

**Definition** (Directed set). A set  $S \subseteq \mathbf{P}$  is *directed* if each pair of elements in  $S$  has an upper bound: for all  $a, b \in S$ , there exists  $c \in S$  such that  $a \leq c$  and  $b \leq c$ .