Definition (Product of posets). Given two posets $\langle \mathbf{P}, \leq_{\mathbf{P}} \rangle$ and $\langle \mathbf{Q}, \leq_{\mathbf{Q}} \rangle$, the *product poset* is $\langle \mathbf{P} \times \mathbf{Q}, \leq_{\mathbf{P} \times \mathbf{Q}} \rangle$, where $\mathbf{P} \times \mathbf{Q}$ is the Cartesian product of two sets (**??**) and the order $\leq_{\mathbf{P} \times \mathbf{Q}}$ is given by:

$$\langle p_1, q_1 \rangle \leq_{\mathbf{P} \times \mathbf{Q}} \langle p_2, q_2 \rangle \quad \Leftrightarrow \quad (p_1 \leq_{\mathbf{P}} p_2) \wedge (q_1 \leq_{\mathbf{Q}} q_2).$$