

## Definition ( $\mathcal{U}$ monad)

The  $\mathcal{U}$  monad on **Pos** consists of:

1. The  $\mathcal{U}$  endofunctor (??).
2. The unit natural transformation  $\text{un}_{\mathcal{U}} : \text{Id}_{\mathbf{Pos}} \Rightarrow \mathcal{U}$ , which associates to every object  $\mathbf{P} \in \text{Ob}_{\mathbf{Pos}}$  a morphism in **Pos** given by:

$$\begin{aligned} \text{un}_{\mathcal{U}}^{\mathbf{P}} : \mathbf{P} &\rightarrow \mathcal{U}\mathbf{P} \\ p &\mapsto \uparrow \{p\}. \end{aligned}$$

3. The compositional natural transformation  $\text{mu}_{\mathcal{U}} : \mathcal{U} \circ \mathcal{U} \Rightarrow \mathcal{U}$ , which associates to every  $\mathbf{P} \in \text{Ob}_{\mathbf{Pos}}$  the morphism in **Pos** given by:

$$\begin{aligned} \text{mu}_{\mathcal{U}}^{\mathbf{P}} : \mathcal{U}(\mathcal{U}\mathbf{P}) &\rightarrow \mathcal{U}\mathbf{P} \\ \mathbf{P}'' &\mapsto \bigcup_{\mathbf{P}' \in \mathbf{P}''} \mathbf{P}'. \end{aligned}$$