

**Definition** (Metric space). A *metric space*  $\langle \mathbf{A}, d \rangle$  consists of:

1. A set  $\mathbf{A}$ , elements of which are called *points*;
2. A map  $d : \mathbf{A} \times \mathbf{A} \rightarrow \mathbb{R}_{\geq 0}$ , called *distance*.

The constituents must satisfy:

- ▷  $d(a, a) = 0$ , for all  $a \in \mathbf{A}$ ;
- ▷ If  $d(a, b) = 0$ , then  $a = b$ , for all  $a, b \in \mathbf{A}$ ;
- ▷  $d(a, b) = d(b, a)$ , for all  $a, b \in \mathbf{A}$ ;
- ▷  $d(a, b) + d(b, c) \geq d(a, c)$ , for all  $a, b, c \in \mathbf{A}$ .