# SVAR: An application to the study of the interdependence between monetary and stock price shocks in Brazil


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## Abstract

Using both short and long-run restrictions, just as Bjørnland and Leitemo (2009) identification strategy, I find that there is non-trivial simultaneous interdependence between monetary policy and real stock price shocks for Brazilian economy. Real stock prices immediately fall around fifteen percent due to an one percent exogenous increase in interest rate-SELIC. On the other hand, a stock price shock that raises real stock prices by one percent leads to an increase in interest rate-SELIC of around three basic points.


*This document does not have any relation with the university beyond the fact that I study there. Errors are my own.*

# Contents

# 1 Introduction: simultaneity problem

Monetary policy and private-sector decisions are closely related. If there are nominal rigidities, which is a widely accepted fact, then central banks can influence real economic activity in the short run and play a key role over macroeconomic stability. At the same time, in order to attain stability, central banks must respond to and influence private-sector decisions appropriately. This interdependence is higher as faster private-sector response to central bank decisions. In particular, financial markets commonly react immediately to economy shocks due to the speed of information availability in those markets. Consequently, we would expect presence of *simultaneity between financial markets and monetary policy*.

Economic theory suggests several reasons for this interdependence, in particular, between monetary policy with stock prices. First, there are three main channels through monetary policy affects stock prices: a) interest rate channel, by affecting discount rate for expected dividends; b) by influencing determinants of dividends; and c) risk premium channel, by influencing the degree of uncertainty faced by agents. On the other hand, since central banks react to measures of demand pressures, in particular, output gap and inflation rate, stock prices influence monetary policy by: a) wealth effects (consumption); b) affecting investment decisions (Tobin's Q theory); and c) by relaxing private credit conditions (credit channels).

In spite of economic theory suggestions relatively few empirical studies attempt to model the cited interdependence. There is a first set of studies using vector autoregressive (VAR, hereafter) approach with Cholesky decomposition [Lee (1992), Patelis (1997), Thorbecke (1997), Millard and Wells (2003), and Neri (2004)] and have two main findings: a) little degree of interdependence between monetary policy and stock prices, and b) impulse responses are counterintuitive. Lastrapes (1998) and Rapach (2001), on the contrary, use long-run identification restrictions to identify monetary policy shocks and find strong effects over stock prices; even though, they ignore contemporaneous relation with stock prices. Finally, Rigobon and Sack (2003) and Rigobon and Sack (2004) use high frequency data and identify strong causality from both sides, even though, they disregard dynamically adjustment following initial shock and do not provide two-way causation at the same time.

Contrary to these studies, Bjørnland and Leitemo (2009) (BL, hereafter) propose using both short and long run restrictions to solve the *simultaneity problem*. Their main findings are: a) great interdependence between interest rate and real stock prices, and b) expected directions for responses to both interest rate and stock prices shock.

The objective of the present document is to give empirical evidence about the measure of the considered interdependence. I use BL identification strategy and find that: a) results change drastically in compare with Cholesky decomposition, b) there is non-trivial degree of interdependence between monetary policy and stock prices. Finally, I discuss some alternatives identification strategies that could let richer contemporaneous relation between interest rate and real stock prices.

## 2 Reduced form model and data description

As in BL, I use a VAR approach with $m = 5$ variables: output gap ($y_t$), inflation ($\pi_t$), commodities inflation ($\Delta c_t$), real stock price variation ($\Delta s_t$) and interest rate ($i_t$). Let $z_t = [y_t, \pi_t, \Delta c_t, \Delta s_t, i_t]'$, and assuming stationarity the process can be written as a infinite moving-average process:

$$\mathbf{z}_t = \boldsymbol{\Theta}(L)\mathbf{v}_t, \qquad \mathbf{v}_t \sim iid(0, \Sigma_v) \tag{2.1}$$

$$\mathbf{z}_t = \boldsymbol{\Psi}(L)\boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim iid(0, \mathbf{I}_m) \tag{2.2}$$

$$\mathbf{v}_t = \mathbf{A}\boldsymbol{\varepsilon}_t \tag{2.3}$$

$$\boldsymbol{\Psi}(L) = \boldsymbol{\Theta}(L)\mathbf{A} \tag{2.4}$$

where $\boldsymbol{\varepsilon}_t = [\varepsilon_t^y, \varepsilon_t^\pi, \varepsilon_t^c, \varepsilon_t^{sp}, \varepsilon_t^{mp}]'$ and $\boldsymbol{\Theta}(L), \boldsymbol{\Psi}(L)$ are infinite lag operators. The main objective of the present document is identifying $\varepsilon_t^{mp}$ and $\varepsilon_t^{sp}$ and the dynamic effects of both of them. The infinite moving-average model is approximated by a finite VAR model of order p:

$$\mathbf{z}_t = \mathbf{C_0} + \mathbf{C_1}\mathbf{z}_{t-1} + \cdots + \mathbf{C_p}\mathbf{z}_{t-p} + \mathbf{v}_t \tag{2.5}$$

In order to identify monetary policy and real stock price shocks and the effects of them, we need to identify the matrix $\mathbf{A}$. The next section discusses the identification strategy followed in BL. The rest of this section analyses data used for estimation.

Monthly Brazilian data from January 2002 to November 2017 is employed for model estimation. Hodrick-Prescott (HP) cycle component of the log Industrial Production Index ($\mathsf{Y}_t$) is taken as an output gap measure, annual change in the log of consumer price index ($\mathsf{P}_t$) as inflation, and annual change in the log of Brazilian commodity price index ($\mathsf{C}_t$) is used as commodity inflation. I take IBOVESPA ($\mathsf{S}_t$) as a measure of stock price, then real stock price variation is obtained deflating $\mathsf{S}_t$ by $\mathsf{P}_t$ and then taking the first difference (monthly variation) in the log of the deflated variable. Finally, interest rate is directly measured by SELIC. Table 1 summarizes data used in estimation while Fig. 1 plots the historical path for each serie.

