Lemma. $\mathbf{Pos}_{\mathscr{U}}$ and $\mathbf{Pos}_{\mathscr{L}}$ are equivalent: there exists a pair of functors

$$\swarrow : \mathbf{Pos}_{\mathscr{U}} \to \mathbf{Pos}_{\mathscr{L}},$$
$$\nearrow : \mathbf{Pos}_{\mathscr{L}} \to \mathbf{Pos}_{\mathscr{U}},$$

such that $\swarrow \mathbin{⨟} \nearrow = \mathrm{Id}_{\mathbf{Pos}_{\mathscr{U}}}$ and $\nearrow \mathbin{⨟} \swarrow = \mathrm{Id}_{\mathbf{Pos}_{\mathscr{L}}}$, where $\mathrm{Id}_{\mathbf{Pos}_{\mathscr{U}}}$ and $\mathrm{Id}_{\mathbf{Pos}_{\mathscr{L}}}$ are the identity functors on $\mathbf{Pos}_{\mathscr{U}}$ and $\mathbf{Pos}_{\mathscr{L}}$, respectively.