

Definition (Categorical Product)

Let \mathbf{C} be a category and let $X, Y \in \mathbf{Ob}_{\mathbf{C}}$ be objects. The *product* of X and Y is:
Constituents

1. an object $Z \in \mathbf{Ob}_{\mathbf{C}}$ (this is “the product of X and Y ”);
2. *projection morphisms* $\pi_1 : Z \rightarrow X$ and $\pi_2 : Z \rightarrow Y$,

Conditions

1. For any $T \in \mathbf{Ob}_{\mathbf{C}}$ and any morphisms $f : T \rightarrow X, g : T \rightarrow Y$, there exists a *unique* morphism $\phi_{f,g} : T \rightarrow Z$ such that $f = (\phi_{f,g}) \circ \pi_1$ and $g = (\phi_{f,g}) \circ \pi_2$.