**Lemma.** The lower closure operator  $\downarrow$  is a monotone map.