Definition (Free category on a graph). Let G = (V, A, s, t) be a graph. The *free* category on G, denoted $\mathbf{Free}(G)$, has as objects the vertices V of G, and given vertices $x \in V$ and $y \in V$, the morphisms $\mathbf{Free}(G)(x,y)$ are the paths from x to y. The composition of morphisms is given by concatenation of paths, and for any object $x \in V$, the associated identity morphism id is the trivial path which starts and ends at x.