

**Definition** (Trace in  $\mathbf{Pos}_{\mathcal{U}}$ ). Given a morphism  $f : X \otimes Z \rightarrow \mathcal{U}(Y \otimes Z)$  in  $\mathbf{Pos}_{\mathcal{U}}$ , its trace in is defined as:

$$\mathrm{Tr}_{X,Y}^Z(f) : X \rightarrow \mathcal{U}Y$$

$$x \mapsto \{y \in Y \mid \bigvee_{z \in Z} \langle y, z \rangle \in f(x, z)\}.$$