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Fill cover letter

Yours sincerely,

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1 The prospectus

Title of the book: Applied Category Theory for Engineering

1.1 Brief description

In many domains of engineering and applied sciences, it is advantageous to think explicitly about abstraction and compositionality, to improve both the understanding of problems, and the design of solutions. However, the type of math which could be useful to applications is not traditionally taught. Applied Category Theory is a recent field of mathematics which could help a lot, but it is quite unreachable by non-mathematicians. While several valuable resources for learning applied category theory have emerged recently, they fail to meet the two essential criteria of a) being approachable from an engineering background and b) highlighting how applied category theory can be used to formalize and solve concrete problems.

This book will fill this gap, by illustrating how applied category theory can be effectively leveraged to address the design and analysis of complex systems, through the "compositionality way of thinking". The book adopts a computational and constructive approach: each concept is accompanied by pen-and-paper exercises, as well as computer-based (Python-based) ones. Furthermore, each section provides exhaustive video content (in the style of MOOCs), explaining concepts and providing further examples.

1.2 Outstanding features

The book is unique in at least five aspects.

- It is specifically written **for engineers and practitioners** (unlike existing efforts, as described in section 1.3), and applied category theory is just the means toward practical tools for complex system analysis and design. This implies that the presentation of materials sometimes diverges from the usual way category theory is taught, and some common concepts will be de-emphasized in favor of more obscure concepts that find extensive use in applications. Furthermore, the book is **self-contained** for readers with a basic background in engineering. The book culminates presenting a success story of the aforementioned "compositionality way of thinking", presenting a monotone theory of co-design and its application to engineering design optimization problems.
- It uses an **inductive exposition** style, rather than a deductive explanation style. Indeed, in a typical mathematical exposition of category theory, one usually defines a general mathematical structure, and then gives several specific examples. Instead, in this book, we first build up the examples as something that is interesting per se, and then we show how they can all be instances of the same general concept. This way, the general concept is well motivated. The path laid by the book is one of *spiral learning*.
- The book follows a **computational approach**: in addition to classic pen-and-paper exercises, the reader can implement various constructions in Python, and leverage an auto-grading system to get feedback. Once more, this allows one to fully grasp the practical advantages promoted by the presented "compositionality way of thinking".
- The book uses a **new pedagogy of colors** to aid in the parsing of formulas and diagrams (see attached example). We also color the various composition operations. Typically, applied category theory features many different quantities, which are used in different contexts: our approach, which is colorblind-friendly (one of the authors is colorblind), helps the reader understanding the various concepts introduced.

add use of colors table

- Every section of the book features **dedicated video content** (e.g., explanations of concepts and step-by-step solutions of exercises) in the style of MOOCs. We are in the process of creating exhaustive content for a MOOC.

1.3 Competition

Our book stands, to some degree, in competition with books which are general introductions to category theory or to applied category theory. Below we discuss the main titles of this kind which exist today, in order to compare and contrast them to our book. Our book is unique among these existing books because it has the following two qualities simultaneously:

1. it is slow-paced, self-contained, and written in an accessible style oriented toward people who do not have a formal training in pure mathematics, including, in particular, engineering students and practitioners;

2. it's contents and style are oriented expressly towards applying category theory in engineering. In particular, we often use engineering examples to motivate and introduce concepts, and our choices in topics and exposition sometimes diverge from more classical mathematical treatments, in order to address the particular relevance of certain topics for engineering applications.

Categories for the Working Mathematician, *S. MacLane, Graduate Texts in Mathematics, Springer, 1978*
This was the early major expository monograph on category theory and for a very long time the go-to book for mathematicians to learn category theory. Written in a terse and dense style, it is comprehensive, insightful, and a slow and challenging read, even for mathematicians. As far as introductory category theory texts go, this is likely the least accessible one. It focuses on only on pure mathematics, and assumes a good deal of prior mathematical experience.

Conceptual Mathematics, 2nd Ed., *S. H. Schanuel, F. W. Lawvere, Cambridge University Press, 2009*
This is one of the early expository monographs on category theory which aims to be broadly accessible and convey not only definitions and theorems, but also the spirit and ways of thinking used in category theory. It uses a non-traditional, conversational, lecture-based style, and focuses on basic conceptual insights rather than covering a large amount of technical material in a formally structured way. The book uses simple and insightful examples, both of a 'pure' and 'applied' flavor. However, it is still not a text which is mainly focused on *applied* category theory, let alone one that centers engineering. Nor does it cover the amount and scope of material that our book does.

