

**Lemma.** Given  $f_1, f_2 \in \text{Hom}_{\mathbf{DP}}(C \times A; C \times B)$ , one has:

$$\frac{f_1 \leq_{\mathbf{DP}} f_2}{\text{Tr}_{A,B}^C(f_1) \leq_{\mathbf{DP}} \text{Tr}_{A,B}^C(f_2)}$$

In other words, trace preserves order on  $\mathbf{DP}$ .