Inference of Hypersonic Vehicle State and Configuration Based on IR Signatures using Surrogate Optimization

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1 Introduction

High aerodynamic heating loads developed during hypersonic flight result in high temperatures at the vehicle surface and in the surrounding flow. These result in radiative emission of energy, proportional to the wall temperature, which is influenced by vehicle configuration and state. One particularly useful band of radiation that has been studied in the literature is the IR emission spectrum, which spans wavelengths 7.8E-7 - 1E-3 m. IR detectors can be used to characterize thermal signatures, and detect properties about hypersonic vehicles in flight. However, inferring accurate state estimation from IR signatures, particularly in real-time applications, remains a challenge. This paper explores the use of a high-fidelity CFD generated surrogate, spanning the space of vehicle states and parameters, that can be effectively used to solve the inverse identification problem for real-time applications.

2 Problem Formulation

Let y be a measurement of an IR signature obtained by instrumentation, and $f(x;\mu)$ be a predicted IR signature, where $x \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^n$ are the vehicle state and parameter vectors, respectively. The problem of inferring information regarding the vehicle from the observed and predicted values can be posed as a nonlinear least-squares optimization problem as

$$\min_{x,\mu} \quad (y - f(x;\mu))^2. \tag{1}$$

The optimal combined state/parameter vector can be expressed as

$$x^*, \mu^* = \arg\min_{x,\mu} \quad (y - f(x;\mu))^2. \tag{2}$$

Assume that y is a noiseless measurement. The functional f is a map $\mathbb{R}^{m+n} \mapsto \mathbb{R}$ from state/parameter space to a real-valued output the radiant intensity [W/sr] observed from the vehicle.

The functional can be defined as

$$f = \iint\limits_{S,\lambda_{IR}} L(T_w, \lambda) \ \mathrm{d}\lambda \mathrm{d}A. \tag{3}$$

$L(T_w, \lambda)$ is defined by Planck's Law as the spectral radiance of a blackbody at a specified temperature and wavelength, defined as

$$L = \frac{2hc^2}{\lambda^2} \left(\frac{1}{\exp \frac{hc}{k_b T_w \lambda} - 1} \right). \tag{4}$$

The wavelength spectrum to be observed is a feature of the detection equipment, in this case the IR spectrum is used as an example. The area to be integrated is the vehicle surface in view of the detector, and is a function of vehicle parameters (size, geometry), and state.

We assume that the IR signatures is due to the surface of the vehicle at some radiative equilibrium wall temperature, T_w. The radiative equilibrium wall temperature can be solved equating contributions of radiative and convective heating at the vehicle surface

$$q_{conv} = q_{rad}. \tag{5}$$

The convective and diffusive flux at the surface can be expressed compactly as

$$q_{conv} = \sum_k \kappa^k \frac{\partial T_w^k}{\partial \mathbf{n}}. \tag{6}$$

And the radiative flux can be approximated as a grey body as

$$q_{rad} = \epsilon \sigma T_w^4. \tag{7}$$

Eqn. 5 can then be solved iteratively to obtain a wall temperature at each point on the surface.

3 Optimization

The primary difficulty in minimization of the objective listed in Eqn. 1 is the computational expense of evaluating the model-predicted value of radiant intensity. A gradient-free approach would require atleat one function evaluation at each optimization step. Current capabilities to produce high-fidelity CFD simulations for a representative vehicle geometry in SU2 would require on the order of hours wall-clock time, even for a highly-parallelized simulation.

A gradient-based approach would require evaluation of the gradient of the objective which is similarly computationally expensive, prohibitive to rapid evaluation.

This motivates the application of function approximation methods, particularly projection-based reduced order modeling (ROMs).

Offline development of a ROM over the vehicle parameter space would allow rapid evaluation of the objective. It could also be used to construct a differentiable surrogate, $\hat{f}$.

