Definition (Series composition of design problems with implementation). Given two design problems with implementation $\langle \mathbf{I}_f, \mathsf{prov}_f, \mathsf{req}_f \rangle : A \longrightarrow B$ and $\langle \mathbf{I}_g, \mathsf{prov}_g, \mathsf{req}_g \rangle : B \longrightarrow C$, we can define their series interconnection

$$\langle \mathbf{I}_{f \circ g}, \mathsf{prov}_{f \circ g}, \mathsf{req}_{f \circ g} \rangle : A \longrightarrow C.$$

as follows. With reference to this diagram:

$$A \xleftarrow{\mathsf{prov}_f} \mathbf{I}_f \xrightarrow{\mathsf{req}_f} B \xleftarrow{\mathsf{prov}_g} \mathbf{I}_g \xrightarrow{\mathsf{req}_g} C$$

we let the implementation space be the *pullback*

$$\mathbf{I}_{f \nmid g} = \mathbf{I}_f \times_B \mathbf{I}_g := \{ \langle i_f, i_g \rangle \in \mathbf{I}_f \times \mathbf{I}_g : \operatorname{req}_f(i_f) \leq_B \operatorname{prov}_g(i_g) \},$$

and the two maps prov, req defined as:

$$\operatorname{req}: \left\langle i_f, i_g \right\rangle \mapsto \operatorname{req}_2(i_g)$$
 $\operatorname{prov}: \left\langle i_f, i_g \right\rangle \mapsto \operatorname{prov}_1(i_f).$

In terms of the profunctors, one has

$$\langle \mathbf{I}_{f \nmid g}, \mathsf{prov}_{f \nmid g}, \mathsf{req}_{f \nmid g} \rangle : A \times C \to_{\mathbf{Pos}} \mathscr{P}(\mathbf{I}_f \times_B \mathbf{I}_g)$$

$$\langle a^*, c \rangle \mapsto \bigcup_{\substack{(b,b') \in B^{\mathrm{op}} \times B^{\mathrm{op}} \\ b \leq_{p} b'}} \left[\langle \mathbf{I}_f, \mathsf{prov}_f, \mathsf{req}_f \rangle (a^*, b) \times \langle \mathbf{I}_g, \mathsf{prov}_g, \mathsf{req}_g \rangle (b'^*, c) \right].$$