

**Lemma.** Given a set  $\mathbf{A}$  and a family of subsets of  $\mathbf{A}$ , denoted  $\{\mathbf{A}_i\}_{i \in \mathbf{I}}$  (with index set  $\mathbf{I}$ ), one has:

$$\bigcup_{i \in \mathbf{I}} \bigcup \{\mathbf{A}_i\} = \bigcup_{i \in \mathbf{I}} \{\mathbf{A}_i\}.$$