Lemma. Given a set A and a family of subsets of A, denoted $\{A_i\}_{i \in I}$ (with index set I), one has:

$$\bigcup \bigcup_{i \in I} \{A_i\} = \bigcup_{i \in I} \{A_i\}.$$