**Table 1.** *Data description*

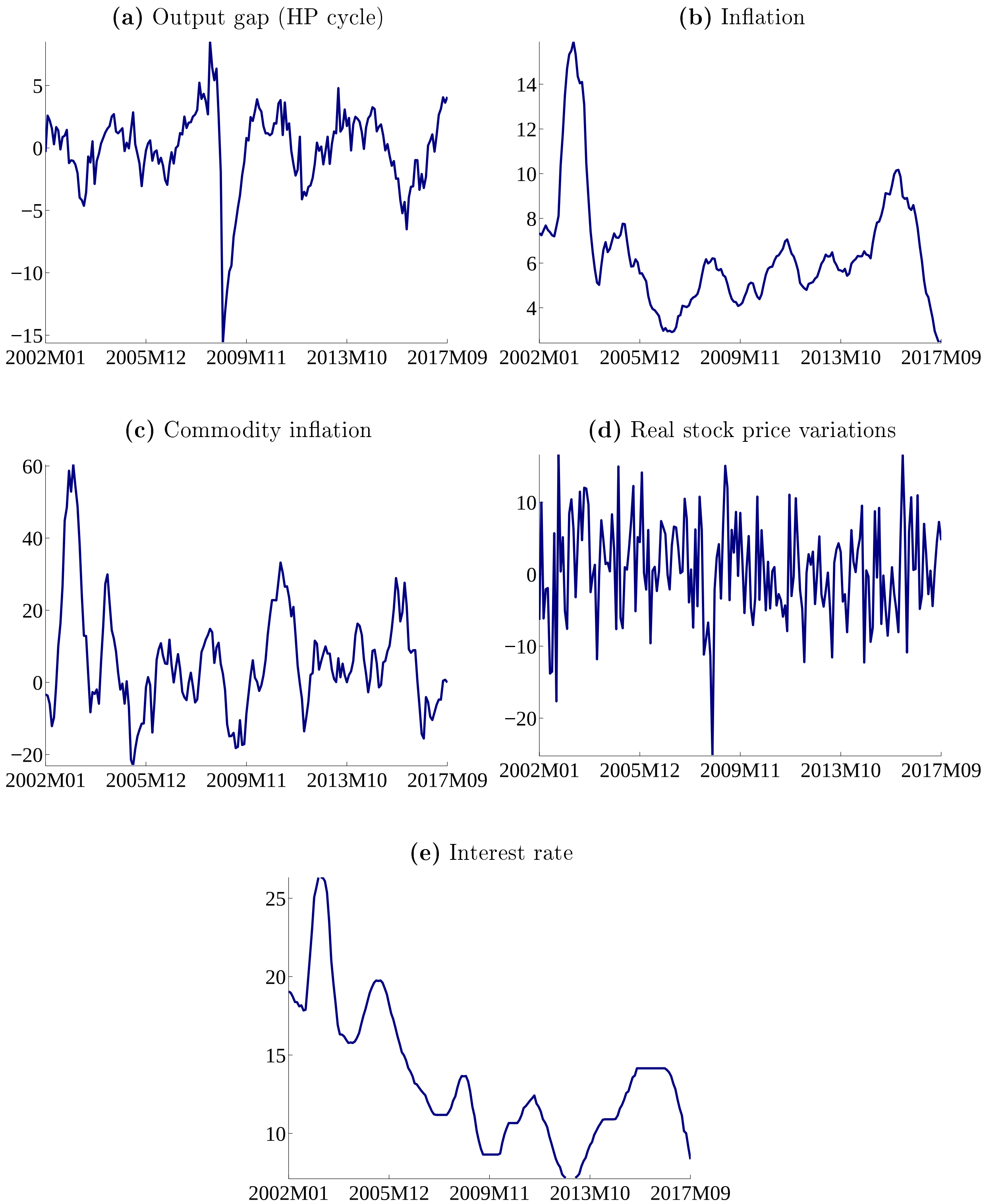
| Variable | Data | Measured by |
|---|---|---|
| Output gap ($y_t$) | General Industrial Production Index ($\mathsf{Y}_t$) | HP cycle of $\ln \mathsf{Y}_t$ |
| Inflation ($\pi_t$) | Consumer price index ($\mathsf{P}_t$) | $\ln \mathsf{P}_t - \ln \mathsf{P}_{t-12}$ |
| Commodity inflation ($\Delta c_t$) | Brazil's Commodity Price Index ($\mathsf{C}_t$) | $\ln \mathsf{C}_t - \ln \mathsf{C}_{t-12}$ |
| Real stock price var. ($\Delta s_t$) | IBOVESPA ($\mathsf{S}_t$) | $\ln \frac{\mathsf{S}_t}{\mathsf{P}_t} - \ln \frac{\mathsf{S}_{t-1}}{\mathsf{P}_{t-1}}$ |
| Interest rate ($i_t$) | SELIC | none |

**Note.** Source: Brazilian central bank (BCB) and Brazilian Geographic and Statistic Institute (IBGE) database. Brazil's commodity price index (IC-Br) is built by BCB using appropriate weighting structure to measure the impact on Brazilian consumer inflation. IBOVESPA is the stock price index for São Paulo's stock market. The interest rate-SELIC is taken accumulated in the month in annual terms.

For the sake of validating the use of a stationary VAR(p), I perform univariate unit root tests for each series. In general, there are sufficient evidence that series used in estimation are stationary, see Table 2. There is only one observation about these results, using estimation sample the tests suggest that SELIC is trend stationary which implies that a linear detrended series of SELIC must be used in estimation; however, when I use a reasonable larger sample, null hypothesis is rejected. Hence, reduced form model is estimated using the proposed time series (see Appendix B.1 for estimation details).

The lag order for the VAR model is estimated using Schwarz Bayesian and Hannan-Quinn

**Figure 1.** *Time series used in VAR model*




**Source.** Brazilian central bank (BCB) and Brazilian Geographic and Statistic Institute (IBGE) database.

information statistic: $\hat{p} = 2$. See Appendix A for details.

