Definition (Semicategory)

A *semicategory* **C** is specified by:

<u>Constituents</u>

1. Objects: A collection $\mathrm{Ob}_\mathbf{C}$ whose elements are called *objects*.
2. Morphisms: For every pair of objects X, Y in $\mathrm{Ob}_\mathbf{C}$, there exists a set $\mathrm{Hom}_\mathbf{C}(X;Y)$, elements of which are called *morphisms*. We write

$$f : X \to_\mathbf{C} Y$$

   to indicate

$$f \in \mathrm{Hom}_\mathbf{C}(X;Y).$$

3. Composition operations: For every three objects X, Y, Z in $\mathrm{Ob}_\mathbf{C}$ there is a composition map

$$\mathring{,}_{X,Y,Z} : \mathrm{Hom}_\mathbf{C}(X;Y) \times \mathrm{Hom}_\mathbf{C}(Y;Z) \to \mathrm{Hom}_\mathbf{C}(X;Z).$$

   We usually just write $\mathring{,}$ instead of $\mathring{,}_{X,Y,Z}$:

$$\frac{f : X \to Y \quad g : Y \to Z}{f \mathring{,} g : X \to Z}$$

   The morphism $f \mathring{,} g$ is called the *composition* of f and g.

<u>Conditions</u>

1. Associativity: it holds that

$$\frac{f : X \to Y \quad g : Y \to Z \quad h : Z \to U}{(f \mathring{,} g) \mathring{,} h = f \mathring{,} (g \mathring{,} h)}$$