Definition (The \mathcal{U} monad). The \mathcal{U} monad on **DP** consists of:

- 1. The functor Unc : $\mathbf{DP} \rightarrow \mathbf{DP}$;
- 2. The natural transformation $un_{\mathcal{U}}: Id_{\mathbf{DP}} \Rightarrow Unc$, specified as

$$\operatorname{un}_{\mathcal{U}}^{\mathbf{A}} \colon \mathbf{A} \longrightarrow \operatorname{Int}(\mathbf{A})$$
$$\langle a^*, [x, y] \rangle \mapsto a \leq x.$$

3. The natural transformation $mu_{\mathcal{U}}$: UncUnc \Rightarrow Unc, specified as:

$$\operatorname{mu}_{\mathcal{U}}^{\mathbf{A}} : \operatorname{Int}(\operatorname{Int}(\mathbf{A}))^{\operatorname{op}} \times \operatorname{Int}(\mathbf{A}) \to_{\operatorname{Pos}} \operatorname{Bool}$$
$$\left\langle \left[[a,b], [c,d] \right]^*, \left[e,f \right] \right\rangle \mapsto (a \leq e) \wedge (b \leq e) \wedge (c \leq f) \wedge (d \leq f).$$