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Dear editor,

We are pleased to submit a proposal for the book titled “Applied Category Theory for Engineering”, authored by Dr. Andrea Censi, Dr. Jonathan Lorand, and Gioele Zardini¹, currently employed at the Institute for Dynamic Systems and Control, ETH Zurich. We believe that this book presents a unique and valuable contribution to the fields of applied category theory and engineering, providing an accessible resource for a broad audience from engineering and applied mathematics, both in academia and in industry.

The book aims to bridge the gap between the need for abstract, compositional mathematical tools to address the complex challenges of contemporary engineering, and the lack of accessible, comprehensive resources which teach such tools in a way that is tailored to engineering. Besides providing a self-contained, pedagogical exposition of the essentials of category theory, our text covers various more advanced topics which are particularly relevant for engineering, as well as extended examples of applications, particularly in the area of complex systems analysis and design.

Features of our book include the innovative use of colors in mathematical notation, a wealth of examples, exercises and quizzes, companion code exercises in Python to highlight the computational side of the theory, and dedicated external multimedial resources which are linked with the book via QR-codes. A first-draft version of the manuscript is currently complete, and has a length of approximately 200,000 words. Considering the growing demand for applied category theory resources in engineering, we believe this book will be highly valuable both in university engineering programs and as a resource for practitioners.

We look forward to the opportunity to discuss this project further with you and explore a potential collaboration with your publishing house. Please do not hesitate to reach out if you require any additional information or have any questions.

Yours sincerely,

Andrea Censi, Jonathan Lorand, Gioele Zardini

¹Gioele Zardini is an incoming faculty at MIT, and Andrea Censi was a PI at MIT.

1 The prospectus

Title of the book: **Applied Category Theory for Engineering**

1.1 Brief description

In many engineering and applied science domains, it is advantageous to think in different levels of abstraction about modular and hierarchical decompositions, about the ways in which systems can compose and interoperate, and how patterns and insights can be translated from one context to another.

However, category theory, the mathematics particularly useful for this kind of thinking and analysis, is not traditionally taught in engineering programs, and most courses and textbooks on category theory are aimed at a mathematics audience. Several valuable resources for learning applied category theory have emerged recently, however to date there are no resources which are a) accessible to those with an engineering background, and b) focus on how applied category theory can be employed to formalize and solve real-world engineering problems.

This book aims to bridge this gap. It offers a slow-paced, accessible, rigorous, and comprehensive introduction to category theory and the ways it can be applied in various engineering contexts. It is designed to offer an engaging, multi-modal learning experience which is valuable for university lecture courses, self-learners, as well as for practitioners and experts who will use the book as a reference. Below we discuss the numerous features of the book which are all aimed at making applied category theory available to engineers, empowering them to tackle complex problems, connect insights across disciplines, and foster innovative solutions.

1.2 Outstanding features

We highlight five outstanding features:

1. As emphasized above, the book is specifically **tailored to engineers and practitioners**, setting it apart from existing efforts in the field (as described in detail in section 1.3 about competition to our book). In this context, applied category theory is a means toward analysis, design, and problem-solving in engineering. In order to emphasize concepts of particular relevance for applications, the presentation of materials diverges in some ways from many typical texts on category theory. In addition to the many example applications discussed along the way, the book culminates with an in-depth success story of the “compositional way of thinking”, presenting a theory of co-design and its application to engineering design optimization problems.
2. The book is designed to facilitate **spiral learning**, which means that, as readers progress, they incrementally and periodically encounter previous material framed in a new light which deepens their understanding. As the book becomes increasingly abstract, general, and interwoven, this provides an integrative, gradual path of learning which allows for a comprehensive and interconnected understanding of the subject matter.

The spiral learning is also supported by an **inductive exposition** style, which differs from the deductive style often used in mathematical expositions where a general mathematical structure is defined first and then afterward specific examples are given. Instead, in this book, examples are presented initially as intriguing and valuable in their own right, and as motivation for the general concepts that follow, which are then ultimately connected to the broader, overarching theory.

3. The book emphasizes not only theory but also **computation**. In particular, the book discusses how to implement various constructions in Python and provides extensive programming exercises which reinforce readers’ understanding of the concepts discussed and offers them an opportunity to gain hands-on, practical experience. The programming exercises are accompanied by an auto-grading system, which offers valuable feedback on the implemented code, allowing readers to assess their

progress and ensure their solutions meet the intended objectives. By combining theory with practical implementation, the book fosters a **comprehensive understanding** of the practical benefits that arise from adopting the “compositional way of thinking”.

