Definition (Lower closure operator). The *lower closure operator* ↓ maps a subset to the smallest lower set that includes it:

 $S \mapsto \{ y \in \mathbf{P} \mid \exists x \in S : y \leq_{\mathbf{P}} x \}.$

 $\downarrow : \mathscr{P}P \to \mathscr{L}P$