

Definition (Trace of a design problem). Given a design problem $\mathbf{d} : \mathbf{P} \times \mathbf{R} \dashrightarrow \mathbf{Q} \times \mathbf{R}$, we can define its *trace* $\text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}) : \mathbf{P} \dashrightarrow \mathbf{Q}$ as follows:

$$\text{Tr}_{\mathbf{P}, \mathbf{Q}}^{\mathbf{R}}(\mathbf{d}) : \mathbf{P}^{\text{op}} \times \mathbf{Q} \rightarrow_{\text{Pos}} \mathbf{Bool}$$

$$\langle a^*, b \rangle \mapsto \bigvee_{c \in \mathbf{R}} \mathbf{d}(\langle a, c \rangle^*, \langle b, c \rangle).$$