Inference of Hypersonic Vehicle State Based on IR Signatures using Surrogate Optimization

Jacob Needels

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1 Introduction

Let y be a measurement of an IR signature obtained by instrumentation, and $f(x;\mu)$ be a predicted IR signature, where $x \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^n$ are the vehicle state and parameter vectors, respectively. The problem of inferring information regarding the vehicle from the observed and predicted values can be posed as a nonlinear least-squares optimization problem as

$$\min_{x,\mu} \quad \tfrac{1}{2}(y - f(x;\mu))^2. \tag{1}$$

The optimal combined state/parameter vector can be expressed as

$$x^*, \mu^* = \arg\min_{x,\mu} \quad \tfrac{1}{2}(y - f(x;\mu))^2. \tag{2}$$

Assume that y is a noiseless measurement. The functional f is a map $\mathbb{R}^{m+n} \mapsto \mathbb{R}$ from state/parameter space to a real-valued output. the radiant intensity [W/sr] observed from the vehicle.

The functional can be defined as

$$f = \iint_{S,\lambda_{IR}} L(T_w, \lambda)\ \mathrm{d}\lambda \mathrm{d}A. \tag{3}$$

$L(T_w, \lambda)$ is defined by Planck's Law as the spectral radiance of a blackbody at a specified temperature and wavelength, defined as

$$L = \frac{2hc^2}{\lambda^2}\left(\frac{1}{\exp\frac{hc}{k_b T_w \lambda} - 1}\right). \tag{4}$$

The wavelength spectrum to be observed is a feature of the detection equipment, in this case the IR spectrum is used as an example. The area to be integrated is the vehicle surface in view of the detector, and is a function of vehicle parameters (size, geometry), and state.

In this case we assume that the surface generating radiation is the surface of the vehicle at some radiative equilibrium wall temperature, T_w.

2 Optimization

Several methods exist for the optimization of the objective function listed in Eqn. 1. One approach involves the use of gradient-based methods, which requires differentiation of the prediction function f. A computationally efficient method is to construct a differentiable surrogate, $\hat{f}$, through offline evaluation of a database of multi-physics simulations, spanning the combined state/parameter space.

An approximation of the unconstrained problem then becomes

$$x^*, \mu^* \approx \underset{x,\mu}{\arg\min} \quad \tfrac{1}{2}(y - \hat{f}(x;\mu))^2. \tag{5}$$

A gradient based optimization strategy can then be employed. The gradient of the objective function is

$$\nabla \frac{1}{2}(y - \hat{f}(x;\mu))^2 = (y - \hat{f}(x;\mu))\nabla \hat{f}(x;\mu). \tag{6}$$

A major challenge in this problem is the likelihood of large feasible regions, which makes selection of a single point challenging. This is particularly challenging for disconnected feasible regions, which may represent widely divergent state/parameter tuples, and therefore provide little value in terms of evidence. One way to ameliorate this problem is through the application of constraints to optimization problem.

This likely necessitates the use of a more complex optimization procedure, like stochastic gradient descent, in order to locate the various local minio

3 Constraints

In general, this solution is not unique, and may define a manifold in $\mathbb{R}^{m+n}$. Constraints can be imposed on the optimization on the solution space, based on additional information about the vehicle. For example, information about vehicle altitude limitations can constrain set of possible altitudes. The constrained problem can then be expressed as

$$\begin{aligned} \min_{x,\mu} \quad & \frac{1}{2}(y - f(x;\mu))^2 \\ \text{s.t.} \quad & g_i(x;\mu) = c_i \\ & h_j(x;\mu) \geq d_j. \end{aligned} \tag{7}$$

The set of feasible points will likely consist of some subspace of $\mathbb{R}^{m+n}$. The extent of this subspace is controlled by constraints on the feasible values of both the state and parameters through two mechanisms: additional in situ observations on the state/parameters of the vehicle (e.g. velocity measurements) and prior knowledge regarding design and operating characteristics.

For example, if flight velocity is measured through another source, it assigns an equality constraint on that component of the state. If the velocity is not measurable but assigned to some uncertainty bound, it is possible to assign inequality constraints.