

**Lemma.** Given a poset  $P$  and a family of upper sets of  $P$ , denoted  $\{P_i\}_{i \in \mathbf{I}}$  (with index set  $\mathbf{I}$ ), their union is also an upper set.