Definition (Monoid). A *monoid* $\mathbf{M}$ is:

Constituents

1. a set $\mathbf{M}$;
2. a binary operation $\fatsemi : \mathbf{M} \times \mathbf{M} \to \mathbf{M}$;
3. a specified element $\mathrm{id} \in \mathbf{M}$, called *neutral element*.

Conditions

1. Associative law: $(x \fatsemi y) \fatsemi z = x \fatsemi (y \fatsemi z)$ $\quad \forall\, x, y, z \in \mathbf{M}$;
2. Neutrality Laws: $\mathrm{id} \fatsemi x = x = x \fatsemi \mathrm{id}$ $\quad \forall\, x \in \mathbf{M}$.