4. The book employs an **innovative pedagogical use of color**, which facilitates the readability and understanding of formulas and diagrams. This visual approach enhances the reader’s comprehension by applying distinct colors to represent different elements and operations within applied category theory. In traditional treatments of this subject, a wide range of concepts and notations is introduced, each with specific contexts of use. Our color scheme addresses this potential complexity and allows for easy parsing and type-checking of formulas. We also make sure that it is accessible for those with colorblindness. A small taste of the color scheme is shown in fig. 1 below.

sets	A, B
posets	P, Q
categories	C, D
objects	<i>X, Y</i>
morphisms	<i>f</i> : <i>X</i> → <i>Y</i>
functors	<i>F</i> : C → D
natural transformations	<i>α</i> : <i>F</i> ⇒ <i>G</i>

Figure 1: Colors used to represent different concepts.

5. The book goes beyond traditional written content by incorporating **dedicated video resources** which are hosted online cross-linked from every part of the book via QR-code from. These videos serve as supplementary materials, offering dynamic explanations of concepts and step-by-step solutions to exercises. This provides additional support and guidance to readers, as well an engaging human element. Recognizing the value of multimedia in enhancing the learning experience, the authors have invested significant effort into creating this content which can also be employed in lecture courses.

1.3 Competition

Our book stands, to some degree, in competition with books which are general introductions to category theory or to applied category theory. Below we discuss the main titles of this kind which exist today, in order to compare and contrast them to our book. Our book is unique among these existing books because it has the following two qualities simultaneously:

1. **Approachable and self-contained:** Our book takes a deliberate, slow-paced approach that is accessible to readers without formal training in pure mathematics. It is written in a clear and engaging style, catering specifically to engineering students and practitioners. In particular, we often use engineering examples to motivate and introduce concepts, and our choices in topics and exposition sometimes diverge from more classical mathematical treatments, in order to address the particular relevance of certain topics for engineering applications. By presenting the material in a self-contained manner, we ensure that readers can follow along without the need of extensive mathematical background.
2. **Application-oriented for engineering:** Our book is distinct in its focus on applying category theory to engineering. We go beyond abstract concepts by incorporating real-world engineering examples to motivate and illustrate the theory.

Categories for the Working Mathematician, S. MacLane, *Graduate Texts in Mathematics*, Springer, 1978 [1]

This was the early major expository monograph on category theory and for a very long time the go-to book for mathematicians to learn category theory. Written in a terse and dense style, it is comprehensive, insightful, and a slow and challenging read, even for mathematicians. As far as introductory category theory texts go, this is likely the least accessible one. It focuses on only on pure mathematics, and assumes a good deal of prior mathematical experience.

Conceptual Mathematics, 2nd Ed., S. H. Schanuel, F. W. Lawvere, *Cambridge University Press*, 2009 [2]

This is one of the early expository monographs on category theory which aims to be broadly accessible and convey not only definitions and theorems, but also the spirit and ways of thinking used in category theory. It uses a non-traditional, conversational, lecture-based style, and focuses on basic conceptual insights rather than covering a large amount of technical material in a formally structured way. The book uses simple and insightful examples, both of a ‘pure’ and ‘applied’ flavor. However, it is still not a text which is mainly focused on *applied* category theory, let alone one that centers engineering. Nor does it cover the amount and scope of material that our book does.

Category Theory, 2nd Ed., S. Awodey, *Oxford University Press*, 2010 [3]

This was the first major ‘mathematics-style’ textbook after MacLane’s *Categories for the working mathematician*. It covers a good deal of standard category theory material, and it aims to be much more accessible than MacLane’s book, both in terms of prerequisites needed and in terms of its pedagogical style. It is addressed not only to mathematics students, but also expressly to readers with a background in computer science, logic, linguistics, or philosophy. For this reason, the book covers material for example on cartesian closed categories and the lambda calculus. Despite its broader audience, the book remains, however, one written in a mathematical style. Furthermore, the applied areas that it does address are not in engineering. Our book differs both in scope and in style.

Basic Category Theory, T. Leinster, *Cambridge University Press*, 2014 [4]

This is an excellently-written, introductory book on category theory. Relatively short in length and covering less material than, for example, Riehl’s *Category theory in context* (see below), this book is aimed at mathematicians and provides a concise, pedagogical, and elegant exposition of core category theory topics for such an audience. It does not however cover some theoretical topics which are essential for applied category theory (for example, monoidal categories), nor does it aim to discuss examples and applications outside of mathematics.

