Definition ($\mathcal{U}$ monad). The $\mathcal{U}$ *monad* on **Pos** consists of:

1. The U endofunctor (**??**).

2. The unit natural transformation $\text{un}_\mathcal{U} : \text{Id}_{\textbf{Pos}} \Rightarrow U$, which associates to every object $\mathbf{P} \in \text{Ob}_{\textbf{Pos}}$ a morphisms in **Pos** given by:

$$\text{un}_\mathcal{U}^{\mathbf{P}} : \mathbf{P} \to \mathcal{U}\mathbf{P}$$

$$p \mapsto \uparrow \{p\}.$$

3. The compositional natural transformation $\text{mu}_\mathcal{U} : U \mathbin{\mathring{,}} U \Rightarrow U$, which associates to every $\mathbf{P} \in \text{Ob}_{\textbf{Pos}}$ the morphism in **Pos** given by:

$$\text{mu}_\mathcal{U}^{\mathbf{P}} : \mathcal{U}(\mathcal{U}\mathbf{P}) \to \mathcal{U}\mathbf{P}$$

$$\mathbf{P}'' \mapsto \bigcup_{\mathbf{P}' \in \mathbf{P}''} \mathbf{P}'.$$