

**Definition** (Identity homomorphism). Let  $\mathbf{S}$  be a semigroup. The identity function  $\text{Id}_{\mathbf{S}} : \mathbf{S} \rightarrow \mathbf{S}$  is always a morphism of semigroups. Indeed, the condition

$$\text{Id}(s_1 \circ_{\mathbf{S}} s_2) = \text{Id}(s_1) \circ_{\mathbf{S}} \text{Id}(s_2) \quad \forall s_1, s_2 \in \mathbf{S}$$

is satisfied. We call this the *identity homomorphism* of  $\mathbf{S}$ .