Definition

A *category* **C** is specified by:

<u>Constituents</u>

1. Objects: A set $\mathrm{Ob}_\mathbf{C}$ whose elements are called *objects*.

2. Morphisms: For every pair of objects X, Y in $\mathrm{Ob}_\mathbf{C}$, there is a set $\mathrm{Hom}_\mathbf{C}(X; Y)$, elements of which are called *morphisms* from X to Y. A morphism $f \in \mathrm{Hom}_\mathbf{C}(X; Y)$ is often indicated by writing

$$f : X \to Y.$$

3. Identity morphisms: for each object X, there is a morphism $\mathrm{Id}_X : X \to X$ called *the identity morphism of X*.

4. Composition operations: For every three objects X, Y, Z in $\mathrm{Ob}_\mathbf{C}$ there is a composition function

$$\mathring{\mathbf{9}}_{X,Y,Z} : \mathrm{Hom}_\mathbf{C}(X; Y) \times \mathrm{Hom}_\mathbf{C}(Y; Z) \to \mathrm{Hom}_\mathbf{C}(X; Z)$$

We denote the *composition* of composable morphisms f and g by $f \mathbin{\mathring{\mathbf{9}}} g$. (Traditionally, the typical notation would be $g \circ f$).

<u>Conditions</u>

1. Associativity: for all composable morphisms f, g, h,

$$(f \mathbin{\mathring{\mathbf{9}}} g) \mathbin{\mathring{\mathbf{9}}} h = f \mathbin{\mathring{\mathbf{9}}} (g \mathbin{\mathring{\mathbf{9}}} h).$$

2. Unitality: for every morphism $f : X \to Y$,

$$\mathrm{Id}_X \mathbin{\mathring{\mathbf{9}}} f = f \qquad \text{and} \qquad f \mathbin{\mathring{\mathbf{9}}} \mathrm{Id}_Y = f.$$