**Definition** (Group). A *group* is a monoid together with an "inverse" operation. In more detail, a group **G** is

## Constituents

- 1. a set **G**;
- 2. a binary operation  $\S: \mathbf{G} \times \mathbf{G} \to \mathbf{G}$ , called *composition*;
- 3. a specified element  $id \in G$ ;
- 4. a map inv :  $\mathbf{G} \to \mathbf{G}$ , called *inverse*.

## Conditions

- 1. Associative law:  $(x \circ y) \circ z = x \circ (y \circ z), \forall x, y, z \in G$ ;
- 2. Neutrality laws: id  $\S x = x = x \S id$ ,  $\forall x \in G$ ;
- 3. Inverse laws:

$$\operatorname{inv}(x) \circ x = \operatorname{id} = x \circ \operatorname{inv}(x), \quad \forall x \in \mathbf{G}.$$