

Definition (Order on morphisms in $\mathbf{Pos}_{\mathcal{L}}$)

Given any two morphisms $f, g : X \rightarrow Y$ in $\mathbf{Pos}_{\mathcal{L}}$, we define an order between them as

$$f \leq_{\mathbf{Pos}_{\mathcal{L}}} g$$

$$f^{\star}(x) \leq_{\mathcal{L}Y} g^{\star}(x), \quad \forall x \in X$$