Definition (Functions as relations)

Let $\mathbf{A}$ and $\mathbf{B}$ be sets. A relation $R \subseteq \mathbf{A} \times \mathbf{B}$ is a *function* if it satisfies the following two conditions:

1. for all $x \in \mathbf{A} \quad \exists\, y \in \mathbf{B} : \; x R y$;
2. for all $x_1 R y_1, x_2 R y_2$ holds:

$$\frac{x_1 = x_2}{y_1 = y_2}.$$