

Definition (Identity morphism). Let \mathbf{S} be a semigroup. The identity function $\text{Id}_{\mathbf{S}} : \mathbf{S} \rightarrow \mathbf{S}$ is always a morphism of semigroups. Indeed, the condition

$$\text{Id}(x \circ_{\mathbf{S}} y) = \text{Id}(x) \circ_{\mathbf{S}} \text{Id}(y)$$

is satisfied for all $s_1, s_2 \in \mathbf{S}$. We call this the *identity morphism* of \mathbf{S} .