

**Definition** ( $\mathcal{U}$  endofunctor). The  $\mathcal{U}$  *endofunctor* has the form  $\mathcal{U} : \mathbf{Pos} \rightarrow \mathbf{Pos}$  and acts on objects and morphisms as follows:

1. *On objects*: Given a poset  $\mathbf{P} \in \mathbf{Ob}_{\mathbf{Pos}}$ ,  $\mathcal{U}$  maps  $\mathbf{P}$  to its upper set. {bhf:1}
2. *On morphisms*: Given posets  $\mathbf{P}, \mathbf{Q}$ , and a monotone map  $f : \mathbf{P} \rightarrow \mathbf{Q}$ , the  $\mathcal{U}$  endofunctor acts as:

$$\mathcal{U}(f) : \mathcal{U}\mathbf{P} \rightarrow \mathcal{U}\mathbf{Q}$$

$$\mathbf{P}' \mapsto \uparrow \left( \bigcup_{p \in \mathbf{P}'} \{f(p)\} \right).$$

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Recall that in ?? we proved that the upper set is itself an object of  $\mathbf{Pos}$ .