$$\operatorname{Tr}^X_{C\otimes A,C\otimes B}(\operatorname{id}_C\otimes f)=\operatorname{id}_C\otimes\operatorname{Tr}^X_{A,B}(f)$$