Lemma. Given a poset $\langle \mathbf{P}, \preceq_{\mathbf{P}} \rangle$, $\langle \mathcal{A}\mathbf{P}, \preceq_{\mathcal{A}\mathbf{P}} \rangle$ is a poset with

$$A \preceq_{\mathcal{A}\mathbf{P}} B \text{ if and only if } \uparrow A \supseteq \ \uparrow B.$$

Furthermore, it is bounded by the top $\top_{\mathcal{A}\mathbf{P}} = \emptyset$ and the bottom $\bot_{\mathcal{A}\mathbf{P}} = \{\bot_{\mathbf{P}}\}$.