Lemma. The metrics P_{track} and P_{effort} can be written as

$$\lim_{t\to\infty} \mathbb{E}\{\mathbf{x}_t^\intercal \mathbf{Q}_0 \mathbf{x}_t\} = \operatorname{Tr}(\mathbf{Q}_0 (\boldsymbol{\Sigma} + \mathbf{F})),$$

$$\lim_{t\to\infty} \mathbb{E}\{\mathbf{u}_t^\intercal \mathbf{R}_0 \mathbf{u}_t\} = \operatorname{Tr}(\mathbf{S}\mathbf{B}^* \mathbf{R}^{-1} \mathbf{R}_0 \mathbf{R}^{-1} \mathbf{B}\mathbf{S}\mathbf{F}),$$

where $\boldsymbol{\Sigma}$ solves the Riccati equation for estimation, $\mathbf{F}$ solves the Lyapunov equation

$$(\mathbf{A} - \mathbf{B}\mathbf{K})\,\mathbf{F} + \mathbf{F}\,(\mathbf{A} - \mathbf{B}\mathbf{K})^* + \mathbf{L}\mathbf{V}\mathbf{L}^* = \mathbf{0},$$

$\mathbf{S}$ solves the Riccati equation for control, and $\mathbf{L} = \boldsymbol{\Sigma}\mathbf{C}^*\mathbf{V}^{-1}$ is the Kalman gain.