Definition (Lower closure operator)

The *lower closure operator* ↓ maps a subset to the smallest lower set that includes it:

$$\downarrow \; : \; \mathscr{P}\,\mathbf{P} \to \mathscr{L}\,\mathbf{P}$$

$$\mathbf{S} \mapsto \{y \in \mathbf{P} \mid \exists x \in \mathbf{S} : y \preceq_{\mathbf{P}} x\}.$$