**TABLE 2.** *Unit Root test results*

| VARIABLE | ADF TEST - $\mathbb{H}_0$: UR | | | | PP TEST - $\mathbb{H}_0$: UR | | | |
|---|---|---|---|---|---|---|---|---|
| | $\mathbb{H}_1$: S+C | | $\mathbb{H}_1$: TS | | $\mathbb{H}_1$: S+C | | $\mathbb{H}_1$: TS | |
| | *Reject $\mathbb{H}_0$?* | *Lag* | *Reject $\mathbb{H}_0$?* | *Lag* | *Reject $\mathbb{H}_0$?* | *Lag* | *Reject $\mathbb{H}_0$?* | *Lag* |
| *Output (HP cycle)* | Yes | 8 | Yes | 8 | Yes | 0 | Yes | 0 |
| *Inflation* | Yes | 1 | Yes | 4 | No | 0 | No | 0 |
| *Commodity inflation* | Yes | 8 | Yes | 8 | Yes | 1 | Yes | 3 |
| *Real stock prices var.* | Yes | 8 | Yes | 8 | Yes | 0 | Yes | 0 |
| *Interest rate* | No | 4 | Yes | 1 | No | 0 | No | 0 |
| *Interest rate*$^\dagger$ | Yes | 2 | Yes | 1 | Yes | 0 | Yes | 0 |

**Note.** 'UR': presence of unit root, 'S+C': Stationary with a constant, and 'TS': trend stationary. All test were evaluated using 5% of significance level and the lag order was selected using Bayesian information statistic. $^\dagger$: means that it was used an extended sample [from 1996M1 to 2017M9] for testing unit root presence.

## 3 IDENTIFICATION STRATEGY

I follow BL identification strategy which consists in both short and long-run restrictions. Since I have already specified identity variance matrix for $\boldsymbol{\varepsilon}_t$, then $(m(m-1)/2 = 10)$ more restrictions must be imposed.

1. **Short run restrictions**. Recursivity of monetary policy and real stock price shows over the rest of the variables is assumed, i.e.,

$$\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \tag{3.1}$$

   The main difference from related studies is that BL does not assume $a_{45} = 0$ permitting simultaneity between interest rate and real stock price shocks. In this way, BL impose nine zero restrictions.

2. **Long-run restrictions**. Based on conventional long-run neutrality for monetary policy, it is imposed that interest rate shocks have no long-run effects on the level of real stock prices ($\mathsf{s}_t = \ln \frac{\mathsf{S}_t}{\mathsf{P}_t}$). The cumulated long-run effect of $\varepsilon_t^{mp}$ on $\Delta s_t$ (or, equivalently, the long-run effect on $s_t$) is quantified by the $(4,5)$ element of the matrix $\boldsymbol{\Psi}(1)$, i.e.,

$$\boldsymbol{\Psi}_{45}(1) = \boldsymbol{\Theta}_{44}(1)a_{45} + \boldsymbol{\Theta}_{45}(1)a_{55} \tag{3.2}$$

   Hence, the last restriction is a linear dependency between $a_{45}$ and $a_{55}$:

$$a_{55} = -\frac{\boldsymbol{\Theta}_{44}}{\boldsymbol{\Theta}_{45}}a_{45} \tag{3.3}$$

Further details about the estimation of $\mathbf{A}$ is given in Appendix B.

