

**Definition** (Order on morphisms in  $\mathbf{Pos}_{\mathcal{L}}$ ). Given any two morphisms  $f, g : X \rightarrow Y$  in  $\mathbf{Pos}_{\mathcal{L}}$ , we define an order between them as

$$\frac{f \leq_{\mathbf{Pos}_{\mathcal{L}}} g}{f^{\star}(x) \leq_{\mathcal{L}Y} g^{\star}(x), \quad \forall x \in X.}$$