Sheet 1 Convolution Matrix Solutions

Notes:

- The matrix colored in **red** Is the **original matrix** and the pixel colored in **purple** is the **current pixel** which I will put the kernel on it and apply mean filter or median filter and the matrix colored in **black** is the one which have the kernel on it and have the current pixel in the **center**

- How do we calculate the **mean**?

  Sum of numbers/their count so it's the same if we put a 3*3 kernel on the first pixel for example so we will sum up all the numbers in the matrix then divide by 9 because the count of numbers is 9

Example: we have matrix that has the following numbers 1, 2, 3, 4, 5, 6, 7, 8,9 so the mean is 1+2+3+4+5+6+7+8+9/9 = 5

- How do we calculate the **median**?
  1. Sort the numbers from smallest to greatest
  2. If the count of numbers is **odd**, then the median is the middle one (for example we've got 9 numbers) so 9/2 = 4.5 → 5 so the 5^{th} position is the median
  3. If the count of numbers is **even,** we get the 2 middle numbers and add them and divide by 2

Example1: we have a list of the following numbers 9,8,7,6,5,4,3,2,1 it has 9 numbers(**odd**) so first we must sort the numbers 1,2,3,4,**5**,6,7,8,9 then the median will be the 5^{th} element which is the **5**

Example2: we have matrix of the following numbers 9,8,7,6,5,4,3,2 it has 8 numbers(**even**) so first we must sort the numbers 2,3,4,**5,6**,7,8,9 then 8/2 = 4 so we will take 5, 6 and add them together and divide by 2 → 5+6/2 = 5.5 → 6

2. Given the following 2-D array representing a 3-bit gray-level image:

3	1	3	6
3	4	7	5
2	1	4	3

- Apply the following operations separately:

