

**Definition** (Trace of a generalized endomorphism). Let  $\langle \mathbf{C}, \otimes, \mathbf{1}, \text{br} \rangle$  be a symmetric monoidal category. Let  $X \in \text{Ob}_{\mathbf{C}}$  be dualizable and let  $f \in \text{Hom}_{\mathbf{C}}(Y \otimes X; Z \otimes X)$ . The *trace over  $X$*  of  $f$  is the morphism  $\text{Tr}_{Y,Z}^X(f) \in \text{Hom}(Y, Z)$  defined by

$$Y \xrightarrow{\text{id}_Y \otimes \eta_X} Y \otimes X \otimes X^\vee \xrightarrow{f \otimes \text{id}_{X^\vee}} Z \otimes X \otimes X^\vee \xrightarrow{\text{id}_Z \otimes \text{br}} Z \otimes X^\vee \otimes X \xrightarrow{\text{id}_Z \otimes \epsilon_X} Z$$