

Definition (Intersection of DPIs). Given two DPIs with same functionality and resources $\mathbf{f} = \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_1, \text{prov}_1, \text{req}_1 \rangle$ and $\mathbf{g} = \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_2, \text{prov}_2, \text{req}_2 \rangle$, define their intersection as

$$\mathbf{f} \sqcap \mathbf{g} := \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_1 \cap \mathbf{I}_2, \text{prov}, \text{req} \rangle,$$

where

$$\text{prov} : i \mapsto \begin{cases} \text{prov}_1(i), & \text{if } i \in \mathbf{I}_1 \cap \mathbf{I}_2 \text{ and } \text{prov}_1(i) \leq \text{prov}_2(i) \\ \text{prov}_2(i), & \text{if } i \in \mathbf{I}_1 \cap \mathbf{I}_2 \text{ and } \text{prov}_2(i) \leq \text{prov}_1(i) \\ \perp_{\mathbf{F}}, & \text{else.} \end{cases} \quad (0.1)$$

$$\text{req} : i \mapsto \begin{cases} \text{req}_1(i), & \text{if } i \in \mathbf{I}_1 \cap \mathbf{I}_2 \text{ and } \text{req}_1(i) \geq \text{req}_2(i) \\ \text{req}_2(i), & \text{if } i \in \mathbf{I}_1 \cap \mathbf{I}_2 \text{ and } \text{req}_2(i) \geq \text{req}_1(i) \\ \top_{\mathbf{R}}, & \text{else.} \end{cases}$$