Definition (Cartesian product of categories)
Given two categories $\mathbf{C}$ and $\mathbf{D}$, their *cartesian product* $\mathbf{C} \times \mathbf{D}$ is the category specified as follows:

1. *Objects*: Objects are pairs $\langle X, Y \rangle$, with $X \in \mathrm{Ob}_{\mathbf{C}}$ and $Y \in \mathrm{Ob}_{\mathbf{D}}$.
2. *Morphisms*: Morphisms are pairs of morphisms $\langle f, g \rangle : \langle X, Z \rangle \to \langle Y, U \rangle$, with $f : X \to Y$, $g : Z \to U$.
3. *Identity morphisms*: Given objects $X \in \mathrm{Ob}_{\mathbf{C}}$ and $Y \in \mathrm{Ob}_{\mathbf{D}}$, the identity morphism on $\langle X, Y \rangle$ is the pair $\langle \mathrm{Id}_X, \mathrm{Id}_Y \rangle$.
4. *Composition of morphisms*: The composition of morphisms is given by composing each component of the pair separately:

$$\langle f, g \rangle \fatsemi_{\mathbf{C}\times\mathbf{D}} \langle h, i \rangle = \langle f \fatsemi_{\mathbf{C}} h, g \fatsemi_{\mathbf{D}} i \rangle.$$