Lemma. Given a poset **P** and a family of upper sets of **P**, denoted $\{P_i\}_{i\in I}$ (with index set I), their union is also an upper set.