

Definition (Category). A *category* \mathbf{C} is specified by four components:

1. **Objects:** a collection $\text{Ob}_{\mathbf{C}}$, whose elements are called *objects*.
2. **Morphisms:** for every pair of objects $X, Y \in \text{Ob}_{\mathbf{C}}$, there is a set $\text{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms* from X to Y . The set is called the “hom-set from X to Y ”.
3. **Identity morphisms:** for each object X , there is an element $\text{Id}_X \in \text{Hom}_{\mathbf{C}}(X; X)$ which is called *the identity morphism of* X .
4. **Composition rules:** given any morphism $f \in \text{Hom}_{\mathbf{C}}(X; Y)$ and any morphism $g \in \text{Hom}_{\mathbf{C}}(Y; Z)$, there exists a morphism $f \circ g \in \text{Hom}_{\mathbf{C}}(X; Z)$ which is the *composition of* f and g .

Furthermore, the constituents are required to satisfy the following conditions:

- a) *Unitality:* for any morphism $f \in \text{Hom}_{\mathbf{C}}(X; Y)$:

$$\text{Id}_X \circ f = f = f \circ \text{Id}_Y .$$

- b) *Associativity:* for morphisms $f \in \text{Hom}_{\mathbf{C}}(X; Y)$, $g \in \text{Hom}_{\mathbf{C}}(Y; Z)$, and $h \in \text{Hom}_{\mathbf{C}}(Z; W)$,

$$(f \circ g) \circ h = f \circ (g \circ h) .$$