

Lemma. Given $\mathbf{f}_1, \mathbf{f}_2 \in \text{Hom}_{\mathbf{DP}}(\mathbf{A}; \mathbf{B})$ and $\mathbf{g}_1, \mathbf{g}_2 \in \text{Hom}_{\mathbf{DP}}(\mathbf{B}; \mathbf{C})$ one has:

$$\frac{\mathbf{f}_1 \leq_{\mathbf{DP}} \mathbf{f}_2 \quad \mathbf{g}_1 \leq_{\mathbf{DP}} \mathbf{g}_2}{\mathbf{f}_1 \circ \mathbf{g}_1 \leq_{\mathbf{DP}} \mathbf{f}_2 \circ \mathbf{g}_2}$$

In other words, series composition is order-preserving on \mathbf{DP} .