

**Definition** (Semi-category action). A *semi-category action* of a semi-category  $\mathbf{C}$  is defined by

- ▷ a map  $\varphi$  that associates, to each object  $X \in \mathbf{Ob}_{\mathbf{C}}$ , a set  $\varphi(X)$ :

$$\varphi : \mathbf{Ob}_{\mathbf{C}} \rightarrow \mathbf{Ob}_{\mathbf{Set}};$$

- ▷ a map  $\gamma$  that associates, to each morphism in  $\mathbf{C}$ , a function:

$$\gamma : \mathbf{Hom}_{\mathbf{C}}(X; Y) \rightarrow \mathbf{Hom}_{\mathbf{Set}}(\varphi(X); \varphi(Y));$$

Moreover, this condition must hold:

$$\gamma(f \circ g) = \gamma(f) \circ \gamma(g).$$