Definition (Category $\mathbf{Pos}_{\mathcal{U}}$). The category $\mathbf{Pos}_{\mathcal{U}}$ consists of:

1. *Objects*: objects are posets;
2. *Morphisms*: given objects $X, Y \in \mathrm{Ob}_{\mathbf{Pos}_{\mathcal{U}}}$, morphisms from $f : X \to Y$ are monotone maps of the form $f^\star : X \to_{\mathbf{Pos}} \mathcal{U}Y$.
3. *Composition of morphisms*: Given morphisms $f : X \to Y, g : Y \to Z$, their composition $f \fatsemi g : X \to Z$ is given by

$$(f \fatsemi g)^\star : X \to_{\mathbf{Pos}} \mathcal{U}Z$$
$$x \mapsto \bigcup_{y \in f^\star(x)} g^\star(y);$$

4. *Identity morphism*: given an object $X \in \mathrm{Ob}_{\mathbf{Pos}_{\mathcal{U}}}$, the identity morphism $\mathrm{Id}_X : X \to X$ is given by the application of the upper closure operator: $\mathrm{Id}_X^\star(x) := \uparrow\{x\}$.