## **Definition** (Opposite category)

Given a category C, its opposite category Cop is specified by:

- 1. *Objects*:  $Ob_{\mathbb{C}^{op}} = Ob_{\mathbb{C}}$ . Given  $X \in Ob_{\mathbb{C}}$ , we will sometimes (though not always) write  $X^{op}$  to signify when we are thinking of X as an object of  $Ob_{\mathbb{C}}^{op}$ .
- 2. Morphisms: Given objects  $X^{op}$ ,  $Y^{op} \in Ob_{\mathbb{C}^{op}} = Ob_{\mathbb{C}}$ ,

$$\operatorname{Hom}_{\mathbf{C}^{\operatorname{op}}}(X^{\operatorname{op}};Y^{\operatorname{op}}) := \operatorname{Hom}_{\mathbf{C}}(Y;X).$$

Given  $f \in \text{Hom}_{\mathbf{C}}(Y;X)$ , when we are thinking of it as an element of  $\text{Hom}_{\mathbf{C}^{op}}(X^{op};Y^{op})$ , we will sometimes write  $f^{op}$ .

3. *Identity morphisms*: Given  $X^{op} \in Ob_{\mathbb{C}^{op}}$ , its identity morphism is

$$\operatorname{Id}_{X^{\operatorname{op}}} := \operatorname{Id}_{X}^{\operatorname{op}}.$$

4. Composition: Let morphisms  $f^{op} \in \text{Hom}_{\mathbb{C}^{op}}(X^{op}; Y^{op})$  and  $g^{op} \in \text{Hom}_{\mathbb{C}^{op}}(Y^{op}; Z^{op})$ , then

$$f^{\mathrm{op}} \circ_{\mathbf{C}^{\mathrm{op}}} g^{\mathrm{op}} := (g \circ_{\mathbf{C}} f)^{\mathrm{op}}.$$