Definition (Identity homomorphism). Let $\mathbf{S}$ be a semigroup. The identity function $\mathrm{Id}_{\mathbf{S}} : \mathbf{S} \to \mathbf{S}$ is always a morphism of semigroups. Indeed, the condition

$$\mathrm{Id}(s_1 \mathbin{\circ_{\mathbf{S}}} s_2) = \mathrm{Id}(s_1) \mathbin{\circ_{\mathbf{S}}} \mathrm{Id}(s_2) \quad \forall s_1, s_2 \in \mathbf{S}$$

is satisfied. We call this the *identity homomorphism* of $\mathbf{S}$.