

Definition (Subcategory)

A *sub(semi)category* \mathbf{D} of a (semi)category \mathbf{C} is a category for which:

1. All the objects in $\mathbf{Ob}_{\mathbf{D}}$ are in $\mathbf{Ob}_{\mathbf{C}}$;
2. For any two objects $X, Y \in \mathbf{Ob}_{\mathbf{D}}$, the morphisms of \mathbf{D} between them are a subsets of the morphisms of \mathbf{C} :

$$\mathbf{Hom}_{\mathbf{D}}(X; Y) \subseteq \mathbf{Hom}_{\mathbf{C}}(X; Y);$$

3. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ in \mathbf{D} , then the composite $f \circ g$ in \mathbf{C} is in \mathbf{D} and represents the composite in \mathbf{D} .
4. (Categories) If $X \in \mathbf{Ob}_{\mathbf{D}}$, then the identity Id_X in $\mathbf{Hom}_{\mathbf{C}}(X; X)$ is also in $\mathbf{Hom}_{\mathbf{D}}(X; X)$ and acts as its identity morphism.