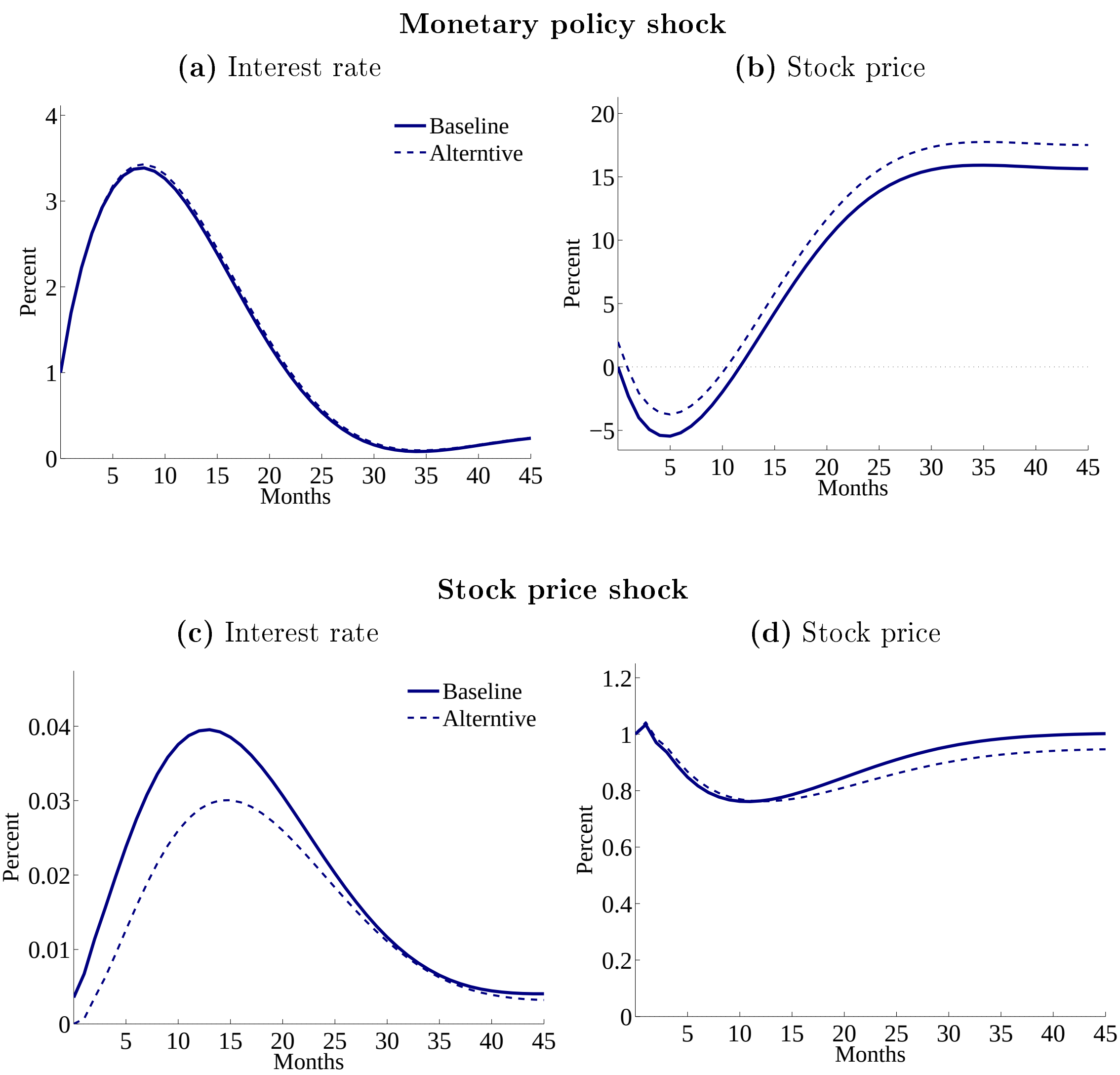
# 4 Empirical results

## 4.1 Cholesky decomposition

In the interest of motivating the structural identification scheme, just as BL, I compute the impulse response functions (IRF) of interest rates and real stock prices to both monetary policy shock and a stock price shock under recursive identification. Fig. 2 shows the results for two different ordering of variables, with the interest rate and the real stock price alternating as the ultimate and penultimate variables in the specification of $\mathbf{z}_t$: *Baseline* model use the variables order presented in the previous section and the *Alternative* model is built changing the orders of $i_t$ and $\Delta s_t$. As BL does, the shocks are normalized so that monetary policy shock increases interest rate by one percent at the moment of the impulse, while the stock price shock increases the real stock prices with one percent instantaneously.

Note that both ordering specifications produce similar effects, and, moreover, in line with related Cholesky identification literature I find: a) low degree of interdependence between monetary policy and stock prices, and b) unrealistic permanent effect of monetary policy on stock price level. Hence, there is a evidence that Cholesky identification distorts the estimation in such a way that degree of interdependence will seem trivial.

**FIGURE 2.** *Impulse-response functions of interest rate and real stock price*



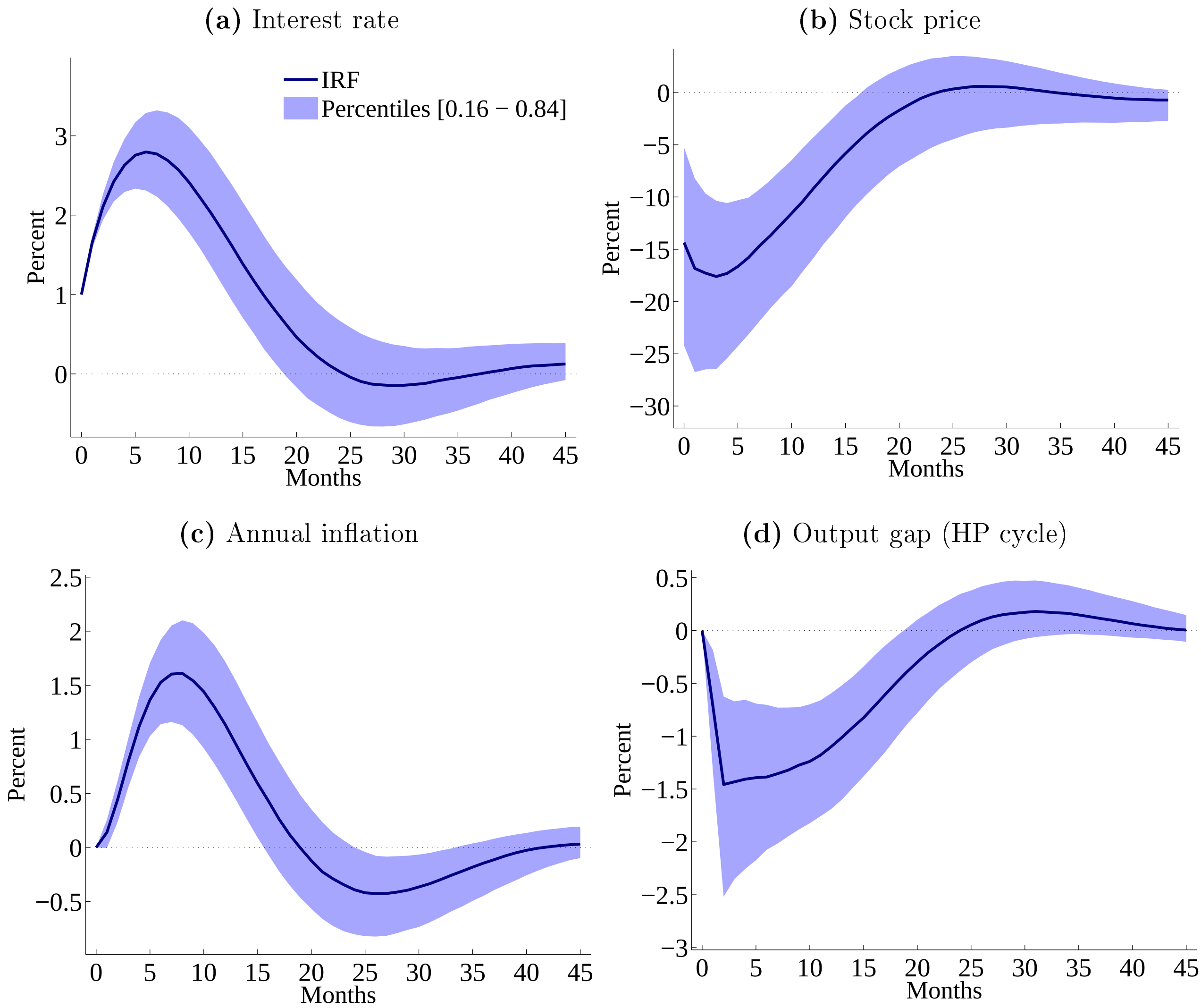

**Note.** 'Baseline' model use the order for $\mathbf{z}_t$ suggested along the document, while 'Alternative' model alternates interest rate and the real stock price as the ultimate and penultimate variables in the specification of $\mathbf{z}_t$. Shocks are normalized.

### 4.2 Short and long-run identification

Figure 3 and Fig. 4 report impulse responses for the interest rate, real stock price, annual inflation, and output gap from monetary policy shock and stock price shock obtained using BL identification strategy. Shocks are again normalized and each impulse response is design with probability bands representing 16th and 84th percentiles.

Figure 3 shows that monetary policy shock has a strong effect over real stock prices, which immediately falls about fifteen (15%) percent for each 100 basis-point increase in SELIC. This is consistent with results found by BL and also with an increase in discount rate of future dividends and lower dividends expectations. Moreover, as is generally found in the literature, output falls temporarily (contraction lasts around two-years) and annual inflation increases for the first nine months ("*price puzzle*") and then starts to decline, so that, after two years inflation is below its initial level.

