

**Definition** (Category). A *category*  $\mathbf{C}$  is:

### Constituents

1. Objects: a collection  $\mathbf{Ob}_{\mathbf{C}}$ , whose elements are called *objects*.
2. Morphisms: for every pair of objects  $X, Y \in \mathbf{Ob}_{\mathbf{C}}$ , there is a set  $\mathbf{Hom}_{\mathbf{C}}(X; Y)$ , elements of which are called *morphisms* from  $X$  to  $Y$ . The set is called the “hom-set from  $X$  to  $Y$ ”.
3. Identity morphisms: for each object  $X$ , there is an element  $\text{Id}_X \in \mathbf{Hom}_{\mathbf{C}}(X; X)$  which is called *the identity morphism of  $X$* .
4. Composition operations: given any morphism  $f \in \mathbf{Hom}_{\mathbf{C}}(X; Y)$  and any morphism  $g \in \mathbf{Hom}_{\mathbf{C}}(Y; Z)$ , there exists a morphism  $f \circ g \in \mathbf{Hom}_{\mathbf{C}}(X; Z)$  which is the *composition of  $f$  and  $g$* .

### Conditions

1. Unitality: for any morphism  $f \in \mathbf{Hom}_{\mathbf{C}}(X; Y)$ ,

$$\text{Id}_X \circ f = f = f \circ \text{Id}_Y.$$

2. Associativity: for any morphisms  $f \in \mathbf{Hom}_{\mathbf{C}}(X; Y)$ ,  $g \in \mathbf{Hom}_{\mathbf{C}}(Y; Z)$ , and  $h \in \mathbf{Hom}_{\mathbf{C}}(Z; W)$ ,

$$(f \circ g) \circ h = f \circ (g \circ h).$$