$$\alpha \geq_{\mathcal{AR}} \operatorname*{Min}_{\leq_R} \bigcup_{r \in \alpha} h_{f_1}(r) \cap \uparrow r.$$