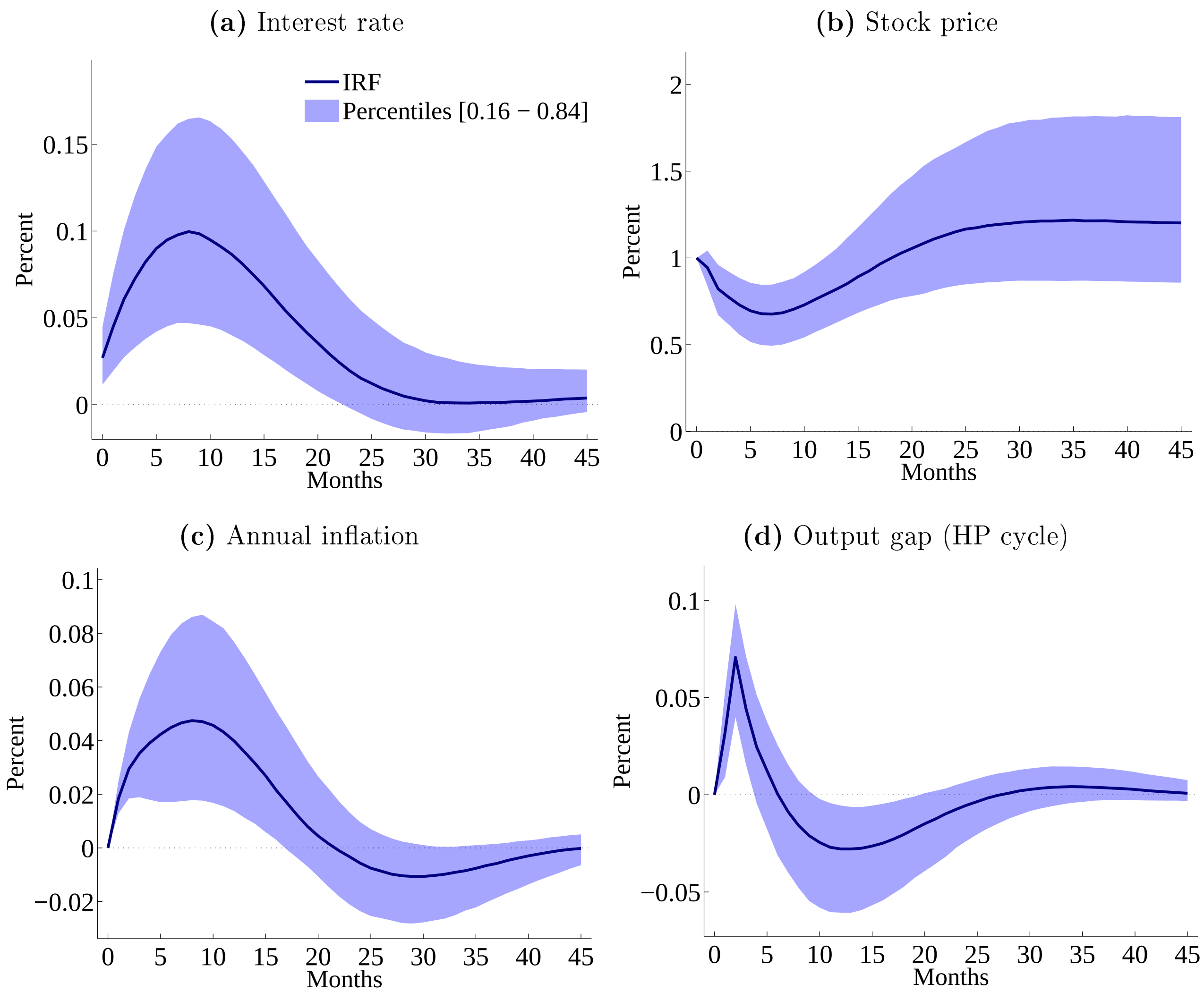
**Figure 3.** *Impulse-responses to monetary policy shock, SVAR*




**Note.** *Residual wild bootstrap technique* with 2000 replications is employed to compute bands and central (median) IRF estimations. Shocks are normalized.

Turning to real stock price shock, there are evidences that that a positive shock increases both inflation and output in the short run as is shown in Fig. 4. This results are consistent with increases in consumption and investment, due to a wealth effect, Tobin Q effect, and with the presence of nominal rigidities. In order to reduce inflation and output volatility, the central bank reacts increasing interest rates around three (3) basic points for each one percent exogenous increase in stock prices.

**Figure 4.** *Impulse-responses to stock price shock, SVAR*




**Note.** *Residual wild bootstrap technique* with 2000 replications is employed to compute bands and central (median) IRF estimations. Shocks are normalized.

### 4.3 Alternative identification strategies

We can improve the model specification by introducing dummy variables which permit us capture effects of: seasonality, international financial crisis, Brazil political crisis, and etcetera; however, the main findings will not change as we continue identifying **A** with the strategy proposed along the document. In this subsection I discuss some alternative identification strategies that could bring us richer contemporaneous relations.

As Carlstrom et al. (2009) argument Cholesky identification can severely distort the impulse response functions for monetary policy shocks. To identify monetary policy and stock price shocks, it was assumed that both shocks does not affect macroeconomic variables contemporaneously, nonetheless this restriction is at odds with New Keynesian (NK) models where all variables respond to all shocks simultaneously. Hence, if observed data is generated by a probabilistic model with this property, then using recursive identification can distorts the estimated impulse response functions. Additionally, Chari et al. (2008) argue that long-run identification strategy leads to truncation bias due to the observed finite sample only admits a finite order VAR approximation.

In exchange for attempt some of these possible drawbacks I briefly propose two alternative identification strategies that can be used to identify interdependence of monetary and stock prices:

1. **A-B Model**. Relation between innovations and structural shocks is based on the commonly used $A - model$:

   $$\mathbf{v}_t = \mathbf{A}\boldsymbol{\varepsilon}_t$$

   Instead of assuming this relation for both type of disturbances I proposed to use an $A - B \;\; Model$

   $$\mathbf{B}\mathbf{v}_t = \mathbf{A}\boldsymbol{\varepsilon}_t$$

   where matrix $\mathbf{B}$ is the contemporaneous relation matrix. Although with this specification we need more restrictions ($2m^2 - m(m+1)/2$ more), it permits richer contemporaneous relation among variables.