An approximation of the unconstrained problem, which can be approached with iterative gradient-based methods, then becomes

$$x^*, \mu^* \approx \underset{x,\mu}{\arg\min} \quad (y - \hat{f}(x;\mu))^2. \tag{8}$$

4 Constraints

An anticipated challenge is a lack of a global minimum of the objective function. The combination of state and parameters that result in the optimal value of radiant intensity is likely not unique. Constraints can be imposed on the optimization on the solution space, based on additional information about the vehicle. For example, information about vehicle altitude limitations can constrain set of possible altitudes. The constrained problem can then be expressed as

$$\begin{aligned} \min_{x,\mu} \quad & (y - f(x;\mu))^2 \\ \text{s.t.} \quad & g_i(x;\mu) = c_i \\ & h_j(x;\mu) \geq d_j. \end{aligned} \tag{9}$$

The result is that the solution is constrained to some feasible subspace $\mathcal{F} \subseteq \mathbb{R}^{m+n}$. The extent of this subspace is controlled by constraints on the feasible values of both the state and parameters through two mechanisms: additional in situ observations on the state/parameters of the vehicle (e.g. velocity measurements) and prior knowledge regarding design and operating characteristics.

For example, if flight velocity is measured through another source, it assigns an equality constraint on that component of the state. If the velocity is not measurable but assigned to some uncertainty bound, it is possible to assign inequality constraints.

5 Preliminary Results

As an initial test of method, we look at solving the inverse problem over a severely reduced state-parameter space using correlations to predict vehicle surface temperature. We consider a 1-D vehicle parameterized by its length, s, with variation in it's velocity magnitude, v. We utilize correlations due to Tauber for stagnation point and flat plate heating for hypersonic flows, expressed as

$$q_{sp} = \frac{1.83E-4}{\sqrt{R_n}}\left(1-\frac{h_w}{h_o}\right)\rho^{0.5}v^3, \tag{10}$$

$$q_{fp} = \begin{cases} v > 4kps & 2.2E-5\left(\frac{cos^{2.08}\theta sin^{1.6}\theta}{x^0.2}\right)\left(1-\frac{1.11h_w}{h_o}\right)\rho^{0.8}v^{3.7} \\ v \leq 4kps & 3.89E-4\left(\frac{cos^{1.76}\theta sin^{1.6}\theta}{x^0.2}\right)\left(1-\frac{1.11h_w}{h_o}\right)\left(\frac{556}{T_w}\right)^{0.25}\rho^{0.8}v^{3.37}. \end{cases} \tag{11}$$

Blackbody (ϵ=1) radiation to solve for the equilibrium wall temperature along the axial dimension of the vehicle. While the analytic correlations are computationally efficient to evaluate, performing the double integral to determine the radiant intensity over the vehicle surface numerically results in a wall-clock time of 3 mins per simulation computed in serial on a laptop. While performance could certainly be improved, for this simple test case 9 uniformly sampled points were selected as v = [2500, 3000, 3500] and s = [0.5, 1, 1.5]. A 2nd order polynomial surrogate was fit with the form

$$\hat{f}(x;\mu) = 203.4760sv - 593.64v - 4.47E5s - 8.35E - 10s^2 + 9.89E - 2v^2 + 8.74E6 \tag{12}$$

A test observation value of radiant intensity y = 3.5E5 W/sr was selected. The resulting least squares problem was solved using the SciPy Optimize package in Python, using the L-BFGS-B algorithm. The optimization was constrained to the sampled domain.

