

Definition (Arrow category)

Given any category \mathbf{C} , its *arrow category* **Arr** \mathbf{C} is the category in which:

1. *Objects*: An object $X \in \mathbf{Arr}(\mathbf{C})$ is a morphism $X : X_0 \rightarrow X_1$ of \mathbf{C} ;
2. *Morphisms*: A morphism $f : X \rightarrow Y$ in **Arr** \mathbf{C} is a commutative square

$$\begin{array}{ccc} X_0 & \xrightarrow{f_0} & Y_0 \\ X \downarrow & & \downarrow Y \\ X_1 & \xrightarrow{f_1} & Y_1 \end{array}$$

in \mathbf{C} ;

3. *Composition*: Composition in **Arr** \mathbf{C} is given by playing commutative squares side by side.