

**Definition** (Series composition). Let  $f : A \multimap B$  and  $g : B \multimap C$  be design problems. We define their *series composition*  $(f \circ g) : A \multimap C$  as:

$$(f \circ g) : A^{\text{op}} \times C \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle a^*, c \rangle \mapsto \bigvee_{b \in B} f(a^*, b) \wedge g(b^*, c).$$

Alternatively:

$$(f \circ g) : A^{\text{op}} \times C \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle a^*, c \rangle \mapsto \bigvee_{b_1 \leq b_2, b_1, b_2 \in B} f(a^*, b_1) \wedge g(b_2^*, c).$$