

Definition (Disjoint union category). Given two categories \mathbf{C} and \mathbf{D} , their *disjoint union* $\mathbf{C} + \mathbf{D}$ is the category specified as follows:

1. *Objects*: Objects are elements of $\text{Ob}_{\mathbf{C}} + \text{Ob}_{\mathbf{D}}$; that is, objects are tuples of the form $\langle X, i \rangle$, with $i = 1$ or $i = 2$, depending on whether $X \in \text{Ob}_{\mathbf{C}}$ or $X \in \text{Ob}_{\mathbf{D}}$.
2. *Morphisms*: Given objects $\langle X, i \rangle, \langle Y, j \rangle \in \text{Ob}_{\mathbf{C} + \mathbf{D}}$,

$$\text{Hom}_{\mathbf{C} + \mathbf{D}}(\langle X, i \rangle, \langle Y, j \rangle) := \begin{cases} \text{Hom}_{\mathbf{C}}(X, Y) & \text{if } i = j = 1, \\ \text{Hom}_{\mathbf{D}}(X, Y) & \text{if } i = j = 2, \\ \emptyset & \text{else.} \end{cases}$$

3. *Identity morphisms*:
4. *Composition of morphisms*: