

**Definition** (Trace of a design problem). Given a design problem  $\mathbf{f} : \mathbf{C} \times \mathbf{A} \dashrightarrow \mathbf{C} \times \mathbf{B}$ , we can define its *trace*  $\text{Tr}_{\mathbf{A}, \mathbf{B}}^{\mathbf{C}}(\mathbf{f}) : \mathbf{A} \dashrightarrow \mathbf{B}$  as follows:

$$\text{Tr}_{\mathbf{A}, \mathbf{B}}^{\mathbf{C}}(\mathbf{f}) : \mathbf{A}^{\text{op}} \times \mathbf{B} \xrightarrow{\text{Pos}} \mathbf{Bool}$$

$$\langle a^*, b \rangle \mapsto \bigvee_{c \in \mathbf{C}} \mathbf{f}(\langle a^*, c \rangle, \langle b^*, c \rangle).$$