Definition. A *category* $\mathbf{C}$ is specified by:

Constituents

1. Objects: A set $\mathrm{Ob}_{\mathbf{C}}$ whose elements are called *objects*.
2. Morphisms: For every pair of objects X, Y in $\mathrm{Ob}_{\mathbf{C}}$, there is a set $\mathrm{Hom}_{\mathbf{C}}(X; Y)$, elements of which are called *morphisms* from X to Y. A morphism $f \in \mathrm{Hom}_{\mathbf{C}}(X; Y)$ is often indicated by writing

$$f : X \to Y.$$

3. Identity morphisms: for each object X, there is a morphism $\mathrm{Id}_X : X \to X$ called *the identity morphism of* X.
4. Composition operations: For every three objects X, Y, Z in $\mathrm{Ob}_{\mathbf{C}}$ there is a composition function

$$\fatsemi_{X,Y,Z} : \mathrm{Hom}_{\mathbf{C}}(X; Y) \times \mathrm{Hom}_{\mathbf{C}}(Y; Z) \to \mathrm{Hom}_{\mathbf{C}}(X; Z)$$

We denote the *composition* of composable morphisms f and g by $f \fatsemi g$. (Traditionally, the typical notation would be $g \circ f$).

Conditions

1. Associativity: for all composable morphisms f, g, h,

$$(f \fatsemi g) \fatsemi h = f \fatsemi (g \fatsemi h).$$

2. Unitality: for every morphism $f : X \to Y$,

$$\mathrm{Id}_X \fatsemi f = f \qquad \text{and} \qquad f \fatsemi \mathrm{Id}_Y = f.$$