

Definition (Graph homomorphism)

Given graphs $\mathcal{G} = \langle \mathbf{V}, \mathbf{A}, \text{src}, \text{tgt} \rangle$ and $\mathcal{G}' = \langle \mathbf{V}', \mathbf{A}', \text{src}', \text{tgt}' \rangle$, a graph homomorphism $f : \mathcal{G} \rightarrow \mathcal{G}'$ is given by maps $f_0 : \mathbf{V} \rightarrow \mathbf{V}'$ and $f_1 : \mathbf{A} \rightarrow \mathbf{A}'$, such that the following diagrams commute:

$$\begin{array}{ccc} \mathbf{A} & \xrightarrow{f_1} & \mathbf{A}' \\ \downarrow \text{src} & & \downarrow \text{src}' \\ \mathbf{V} & \xrightarrow{f_0} & \mathbf{V}' \end{array}$$

$$\begin{array}{ccc} \mathbf{A} & \xrightarrow{f_1} & \mathbf{A}' \\ \downarrow \text{tgt} & & \downarrow \text{tgt}' \\ \mathbf{V} & \xrightarrow{f_0} & \mathbf{V}' \end{array}$$