

**Definition** (Loop operator  $\dagger$ )

For a map  $h : \mathbf{F}_1 \times \mathbf{F}_2 \rightarrow \mathcal{AR}$ , define

$$h^\dagger : \mathbf{F}_1 \rightarrow \mathcal{AR},$$

$$f_1 \mapsto \text{lfp} \left( \Psi_{f_1}^h \right),$$

where lfp is the least-fixed point operator, and  $\Psi_{f_1}^h$  is defined as

$$\Psi_{f_1}^h : \mathcal{AR} \rightarrow \mathcal{AR},$$

$$R \mapsto \text{Min}_{\leq_R} \bigcup_{r \in R} h(f_1, r) \cap \uparrow r.$$