A. Perform 3*3 – noise filtering using the mean filter.

1

sheet 1 Convolution matrix steps

① original matrix →

3 x 4 matrix

3	1	3	6
3	4	7	5
2	1	4	3

* first step duplicate the edges

3	3	1	3	6	6
3	3	1	3	6	6
3	3	4	7	5	5
2	2	1	4	3	3
2	2	1	4	3	3

real matrix

* second step apply the mean filter to the image

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

→ 3 x 3 kernel size

* multiply this kernel to the first pixil

first pixil

first row

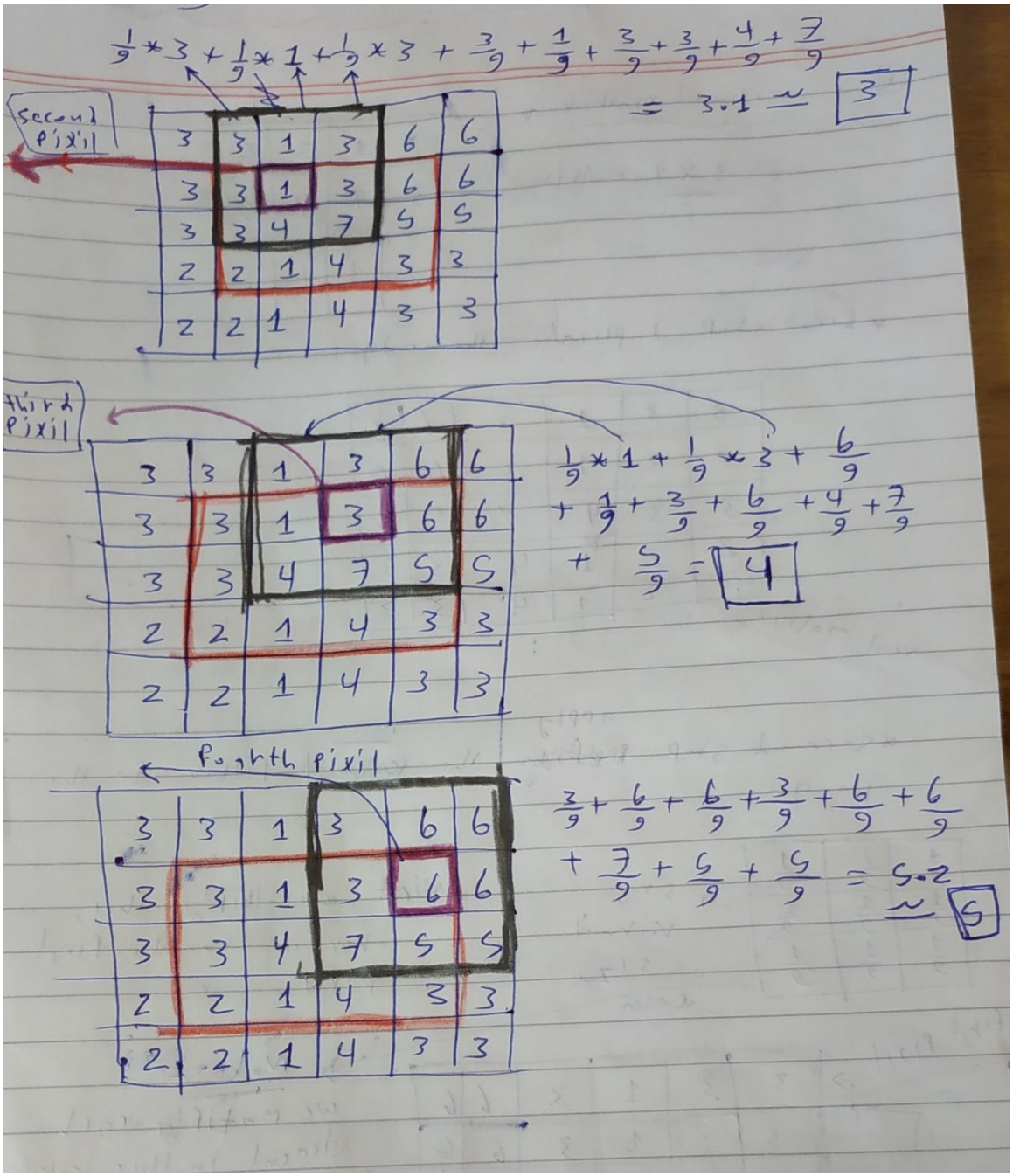
3	3	1	3	6	6
3	3	1	3	6	6
3	3	4	7	5	5
2	2	1	4	3	3
2	2	1	4	3	3

Note:
we multiply each element in this kernel by the matrix highlighted in black for example first * first + second * ...

$$\frac{1}{9}\times 3 + \frac{1}{9}\times 3 + \frac{1}{9}\times 1 + \frac{3}{9} + \frac{3}{9} + \frac{1}{9} + \frac{3}{9} + \frac{3}{9} + \frac{4}{9}$$

$$= 2.6 \simeq \boxed{3}$$

Note: I could've just sum up the numbers then divide by 9(taking 1/9 common factor) like this 3+3+1+3+3+1+3+3+4/9 = 2.6 -> 3 and you could do the same for the rest of the pixels instead of 1/9*3+1/9*3+1/9*1 + 3/9+3/9+1/9+3/9+3/9+4/



3

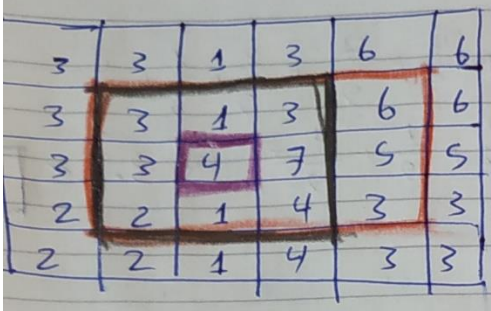
Fifth Pixil

3	3	1	3	6	6
3	3	1	3	6	6
3	3	4	7	5	5
2	2	1	4	3	3
2	2	1	4	3	3

$\frac{3}{9}+\frac{3}{9}+\frac{1}{9}+\frac{3}{9}+\frac{3}{9}+\frac{4}{9}$
$+\frac{2}{9}+\frac{2}{9}+\frac{1}{9}=2.4$
$\simeq \boxed{2}$

