

## Definition (Opposite category)

Given a category  $\mathbf{C}$ , its *opposite category*  $\mathbf{C}^{\text{op}}$  is specified by:

1. *Objects*:  $\text{Ob}_{\mathbf{C}^{\text{op}}} = \text{Ob}_{\mathbf{C}}$ . Given  $X \in \text{Ob}_{\mathbf{C}}$ , we will sometimes (though not always) write  $X^{\text{op}}$  to signify when we are thinking of  $X$  as an object of  $\text{Ob}_{\mathbf{C}^{\text{op}}}$ .
2. *Morphisms*: Given objects  $X^{\text{op}}, Y^{\text{op}} \in \text{Ob}_{\mathbf{C}^{\text{op}}} = \text{Ob}_{\mathbf{C}}$ ,

$$\text{Hom}_{\mathbf{C}^{\text{op}}}(X^{\text{op}}; Y^{\text{op}}) := \text{Hom}_{\mathbf{C}}(Y; X).$$

Given  $f \in \text{Hom}_{\mathbf{C}}(Y; X)$ , when we are thinking of it as an element of  $\text{Hom}_{\mathbf{C}^{\text{op}}}(X^{\text{op}}; Y^{\text{op}})$ , we will sometimes write  $f^{\text{op}}$ .

3. *Identity morphisms*: Given  $X^{\text{op}} \in \text{Ob}_{\mathbf{C}^{\text{op}}}$ , its identity morphism is

$$\text{Id}_{X^{\text{op}}} := \text{Id}_X^{\text{op}}.$$

4. *Composition*: Let morphisms  $f^{\text{op}} \in \text{Hom}_{\mathbf{C}^{\text{op}}}(X^{\text{op}}; Y^{\text{op}})$  and  $g^{\text{op}} \in \text{Hom}_{\mathbf{C}^{\text{op}}}(Y^{\text{op}}; Z^{\text{op}})$ , then

$$f^{\text{op}} \circ_{\mathbf{C}^{\text{op}}} g^{\text{op}} := (g \circ_{\mathbf{C}} f)^{\text{op}}.$$