

**Definition** (Semicategory). A *semicategory*  $\mathbf{C}$  is specified by:

### Constituents

1. Objects: A collection  $\mathbf{Ob}_{\mathbf{C}}$  whose elements are called *objects*.
2. Morphisms: For every pair of objects  $X, Y$  in  $\mathbf{Ob}_{\mathbf{C}}$ , there is a set  $\mathbf{Hom}_{\mathbf{C}}(X; Y)$ , elements of which are called *morphisms*. We write

$$f : X \rightarrow_{\mathbf{C}} Y$$

to indicate

$$f \in \mathbf{Hom}_{\mathbf{C}}(X; Y).$$

3. Composition operations: For every three objects  $X, Y, Z$  in  $\mathbf{Ob}_{\mathbf{C}}$  there is a composition map

$$\circ_{X,Y,Z} : \mathbf{Hom}_{\mathbf{C}}(X; Y) \times \mathbf{Hom}_{\mathbf{C}}(Y; Z) \rightarrow \mathbf{Hom}_{\mathbf{C}}(X; Z)$$

We omit mentioning the three objects and just write

$$\frac{f : X \rightarrow Y \quad g : Y \rightarrow Z}{f \circ g : X \rightarrow Z}$$

The morphism  $f \circ g$  is called the *composition* of  $f$  and  $g$ .

### Conditions

1. Associativity: it holds that

$$\frac{f : X \rightarrow Y \quad g : Y \rightarrow Z \quad h : Z \rightarrow W}{(f \circ g) \circ h = f \circ (g \circ h)}$$