Definition (Traced monoidal category)
A symmetric monoidal category $\langle \mathbf{C}, \otimes, \mathbf{1}, \mathrm{br} \rangle$ is said to be *traced* if it is equipped with a family of functions

$$\mathrm{Tr}^{Z}_{X,Y} : \mathrm{Hom}_{\mathbf{C}}(X \otimes Z; Y \otimes Z) \to \mathrm{Hom}_{\mathbf{C}}(X; Y),$$

satisfying the following axioms:

1. *Naturality in X:* For all morphisms $f : X \otimes Z \to Y \otimes Z$ and $g : X' \to X$,
$$\mathrm{Tr}^{Z}_{X',Y}((g \otimes \mathrm{Id}_Z) \mathbin{⨟} f) = g \mathbin{⨟} \mathrm{Tr}^{Z}_{X,Y}(f)$$

2. *Naturality in Y:* For all morphisms $f : X \otimes Z \to Y \otimes Z$ and $g : Y \to Y'$,
$$\mathrm{Tr}^{Z}_{X,Y'}(f \mathbin{⨟} (g \otimes \mathrm{Id}_Z)) = \mathrm{Tr}^{Z}_{X,Y}(f) \mathbin{⨟} g$$

3. *Vanishing:* For all morphisms $f : X \to Y$ in $\mathbf{C}$,
$$\mathrm{Tr}^{\mathbf{1}}_{X,Y}(f) = f.$$
Furthermore, for all morphisms $f : X \otimes Z \otimes U \to Y \otimes Z \otimes U$ in $\mathbf{C}$,
$$\mathrm{Tr}^{Z \otimes U}_{X,Y}(f) = \mathrm{Tr}^{Z}_{X,Y}\left(\mathrm{Tr}^{U}_{X \otimes Z, Y \otimes Z}(f)\right).$$

4. *Superposing:* For all morphisms $f : X \otimes Z \to Y \otimes Z$ in $\mathbf{C}$,
$$\mathrm{Tr}^{Z}_{V \otimes X, V \otimes Y}(\mathrm{Id}_V \otimes f) = \mathrm{Id}_V \otimes \mathrm{Tr}^{Z}_{X,Y}(f).$$

5. *Yanking:*
$$\mathrm{Tr}^{Z}_{Z,Z}(\mathrm{br}_{Z,Z}) = \mathrm{Id}_Z.$$