

**Definition** (Series composition). Let  $f : \mathbf{A} \multimap \mathbf{B}$  and  $g : \mathbf{B} \multimap \mathbf{C}$  be design problems. We define their *series composition*  $(f \circ g) : \mathbf{A} \multimap \mathbf{C}$  as:

$$(f \circ g) : \mathbf{A}^{\text{op}} \times \mathbf{C} \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle a^*, c \rangle \mapsto \bigvee_{b \in \mathbf{B}} f(a^*, b) \wedge g(b^*, c).$$

Alternatively:

$$(f \circ g) : \mathbf{A}^{\text{op}} \times \mathbf{C} \rightarrow_{\text{Pos}} \mathbf{Bool},$$

$$\langle a^*, c \rangle \mapsto \bigvee_{b_1 \leq b_2, b_1, b_2 \in \mathbf{B}} f(a^*, b_1) \wedge g(b_2^*, c).$$