Definition (Categorical Product). Let $\mathbf{C}$ be a category and let $X, Y \in \mathrm{Ob}_{\mathbf{C}}$ be objects. The *product* of X and Y is:

Constituents

1. an object $Z \in \mathrm{Ob}_{\mathbf{C}}$ (this is "the product" of X and Y);
2. *projection morphisms* $\pi_1 : Z \to X$ and $\pi_2 : Z \to Y$,

Conditions

1. For any $T \in \mathrm{Ob}_{\mathbf{C}}$ and any morphisms $f : T \to X, g : T \to Y$, there exists a *unique* morphism $\phi_{f,g} : T \to Z$ such that $f = (\phi_{f,g}) \fatsemi \pi_1$ and $g = (\phi_{f,g}) \fatsemi \pi_2$.