Lemma. **UPos** is a monoidal category with the following additional structure:

1. *Tensor product* $⨟$: On objects, the tensor product corresponds to the product of posets. Given two morphisms $f : X \to \mathcal{U}Y$ and $g : Z \to \mathcal{U}W$, we have:

$$\begin{aligned} f ⨟ g : X \times Z &\to \mathcal{U}(Y \times W) \\ \langle x, z \rangle &\mapsto f(x) \times g(z). \end{aligned}$$

   Note that the Cartesian product of upper sets is an upper set.
2. *Unit*: The unit is the identity poset.
3. *Left unitor*: The left unitor is given by the pair of morphisms

$$\begin{aligned} \mathrm{lu}_{\mathbf{P}} : \{\bullet\} ⨟ X &\to \mathcal{U}X \\ \langle \bullet, x \rangle &\mapsto \uparrow\{x\}, \end{aligned}$$

   and

$$\begin{aligned} \mathrm{lu}_{\mathbf{P}}{}^{-1} : X &\to \mathcal{U}(\{\bullet\} ⨟ X) \\ x &\mapsto \{\bullet\} \times \uparrow\{x\}. \end{aligned}$$

4. *Right unitor*: The right unitor is given by the pair of morphisms

$$\begin{aligned} \mathrm{ru}_{\mathbf{P}} : X ⨟ \{\bullet\} &\to \mathcal{U}X \\ \langle x, \bullet \rangle &\mapsto \uparrow\{x\}, \end{aligned}$$

   and

$$\begin{aligned} \mathrm{ru}_{\mathbf{P}}{}^{-1} : X &\to \mathcal{U}(X ⨟ \{\bullet\}) \\ x &\mapsto \uparrow\{x\} \times \{\bullet\}. \end{aligned}$$

5. *Associator*: The associator is given by the pair of morphisms:

$$\begin{aligned} \mathrm{as}_{XY,Z} : (X ⨟ Y) ⨟ Z &\to \mathcal{U}X \times (\mathcal{U}Y \times \mathcal{U}Z) \\ \langle\langle x, y \rangle, z \rangle &\mapsto \uparrow\{x\} \times (\uparrow\{y\} \times \uparrow\{z\}), \end{aligned}$$

   and

$$\begin{aligned} \mathrm{as}_{X,YZ} : X ⨟ (Y ⨟ Z) &\to (\mathcal{U}X \times \mathcal{U}Y) \times \mathcal{U}Z \\ \langle x, \langle y, z \rangle\rangle &\mapsto (\uparrow\{x\} \times \uparrow\{y\}) \times \uparrow\{z\}. \end{aligned}$$