Category Theory for the Sciences, D. I. Spivak, *MIT Press*, 2014 [5]

This text is arguably the first book on applied category theory proper and a pioneering contribution to the field. It is aimed both at scientists and at mathematicians interested in using category theory in applied domains. It is written to be accessible to a broad audience, and does so by teaching via examples and exercises, rather than theorems and proofs. The main differences with our book are scope – our book aims to cover a much larger amount of category theory material and applications thereof – and the fact that our book is focused expressly on engineering. Furthermore, although we provide many examples and exercises, our book has theorems and proofs, too. This is because some of our intended readers will want to understand the theory in this level of formal depth, and it is important for their work to learn to do so, to some degree. Nevertheless, our book remains accessible for those who wish to skip over proofs.

Category Theory in Context, E. Riehl, *Aurora: Dover Modern Math Originals*, 2016 [6]

This is another well-known textbook for (pure) category theory, written in a modern style and conveying some of the categorical ‘way of thinking’ alongside the formal mathematics. Although certainly more accessible than MacLane, it is nevertheless aimed at readers with a moderate level of mathematical training and maturity, and is also comparatively dense, covering a good deal of material relative to its length. It

is a wonderful book for mathematicians which emphasizes how to use category theory fruitfully inside of mathematics; however it is not realistically accessible for readers without mathematical training, nor does it emphasize applications outside of mathematics.

An Invitation to Applied Category Theory: Seven Sketches in Compositionality, B. Fong, D. I. Spivak, Cambridge University Press, 2019 [7]

This book is, in two respects, very close to fulfilling our aims: it is written in an accessible style intended for a very broad audience and it is focused on applications of category theory outside of mathematics. The two main differences to our book are that this one is not focused mainly on engineering applications, and it is not as comprehensive as ours. As the word ‘sketch’ suggests, some of the topics in Fong and Spivak’s book are only touched upon briefly, in order to introduce an important idea and whet the readers appetite to learn more through other resources. In contrast, our book offers more thorough treatments of many important topics, illustrating and spelling out details in depth so that readers without much mathematical training are able to follow even technical aspects step-by-step and connect the technical theory with examples and applications.

The Joy of Abstraction: An Exploration of Math, Category Theory, and Life, E. Cheng, Cambridge University Press, 2022 [8]

This book is aimed at a very broad audience – it does not assume any university-level mathematical training – yet it builds up to introducing the core essential material of category theory, including technical details, so that readers might then be able to read other introductory books on category theory without struggling too much. It is accessible and pedagogical, and conveys also a conceptual and intuitive understanding of the categorical way of thinking. A main difference to our book is that, while Cheng’s book uses examples that include various aspects of daily life, it is not focused on applying category theory in the sciences or in engineering specifically. Our book, in contrast, covers more – and other – kinds of material, both in terms of theory and in terms of examples and exercises.

We are aware of a few further books [9, 10, 11] which are also mathematics-focused introductions to basic category theory. Because they are subsumed in content and expository nature by the books discussed above, we do not see the need to describe these in detail. However, we mention them for completeness’ sake.

Other books that are related to ours, but do not stand in competition, are books which focus explicitly on applications of category theory in specific domains other than engineering, and include a somewhat accessible introduction to category theory (most specialized books do not). There are books, for example, which cover topics of category theory in computer science [12, 13, 14] and physics [15].

1.4 Apparatus

The book encompasses a comprehensive set of learning resources to enhance understanding and engagement. It features a range of examples, pen-and-paper exercises and quizzes, as well as computational exercises implemented in Python. Solutions to the pen-and-paper exercises and quizzes will be readily available, while the computational exercises will be automatically graded using a system that we have developed and refined while teaching lectures on this topic at ETH Zurich. In addition to these learning components, short expository videos will be available online and cross-linked with the book via QR-codes. Finally, the book also includes glossaries, a bibliography, and appendices to further support readers in their exploration of the subject matter.

To cater to instructors and their teaching needs, we have developed an “instructors” version of the book which provides additional resources to those who wish to teach a course based on materials in the book. Through the combination of examples, exercises, automated grading, and supplementary resources, our

book aims to create a comprehensive and multimodal learning experience that fosters engagement and facilitates effective instruction.

1.5 Audience

The book is specifically tailored to engineering students, academic and professional engineers, and anyone interested in using category theory in engineering contexts. As mentioned in the opening description, our text is intended equally for university lecture courses, self-learning, and as a go-to reference for practitioners and experts. The book is self-contained because it introduces the essentials of logic, set-theory, algebra, and order-theory that are needed to then learn the basics of category theory and the applications we present. However a foundational understanding of linear algebra and algebra at a level typical for a bachelor's degree in engineering or computer science is nevertheless advantageous as prior knowledge, in order to more easily dive into the material and appreciate its depth, breadth, and value.

In the past, we have successfully taught several year's worth of lectures at ETH Zurich using a subset of the material of the book, and we also organized an online course that reached a large international audience. Through these experiences we have observed a strong interest from students and professionals alike for the topics covered in the book.

