Definition (**Moo**). The *semi-category of Moore machines* **Moo** is given by:

1. *Objects:* objects of **Set**$*$.
2. *Morphisms:* A morphism is a tuple

$$f = \langle \mathbf{U}_f, \mathbf{X}_f, \mathbf{Y}_f, \mathrm{dyn}_f, \mathrm{ro}_f, \mathrm{start}_f \rangle,$$

   where:

   - $\mathbf{U}$, $\mathbf{X}$, $\mathbf{Y}$ are objects of **Set**$*$.
   - $\mathrm{dyn} : \mathbf{U} \to_{\mathbf{Set}*} \mathbf{End}(\mathbf{X})$;
   - $\mathrm{ro} : \mathbf{X} \to_{\mathbf{Set}*} \mathbf{Y}$.
3. *Composition of morphisms:* Composition is given by:

$$\begin{aligned}
\mathbf{U}_{f \fatsemi g} &= \mathbf{X}_f \\
\mathbf{X}_{f \fatsemi g} &= \mathbf{X}_f \fatsemi \mathbf{X}_g \\
\mathrm{start}_{f \fatsemi g} &= [\, \mathrm{start}_f \; ; \mathrm{start}_g \,] \\
\mathbf{Y}_{f \fatsemi g} &= \mathbf{Y}_g,
\end{aligned}$$

   with

$$\begin{aligned}
\mathrm{dyn}_{f \fatsemi g} : (\mathbf{U}_f \fatsemi \mathbf{X}_f \fatsemi \mathbf{X}_g) &\longrightarrow (\mathbf{X}_f \fatsemi \mathbf{X}_g) \\
\langle u, [x_f \; ; x_g] \rangle &\longmapsto [\mathrm{dyn}_f(u, x_f) \; ; \mathrm{dyn}_g(\mathrm{ro}_f(x_f), x_g)],
\end{aligned}$$

   and

$$\begin{aligned}
\mathrm{ro}_{f \fatsemi g} : (\mathbf{X}_f \fatsemi \mathbf{X}_g) &\longrightarrow \mathbf{Y}_g \\
[x_f \; ; x_g] &\longmapsto \mathrm{ro}_g(x_g)
\end{aligned}$$