Definition (DPI composition). The series composition of two DPIs

$$\begin{aligned}\mathbf{f}_1 &= \langle \mathbf{F}_1, \mathbf{R}_1, \mathbf{I}_1, \mathsf{prov}_1, \mathsf{req}_1 \rangle, \\ \mathbf{f}_2 &= \langle \mathbf{F}_2, \mathbf{R}_2, \mathbf{I}_2, \mathsf{prov}_2, \mathsf{req}_2 \rangle,\end{aligned}$$

for which $\mathbf{F}_2 = \mathbf{R}_1$, is defined as

$$(\mathbf{f}_1 \fatsemi \mathbf{f}_2) := \langle \mathbf{F}_1, \mathbf{R}_2, \mathbf{I}, \mathsf{prov}, \mathsf{req} \rangle,$$

where:

$$\mathbf{I} = \{[i_1 \,;\, i_2] \in (\mathbf{I}_1 \fatsemi \mathbf{I}_2) \mid \mathsf{req}_1(i_1) \preceq_{\mathbf{R}_1} \mathsf{prov}_2(i_2)\},$$

$$\begin{aligned}\mathsf{prov} &: \quad [i_1 \,;\, i_2] \mapsto \mathsf{prov}_1(i_1), \\ \mathsf{req} &: \quad [i_1 \,;\, i_2] \mapsto \mathsf{req}_2(i_2).\end{aligned}$$