

**Definition** (Category **UPos**). The category **UPos** consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects  $X, Y \in \mathbf{Ob}_{\mathbf{UPos}}$ , morphisms from  $X$  to  $Y$  are monotone maps of the form  $f : X \rightarrow \mathcal{U}Y$ .
3. *Composition of morphisms*: Given morphisms  $f : X \rightarrow \mathcal{U}Y$   $g : Y \rightarrow \mathcal{U}Z$ , their composition is given as

$$f \circ g : X \rightarrow \mathcal{U}Z$$
$$x \mapsto \bigcup_{y \in f(x)} g(y);$$

4. *Identity morphism*: given an object  $X \in \mathbf{Ob}_{\mathbf{UPos}}$ , the identity morphism is given by the application of the upper closure operator:  $\text{Id}_X(x) := \uparrow \{x\}$ .