2. **SVAR with instruments**. Contrary to face the identification problem by imposing restrictions on the relation between innovations and structural shocks, we can impose restrictions on the impacts of predetermined variables. Consider, the following structural VARX model:

   $$\mathbf{B}\mathbf{z}_t = \mathbf{A}_0 + \mathbf{A}_1\mathbf{z}_{t-1} + \cdots + \mathbf{A}_p\mathbf{z}_{t-p} + \boldsymbol{\Gamma}_0\mathbf{x}_t + \boldsymbol{\Gamma}_1\mathbf{x}_{t-1} + \cdots + \boldsymbol{\Gamma}_q\mathbf{x}_{t-q} + \boldsymbol{\varepsilon}_t \tag{4.1}$$

   where $\mathbf{x}_t$ is a vector of exogenous variables (e.g., foreign economy variables). Then, this structural model can be written as

   $$\mathbf{B}\mathbf{z}_t = \mathbf{A}_0 + \boldsymbol{\Pi}\mathbf{w}_t + \boldsymbol{\varepsilon}_t \tag{4.2}$$

   where $\boldsymbol{\Pi} = [\mathbf{A}_1 \ldots \mathbf{A}_p \;\; \boldsymbol{\Gamma}_0 \ldots \boldsymbol{\Gamma}_q]$ and $\mathbf{w}_t = [\mathbf{z}'_{t-1} \ldots \mathbf{z}'_{t-p} \;\; \mathbf{x}'_t \ldots \mathbf{x}'_{t-q}]'$. Hence, instead of enforcing restrictions on the matrix $\mathbf{B}$, we can identify this model by imposing restrictions over $\boldsymbol{\Pi}$.

There are other alternative models that can be used to identify monetary policy shocks (augmenting the number of endogenous variables, FAVAR systems, etcetera), however I consider that they imply a big change in modelling so it is less clear for me how to compare identification strategies among these alternatives.

# 5 Conclusion

Using Bjørnland and Leitemo (2009) identification strategy, I find that there is non-trivial simultaneous interdependence between monetary policy and real stock price shocks for Brazilian economy. Additionally, some other identification strategies that lead to richer contemporaneous relation among variables were discussed and they can be used for further studies in this area.

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# Appendix

## A Model selection

To select the lag order for the VAR model, I used four different information criterion. Being $p$ the order candidate, these indicators were computed using

$$\text{[Final Prediction Error]} \quad FPE(p) = \left[\frac{T+mp+1}{T-mp-1}\right]^m |\hat{\Sigma}_v(p)| \tag{A.1}$$

$$\text{[Akaike's Information Criterion]} \quad AIC(p) = \ln|\hat{\Sigma}_v(p)| + \frac{2}{T}pm^2 \tag{A.2}$$

$$\text{[Schwarz's Information Criterion]} \quad BIC(p) = \ln|\hat{\Sigma}_v(p)| + \frac{\ln T}{T}pm^2 \tag{A.3}$$

$$\text{[Hannan - Quinn Information Criterion]} \quad BIC(p) = \ln|\hat{\Sigma}_v(p)| + 2\frac{\ln(\ln T)}{T}pm^2 \tag{A.4}$$

where $\hat{\Sigma}_v(p)$ is the estimator for variance matrix of $\mathbf{v}_t$ using $p$ lags, and $m$ is the number of variables of the model. Comparisons among different order models is given at Table 3.

**Table 3.** *Estimation of lag order, p*

| Lag | Information criteria | | | |
|---|---|---|---|---|
| | FPE | AIC | BIC | HQ |
| 1 | 91.3028 | 4.4612 | 4.8900 | 4.6349 |
| 2 | 23.1772 | 3.0896 | 3.9472* | 3.4370* |
| 3 | 26.0244 | 3.2041 | 4.4905 | 3.7252 |
| 4 | 25.1507 | 3.1674 | 4.8826 | 3.8622 |
| 5 | 23.1664 | 3.0810 | 5.2250 | 3.9496 |
| 6 | 22.6766 | 3.0535 | 5.6263 | 4.0958 |
| 7 | 22.4482* | 3.0348* | 6.0364 | 4.2508 |
| 8 | 24.6095 | 3.1152 | 6.5456 | 4.5049 |

* Minimum

## B Structural model estimation

### B.1 Reduced form model estimation

I approximate the MA($\infty$) model with the parsimonious AR(p) model:

$$\mathbf{z}_t = \mathbf{C_0} + \mathbf{C_1}\mathbf{z}_{t-1} + \cdots + \mathbf{C_p}\mathbf{z}_{t-p} + \mathbf{v}_t \tag{B.1}$$

where $\{\mathbf{v}_t\}$ is serially uncorrelated and $\mathbb{E}[\mathbf{v}_t\mathbf{v}_t'] = \Sigma_v$. The reduced form is estimated by ordinary least square (OLS): define $\mathbf{C} \equiv \begin{bmatrix}\mathbf{C_0} & \mathbf{C_1} & \ldots & \mathbf{C_p}\end{bmatrix}$, $\mathbf{Z} \equiv \begin{bmatrix}\mathbf{z}_1 & \mathbf{z}_2 & \ldots & \mathbf{z}_T\end{bmatrix}$, $\mathbf{X} \equiv \begin{bmatrix}\mathbf{Z}_1 & \mathbf{Z}_2 & \ldots & \mathbf{Z}_T\end{bmatrix}$, and $\mathbf{Z}_t \equiv \begin{bmatrix}1 & \mathbf{z}_1' & \mathbf{z}_2' & \ldots & \mathbf{z}_{t-p+1}'\end{bmatrix}'$. Estimators are:

$$\hat{\mathbf{C}} = \mathbf{Z}\mathbf{X}'(\mathbf{X}\mathbf{X}')^{-1} \tag{B.2}$$

$$\hat{\Sigma}_v = T^{-1}(\mathbf{Z} - \hat{\mathbf{C}}\mathbf{X})(\mathbf{Z} - \hat{\mathbf{C}}\mathbf{X}) \quad \text{[Maximum Likelihood Estimator]} \tag{B.3}$$

Based on Amisano and Gianini (1997), the structural model is estimated by quasi-maximum likelihood (QML) subject to identification restrictions[1], i.e.:

$$\max_{\mathbf{A}} \quad \ln \mathcal{L}(\mathbf{A}) = \mathsf{constant} - T \ln |\mathbf{A}| - \frac{T}{2} tr\{(\mathbf{A}')^{-1}\mathbf{A}^{-1}\hat{\Sigma}_v\} \tag{B.4}$$

subject to

$$vec(\mathbf{A}) = \mathbf{R_A}\boldsymbol{\gamma}_A \tag{B.5}$$

where $\hat{\Sigma}_v$ is the maximum likelihood estimator for the variance matrix for $\mathbf{z}_t$.

