Definition (Monoid morphism). Consider two monoids

$$\mathbf{M} = \langle \mathbf{M}, \circ_{\mathbf{M}}, \mathrm{id}_{\mathbf{M}} \rangle$$

and

$$\mathbf{N} = \langle \mathbf{N}, \circ_{\mathbf{N}}, \mathrm{id}_{\mathbf{N}} \rangle$$

A morphism of monoids from $\mathbf{M}$ to $\mathbf{N}$ is a function $F : \mathbf{M} \to \mathbf{N}$ such that $\forall x, y \in \mathbf{M}$,

$$F(x \circ_{\mathbf{M}} y) = F(x) \circ_{\mathbf{N}} F(y)$$

and

$$F(\mathrm{id}_{\mathbf{M}}) = \mathrm{id}_{\mathbf{N}}$$