

Definition (Trace of a design problem). Given a design problem $\mathbf{f} : \mathbf{A} \times \mathbf{C} \rightarrow \mathbf{B} \times \mathbf{C}$, we can define its *trace* $\text{Tr}_{\mathbf{A}, \mathbf{B}}^{\mathbf{C}}(\mathbf{f}) : \mathbf{A} \rightarrow \mathbf{B}$ as follows:

$$\text{Tr}_{\mathbf{A}, \mathbf{B}}^{\mathbf{C}}(\mathbf{f}) : \mathbf{A}^{\text{op}} \times \mathbf{B} \rightarrow_{\text{Pos}} \mathbf{Bool}$$

$$\langle a^*, b \rangle \mapsto \bigvee_{c \in \mathbf{C}} \mathbf{f}(\langle a, c \rangle^*, \langle b, c \rangle).$$