### B.2 Short and long-run restrictions: the matrix $\mathbf{R_A}$

The identification restrictions have already been specified along the document, now I would like to show how express these as in (B.5). The vector $\boldsymbol{\gamma}_A$ is

$$\boldsymbol{\gamma}_A = \begin{bmatrix} a_{11} & \dots & a_{51} & a_{22} & \dots & a_{52} & a_{33} & \dots & a_{53} & a_{44} & a_{54} & a_{45} \end{bmatrix}'$$

and $\mathbf{R_A}$ is an $m^2 \times m(m+1)/2$ matrix (where $m = 5$) which imposes zero and long run restrictions. Note that, due to the long-run restriction, $a_{45}$ and $a_{55}$ are linear dependent and one of them is actually free to be chose in the estimation, then

$$\mathbf{R_A'} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \mathsf{LR}
\end{pmatrix} \tag{B.6}$$

where $\mathsf{LR} = -\frac{\hat{\boldsymbol{\Theta}}_{44}(1)}{\hat{\boldsymbol{\Theta}}_{45}(1)}$. As you must not the last two restrictions are:

$$a_{45} = a_{45}$$
$$a_{55} = -\frac{\hat{\boldsymbol{\Theta}}_{44}(1)}{\hat{\boldsymbol{\Theta}}_{45}(1)} a_{45}$$

Hence, $\mathbf{R_A}$ was constructed to include both short and long-run restrictions

[1]Provided by local identification the QML estimator has the well-known properties: consistency, asymptotic normality, and asymptotic efficiency. In the present context (Model A), the sufficient condition for local identification is

$$rank \begin{bmatrix} 2\mathbf{D}_m^+(\mathbf{A} \otimes I_m) \\ \mathbf{C}_A \end{bmatrix} = m^2$$

where $\mathbf{D}_m^+ \equiv (\mathbf{D}_m'\mathbf{D}_m)^{-1}\mathbf{D}_m'$, $\mathbf{D}_m$ is a duplication matrix [i.e., $\mathbf{D}_m vech(\mathbf{A}) = vec(\mathbf{A})$], and $\mathbf{C}_A$ is the orthogonal complement of $\mathbf{R_A}$.

### B.3 Algorithm for computing $\hat{\boldsymbol{\gamma}}_A$

Following Amisano and Gianini (1997), I use scoring algorithm to compute the QML estimator for $\hat{\boldsymbol{\gamma}}_A$. Start with and initial guess for $\hat{\boldsymbol{\gamma}}_A^0$ and a fixed number for $k$, then for $i = 1$ to maxiter [or until we attain a convergence criteria] compute:

$$\hat{\boldsymbol{\gamma}}_A^i = \hat{\boldsymbol{\gamma}}_A^{i-1} + k[\hat{\mathcal{I}}_T(\hat{\boldsymbol{\gamma}}_A^{i-1})]^{-1}\mathcal{S}(\hat{\boldsymbol{\gamma}}_A^{i-1}) \tag{B.7}$$

where $\hat{\mathcal{I}}_T(.)$ is an estimator for the information matrix for free parameters, $\hat{\boldsymbol{\gamma}}_A$ and $\mathcal{S}(.)$ is the score vector:

$$\mathcal{S}(\hat{\boldsymbol{\gamma}}_A) \equiv \frac{\partial \ln \mathcal{L}(\mathbf{A})}{\partial \gamma_A}$$
$$\mathcal{I}_T(\hat{\boldsymbol{\gamma}}_A) \equiv \mathbb{E}\Big[-\frac{\partial^2 \ln \mathcal{L}(\mathbf{A})}{\partial \gamma_A \partial \gamma_A'}\Big]$$

I use an optimization numeric algorithm using analytical expressions for these derivatives:

1. *Score vector*: Let define $\ln \tilde{\mathcal{L}}(\tilde{\mathbf{A}}) \equiv T \ln |\tilde{\mathbf{A}}| - \frac{T}{2} tr\{\tilde{\mathbf{A}}'\tilde{\mathbf{A}}\hat{\Sigma}_v\}$ and $\tilde{\mathbf{A}} \equiv \mathbf{A}^{-1}$

$$[*] \quad \frac{\partial \ln \tilde{\mathcal{L}}(\tilde{\mathbf{A}})}{\partial \tilde{\mathbf{A}}} = T\tilde{\mathbf{A}}'^{-1} - T\tilde{\mathbf{A}}\hat{\Sigma}_v$$
$$\Rightarrow \frac{\partial \ln \tilde{\mathcal{L}}(\tilde{\mathbf{A}})}{\partial vec(\tilde{\mathbf{A}})} = Tvec(\tilde{\mathbf{A}}'^{-1}) - T(\hat{\Sigma}_v \otimes \mathbf{I}_m)vec(\tilde{\mathbf{A}})$$
$$[**] \quad \frac{\partial vec(\tilde{\mathbf{A}})'}{\partial vec(\mathbf{A})} = -\mathbf{A}^{-1} \otimes (\mathbf{A}')^{-1}$$
$$\Rightarrow \frac{\partial \ln \mathcal{L}(\mathbf{A})}{\partial vec(\mathbf{A})} = -(\mathbf{A}^{-1} \otimes (\mathbf{A}')^{-1})[Tvec(\mathbf{A}') - T(\tilde{\Sigma}_v \otimes I_m)vec(\mathbf{A}^{-1})]$$

