

Definition (Product of posets)

Given two posets $\mathbf{P} = \langle \mathbf{P}, \leq_{\mathbf{P}} \rangle$ and $\mathbf{Q} = \langle \mathbf{Q}, \leq_{\mathbf{Q}} \rangle$, the *product poset* is $\mathbf{P} \times \mathbf{Q} = \langle \mathbf{P} \times \mathbf{Q}, \leq_{\mathbf{P} \times \mathbf{Q}} \rangle$, where $\mathbf{P} \times \mathbf{Q}$ is the Cartesian product of the sets \mathbf{P} and \mathbf{Q} (??), and the order $\leq_{\mathbf{P} \times \mathbf{Q}}$ is given by:

$$\frac{\langle p_1, q_1 \rangle \leq_{\mathbf{P} \times \mathbf{Q}} \langle p_2, q_2 \rangle}{(p_1 \leq_{\mathbf{P}} p_2) \wedge (q_1 \leq_{\mathbf{Q}} q_2)}.$$