

Definition (Categorical Coproduct)

Let \mathbf{C} be a category and let $X, Y \in \mathbf{Ob}_{\mathbf{C}}$ be objects. The *coproduct* of X and Y is:

Constituents

1. an object $Z \in \mathbf{Ob}_{\mathbf{C}}$ (“the coproduct of X and Y ”)
2. *injection morphisms* $\iota_1 : X \rightarrow Z$ and $\iota_2 : Y \rightarrow Z$

Conditions

1. For any $T \in \mathbf{Ob}_{\mathbf{C}}$ and any morphisms $f : X \rightarrow T, g : Y \rightarrow T$, there exists a *unique* morphism $\psi_{f,g} : Z \rightarrow T$ such that $f = \iota_1 \circ \psi_{f,g}$ and $g = \iota_2 \circ \psi_{f,g}$.