{Id}_{D^*}, \beta \rangle)(t) \rangle \end{cases}\end{aligned}$$

In the formulas:

$$\alpha : C_2 \to C_1, \qquad \beta : E_1 \to E_2,$$

and $\langle s, t \rangle$ is a pair of elements for which there exists a $D \in \mathrm{Ob}\mathbf{D}$ such that

$$s \in F_{\mathrm{ob}}(C_1^*, D), \qquad t \in G_{\mathrm{ob}}(D^*, E).$$