

$$\begin{aligned}
 (\mathbf{br}_{U,Y} \circ (f \otimes g))^{\star}(x,z) &= \langle \uparrow\{z\}, \uparrow\{x\} \rangle \circ (f \otimes g)^{\star} \\
 &= \left\langle \bigcup_{z' \in \uparrow\{z\}} g^{\star}(z'), \bigcup_{x' \in \uparrow\{x\}} f^{\star}(x') \right\rangle.
 \end{aligned}$$