Definition (Coproduct of DPIs)

Given two DPIs with same functionality and resources $\mathbf{d} = \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_1, \mathrm{prov}_1, \mathrm{req}_1 \rangle$ and $\mathbf{e} = \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_2, \mathrm{prov}_2, \mathrm{req}_2 \rangle$, define their co-product as

$$\mathbf{d} \sqcup \mathbf{e} := \langle \mathbf{F}, \mathbf{R}, \mathbf{I}_1 \sqcup \mathbf{I}_2, \mathrm{prov}, \mathrm{req} \rangle,$$

where

$$
\begin{aligned}
\mathrm{prov} \quad &: \quad i \mapsto \begin{cases} \mathrm{prov}_1(i), & \text{if } i \in \mathbf{I}_1, \\ \mathrm{prov}_2(i), & \text{if } i \in \mathbf{I}_2, \end{cases} \\[2em]
\mathrm{req} \quad &: \quad i \mapsto \begin{cases} \mathrm{req}_1(i), & \text{if } i \in \mathbf{I}_1, \\ \mathrm{req}_2(i), & \text{if } i \in \mathbf{I}_2. \end{cases}
\end{aligned}
\tag{0.1}
$$