

## Definition (Pre-ordered set)

A *pre-ordered* set is a tuple  $\mathbf{P} = \langle \mathbf{P}, \leq_{\mathbf{P}} \rangle$ , where  $\mathbf{P}$  is a set (also called the *carrier set* or *underlying set*), together with a relation  $\leq_{\mathbf{P}}$  that is:

▷ *Reflexive*:

$$\frac{\top}{p \leq_{\mathbf{P}} p}.$$

▷ *Transitive*:

$$\frac{p \leq_{\mathbf{P}} q \quad q \leq_{\mathbf{P}} r}{p \leq_{\mathbf{P}} r}.$$