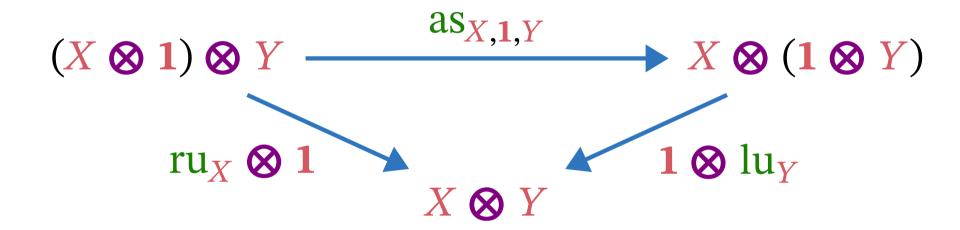
**Definition** (Monoidal category). A *monoidal structure* on a category **C** consists of:

- 1. An object  $1 \in Ob_{\mathbb{C}}$  called the monoidal unit.
- 2. A functor  $\otimes$ :  $\mathbf{C} \times \mathbf{C} \to \mathbf{C}$ , called the *monoidal product*.

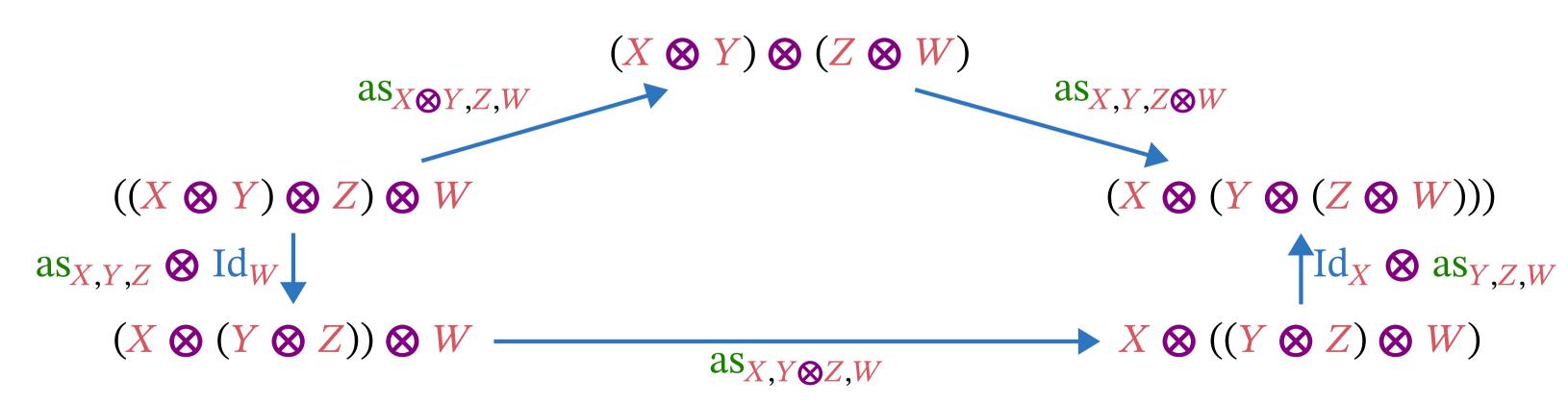
The two constituents are subject to the natural isomorphisms:

- a) (Left unitor)  $lu_X : \mathbf{1} \otimes X \xrightarrow{\cong} X$  for every  $X \in Ob_{\mathbf{C}}$ ,
- b) (Right unitor)  $\operatorname{ru}_X: Z \otimes 1 \xrightarrow{\cong} X$  for every  $X \in \operatorname{Ob}_{\mathbb{C}}$ ,
- c) (Associator)  $\operatorname{as}_{X,Y,Z}$ :  $(X \otimes Y) \otimes Z \xrightarrow{\cong} X \otimes (Y \otimes Z)$  for every  $X,Y,Z \in \operatorname{Ob}_{\mathbf{C}}$ .

These isomorphisms are themselves required to satisfy the triangle identity



and the pentagon identity



for  $X, Y, Z, W \in Ob_{\mathbb{C}}$ . A category equipped with a monoidal structure is called a *monoidal category*. If the isomorphisms in a), b), and c) are equivalences, one calls the category *strict* monoidal.