Definition (Series composition of design problems with implementation)
Given two design problems with implementation $\langle \mathbf{I}_f, \mathsf{prov}_f, \mathsf{req}_f \rangle : A \nrightarrow B$ and $\langle \mathbf{I}_g, \mathsf{prov}_g, \mathsf{req}_g \rangle : B \nrightarrow C$, we can define their series interconnection

$$\langle \mathbf{I}_{f ⨾ g}, \mathsf{prov}_{f ⨾ g}, \mathsf{req}_{f ⨾ g} \rangle : A \nrightarrow C.$$

as follows. With reference to this diagram:

$$A \xleftarrow{\mathsf{prov}_f} \mathbf{I}_f \xrightarrow{\mathsf{req}_f} B \xleftarrow{\mathsf{prov}_g} \mathbf{I}_g \xrightarrow{\mathsf{req}_g} C$$

we let the implementation space be the *pullback*

$$\mathbf{I}_{f ⨾ g} = \mathbf{I}_f \times_B \mathbf{I}_g := \{\langle i_f, i_g \rangle \in \mathbf{I}_f \times \mathbf{I}_g : \mathsf{req}_f(i_f) \leq_B \mathsf{prov}_g(i_g)\},$$

and the two maps prov, req defined as:

$$\begin{aligned} \mathsf{req} : \ & \langle i_f, i_g \rangle \mapsto \mathsf{req}_2(i_g) \\ \mathsf{prov} : \ & \langle i_f, i_g \rangle \mapsto \mathsf{prov}_1(i_f). \end{aligned}$$

In terms of the profunctors, one has

$$\begin{aligned} \langle \mathbf{I}_{f ⨾ g}, \mathsf{prov}_{f ⨾ g}, \mathsf{req}_{f ⨾ g} \rangle : \ & A \times C \to_{\mathbf{Pos}} \mathscr{P}(\mathbf{I}_f \times_B \mathbf{I}_g) \\ \langle a^*, c \rangle \mapsto & \bigcup_{\substack{(b,b') \in B^{\mathrm{op}} \times B^{\mathrm{op}} \\ b \leq_B b'}} \left[\langle \mathbf{I}_f, \mathsf{prov}_f, \mathsf{req}_f \rangle (a^*, b) \times \langle \mathbf{I}_g, \mathsf{prov}_g, \mathsf{req}_g \rangle (b'^*, c) \right]. \end{aligned}$$