

Definition (Companion and conjoint). Let \mathbf{P} and \mathbf{Q} be posets, and suppose that $f : \mathbf{P} \rightarrow_{\mathbf{Pos}} \mathbf{Q}$ is a monotone map. We define its *companion* in \mathbf{DP} , denoted $\hat{f} : \mathbf{P} \rightarrowtail \mathbf{Q}$, and its *conjoint*, denoted $\check{f} : \mathbf{Q} \rightarrowtail \mathbf{P}$ as

$$\hat{f}(p^*, q) \doteq f(p) \leq_{\mathbf{Q}} q \quad \text{and} \quad \check{f}(q^*, p) \doteq q \leq_{\mathbf{P}} f(p).$$