

Definition (Series composition of design problems with implementation)

Given two design problems with implementation $\langle \mathbf{I}_f, \text{prov}_f, \text{req}_f \rangle : A \rightharpoonup B$ and $\langle \mathbf{I}_g, \text{prov}_g, \text{req}_g \rangle : B \rightharpoonup C$, we can define their series interconnection

$$\langle \mathbf{I}_{f \circ g}, \text{prov}_{f \circ g}, \text{req}_{f \circ g} \rangle : A \rightharpoonup C.$$

as follows. With reference to this diagram:

$$A \xleftarrow{\text{prov}_f} \mathbf{I}_f \xrightarrow{\text{req}_f} B \xleftarrow{\text{prov}_g} \mathbf{I}_g \xrightarrow{\text{req}_g} C$$

we let the implementation space be the *pullback*

$$\mathbf{I}_{f \circ g} = \mathbf{I}_f \times_B \mathbf{I}_g := \{ \langle i_f, i_g \rangle \in \mathbf{I}_f \times \mathbf{I}_g : \text{req}_f(i_f) \leq_B \text{prov}_g(i_g) \},$$

and the two maps prov, req defined as:

$$\begin{aligned} \text{req} : \langle i_f, i_g \rangle &\mapsto \text{req}_2(i_g) \\ \text{prov} : \langle i_f, i_g \rangle &\mapsto \text{prov}_1(i_f). \end{aligned}$$

In terms of the profunctors, one has

$$\begin{aligned} \langle \mathbf{I}_{f \circ g}, \text{prov}_{f \circ g}, \text{req}_{f \circ g} \rangle : A \times C &\rightarrow_{\mathbf{Pos}} \mathcal{P}(\mathbf{I}_f \times_B \mathbf{I}_g) \\ \langle a^*, c \rangle &\mapsto \bigcup_{\substack{(b, b') \in B^{\text{op}} \times B^{\text{op}} \\ b \leq_B b'}} \left[\langle \mathbf{I}_f, \text{prov}_f, \text{req}_f \rangle(a^*, b) \times \langle \mathbf{I}_g, \text{prov}_g, \text{req}_g \rangle(b'^*, c) \right]. \end{aligned}$$