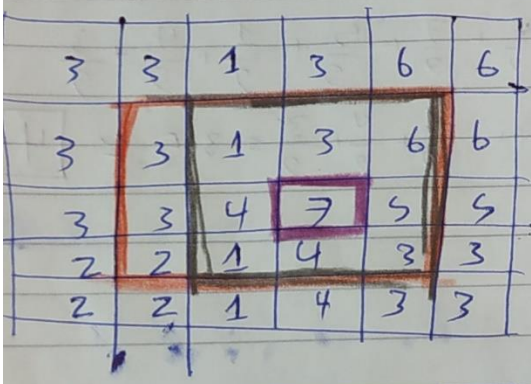
OR

$3+3+1+3+3+4+2+2+1$
$= 22/9 = 2.4 \simeq \boxed{2}$



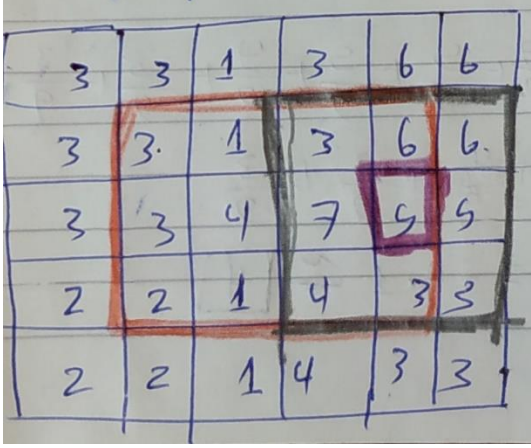
3	3	1	3	6	6
3	3	1	3	6	6
3	3	4	7	5	5
2	2	1	4	3	3
2	2	1	4	3	3

$\frac{3}{9}+\frac{1}{9}+\frac{3}{9}+\frac{3}{9}+\frac{4}{9}+\frac{7}{9}$
$+\frac{2}{9}+\frac{1}{9}+\frac{4}{9}=3.1\simeq\boxed{3}$



3	3	1	3	6	6
3	3	1	3	6	6
3	3	4	7	5	5
2	2	1	4	3	3
2	2	1	4	3	3

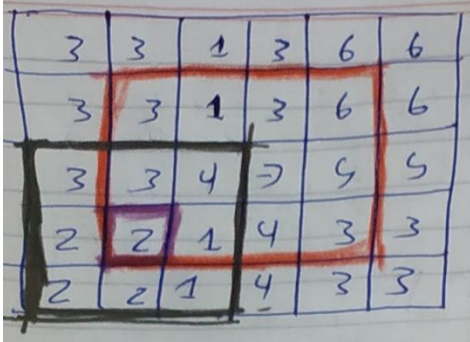
$\frac{1}{9}+\frac{3}{9}+\frac{6}{9}+\frac{4}{9}+\frac{7}{9}+\frac{5}{9}$
$+\frac{1}{9}+\frac{4}{9}+\frac{3}{9}$
$=3.7\simeq\boxed{4}$



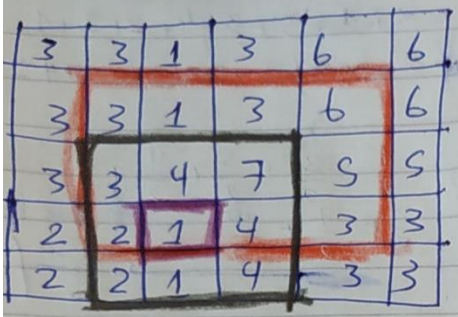
3	3	1	3	6	6
3	3	1	3	6	6
3	3	4	7	5	5
2	2	1	4	3	3
2	2	1	4	3	3

