

**Definition** (Algebra of a monad). Let  $\langle M, \mu, \text{un} \rangle$  be a monad on a category  $\mathbf{C}$ . An algebra of  $M$  (also called an  $M$ -algebra) is specified by:

### Constituents

1. an object  $X$  of  $\mathbf{C}$ ;
2. a morphism  $a : M(X) \rightarrow X$  of  $\mathbf{C}$ .

### Conditions

1. *Composition*: the diagram

$$\begin{array}{ccc}
 (M \circ M)(X) & \xrightarrow{Ma} & M(X) \\
 \mu_X \downarrow & & \downarrow a \\
 M(X) & \xrightarrow{a} & X
 \end{array}$$

commutes.

2. *Unit*: the diagram

$$\begin{array}{ccc}
 X & \xrightarrow{\text{un}_X} & M(X) \\
 & \searrow \text{Id} & \downarrow a \\
 & & X
 \end{array}$$

commutes.