Definition (Traced monoidal category)
A symmetric monoidal category $\langle \mathbf{C}, \otimes, \mathbf{1}, \mathrm{br} \rangle$ is said to be *traced* if it is equipped with a family of functions

$$\mathrm{Tr}^{Z}_{X,Y} : \mathrm{Hom}_{\mathbf{C}}(X \otimes Z; Y \otimes Z) \to \mathrm{Hom}_{\mathbf{C}}(X; Y),$$

satisfying the following axioms:

1. *Naturality in* X: For all morphisms $f : X \otimes Z \to Y \otimes Z$ and $g : X' \to X$,
$$\mathrm{Tr}^{Z}_{X',Y}((g \otimes \mathrm{Id}_Z) \fatsemi f) = g \fatsemi \mathrm{Tr}^{Z}_{X,Y}(f)$$
2. *Naturality in* Y: For all morphisms $f : X \otimes Z \to Y \otimes Z$ and $g : Y \to Y'$,
$$\mathrm{Tr}^{Z}_{X,Y'}(f \fatsemi (g \otimes \mathrm{Id}_Z)) = \mathrm{Tr}^{Z}_{X,Y}(f) \fatsemi g$$
3. *Vanishing*: For all morphisms $f : X \to Y$ in $\mathbf{C}$,
$$\mathrm{Tr}^{\mathbf{1}}_{X,Y}(f) = f.$$
Furthermore, for all morphisms $f : X \otimes Z \otimes U \to Y \otimes Z \otimes U$ in $\mathbf{C}$,
$$\mathrm{Tr}^{Z \otimes U}_{X,Y}(f) = \mathrm{Tr}^{Z}_{X,Y}\left(\mathrm{Tr}^{U}_{X \otimes Z, Y \otimes Z}(f)\right).$$
4. *Superposing*: For all morphisms $f : X \otimes Z \to Y \otimes Z$ in $\mathbf{C}$,
$$\mathrm{Tr}^{Z}_{V \otimes X, V \otimes Y}(\mathrm{Id}_V \otimes f) = \mathrm{Id}_V \otimes \mathrm{Tr}^{Z}_{X,Y}(f).$$
5. *Yanking*:
$$\mathrm{Tr}^{Z}_{Z,Z}(\mathrm{br}_{Z,Z}) = \mathrm{Id}_Z.$$