$\frac{3}{9}+\frac{6}{9}+\frac{6}{9}+\frac{7}{9}+\frac{5}{9}+\frac{5}{9}$
$+\frac{4}{9}+\frac{5}{9}+\frac{3}{9}$
$=4.6\simeq\boxed{5}$

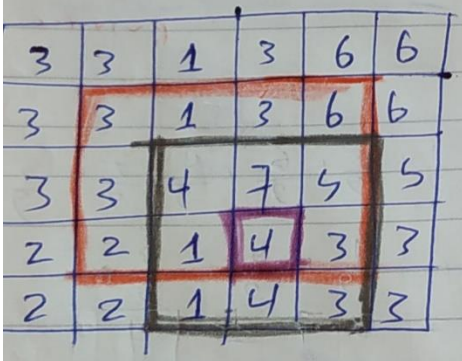
4



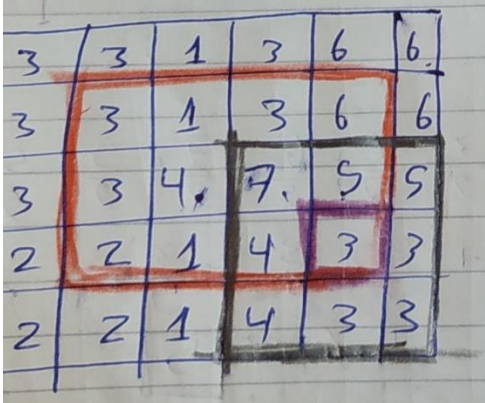
$\frac{3}{9} + \frac{3}{9} + \frac{4}{9} + \frac{2}{9} + \frac{2}{9} + \frac{1}{9}$
$+ \frac{2}{9} + \frac{2}{9} + \frac{1}{9} = 2 \cdot 2 \simeq \boxed{2}$



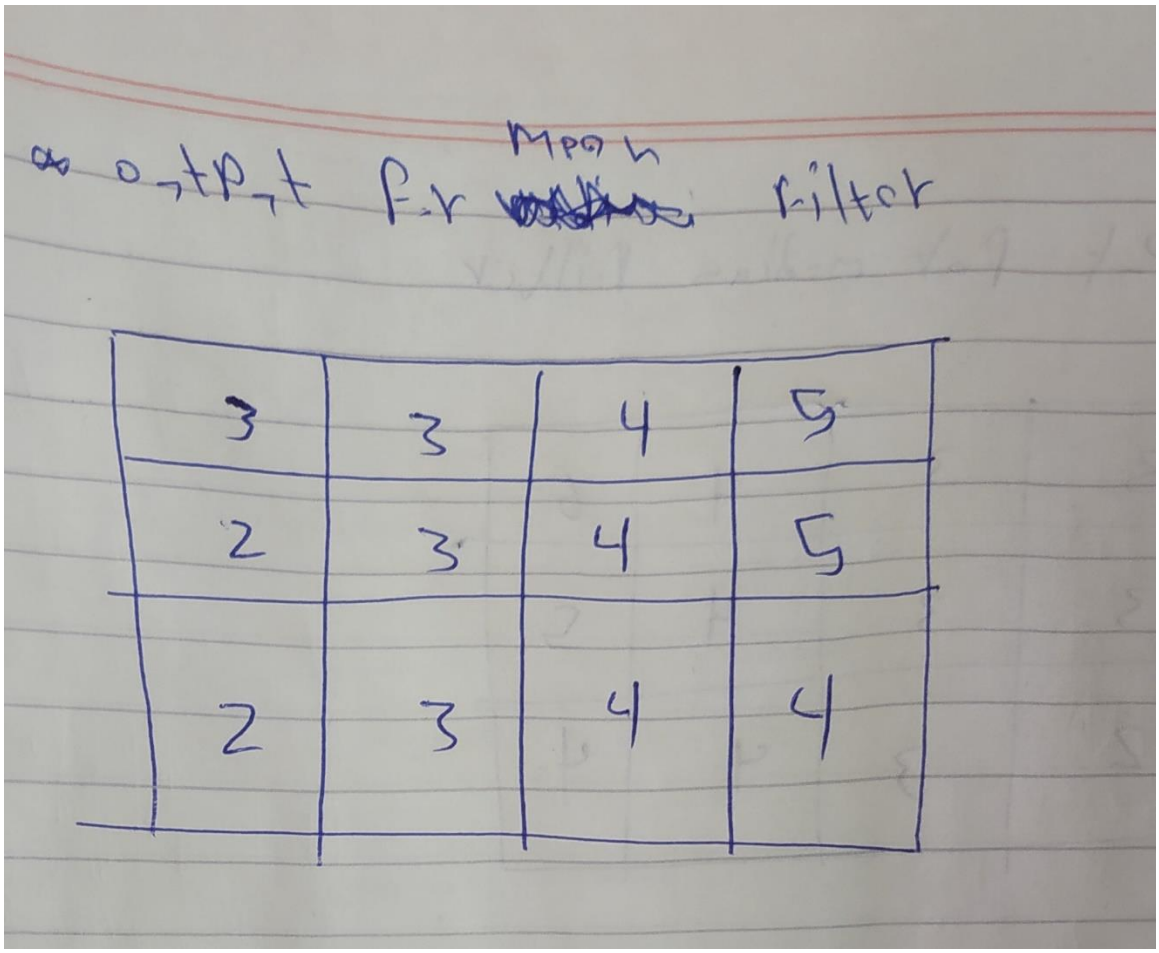
$\frac{3}{9} + \frac{4}{9} + \frac{7}{9} + \frac{2}{9} + \frac{1}{9} + \frac{4}{9}$
$+ \frac{2}{9} + \frac{1}{9} + \frac{4}{9} = 3 \cdot 1 \simeq \boxed{3}$



