

# Definition (Monad)

Let  $\mathbf{C}$  be a category. A *monad* on  $\mathbf{C}$  is specified by:

## Constituents

1. A functor  $M : \mathbf{C} \rightarrow \mathbf{C}$ ;
2. A natural transformation  $\mu : M \circ M \Rightarrow M$ , called the *composition* or *multiplication*;
3. A natural transformation  $\eta : \text{Id}_{\mathbf{C}} \Rightarrow M$ , called the *unit*.

## Conditions

1. *Associativity*: the diagram

$$\begin{array}{ccc} M \circ M \circ M & \xrightarrow{M\mu} & M \circ M \\ \downarrow \mu M & & \downarrow \mu \\ M \circ M & \xrightarrow{\mu} & M \end{array}$$

must commute.

2. *Left and right unitality*: the diagrams

$$\begin{array}{ccc} M & \xrightarrow{\eta M} & M \circ M \\ \searrow \text{Id}_M & & \downarrow \mu \\ & & M \end{array} \qquad \begin{array}{ccc} M & \xrightarrow{M\eta} & M \circ M \\ \searrow \text{Id}_M & & \downarrow \mu \\ & & M \end{array}$$

must commute.