

**Definition** (Category **Rel**). The category of relations **Rel** is given by:

1. *Objects*: The objects of this category are all sets.
2. *Morphisms*: Given sets  $X, Y$ , the homset  $\text{Hom}_{\mathbf{Rel}}(X; Y)$  consists of all relations  $R \subseteq X \times Y$ .
3. *Identity morphisms*: Given a set  $X$ , its identity morphism is

$$\text{Id}_X := \{\langle x, x \rangle \mid x \in X\}.$$

4. *Composition*: Given relations  $R : X \rightarrow Y, S : Y \rightarrow Z$ , their composition is given by

$$R \circ S := \{\langle x, z \rangle \mid \exists y \in Y : (\langle x, y \rangle \in R) \wedge (\langle y, z \rangle \in S)\}.$$