Definition (Category **Rel**). The category of relations **Rel** is given by:

1. *Objects*: The objects of this category are all sets.
2. *Morphisms*: Given sets X, Y, the homset $\mathrm{Hom}_{\mathbf{Rel}}(X; Y)$ consists of all relations $R \subseteq X \times Y$.
3. *Identity morphisms*: Given a set X, its identity morphism is

$$\mathrm{Id}_X := \{\langle x, x' \rangle \in X \times X \mid x = x'\}.$$

4. *Composition*: Given relations $R : X \to Y$, $S : Y \to Z$, their composition is given by

$$R \fatsemi S := \{\langle x, z \rangle \in X \times Z \mid \exists y \in Y : \ (\langle x, y \rangle \in R) \wedge (\langle y, z \rangle \in S)\}.$$