Definition (Category **Rel**). The category of relations **Rel** is given by:

- 1. *Objects*: The objects of this category are all sets.
- 2. *Morphisms*: Given sets X, Y, the homset $\operatorname{Hom}_{\operatorname{Rel}}(X;Y)$ consists of all relations $\mathbb{R} \subseteq X \times Y$.
- 3. *Identity morphisms*: Given a set X, its identity morphism is

$$\operatorname{Id}_{X} := \{\langle x, x \rangle \mid x \in X\}.$$

4. Composition: Given relations $R: X \to Y$, $S: Y \to Z$, their composition is given by

$$\mathbf{R} \, \, \, \, \, \, \, \mathbf{S} \, := \{ \langle x, z \rangle \mid \exists y \in Y : \ \, (\langle x, y \rangle \in \mathbf{R}) \land (\langle y, z \rangle \in \mathbf{S}) \}.$$