Definition (Free category on a graph)
Let $\mathcal{G} = \langle \mathbf{V}, \mathbf{A}, \mathrm{src}, \mathrm{tgt} \rangle$ be a graph. The *free category on* $\mathcal{G}$, denoted $\mathbf{Free}(\mathcal{G})$, has as objects the vertices $\mathbf{V}$ of $\mathcal{G}$, and given vertices $v \in \mathbf{V}$ and $w \in \mathbf{V}$, the morphisms $\mathrm{Hom}_{\mathbf{Free}(\mathcal{G})}(v; w)$ are the paths from v to w. The composition of morphisms is given by concatenation of paths, and for any object $v \in \mathbf{V}$, the associated identity morphism id_v is the trivial path which starts and ends at v.