$\frac{4}{9} + \frac{7}{9} + \frac{5}{9} + \frac{1}{9} + \frac{4}{9} + \frac{3}{9}$
$+ \frac{1}{9} + \frac{4}{9} + \frac{3}{9} = 3 \cdot 5 \simeq \boxed{4}$



$\frac{7}{9} + \frac{5}{9} + \frac{5}{9} + \frac{4}{9} + \frac{3}{9} + \frac{3}{9}$
$+ \frac{4}{9} + \frac{3}{9} + \frac{3}{9}$
$= 4 \cdot 1 \simeq \boxed{4}$

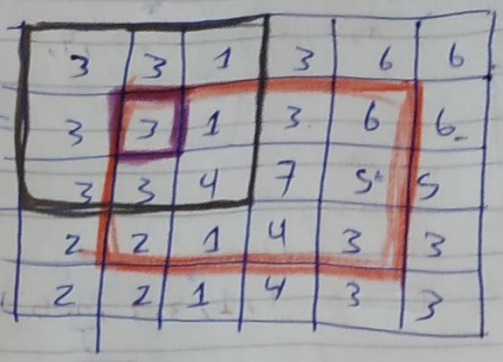


3	3	4	5
2	3	4	5
2	3	4	4

B. Perform 3*3 – noise filtering using the median filter.

3	3	4	6
3	3	4	5
2	3	4	4

(2) using median filter



* first step put the kernel on the matrix that contains the first pixil in the middle

* takes these the matrix numbers and sort them from lowest to ~~biggest~~ highest

* choose the middle number after sorting if the numbers is even pick the two in middle and get their average else if the numbers are odd pick the middle one only which this the case in this (9 numbers)

