

# Definition (Product of posets)

Given two posets  $\mathbf{P} = \langle \mathbf{P}, \leq_{\mathbf{P}} \rangle$  and  $\mathbf{Q} = \langle \mathbf{Q}, \leq_{\mathbf{Q}} \rangle$ , the *product poset* is  $\mathbf{P} \times \mathbf{Q} = \langle \mathbf{P} \times \mathbf{Q}, \leq_{\mathbf{P} \times \mathbf{Q}} \rangle$ , where  $\mathbf{P} \times \mathbf{Q}$  is the Cartesian product of the sets  $\mathbf{P}$  and  $\mathbf{Q}$  (??), and the order  $\leq_{\mathbf{P} \times \mathbf{Q}}$  is given by:

$$\frac{\langle p_1, q_1 \rangle \leq_{\mathbf{P} \times \mathbf{Q}} \langle p_2, q_2 \rangle}{(p_1 \leq_{\mathbf{P}} p_2) \wedge (q_1 \leq_{\mathbf{Q}} q_2)}.$$