Definition (Chain in a poset). Given a poset $P = \langle P, \leq_P \rangle$, a *chain* is a subset $S \subseteq$ **P** such that any two elements of **S** are comparable: if $x, y \in S$, then $x \leq_{\mathbf{P}} y$ or $y \leq_{\mathbf{p}} x$.