

Lemma. The metrics P_{track} and P_{effort} can be written as

$$\lim_{t \rightarrow \infty} \mathbb{E}\{\mathbf{x}_t^\top \mathbf{Q}_0 \mathbf{x}_t\} = \text{Tr}(\mathbf{Q}_0 (\mathbf{\Sigma} + \mathbf{F})),$$

$$\lim_{t \rightarrow \infty} \mathbb{E}\{\mathbf{u}_t^\top \mathbf{R}_0 \mathbf{u}_t\} = \text{Tr}(\mathbf{S} \mathbf{B}^* \mathbf{R}^{-1} \mathbf{R}_0 \mathbf{R}^{-1} \mathbf{B} \mathbf{S} \mathbf{F}),$$

where $\mathbf{\Sigma}$ solves the Riccati equation for estimation, \mathbf{F} solves the Lyapunov equation

$$(\mathbf{A} - \mathbf{B} \mathbf{K}) \mathbf{F} + \mathbf{F} (\mathbf{A} - \mathbf{B} \mathbf{K})^* + \mathbf{L} \mathbf{V} \mathbf{L}^* = \mathbf{0},$$

\mathbf{S} solves the Riccati equation for control, and $\mathbf{L} = \mathbf{\Sigma} \mathbf{C}^* \mathbf{V}^{-1}$ is the Kalman gain.