Definition ($\mathcal{U}$ monad)

The $\mathcal{U}$ *monad* on **Pos** consists of:

1. The U endofunctor (**??**).
2. The unit natural transformation $\mathrm{un}_{\mathcal{U}} : \mathrm{Id}_{\mathbf{Pos}} \Rightarrow U$, which associates to every object $\mathbf{P} \in \mathrm{Ob}_{\mathbf{Pos}}$ a morphisms in **Pos** given by:

$$\mathrm{un}_{\mathcal{U}}^{\mathbf{P}} : \mathbf{P} \to \mathcal{U}\mathbf{P}$$
$$p \mapsto \uparrow \{p\}.$$

3. The compositional natural transformation $\mathrm{mu}_{\mathcal{U}} : U \fatsemi U \Rightarrow U$, which associates to every $\mathbf{P} \in \mathrm{Ob}_{\mathbf{Pos}}$ the morphism in **Pos** given by:

$$\mathrm{mu}_{\mathcal{U}}^{\mathbf{P}} : \mathcal{U}(\mathcal{U}\mathbf{P}) \to \mathcal{U}\mathbf{P}$$
$$\mathbf{P}'' \mapsto \bigcup_{\mathbf{P}' \in \mathbf{P}''} \mathbf{P}'.$$