

Definition (Enriched functor)

Given two categories \mathbf{C} and \mathbf{D} enriched in the same monoidal category \mathbf{V} , an enriched functor $F : \mathbf{C} \rightarrow \mathbf{D}$ consists of:

1. A map $F : \mathbf{Ob}_{\mathbf{C}} \rightarrow \mathbf{Ob}_{\mathbf{D}}$ that maps objects of \mathbf{C} to objects of \mathbf{D} .
2. For each X, Y in $\mathbf{Ob}_{\mathbf{C}}$, there exists a morphism in \mathbf{V} given by

$$F_{X,Y} : \mathrm{Hom}_{\mathbf{C}}(X; Y) \rightarrow \mathrm{Hom}_{\mathbf{D}}(F(X); F(Y)),$$

such that composing maps “across F ” respects the composition in \mathbf{C} and the unit in \mathbf{V} in the obvious ways:

$$\begin{array}{ccc} \mathrm{Hom}_{\mathbf{C}}(X; Y) \otimes \mathrm{Hom}_{\mathbf{C}}(Y; Z) & \xrightarrow{m_{X,Y,Z}} & \mathrm{Hom}_{\mathbf{C}}(X; Z) \\ F_{X,Y} \otimes F_{Y,Z} \downarrow & & \downarrow F_{X,Z} \\ \mathrm{Hom}_{\mathbf{D}}(F(Y); F(Z)) \otimes \mathrm{Hom}_{\mathbf{D}}(F(X); F(Y)) & \xrightarrow{m_{F(X),F(Y),F(Z)}} & \mathrm{Hom}_{\mathbf{D}}(F(X); F(Z)) \end{array}$$

and

$$\begin{array}{ccc} \mathbf{1} & & \\ \downarrow j_X & \searrow j_{F(X)} & \\ \mathrm{Hom}_{\mathbf{C}}(X; X) & \xrightarrow{F_{X,X}} & \mathrm{Hom}_{\mathbf{D}}(F(X); F(X)) \end{array}$$

where \otimes and $\mathbf{1}$ are the monoidal product and monoidal unit in \mathbf{V} .