22

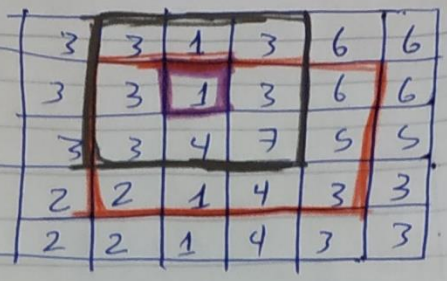
numbers before sorting → 3, 3, 1, 3, 3, 1, 3, 3, 4

numbers after sorting → 1, 1, 3, 3, 3, 3, 3, 3, 4

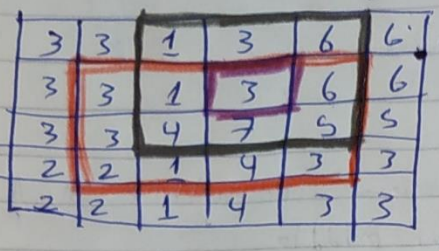
$\frac{9}{2} = 4.5 \approx 5$ → 3

the 5th element is the median

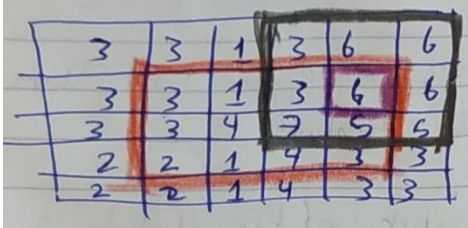
2.



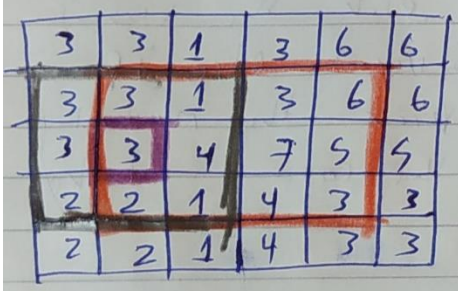
after sorting →
1, 1, 3, 3, 3, 3, 3, 4, 7
↓
[3]



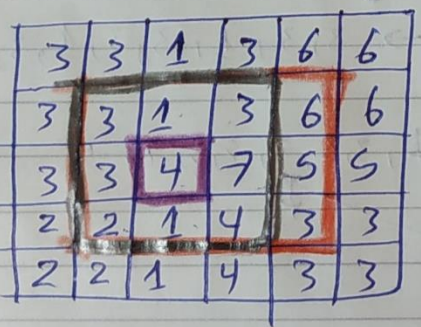
after sorting →
1, 1, 3, 3, 4, 5, 6, 6, 7
↓
[4]



