

Definition (Compact closed category). Let $\langle \mathbf{C}, \otimes, I, \sigma \rangle$ be a symmetric monoidal category. It is called *compact closed* if, for all $C \in \mathbf{C}$ there exists some object $C^* \in \mathbf{C}$ (called the *dual of C*), a morphism $\eta_C : I \rightarrow C^* \otimes C$ (called the *unit for C*), and a morphism $\epsilon_C : C \otimes C^* \rightarrow I$ (called the *counit of C*) such that the following diagrams commute for all $C \in \mathbf{C}$:

$$\begin{array}{c}
 \begin{array}{ccccccc}
 C & \xlongequal{\hspace{10em}} & & & & & C \\
 \rho^{-1} \downarrow & & & & & & \uparrow \lambda \\
 C \otimes I & \xrightarrow{\eta} & C \otimes (C^* \otimes C) & \xrightarrow{\alpha} & (C \otimes C^*) \otimes C & \xrightarrow{\epsilon} & I \otimes C
 \end{array} \\
 \\
 \begin{array}{ccccccc}
 C^* & \xlongequal{\hspace{10em}} & & & & & C^* \\
 \lambda^{-1} \downarrow & & & & & & \uparrow \rho \\
 I \otimes C^* & \xrightarrow{\eta} & (C^* \otimes C) \otimes C^* & \xrightarrow{\alpha} & C \otimes (C \otimes C^*) & \xrightarrow{\epsilon} & C^* \otimes I
 \end{array}
 \end{array}$$