

**Lemma.** Let  $\mathbf{A}$  be any set. Its powerset  $\mathcal{P}\mathbf{A}$ , with the relation of inclusion, is a poset. View this poset as a category (this means there is a single morphism  $\mathbf{S}_1 \rightarrow \mathbf{S}_2$  if and only if  $\mathbf{S}_1 \subseteq \mathbf{S}_2$ ). For any two objects  $\mathbf{S}_1, \mathbf{S}_2 \in \mathcal{P}\mathbf{A}$ , their categorical product exists and is given by  $\mathbf{S}_1 \cap \mathbf{S}_2 \in \mathcal{P}\mathbf{A}$ .