1.6 Authors

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Bio: Andrea Censi is deputy director of the Dynamic Systems and Control chair at ETH Zurich. He obtained a Ph.D. from Caltech. Previously, he has been a research scientist at MIT and Director of Research at Aptiv Mobility (now Motional). He is president of the Duckietown Foundation.

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Bio: Jonathan is a postdoctoral researcher working as a mathematician at the Institute for Dynamic Systems and Control at ETH Zurich. He obtained his PhD in mathematics from the University of Zurich in 2020, working in symplectic geometry and category theory. Since then he is focused on applied category theory, as well on the topic of transdisciplinarity, in which he will obtain an MA in 2024 from the Zurich University of the Arts.

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Bio: Gioele is a Ph.D. candidate at ETH Zurich, and incoming faculty at Massachusetts Institute of Technology. He received his BSc. and MSc. in Mechanical Engineering with focus in Robotics, Systems and Control from ETH Zurich in 2017 and 2019, respectively. He spent time in Singapore as a researcher at nuTonomy (now Motional), at Stanford University, and at MIT.

All three authors have experience in teaching various courses (both at the undergraduate and graduate level) at ETH Zurich and the University of Zurich.

1.7 Market considerations

One of the great engineering challenges of our time revolves around the design of complex systems. However, the conventional mathematics taught traditionally falls short in providing practical solutions and aiding reasoning in this context. In recent years, there has been a remarkable surge of interest in applied category theory. What started as a small community of mathematicians has rapidly evolved into a global movement, with representatives from academia and industry alike. This growing interest underscores the recognition of the potential and importance that applied category theory holds in addressing complex system design challenges. Based on these developments, we believe that lecture courses based on the material in our book will become a staple for at least 50% of the major universities within the next seven years, becoming an integral part of engineering programs worldwide.

In parallel with drafting this book and teaching applied category theory lectures at ETH Zurich, we have organized international online classes and conducted dissemination workshops at prominent international conferences, including the IEEE International Conference on Robotics and Automation and the IEEE International Conference on Intelligent Transportation Systems, focusing on domains such as robotics, systems and control, and intelligent transportation systems. These efforts have enabled us to establish a vast and diverse network of individuals who share a keen interest in this subject. By fostering collaboration, disseminating knowledge, and nurturing a vibrant community, we aim to drive the widespread adoption of applied category theory in engineering, transforming the way complex systems are approached and designed.

1.8 Status of the book

The initial draft version of the book is close to completion and requires adaptation to a standard format by the publisher. We anticipate that the final manuscript will be fully completed within the next twelve months, if not sooner. It is expected to encompass approximately 200,000 words, offering a comprehensive exploration of the subject matter.

Visual aids play a prominent role in the book, with an average of two figures included on each page. These figures are vector-based, typically comprising mathematical diagrams which enhance understanding and visualization. Additionally, each chapter is enriched with a full-color photograph, adding an engaging visual element to complement the textual content. The inclusion of abundant visual elements serves to enhance the reader's comprehension and create an aesthetically pleasing reading experience. By effectively utilizing figures and photographs, the book provides both clarity and visual appeal, contributing to an immersive learning journey.

Finally, we are also in the process of producing the completing the Python code exercises, the quizzes, and the supplementary videos that are all part of the multi-modal learning materials integrated with the book. This work is supported in part by a recent grant we received from ETH Zurich which supports innovative teaching initiatives.

A current PDF version of the book draft may be found [here](#).

1.9 Reviewers

Here is as a list of reviewers who would be able to provide expert, constructive feedback on our manuscript:

- Prof. Michael Johnson, Macquarie University, Michael.Johnson@mq.edu.au;
- Prof. John Baez, University of California, Riverside, and Topos Institute, john.baez@ucr.edu;
- Prof. Daniel Koditschek, University of Pennsylvania, kod@seas.upenn.edu;
- Prof. Aaron Ames, California Institute of Technology, ames@cds.caltech.edu;
- Prof. Paulo Tabuada, University of California, Los Angeles, tabuada@ee.ucla.edu;
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- Prof. Gregory Chirikjian, National University of Singapore, mpegre@nus.edu.sg;
- Dr. Paolo Perrone, University of Oxford, paolo.perrone@cs.ox.ac.uk;
- Dr. David Spivak, Topos Institute, dspivak@gmail.com;
- Dr. Brendan Fong, Topos Institute, brendan@topos.institute;

2 Table of contents

Please find the current table of contents as an attachment.

3 Sample chapters

Sample chapters may be found in the attachment.

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- [1] S. Mac Lane, *Categories for the working mathematician*. Springer Science & Business Media, 1978.
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