

Definition (Design problems with implementation as monotone functions). Suppose that \mathbf{F} , \mathbf{R} are posets. A *design problem with implementation* is a monotone map (a **Set**-enriched profunctor) $\langle \mathbf{I}_d, \text{prov}, \text{req} \rangle : \mathbf{F} \dashv\dashv \mathbf{R}$, where \mathbf{I}_d is a set, prov and req are functions from \mathbf{I}_d to \mathbf{F} and \mathbf{R} , respectively

$$\mathbf{F} \xleftarrow{\text{prov}} I \xrightarrow{\text{req}} \mathbf{R},$$

and $\langle \mathbf{I}_d, \text{prov}, \text{req} \rangle : \mathbf{F} \dashv\dashv \mathbf{R}$ is given by

$$\begin{aligned} \langle \mathbf{I}_d, \text{prov}, \text{req} \rangle : \mathbf{F} \dashv\dashv \mathbf{R} : \mathbf{F}^{\text{op}} \times \mathbf{R} &\rightarrow_{\mathbf{Pos}} \mathcal{P}(\mathbf{I}_d) \\ \langle f^*, r \rangle &\mapsto \{i \in \mathbf{I}_d : (f \leq_{\mathbf{F}} \text{prov}(i)) \wedge (\text{req}(i) \leq_{\mathbf{R}} r)\}, \end{aligned}$$

where the partial order on $\mathcal{P}\mathbf{I}$ is given by subset inclusion.