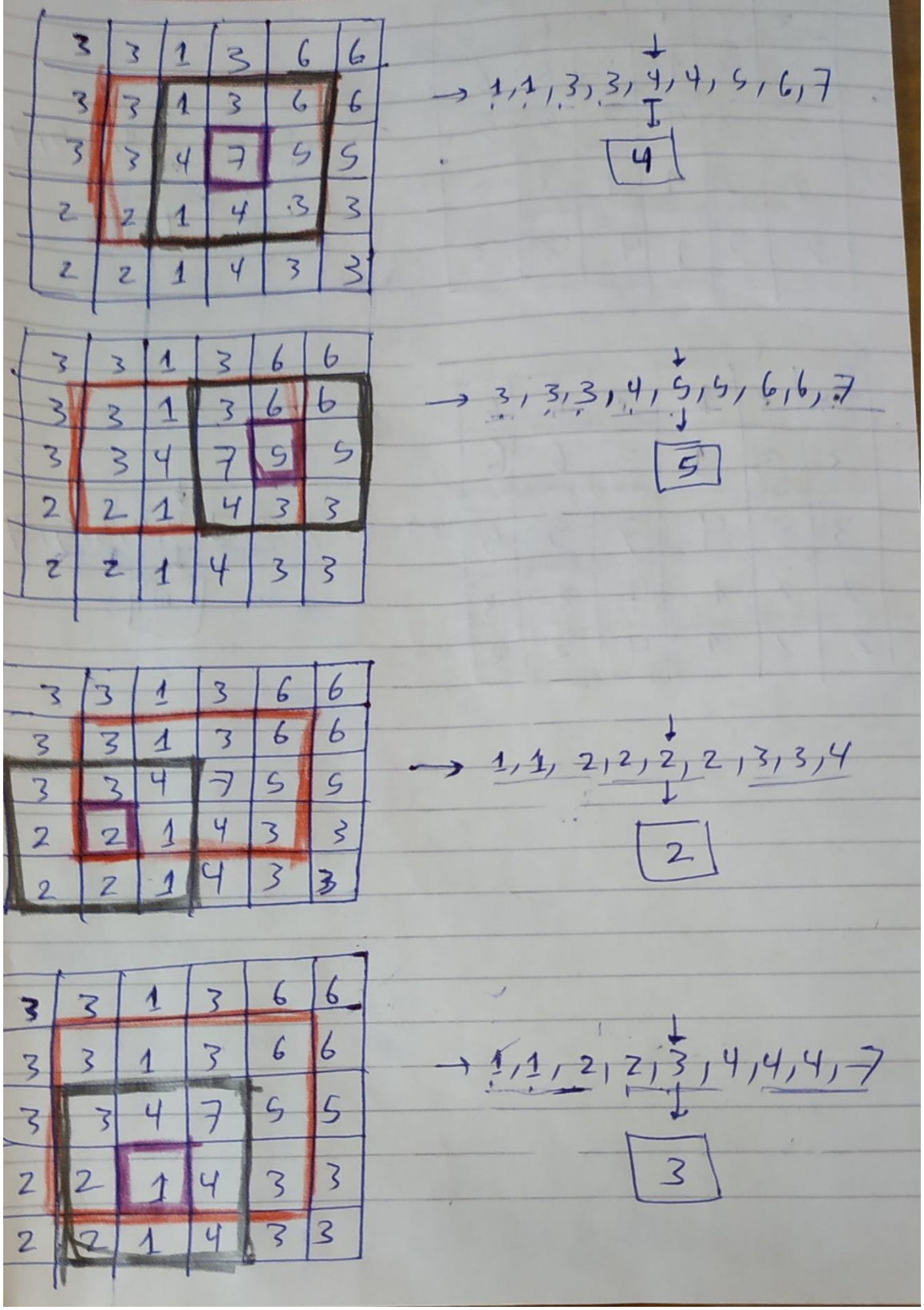
after sorting →
3, 3, 5, 5, 6, 6, 6, 6, 7
↓
[6]

