Definition (Traced monoidal category). A symmetric monoidal category $\langle \mathbf{C}, \otimes, \{\bullet\}, \sigma \rangle$ is said to be *traced* if equipped with a family of functions

$$\mathrm{Tr}^X_{A,B} : \mathbf{C}(A \otimes X, B \otimes X) \to \mathbf{C}(A, B),$$

satisfying the following axioms:

1. *Vanishing:* For all morphisms $f : A \to B$ in $\mathbf{C}$,

$$\mathrm{Tr}^1_{A,B}(f) = f.$$

   Furthermore, for all morphisms $f : A \otimes X \otimes Y \to B \otimes X \otimes Y$ in $\mathbf{C}$:

$$\mathrm{Tr}^{X \otimes Y}_{A,B}(f) = \mathrm{Tr}^X_{A,B}\left(\mathrm{Tr}^Y_{A \otimes X, B \otimes X}(f)\right).$$

2. *Superposing:* For all morphisms $f : A \otimes X \to B \otimes X$ in $\mathbf{C}$:

$$\mathrm{Tr}^X_{C \otimes A, C \otimes B}(\mathrm{id}_C \otimes f) = \mathrm{id}_C \otimes \mathrm{Tr}^X_{A,B}(f).$$

3. *Yanking:*

$$\mathrm{Tr}^X_{X,X}\left(\sigma_{X,X}\right) = \mathrm{id}_X .$$