Lemma. Given any $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_{\mathscr{U}}}$, $\mathrm{Hom}_{\mathbf{Pos}_{\mathscr{L}}}(X; Y)$ is a bounded lattice with intersection $\wedge$ of morphisms in $\mathbf{Pos}_{\mathscr{L}}$ as meet, union $\vee$ of morphisms in $\mathbf{Pos}_{\mathscr{L}}$ as join, least upper bound $\top_{\mathrm{Hom}_{\mathbf{Pos}_{\mathscr{L}}}(X;Y)} : X \to Y$ given by

$$\begin{aligned} \top_{\mathrm{Hom}_{\mathbf{Pos}_{\mathscr{L}}}(X;Y)}^{\star} : X &\to \mathscr{L}Y \\ x &\mapsto Y, \end{aligned}$$

and greatest lower bound $\perp_{\mathrm{Hom}_{\mathbf{Pos}_{\mathscr{U}}}(X;Y)} : X \to Y$ given by

$$\begin{aligned} \perp_{\mathrm{Hom}_{\mathbf{Pos}_{\mathscr{L}}}(X;Y)}^{\star} : X &\to \mathscr{L}Y \\ x &\mapsto \emptyset. \end{aligned}$$