Definition (Semigroup isomorphism). A morphism of semigroups $F : \mathbf{S} \to \mathbf{T}$ is called a *semigroup isomorphism* if there exists a morphism of semigroups $G : \mathbf{T} \to \mathbf{S}$ such that

$$F \fatsemi G = \mathrm{Id}_{\mathbf{S}} \qquad \text{and} \qquad G \fatsemi F = \mathrm{Id}_{\mathbf{T}}.$$