Definition (Identity morphism). Let $\mathbf{S}$ be a semigroup. The identity function $\mathrm{Id}_{\mathbf{S}} : \mathbf{S} \to \mathbf{S}$ is always a morphism of semigroups. Indeed, the condition

$$\mathrm{Id}(x \mathbin{\fatsemi_{\mathbf{S}}} y) = \mathrm{Id}(x) \mathbin{\fatsemi_{\mathbf{S}}} \mathrm{Id}(y)$$

is satisfied for all $s_1, s_2 \in \mathbf{S}$. We call this the *identity morphism* of $\mathbf{S}$.