Definition (Upper closure operator). The *upper closure operator* ↑ maps a subset to the smallest upper set that includes it:

$$\uparrow \; : \; \mathscr{P}\mathbf{P} \to \mathscr{U}\mathbf{P}$$

$$S \mapsto \{y \in \mathbf{P} \mid \exists x \in S : x \leq_{\mathbf{P}} y\}.$$