Definition (Lower closure operator). The *lower closure operator* ↓ maps a subset to the smallest lower set that includes it:

$$\downarrow\; :\; \mathscr{P}\mathbf{P} \to \mathscr{L}\mathbf{P}$$

$$S \mapsto \{y \in \mathbf{P} \mid \exists x \in S : y \leq_{\mathbf{P}} x\}.$$