

Definition (Trace of a generalized endomorphism). Let $\langle \mathbf{C}, \otimes_{\mathbf{C}}, \mathbf{1}_{\mathbf{C}}, \mathbf{br} \rangle$ be a symmetric monoidal category. Let $X \in \mathbf{Ob}_{\mathbf{C}}$ be dualizable and let $f \in \mathbf{Hom}_{\mathbf{C}}(Y \otimes X; Z \otimes X)$. The *trace over X* of f is the morphism $\mathrm{Tr}_{Y,Z}^X(f) \in \mathbf{Hom}(Y, Z)$ defined by

$$Y \xrightarrow{\mathrm{id}_Y \otimes \eta_X} Y \otimes X \otimes X^\vee \xrightarrow{f \otimes \mathrm{id}_{X^\vee}} Z \otimes X \otimes X^\vee \xrightarrow{\mathrm{id}_Z \otimes \mathbf{br}} Z \otimes X^\vee \otimes X \xrightarrow{\mathrm{id}_Z \otimes \epsilon_X} Z$$