Category Theory, 2nd Ed., *S. Awodey, Oxford University Press, 2010*
This was the first major 'mathematics-style' textbook after MacLane's *Categories for the working mathematician*. It covers a good deal of standard category theory material, and it aims to be much much more accessible than MacLane's book, both in terms of prerequisites needed and in terms of its pedagogical style. It is addressed not only to mathematics students, but also expressly to readers with a background in computer science, logic, linguistics, or philosophy. For this reason, the book covers material for example on cartesian closed categories and the lambda calculus. Despite its broader audience, the book remains, however, one written in a mathematical style. Furthermore, the applied areas that it does address are not in engineering. Our book differs both in scope and in style.

Basic Category Theory, *T. Leinster, Cambridge University Press, 2014*
This is an excellently-written, introductory book on category theory. Relatively short in length and covering less material than, for example, Riehl's *Category theory in context* (see below), this book is aimed at mathematicians and provides a concise, pedagogical, and elegant exposition of core category theory topics for such an audience. It does not however cover some theoretical topics which are essential for applied

category theory (for example, monoidal categories), nor does it aim to discuss examples and applications outside of mathematics.

Category Theory for the Sciences, *D. I. Spivak, MIT Press, 2014*

This text is arguably the first book on applied category theory proper and a pioneering contribution to the field. It is aimed both at scientists and at mathematicians interested in using category theory in applied domains. It is written to be accessible to a broad audience, and does so by teaching via examples and exercises, rather than theorems and proofs. The main differences with our book are scope – our book aims to cover a much larger amount of category theory material and applications thereof – and the fact that our book is focused expressly on engineering. Furthermore, although we provide many examples and exercises, our book has theorems and proofs, too. This is because some of our intended readers will want to understand the theory in this level of formal depth, and it is important for their work to learn to do so, to some degree. Nevertheless, our book remains accessible for those who wish to skip over proofs.

Category Theory in Context, *E. Riehl, Aurora: Dover Modern Math Originals, 2016*

This is another well-known textbook for (pure) category theory, written in a modern style and conveying some of the categorical 'way of thinking' alongside the formal mathematics. Although certainly more accessible than MacLane, it is nevertheless aimed at readers with a moderate level of mathematical training and maturity, and is also comparatively dense, covering a good deal of material relative to its length. It is a wonderful book for mathematicians which emphasizes how to use category theory fruitfully inside of mathematics; however it is not realistically accessible for readers without mathematical training, nor does it emphasize applications outside of mathematics.

An Invitation to Applied Category Theory: Seven Sketches in Compositionality, *B. Fong, D. I. Spivak, Cambridge University Press, 2019*

This book is, in two respects, very close to fulfilling our aims: it is written in an accessible style intended for a very broad audience and it is focused on applications of category theory outside of mathematics. The two main differences to our book are that this one is not focused mainly on engineering applications, and it is not as comprehensive as ours. As the word 'sketch' suggests, some of the topics in Fong and Spivak's book are only touched upon briefly, in order to introduce an important idea and whet the readers appetite to learn more through other resources. In contrast, our book offers more thorough treatments of many important topics, illustrating and spelling out details in depth so that readers without much mathematical training are able to follow even technical aspects step-by-step and connect the technical theory with examples and applications.

The Joy of Abstraction: An Exploration of Math, Category Theory, and Life, *E. Cheng, Cambridge University Press, 2022*

This book is aimed at a very broad audience – it does not assume any university-level mathematical training – yet it builds up to introducing the core essential material of category theory, including technical details, so that readers might then be able to read other introductory books on category theory without struggling too much. It is accessible and pedagogical, and conveys also a conceptual and intuitive understanding of the categorical way of thinking. A main difference to our book is that, while Cheng's book uses examples that include various aspects of daily life, it is not focused on applying category theory in the sciences or in engineering specifically. Our book, in contrast, covers more – and other – kinds of material, both in terms of theory and in terms of examples and exercises.

We are aware of a few further books ([2], [3], [4]) which are also mathematics-focused introductions to basic category theory. Because they are subsumed in content and expository nature by the books discussed above, we do not see the need to describe these in detail. However, we mention them for completeness' sake.

Other books that are related to ours, but do not stand in competition, are books which focus explicitly on applications of category theory in specific domains other than engineering, and include a somewhat accessible introduction to category theory (most specialized books do not). The following books, for example, cover topics of category theory in computer science ([7], [5], [6]) and physics ([1]).

