

Definition (Dualizable object). Let $\langle \mathbf{C}, \otimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}} \rangle$ be a monoidal category, and let $X \in \mathbf{Ob}_{\mathbf{C}}$. A *right dual object* of X is specified by:

Constituents

1. an object $X^{\vee} \in \mathbf{Ob}_{\mathbf{C}}$;
2. an evaluation map $\epsilon_X : X^{\vee} \otimes X \rightarrow \mathbf{1}$;
3. a coevaluation map $\eta_X : \mathbf{1} \rightarrow X \otimes X^{\vee}$;

Conditions

1. $\text{lu}_X^{-1} \circ (\eta_X \otimes \text{Id}_X) \circ \text{as}_{X, X^{\vee}, X} \circ (\text{Id}_X \otimes \epsilon_X) \circ \text{ru}_X = \text{Id}_X$;
2. $\text{ru}_{X^{\vee}}^{-1} \circ (\text{Id}_{X^{\vee}} \otimes \eta_X) \circ \text{as}_{X^{\vee}, X, X^{\vee}}^{-1} \circ (\epsilon_X \otimes \text{Id}_{X^{\vee}}) \circ \text{lu}_{X^{\vee}} = \text{Id}_{X^{\vee}}$.