Maths Problems

1)

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$H = \frac{\Omega_x}{2}\sigma_x + \frac{\Omega_y}{2}\sigma_y$$

$$H = \frac{\Omega_x}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{\Omega_y}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\cancel{H = \left(\frac{\Omega_x - i\Omega_y}{} \right.}$$

$$H = \frac{1}{2}\begin{pmatrix} 0 & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & 0 \end{pmatrix}$$

for eigenvalues

$\cancel{\det[H \quad \lambda]}$

$$\det[H - \lambda \mathbb{I}] = 0$$

$$\det\begin{bmatrix} -\lambda & \Omega_x - i\Omega_y \\ \Omega_x + i\Omega_y & -\lambda \end{bmatrix} = 0$$

$$\lambda^2 - (\Omega_x - i\Omega_y)(\Omega_x + i\Omega_y) = 0$$

$$\lambda^2 - (\Omega_x^2 + \Omega_y^2) = 0$$

$$\boxed{\Omega_x^2 - i\Omega_y\Omega_x + i\Omega_y\Omega_x - (-1)\Omega_y^2 = 0}$$

yeh im not crazy

$\cancel{\lambda^2 = \Omega_x^2 + \Omega_y^2}$

$$\lambda^2 = \Omega_x^2 + \Omega_y^2$$

$$\lambda = \pm\sqrt{\Omega_x^2 + \Omega_y^2}$$

$$H \cdot e_1 = \lambda_1 \cdot e_1$$

$$(H - \lambda_1 \mathbb{I})\, \underline{e}_1 = 0$$

$$\cancel{\tfrac{1}{2}} \begin{pmatrix} -\sqrt{\Omega_x^2 + \Omega_y^2} & (\Omega_x - i\Omega_y)/2 \\ \frac{\Omega_x + i\Omega_y}{2} & -\sqrt{\Omega_x^2 + \Omega_y^2} \end{pmatrix} \underline{e}_1 = 0$$

$$\cancel{\tfrac{1}{2}} \begin{pmatrix} -\sqrt{\Omega_x^2 + \Omega_y^2} & (\Omega_x - i\Omega_y)/2 \\ \frac{\Omega_x + i\Omega_y}{2} & -\sqrt{\Omega_x^2 + \Omega_y^2} \end{pmatrix} \begin{pmatrix} e_{1x} \\ e_{1y} \end{pmatrix} = 0$$

$$\begin{cases} -e_{1x}\sqrt{\Omega_x^2 + \Omega_y^2} + \frac{1}{2} e_{1y}(\Omega_x - i\Omega_y) = 0 \\ \frac{1}{2} e_{1x} \frac{\Omega_x + i\Omega_y}{\cancel{2}} - e_{1y}\sqrt{\Omega_x^2 + \Omega_y^2} = 0 \end{cases}$$

I think its to do with the fact that Ω_x & Ω_y are real.

2)
$$\underline{\Psi} = \begin{pmatrix} a \\ b \end{pmatrix} \quad |a|^2 + |b|^2 = 1$$

$$\tilde{e}_n = e_n \cdot e^{i\lambda_n t}$$

$$U(t)\underline{\Psi}$$

Sorry I got nothing