1.4 Apparatus

The book will include examples, pen-and-paper exercises, and computational (Python-based) exercises. Solutions to pen-and-paper exercises will be provided, and the computational exercises will be automatically graded through a system we have developed during dedicated classes at ETH Zurich. The book will include glossaries, bibliography, and appendices. Furthermore, we have created a "developer" version of the book, for instructors to use in their classes.

1.5 Audience

The book is intended for engineers, and we assume basic knowledge of algebra at the level of a bachelor's degree in engineering/computer science and is self-contained. In the past, we have organized both ETH Zurich lectures, as well as international, online ones. From our records, we highlight that professionals were particularly interested in the subject presented in this book. The book has a rigorous character, and provides both descriptive and quantitative explanations of various ideas.

1.6 Authors

See if we need more exhaustive bios

Dr. Andrea Censi, Senior Research Scientist
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Bio: Andrea Censi is deputy director of the Dynamic Systems and Control chair at ETH Zurich. He obtained a Ph.D. from Caltech. Previously, he has been a research scientist at MIT and Director of Research at Aptiv Mobility (now Motional). He is president of the Duckietown Foundation.

Dr. Jonathan Lorand, Postdoctoral researcher
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Bio: Jonathan is a postdoctoral researcher working as a mathematician at the Institute for Dynamic Systems and Control at ETH Zurich. He obtained his PhD in mathematics from the University of Zurich in 2020, working in symplectic geometry and category theory. Since then he is focused on applied category theory, as well on the topic of transdisciplinarity, in which he will obtain an MA in 2024 from the Zurich University of the Arts.

Gioele Zardini, Ph. D. Candidate
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Bio: Gioele is a Ph.D. candidate at ETH Zurich, and incoming faculty at Massachusetts Institute of Technology. He received his BSc. and MSc. in Mechanical Engineering with focus in Robotics, Systems and Control from ETH Zurich in 2017 and 2019, respectively. He spent time in Singapore as a researcher at nuTonomy

(now Motional), at Stanford University and at MIT.

All three authors have experience in teaching various courses (both at the undergraduate and graduate level) at ETH Zurich and University of Zurich.

1.7 Market considerations

One of the great engineering challenges of this century is dealing with the design of complex systems, and, as explained above, the math traditionally thought is not practical, and does not help reasoning about them. The interest for applied category theory has been growing dramatically over the past few years, taking a small community of mathematicians and bringing it to a worldwide movement, with representatives in academia and in industry.

We believe that the course presented by this book will be taught in at least 50% of the major universities in the next 7 years, as part of every engineering program.

Over the past few years, we organized online classes, as well as dissemination workshops at large international conferences (e.g., in robotics, systems and control, and intelligent transportation systems), and we have created a large, diverse network of interested people. We are curating a dedicated mailing list, with people following our steps in this journey.

1.8 Status of the book

A first draft version of the book is mostly complete, and likely needs adaptation to a common format from the publisher. We expect the complete manuscript to be completed within the next six months or less. The manuscript will reach around 200'000 words.

The book features, on average, at least two figures on each page (vector figures, typically mathematical diagrams), and one full color photograph per chapter.

The current version of the book may be found here: https://z7-stage-act4e-book.zuper.ai/sync/ACT4E/ACT4E/alphubel
prod/build/last/build-public/ACT4E-public.pdf.

make the link look nice

1.9 Reviewers

choose order wisely

We suggest the following list of reviewers:

- Prof. Michael Johnson, Macquarie University, Michael.Johnson@mq.edu.au;
- Prof. John Baez, University of California, Riverside and Topos Institute, john.baez@ucr.edu;
- Prof. Daniel Koditschek, University of Pennsylvania, kod@seas.upenn.edu;
- Prof. Aaron Ames, California Institute of Technology, ames@cds.caltech.edu;
- Prof. Paulo Tabuada, University of California, Los Angeles, tabuada@ee.ucla.edu;
- Prof. Hadas Kress-Gazit, Cornell University, hadaskg@cornell.edu;
- Prof. Gregory Chirikjian, National University of Singapore, mpegre@nus.edu.sg;
- Dr. Paolo Perrone, University of Oxford, paolo.perrone@cs.ox.ac.uk;
- Dr. David Spivak, Topos Institute, dspivak@gmail.com;

- Dr. Brendan Fong, Topos Institute, brendan@topos.institute;
- others?

2 Table of contents

Attached, you find the current table of contents.

3 Sample chapters

Attached, you find sample chapters.

The bibliography below needs to be completed and cleaned up... for instance, put in alphabetical order by author

Bibliography

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