

**Lemma.** Given  $f_1, f_2 \in \text{Hom}_{\mathbf{DP}}(A; B)$  and  $g_1, g_2 \in \text{Hom}_{\mathbf{DP}}(B; C)$  one has:

$$\frac{f_1 \leq_{\mathbf{DP}} f_2 \quad g_1 \leq_{\mathbf{DP}} g_2}{f_1 \circ g_1 \leq_{\mathbf{DP}} f_2 \circ g_2}$$

In other words, series composition is order-preserving on  $\mathbf{DP}$ .