**Lemma.** $\mathcal{A}\mathbf{P}$ is a poset with the relation $\preceq_{\mathcal{A}\mathbf{P}}$ defined by

$$A \preceq_{\mathcal{A}\mathbf{P}} B \quad \equiv \quad \uparrow A \supseteq \uparrow B.$$