

Definition (Monoid isomorphism). A morphism of monoids $F : \mathbf{M} \rightarrow \mathbf{N}$ is called a *monoid isomorphism* if there is a morphism of monoids $G : \mathbf{N} \rightarrow \mathbf{M}$ such that

$$F \circ G = \text{Id}_{\mathbf{M}} \quad \text{and} \quad G \circ F = \text{Id}_{\mathbf{N}}.$$