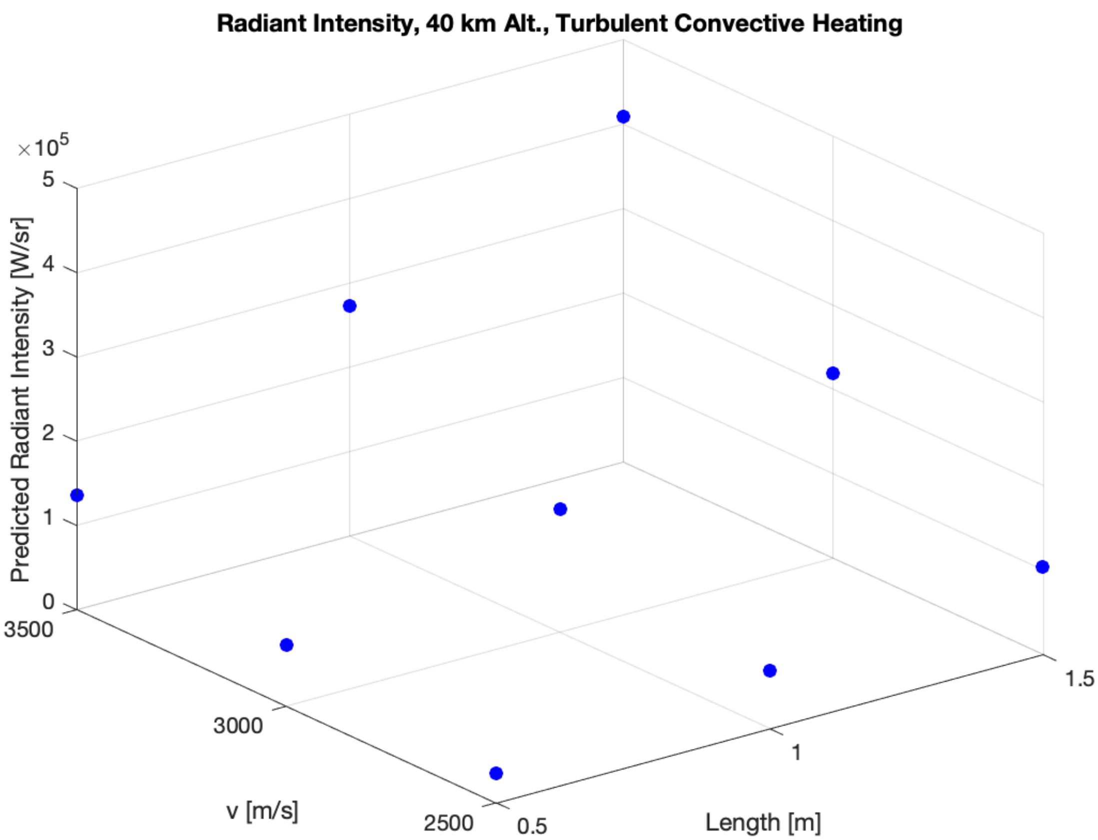

Figure 1: Sample response surface over range of vehicle length and velocity.

