Definition (Semicategory). A *semicategory* **C** is:

<u>Constituents</u>

1. A collection $\mathrm{Ob}_{\mathbf{C}}$ whose elements are called *objects*.
2. For every pair of objects X, Y in $\mathrm{Ob}_{\mathbf{C}}$, there is a set $\mathrm{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms*. We write

$$f : X \to_{\mathbf{C}} Y$$

to indicate

$$f \in \mathrm{Hom}_{\mathbf{C}}(X; Y).$$

3. For every three objects X, Y, Z in $\mathrm{Ob}_{\mathbf{C}}$ there is a composition map

$$\mathbin{⨟}_{X,Y,Z} : \mathrm{Hom}_{\mathbf{C}}(X; Y) \times \mathrm{Hom}_{\mathbf{C}}(Y; Z) \to \mathrm{Hom}_{\mathbf{C}}(X; Z)$$

Given any morphism $f \in \mathrm{Hom}_{\mathbf{C}}(X; Y)$ and any morphism $g \in \mathrm{Hom}_{\mathbf{C}}(Y; Z)$, there exists a morphism $f \mathbin{⨟} g \in \mathrm{Hom}_{\mathbf{C}}(X; Z)$ which is the *composition of* f *and* g.

<u>Conditions</u>

1. Associativity: it holds that

$$\frac{f : X \to Y \quad g : Y \to Z \quad h : Z \to U}{(f \mathbin{⨟} g) \mathbin{⨟} h = f \mathbin{⨟} (g \mathbin{⨟} h)}$$