

**Lemma.** For  $A, B, C, D \in \mathcal{AP}$ ,  $A \leq_{AR} C$  and  $B \leq_{AR} D$  implies  $A \cup B \leq_{AR} C \cup D$ .