Lemma. Let $\langle S, \rangle$ be a semigroup. If there exists elements $1 \in S$ and $1' \in S$

such that $\langle S, 3, 1 \rangle$ and $\langle S, 3, 1' \rangle$ are each monoids, then 1 = 1' must hold. In

other words, the neutral element of a monoid is uniquely determined by the

underlying semigroup structure.