

**Definition** (Cartesian product of categories). Given two categories  $\mathbf{C}$  and  $\mathbf{D}$ , their *cartesian product*  $\mathbf{C} \times \mathbf{D}$  is the category specified as follows:

1. *Objects*: Objects are pairs  $\langle X, Y \rangle$ , with  $X \in \mathbf{Ob}_{\mathbf{C}}$  and  $Y \in \mathbf{Ob}_{\mathbf{D}}$ .
2. *Morphisms*: Morphisms are pairs of morphisms  $\langle f, g \rangle : \langle X, Z \rangle \rightarrow \langle Y, U \rangle$ , with  $f : X \rightarrow Y$ ,  $g : Z \rightarrow U$ .
3. *Identity morphisms*: Given objects  $X \in \mathbf{Ob}_{\mathbf{C}}$  and  $Y \in \mathbf{Ob}_{\mathbf{D}}$ , the identity morphism on  $\langle X, Y \rangle$  is the pair  $\langle \text{Id}_X, \text{Id}_Y \rangle$ .
4. *Composition of morphisms*: The composition of morphisms is given by composing each component of the pair separately:

$$\langle f, g \rangle \circ_{\mathbf{C} \times \mathbf{D}} \langle h, i \rangle = \langle f \circ_{\mathbf{C}} h, g \circ_{\mathbf{D}} i \rangle.$$