$$f_k \in \left\langle \mathbf{F}_k, \preceq_{\mathbf{F}_k} \right\rangle$$