Definition (Monoid morphism). A morphism of monoids $F : \mathbf{M} \to \mathbf{N}$ is called a *monoid isomorphism* if there is a morphism of monoids $G : \mathbf{N} \to \mathbf{M}$ such that

$$F \mathbin{\text{\textsubscript{9}}} G = \mathrm{Id}_{\mathbf{M}} \quad \text{and} \quad G \mathbin{\text{\textsubscript{9}}} F = \mathrm{Id}_{\mathbf{N}}.$$