Definition (Category $\mathbf{Pos}_{\mathscr{L}}$)

The category $\mathbf{Pos}_{\mathscr{L}}$ consists of:

1. *Objects*: objects are posets;

2. *Morphisms*: given objects $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_{\mathscr{L}}}$, morphisms $f : X \to Y$ are monotone maps of the form $f^{\star} : X \to_{\mathbf{Pos}} \mathscr{L}Y$.

3. *Composition of morphisms*: Given morphisms $f : X \to Y$, $g : Y \to Z$, their composition $f \mathbin{\fatsemi} g : X \to Z$ is given by

$$(f \mathbin{\fatsemi} g)^{\star} : X \to_{\mathbf{Pos}} \mathscr{L}Z$$
$$x \mapsto \bigcup_{y \in f^{\star}(x)} g^{\star}(y);$$

4. *Identity morphism*: given an object $X \in \mathrm{Ob}_{\mathbf{Pos}_{\mathscr{L}}}$, the identity morphism $\mathrm{Id}_X : X \to X$ is given by the application of the lower closure operator: $\mathrm{Id}_X^{\star}(x) := \mathop{\downarrow} \{x\}$.