

Lemma. $\text{Pos}_\mathcal{U}$ and $\text{Pos}_\mathcal{L}$ are categories.