Definition (Trace of a generalized endomorphism). Let $\langle \mathbf{C}, \otimes, \mathbf{1}, \mathrm{br} \rangle$ be a symmetric monoidal category. Let $X \in \mathrm{Ob}_{\mathbf{C}}$ be dualizable and let $f \in \mathrm{Hom}_{\mathbf{C}}(Y \otimes X; Z \otimes X)$. The *trace over* X of f is the morphism $\mathrm{Tr}^{X}_{Y,Z}(f) \in \mathrm{Hom}(Y, Z)$ defined by

$$Y \xrightarrow{\mathrm{id}_Y \otimes \mathrm{coev}_X} Y \otimes X \otimes X^\vee \xrightarrow{f \otimes \mathrm{id}_{X^\vee}} Z \otimes X \otimes X^\vee \xrightarrow{\mathrm{id}_Z \otimes \mathrm{br}} Z \otimes X^\vee \otimes X \xrightarrow{\mathrm{id}_Z \otimes \mathrm{ev}_X} Z$$