Definition (Trace of a design problem)

Given a design problem $\mathbf{d} : \mathbf{P} \times \mathbf{R} \nrightarrow \mathbf{Q} \times \mathbf{R}$, its *trace* $\mathrm{Tr}^{\mathbf{R}}_{\mathbf{P},\mathbf{Q}}(\mathbf{d}) : \mathbf{P} \nrightarrow \mathbf{Q}$ is defined as follows:

$$\mathrm{Tr}^{\mathbf{R}}_{\mathbf{P},\mathbf{Q}}(\mathbf{d}) : \mathbf{P}^{\mathrm{op}} \times \mathbf{Q} \rightarrow_{\mathbf{Pos}} \mathbf{Bool},$$

$$\langle p^*, q \rangle \mapsto \bigvee_{r \in \mathbf{R}} \mathbf{d}(\langle p, r \rangle^*, \langle q, r \rangle).$$