Definition (Enriched functor). Given two categories $\mathbf{C}$ and $\mathbf{D}$ enriched in the same monoidal category $\mathbf{V}$, an enriched functor $F : \mathbf{C} \to \mathbf{D}$ consists of:

1. A map $F : \mathrm{Ob}_{\mathbf{C}} \to \mathrm{Ob}_{\mathbf{D}}$ that maps objects of $\mathbf{C}$ to objects of $\mathbf{D}$.
2. For each X, Y in $\mathrm{Ob}_{\mathbf{C}}$, there exists a morphism in $\mathbf{V}$ given by

$$F_{X,Y} : \mathrm{Hom}_{\mathbf{C}}(X;Y) \to \mathrm{Hom}_{\mathbf{D}}(F(X);F(Y)),$$

such that composing maps "across F" respects the composition in $\mathbf{C}$ and the unit in $\mathbf{V}$ in the obvious ways:

$$
\begin{array}{ccc}
\mathrm{Hom}_{\mathbf{C}}(X;Y) \otimes \mathrm{Hom}_{\mathbf{C}}(Y;Z) & \xrightarrow{\ m_{X,Y,Z}\ } & \mathrm{Hom}_{\mathbf{C}}(X;Z) \\
\downarrow{\scriptstyle F_{X,Y} \otimes F_{Y,Z}} & & \downarrow{\scriptstyle F_{X,Z}} \\
\mathrm{Hom}_{\mathbf{D}}(F(Y);F(Z)) \otimes \mathrm{Hom}_{\mathbf{D}}(F(X);F(Y)) & \xrightarrow[m_{F(X),F(Y),F(Z)}]{} & \mathrm{Hom}_{\mathbf{D}}(F(X);F(Z))
\end{array}
$$

and

$$
\begin{array}{ccc}
\mathbf{1} & & \\
\downarrow{\scriptstyle j_X} & \searrow^{j_{F(X)}} & \\
\mathrm{Hom}_{\mathbf{C}}(X;X) & \xrightarrow[F_{X,X}]{} & \mathrm{Hom}_{\mathbf{D}}(F(X);F(X))
\end{array}
$$

where $\otimes$ and $\mathbf{1}$ are the monoidal product and monoidal unit in $\mathbf{V}$.