

**Definition** (Power set as lattice). Given a set  $S$ , its power set  $\mathcal{P}S$  (the set of all subsets) is a lattice where, given  $A, B \in \mathcal{P}S$ :

- ▷ Order is given by inclusion:

$$A \leq B := A \subseteq B;$$

- ▷ The join is given by the union of sets:

$$A \vee B := A \cup B;$$

- ▷ The meet is given by the intersection of sets:

$$A \wedge B := A \cap B;$$

- ▷ The top element is the set  $S$  itself:

$$\top = S;$$

- ▷ The bottom element is the empty set:

$$\perp = \emptyset.$$