Definition (Group). A *group* is a monoid together with an "inverse" operation. In more detail, a group **G** is

Constituents

- 1. a set **G**;
- 2. a binary operation $\S: \mathbf{G} \times \mathbf{G} \to \mathbf{G}$, called *composition*;
- 3. a specified element $id \in G$;
- 4. a map inv : $\mathbf{G} \to \mathbf{G}$, called *inverse*.

Conditions

- 1. Associative law: $(x \ , y) \ , z = x \ , (y \ , z)$ $\forall x, y, z \in G$;
- 2. Neutrality laws: id $\S x = x = x \S id$ $\forall x \in G$;
- 3. Inverse laws:

$$\operatorname{inv}(x) \circ x = \operatorname{id} = \operatorname{inv}(x) \circ x \qquad \forall x \in \mathbf{G}.$$