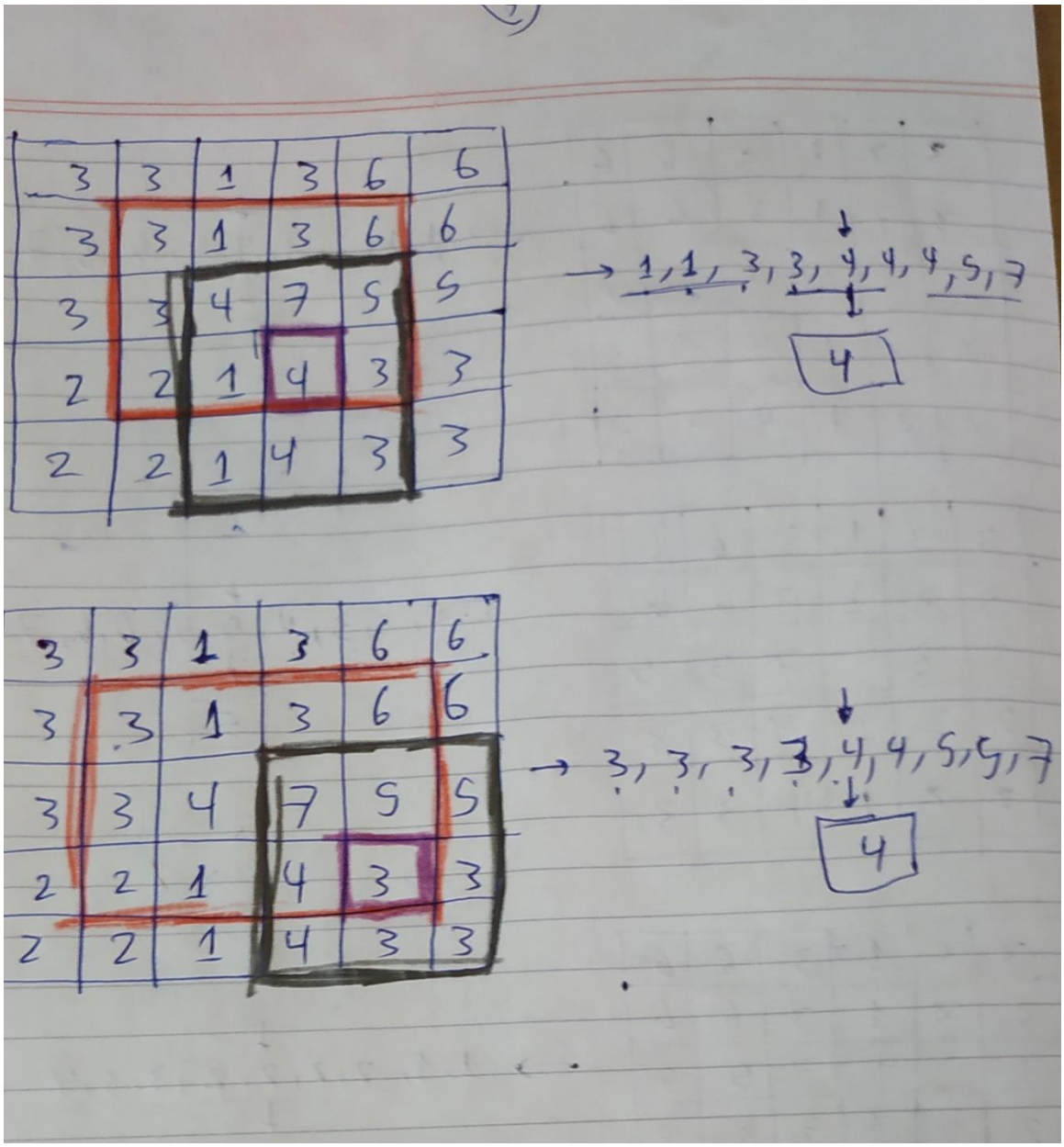


→ 1, 1, 2, 2, 3, 3, 3, 3, 4
↓
[3]



→ 1, 1, 2, 3, 3, 3, 4, 4, 7
↓
[3]


3 3 1 3 6 6
3 3 1 3 6 6
3 3 4 7 5 5
2 2 1 4 3 3
2 2 1 4 3 3
→ 1, 1, 3, 3, 4, 4, 5, 6, 7
4
3 3 1 3 6 6
3 3 1 3 6 6
3 3 4 7 5 5
2 2 1 4 3 3
2 2 1 4 3 3
→ 3, 3, 3, 4, 5, 5, 6, 6, 7
5
3 3 1 3 6 6
3 3 1 3 6 6
3 3 4 7 5 5
2 2 1 4 3 3
2 2 1 4 3 3
→ 1, 1, 2, 2, 2, 2, 3, 3, 4
2
3 3 1 3 6 6
3 3 1 3 6 6
3 3 4 7 5 5
2 2 1 4 3 3
2 2 1 4 3 3
→ 1, 1, 2, 2, 3, 4, 4, 4, 7
3


3 3 1 3 6 6
3 3 1 3 6 6
3 3 4 7 5 5
2 2 1 4 3 3
2 2 1 4 3 3
→ 1, 1, 3, 3, 4, 4, 4, 5, 7
4
3 3 1 3 6 6
3 3 1 3 6 6
3 3 4 7 5 5
2 2 1 4 3 3
2 2 1 4 3 3
→ 3, 3, 3, 3, 4, 4, 5, 5, 7
4

* o/p for median filter

3	3	4	6.
3	3	4	5
2	3	4	4