

Definition (Category)

A *category* \mathbf{C} is a semicategory with an additional constituent and an additional rule:

Constituents

1. Identity morphisms: for each object X , there is a morphism

$$\text{Id}_X : X \rightarrow X$$

called *the identity morphism of* X .

Conditions

1. Unitality: It holds that composing a morphism with a compatible identity leaves the morphism unchanged:

$$\frac{f : X \rightarrow Y}{\text{Id}_X \circ f = f = f \circ \text{Id}_Y}.$$