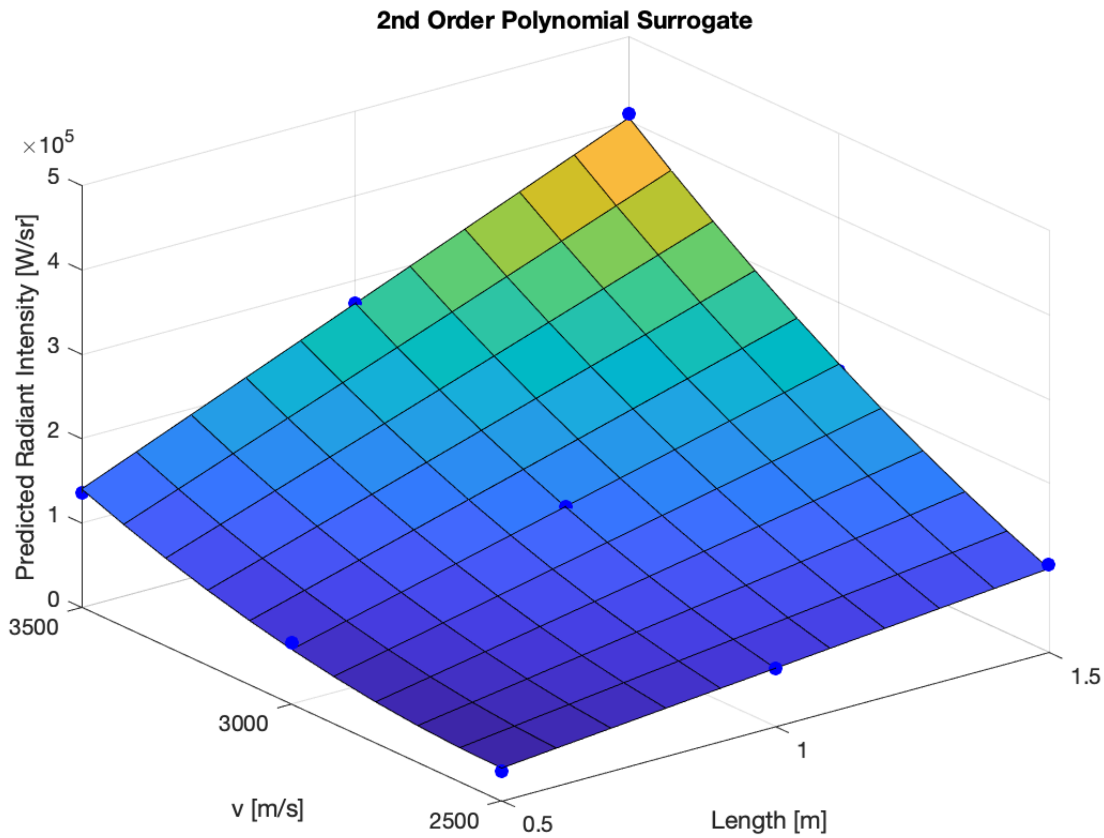

Figure 2: Surrogate fit of sampled points.

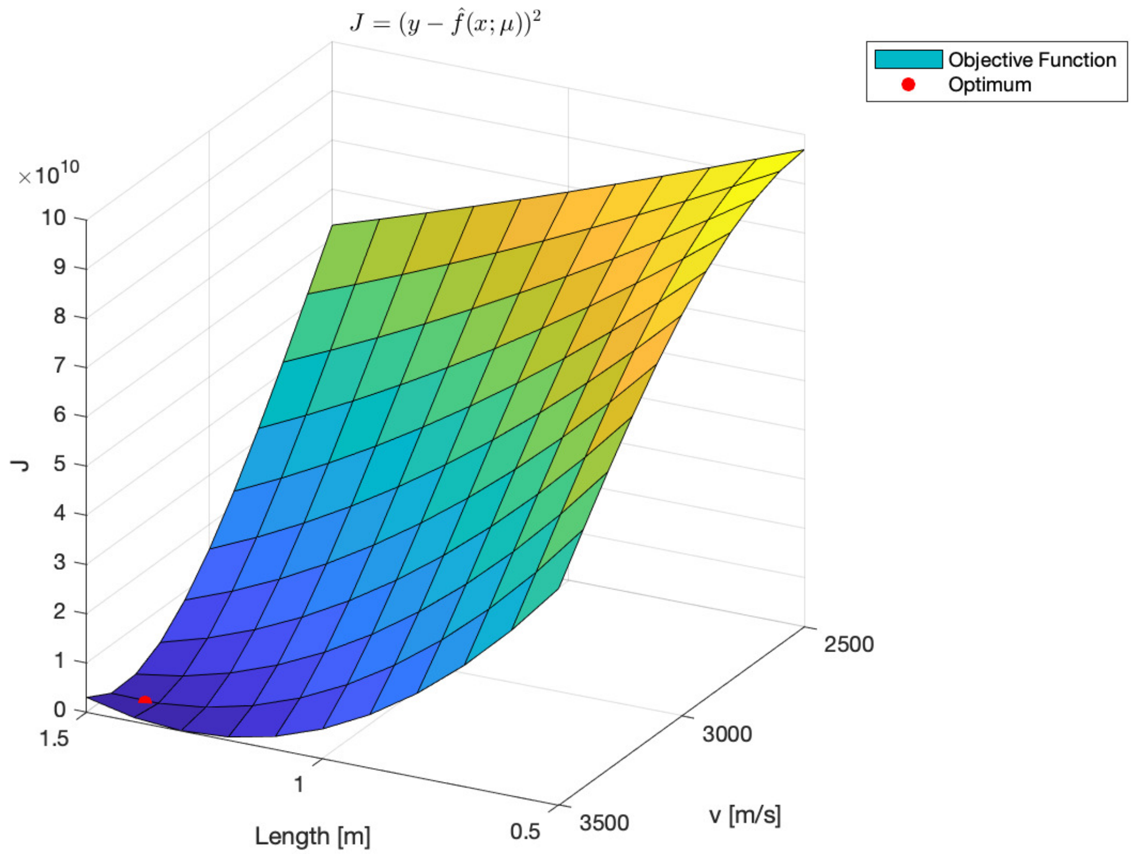

Figure 3: Objective function and optimization results.