Definition (Operad)

An *operad* is defined by:

Constituents

1. *Objects:* A collection $\mathrm{Ob}_{\mathcal{O}}$;
2. *Morphisms:* Let $n \in \mathbb{N}$. For each finite string $[X_1, \dots, X_n]$ of objects and each object Y, one specifies a set $\mathrm{Hom}_{\mathcal{O}}([X_1, \dots, X_n]; Y)$, elements of which are morphisms $[X_1, \dots, X_n] \to Y$;
3. *Identity morphisms:* For each object X, a morphism $\mathrm{Id}_X \in \mathrm{Hom}_{\mathcal{O}}([X_1, \dots, X_n]; Y)$;
4. *Composition operations:*

$$\mathrm{Hom}_{\mathcal{O}}([X^1_1, \dots, X^1_{n_1}]; Y_1) \times \dots \times \mathrm{Hom}_{\mathcal{O}}\left([X^m_1, \dots, X^m_{n_m}]; Y_m\right) \times \mathrm{Hom}_{\mathcal{O}}([Y_1, \dots, Y_m]; Z) \to \mathrm{Hom}_{\mathcal{O}}\left([X^1_1, \dots, X^m_{n_m}]; Z\right)$$
$$\langle f_1, \dots, f_m, g \rangle \mapsto [f_1, \dots, f_m] \fatsemi g.$$

Conditions

1. *Associativity:*

$$[[f^1_1, \dots, f^1_{n_1}] \fatsemi g_1, [f^2_1, \dots, f^2_{n_2}] \fatsemi g_2, \dots, [f^m_1, \dots, f^m_{n_m}] \fatsemi g_m] \fatsemi h = [f^1_1, \dots, f^m_{n_m}] \fatsemi ([g_1, \dots, g_m] \fatsemi h).$$

2. *Unitality*:

$$[\mathrm{Id}_{X_1}, \dots, \mathrm{Id}_{X_n}] \fatsemi f = f = f \fatsemi \mathrm{Id}_Y, \quad \forall f : [X_1, \dots, X_n] \to Y.$$