

Definition (Group). A *group* is a monoid together with an “inverse” operation. In more detail, a group \mathbf{M} is

Constituents

1. a set \mathbf{M} ;
2. a binary operation $\circ : \mathbf{M} \times \mathbf{M} \rightarrow \mathbf{M}$, called *composition*;
3. a specified element $\text{id} \in \mathbf{M}$;
4. a map $\text{inv} : \mathbf{M} \rightarrow \mathbf{M}$ called “inverse”.

Conditions

1. Associative law: $(x \circ y) \circ z = x \circ (y \circ z)$;
2. Neutrality Laws: $\text{id} \circ x = x = x \circ \text{id}$.
3. Inverse law:

$$\text{inv}(x) \circ x = \text{id} = \text{inv}(x) \circ x$$