## **Definition** (Monad). Let **C** be a category. A *monad* on **C** is specified by:

## Constituents

- 1. A functor  $M: \mathbb{C} \to \mathbb{C}$ ;
- 2. A natural transformation mu:  $M ; M \Rightarrow M$ , called the *composition* or *multiplication*;
- 3. A natural transformation un :  $Id_{\mathbb{C}} \Rightarrow M$ , called the *unit*.

## **Conditions**

1. Associativity: the diagram

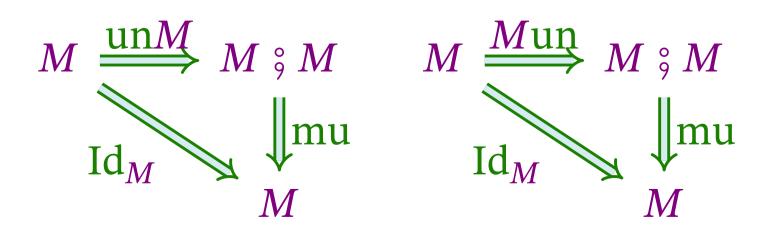
$$M \stackrel{\circ}{\circ} M \stackrel{\circ}{\circ} M \xrightarrow{M} M \stackrel{\circ}{\circ} M$$

$$\downarrow muM \qquad \qquad \downarrow mu$$

$$M \stackrel{\circ}{\circ} M \xrightarrow{mu} M$$

must commute.

2. Left and right unitality: the diagrams



must commute.