

Definition (Adjunction, Version 2)

Let \mathbf{C} and \mathbf{D} be categories. An *adjunction* from \mathbf{C} to \mathbf{D} is given by the following data, satisfying the following conditions.

Data:

1. A functor $L : \mathbf{C} \rightarrow \mathbf{D}$ (the *left adjoint*);
2. A functor $R : \mathbf{D} \rightarrow \mathbf{C}$ (the *right adjoint*);
3. Natural transformations $\text{un} : \text{Id}_{\mathbf{C}} \Rightarrow L \circ R$ and $\text{co} : R \circ L \Rightarrow \text{Id}_{\mathbf{D}}$

Conditions:

1. For all objects X of \mathbf{C} , it holds that

$$L \circ \text{un}_X \circ \text{co}_{LX} = \text{Id}_{LX} \text{ and } \text{un}_{RY} \circ R \circ \text{co}_Y = \text{Id}_{RY}$$

i.e. that the following diagrams commute:



The 2-morphisms un and co are called the *unit* and *counit* of the adjunction. An adjunction is called an *adjoint equivalence* if the unit and counit are natural isomorphisms.