

Lemma. Given $\mathbf{d}, (\mathbf{d} \circ \mathbf{e}) \in \text{Hom}_{\mathbf{DP}} (\mathbf{P}; \mathbf{Q})$ and $\mathbf{g}, \mathbf{h} \in \text{Hom}_{\mathbf{DP}} (\mathbf{Q}; \mathbf{R})$ one has:

$$\frac{\mathbf{d} \leq_{\mathbf{DP}} \mathbf{e} \quad \mathbf{g} \leq_{\mathbf{DP}} \mathbf{h}}{\mathbf{d} \circ \mathbf{g} \leq_{\mathbf{DP}} \mathbf{e} \circ \mathbf{h}} .$$

In other words, series composition is order-preserving on \mathbf{DP} .