Hence

$$\frac{\partial \ln \mathcal{L}(\mathbf{A})}{\partial vec(\boldsymbol{\gamma}_A)} = -\mathbf{R}_{\mathbf{A}}'[\mathbf{A}^{-1} \otimes (\mathbf{A}')^{-1}][Tvec(\mathbf{A}') - T(\hat{\Sigma}_v \otimes I_m)vec(\mathbf{A}^{-1})] \tag{B.8}$$

2. *Information matrix*: Let define $\ln \tilde{\mathcal{L}}(\Sigma_v) \equiv= -\frac{T}{2}\ln|\Sigma_v| - \frac{T}{2} tr\{\Sigma_v^{-1}\hat{\Sigma}_v\}$ and $\Sigma_v \equiv \mathbf{A}\mathbf{A}'$, then

$$[***] \quad \frac{\partial^2 \ln \mathcal{L}(\mathbf{A})}{\partial \boldsymbol{\gamma}_A \partial \boldsymbol{\gamma}_A'} = \frac{\partial vec(\mathbf{A})'}{\partial vec(\boldsymbol{\gamma}_A)}\Big[\frac{\partial^2 \ln \mathcal{L}(\mathbf{A})}{\partial vec(\mathbf{A})\partial vec(\mathbf{A})'}\Big]\frac{\partial vec(\mathbf{A})}{\partial vec(\boldsymbol{\gamma}_A)'}$$
$$= \mathbf{R}_{\mathbf{A}}'\frac{\partial vec(\Sigma_v)'}{\partial vec(\mathbf{A})}\Big[\frac{\partial^2 \ln \tilde{\mathcal{L}}(\Sigma_v)}{\partial vec(\Sigma_v)\partial vec(\Sigma_v)'}\Big]\frac{\partial vec(\Sigma_v)}{\partial vec(\mathbf{A})'}\mathbf{R}_{\mathbf{A}}$$

The expression in brackets is given by:

$$\frac{\partial vec(\Sigma_v)}{\partial vec(\mathbf{A})'} = \frac{\partial vec(\mathbf{A}\mathbf{A}')}{\partial vec(\mathbf{A})'} = (\mathbf{I}_{m^2} + \mathbf{K}_{mm})(\mathbf{A} \otimes \mathbf{I}_m)$$

where $\mathbf{K}_{mm}$ is the commutation matrix of $\mathbf{A}$, i.e., $vec(\mathbf{A}') = \mathbf{K}_{mm}vec(\mathbf{A})$. Defining $X \equiv \Sigma_v\hat{\Sigma}_v^{-1}\Sigma_v$

$$\frac{\partial \ln \tilde{\mathcal{L}}(\Sigma_v)}{\partial vec(\Sigma_v)} = -\frac{T}{2}vec(\Sigma_v^{-1}) + \frac{T}{2}vec(\Sigma_v^{-1}\hat{\Sigma}_v\Sigma_v^{-1})$$
$$\Rightarrow \frac{\partial^2 \ln \mathcal{L}(\tilde{\Sigma}_v)}{\partial vec(\Sigma_v)\partial vec(\Sigma_v)'} = \frac{T}{2}(\Sigma_v^{-1} \otimes \Sigma_v^{-1}) + \frac{T}{2}\frac{\partial vec(X^{-1})}{\partial vec(X)'}\frac{\partial vec(X)}{\partial vec(\Sigma_v)'}$$
$$= \frac{T}{2}(\Sigma_v^{-1} \otimes \Sigma_v^{-1})$$

$$
\begin{aligned}
&+\frac{T}{2}\left[\Sigma_v^{-1}\tilde{\Sigma}_v\Sigma_v^{-1}\otimes\Sigma_v^{-1}\tilde{\Sigma}_v\Sigma_v^{-1}\right]\left[\Sigma_v\tilde{\Sigma}_v^{-1}\otimes I_m + I_m\otimes\Sigma_v\tilde{\Sigma}_v^{-1}\right]\\
&=\frac{T}{2}(\Sigma_v^{-1}\otimes\Sigma_v^{-1})-\frac{T}{2}\left[\Sigma_v^{-1}\otimes\Sigma_v^{-1}\tilde{\Sigma}_v\Sigma_v^{-1}+\Sigma_v^{-1}\tilde{\Sigma}_v\Sigma_v^{-1}\otimes\Sigma_v^{-1}\right]
\end{aligned}
$$

Hence,

$$
\mathcal{I}_T(\boldsymbol{\gamma}_A)=\mathbf{R}_{\mathbf{A}}'\frac{\partial vec(\Sigma_v)'}{\partial vec(\mathbf{A})}\left\{\frac{T}{2}(\Sigma_v^{-1}\otimes\Sigma_v^{-1})\right\}\frac{\partial vec(\Sigma_v)}{\partial vec(\mathbf{A})'}\mathbf{R}_{\mathbf{A}} \tag{B.9}
$$

The estimation for these matrices is made by substituting $(\hat{\boldsymbol{\gamma}}_A,\hat{\Sigma}_A)$ in expressions (B.8) and (B.9).

# C Codes

Reduced form estimation, structural estimation, and confidence intervals for the impulse response functions were all computing in MATLAB. Basically, I have created five script functions that can be used to a wide range of structural VAR models with short and long-run identification restrictions:

1. **mlSVAR.m**: Computes quasi maximum likelihood estimator for a structural VAR with A − model residual structure whenever identification restrictions can be expressed as $\mathbf{R}_{\mathbf{A}}\boldsymbol{\gamma}_A = vec(\mathbf{A})$.
2. **VARest.m**: Computes OLS estimators for parameters in a VAR(p) reduced form model.
3. **VAR_boots.m**: Implements model-based bootstrap method for dependent data.
4. **VAROptLag.m**: Selects the lag order by minimization of information statistics.
5. **VARimpulse.m**: Computes impulse response functions using companion form of the estimated VAR(p) model. This functions needs an estimation for matrix **A**.
6. **uroottest.m**: Simplifies the process of executing ADF and Phillips-Perron unit root test with different lag order for univariate time series, selecting the best model based on BIC statistic, and reporting results (similar to Tab. 2). Econometric Toolbox must be added to MATLAB search path.

Replicating the presented results is easy using **main.m** script.