$$\uparrow \left(\bigcup_{p \in \uparrow \bigcup_{i \in I} S_i} \{ f(p) \} \right) = \uparrow \left(\bigcup_{i \in I} \{ f(s_i) \} \right)$$